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Voltage Rectification by a SQUID Ratchet

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We argue that the phase across an asymmetric dc SQUID threaded by a magnetic flux can experience an effective ratchet (periodic and asymmetric) potential. Under an external ac current, a rocking ratchet mechanism operates whereby one sign of the time derivative of the phase is favored. We show that there exists a range of parameters in which a fixed sign (and, in a narrower range, even a fixed value) of the average voltage across the ring occurs, regardless of the sign of the external current dc component. [S0031-9007(96)01045-9]

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Although the nonequilibrium dynamics of a particle in a ratchet potential (i.e., a periodic potential that lacks reflection symmetry) has long been considered a fundamental problem in statistical physics [1], it has become the object of more intense attention in recent years, because of its newly found relevance in diverse areas of physics, chemistry, and biology. A characteristic effect is that, when the ratchet is subject to a stationary nonequilibrium perturbation, particle motion in one direction is favored. Within this context, an important class of dynamical systems is formed by the so-called rocking ratchets, for which the external perturbation is a time periodic, uniform force [2–4]. The effect of dynamically induced unidirectional motion can overcome the drift effect of a small bias that would push the particle into the nonfavored direction. Thus, for not very strong tilts, uphill movement is possible provided the ratchet structure is conveniently rocked.

In this Letter, we propose a realization of the rocking ratchet mechanism in a new type of superconducting quantum interference device (SQUID) containing a characteristic asymmetry. The system we propose, depicted in Fig. 1, is formed by a ring with two Josephson junctions in series in one of the arms and only one junction in the other arm. We will show that, when the ring is threaded by a flux Φ_{ext} that is not an integer multiple of $\Phi_0/2$ ($\Phi_0 \equiv h/2e$ being the flux quantum), the effective potential experienced by the total phase φ across the ring displays a ratchet structure. As a consequence, when the asymmetric SQUID is "rocked" by an external ac current $I(t)$, one sign of the phase velocity $\dot{\varphi}$ is favored. From the Josephson voltage-phase relation, we conclude that there must be a range of parameters for which a fixed sign of the average voltage $V_0 \equiv \hbar\langle\dot{\varphi}\rangle/2e$ occurs regardless of the sign of the external current dc component I_0.

We focus on SQUID structures formed by conventional Josephson junctions whose phase is a classical variable and which can be adequately described by the "resistively shunted junction" model [5,6]. Thus, the phase φ_i across Josephson junction i on the left arm [see Fig. 1(a)] obeys the equation ($i = 1, 2$)

$$I_l(t) = J_i \sin(\varphi_i) + \frac{\hbar}{2eR_i}\,\dot{\varphi}_i + \frac{\hbar C_i}{2e}\,\ddot{\varphi}_i\,, \qquad (1)$$

where $I_l(t)$ is the current through the left arm, and R_i, C_i, and J_i are the resistance, capacitance, and critical current of junction i. For simplicity, we assume here that the two junctions in series are identical, and will comment later on the case of slightly dissimilar junctions. We take $C_1 = C_2 \equiv 2C_l$, $R_1 = R_2 \equiv R_l/2$, and $J_1 = J_2 \equiv J_l$. The total voltage drop across the two junctions is $V = V_1 + V_2$, where $V_i = (\hbar/2e)\dot{\varphi}_i$.

If $\varphi_1(t)$ is a solution for the first junction, then $\varphi_2(t) = \varphi_1(t) \equiv \varphi_l(t)/2$ is also a solution for the second junction [7]. This implies $V = \dot{\varphi}_l\hbar/2e$, with φ_l satisfying the equation

$$I_l(t) = J_l \sin\!\left(\frac{\varphi_l}{2}\right) + \frac{\hbar}{2eR_l}\,\dot{\varphi}_l + \frac{\hbar C_l}{2e}\,\ddot{\varphi}_l\,. \qquad (2)$$

Hence, a series of two identical Josephson junctions can be described by the same equation as a single junction, with the only difference that in the sine function *the argument* $\varphi/2$ *occurs* [8]. This is a most important feature to build the ratchetlike structure. On the right arm, the phase across the single junction obeys an equation that

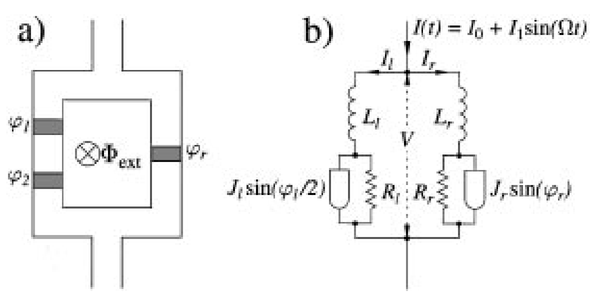

FIG. 1. (a) Schematic picture of an asymmetric SQUID with three junctions threaded by an external flux. (b) Representation of the equivalent circuit: the two junctions in series of the left branch behave like a single junction with φ replaced by $\varphi/2$.

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reads as in Eq. (1) with the labels l and i replaced by r. In the following we assume that each Josephson link operates in the overdamped limit, $(2e/\hbar)J_\alpha R_\alpha^2 C_\alpha \ll 1$ ($\alpha = l, r$), so that the capacitive terms can be neglected in Eq. (2) as well as in its right arm counterpart [6,9].

The total current through the SQUID is $I(t) = I_l(t) + I_r(t)$, and the equivalent circuit [6,10] is shown in Fig. 1(b). In the limit where the contributions L_l and L_r to the total loop inductance $L = L_l + L_r$ are such that $|LI| \ll \Phi_0$, the total flux Φ is approximately the external flux Φ_{ext}. Then, integration of the gauge invariant phase around the loop yields $\varphi_l - \varphi_r = -\varphi_{\text{ext}} + 2\pi n$, with $\varphi_{\text{ext}} \equiv 2\pi\Phi_{\text{ext}}/\Phi_0$. Thus, we see that the application of an external flux provides us with an externally tunable relation between φ_l and φ_r, which, in combination with the freedom to choose the ratio J_l/J_r, allows us to select the shape of the potential experienced by the phase $\varphi \equiv \varphi_l$.

We may include the effect of temperature T by adding Nyquist noise. We assume Gaussian white noise $\eta(t)$ of zero average and correlation $\langle \eta(t)\eta(0)\rangle = (2k_BT/R)\delta(t)$, so that the phase satisfies the equation

$$\frac{\hbar}{eR}\,\dot{\varphi} = -\,J_l \sin\!\left(\frac{\varphi}{2}\right) - J_r \sin(\varphi + \varphi_{\text{ext}}) + I(t) + \eta(t)\,, \qquad (3)$$

where we have set $R_l = R_r \equiv R$. The resulting Fokker-Planck equation is numerically solved by a matrix continued fraction method [11]. The total dc voltage across the SQUID is given by $V_0 = (\hbar/2e)\langle\dot{\varphi}_l\rangle + L_l\langle\dot{I}_l\rangle = (\hbar/2e)\langle\dot{\varphi}_r\rangle + L_r\langle\dot{I}_r\rangle$, where $\langle\cdots\rangle$ stands for time and noise average. Since currents must remain bounded, one has $\langle\dot{I}_l\rangle = \langle\dot{I}_r\rangle = 0$, which leads to $V_0 = (\hbar/2e)\langle\dot{\varphi}\rangle$.

Next we feed the circuit with a current $I(t) = I_0 + I_1\sin(\Omega t)$. In terms of the dimensionless quantities $x \equiv (\varphi + \pi)/2$, $\tau \equiv (eRJ_l/2\hbar)t$, $s \equiv J_r/J_l$, $F \equiv I_0/J_l$, $A \equiv I_1/J_l$, $\omega \equiv 2\hbar\Omega/eRJ_l$, and $D \equiv ek_BT/\hbar J_l$, Eq. (3) reads

$$\frac{dx}{d\tau} = -\frac{\partial}{\partial x}U(x) + F + A\sin(\omega\tau) + \xi(\tau)\,, \qquad (4)$$

where $U(x) = -[\sin(x) + (s/2)\sin(2x + \varphi_{\text{ext}} - \pi/2)]$ is the effective potential and $\xi(\tau)$ is Gaussian noise with $\langle\xi(\tau)\xi(0)\rangle = 2D\delta(\tau)$. The average voltage is now given by $V_0 = (J_lR/2)\langle dx/d\tau\rangle$. Setting, for instance, $s = 1/2$ and $\varphi_{\text{ext}} = \pi/2$, $U(x)$ adopts the form of a ratchet potential with period 2π, as shown in Fig. 2.

We can expect the ratchet structure arising from the combination of asymmetry and $\Phi_{\text{ext}} = \Phi_0/4$ to have major consequences on the device properties. In Fig. 3 we show the dc current-voltage characteristics for a low ac frequency $\omega = 0.01$ and $A = 1$. The resulting dc voltage for the ratchet potential is compared to that obtained for a symmetric potential with the same barrier height. Clearly, the main effect of the ratchet shape of the potential is that of shifting the dc current-voltage characteristics towards more negative values of I_0 or, equivalently, towards more positive values of V_0. Within the present sign convention, we shall refer to this displacement as a shift in the "right" direction, because in it $\langle\dot{\varphi}\rangle$ has the sign that is generally (although not exclusively [3]; see below) favored.

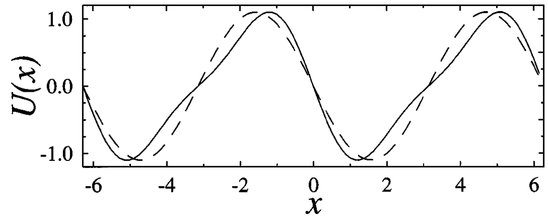

FIG. 2. The ratchet potential (solid line) given after Eq. (4) of the text, which governs the behavior of the three junction SQUID (cf. Fig. 1), is compared to the sine potential (dashed line) $U(x) = 1.1\sin(x)$.

In the deterministic case, the velocity $\langle dx/d\tau\rangle$ is almost quantized at values $n\omega$, $n = 0, \pm 1, \ldots$, corresponding to solutions $x(\tau)$ that are "locked" into the phase of the driving force [12]. For a symmetric potential, these plateaus in the voltage correspond to standard Shapiro steps [6,12–14]. In both the symmetric [15] and asymmetric cases, a small amount of noise ($D = 0.01$) suffices to wipe out the structure of steps. However, the same weak noise does not destroy the ratchet-induced shift in the dc I-V characteristics. As shown in Fig. 3, a stronger noise intensity is needed to appreciably reduce the ratchet effect (note that it still persists with D as large as 0.5) and to lead the system towards a conventional behavior in which $V_0 \propto I_0$. The same trend towards Ohmic response is already shown for weak noise if I_0 is large enough.

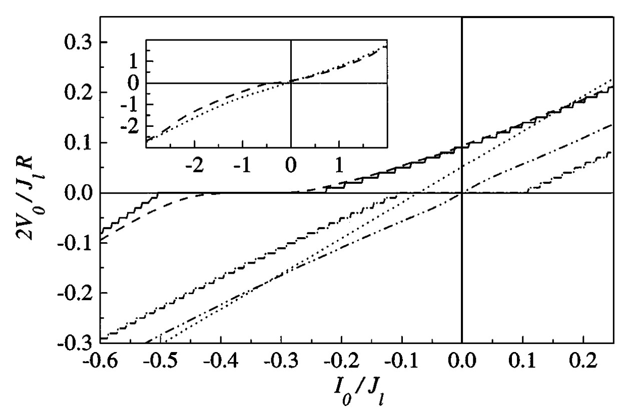

FIG. 3. The dc current-voltage characteristics for the SQUID in Fig. 1 are shown for an adiabatically slow ($\omega = 0.01$) ac contribution of amplitude $A = 1$ at different noise strengths D. The ratchet potential with noise strengths $D = 0$ (solid line), $D = 0.01$ (dashed line), and $D = 0.5$ (dotted line) is compared to the sine potential of Fig. 2 with $D = 0$ (dash-dotted line) and $D = 0.01$ (dash-double dotted line). Inset: Global view of the same I-V curves for the ratchet potential with $D = 0.01$ (dashed line) and $D = 0.5$ (dotted line).

It is remarkable that the ratchet effect can be displayed so clearly at frequencies as low as $\omega = 0.01$. The adiabatic limit ($\omega \to 0$) can actually be studied analytically. For $D = 0$, one obtains $V_0 = (J_l R/2) \int_{-\pi}^{\pi} d\theta / t(\theta)$, where $t(\theta) \equiv \int_{-\pi}^{\pi} dx/[F + A\sin(\theta) - U'(x)]$. For $F = 0$, V_0 is guaranteed to be zero only if $U(x)$ is symmetric, since then $U'(x)$ and thus $t(\theta)$ must be odd functions. On the contrary, if $U(x)$ is not symmetric for any choice of origin (ratchet potential), then one generally has $V_0 \neq 0$ with $I_0 = 0$. For a given amplitude, the ratchet behavior tends to disappear as the frequency grows. On the other hand, for a given frequency, there is an optimum amplitude that maximizes the ratchet effect [3].

In Fig. 4, we show V_0 as a function of I_0 for $\omega = 0.3$ and $A = 1.7$. In the absence of noise, steps at half-integer multiples of ω can be clearly observed. In the inset of Fig. 4, additional steps can be observed at ω/m. They are also present for $\omega = 0.01$, although they cannot be resolved in the scale of Fig. 3. We note that these noninteger steps are not due to the ratchet structure itself but to the deviation of $U(x)$ from a simple $\sin(x)$ law, which is the sole case for which steps lie only at integer values $n\omega$ [16]. Under weak noise ($D = 0.01$), the fractional Shapiro steps disappear, but the structure of integer plateaus is still somewhat preserved. More intense noise ($D = 0.5$) destroys the voltage quantization totally and, as in the adiabatic case, considerably reduces the ratchet effect.

So far we have assumed that ac current sources are applied to the device. It is interesting to analyze what happens when a voltage source of the type $V(t) = V_0 + V_1 \sin(\Omega t)$ is applied instead. Then the phase evolves as

$$x(\tau) = x_0 + \langle \dot{x} \rangle \tau - (A/\omega)\cos(\omega\tau), \qquad (5)$$

where $\langle \dot{x} \rangle / V_0 = A/V_1 = 2/J_l R$. Inserting (5) into Eq. (3) and averaging over time, one obtains that, for $\langle \dot{x} \rangle = n\omega$ or $\langle \dot{x} \rangle = (2n+1)\omega/2$, a continuous interval of dc current values F is possible. For $\varphi_{\text{ext}} = \pi/2$ one obtains, respectively,

$$F = n\omega + J_n\left(\frac{A}{\omega}\right)\cos(x_1) - sJ_{2n}\left(\frac{2A}{\omega}\right)\cos(2x_1),$$
$$F = \frac{2n+1}{2}\omega - sJ_{2n+1}\left(\frac{2A}{\omega}\right)\sin(2x_1), \qquad (6)$$

where $J_n(z)$ is the nth order Bessel function [17]. The finite range of F values spanning a voltage plateau for a given value of n is obtained by letting x_1 take any real value. After comparing the structure of plateaus predicted by Eq. (6) with that obtained numerically for the case of current sources, we have found that, as in the symmetric case [15], similar results are obtained for $\omega \gg 1$ or $A \gg 1$, provided that $A/\omega \lesssim 1$. Inspection of Eq. (6) shows that the resulting structure of steps, although not entirely symmetric, does not exhibit a proper ratchet effect in any range of parameters, since there is always a $I_0 = 0, V_0 = 0$ solution. This can be proved by noting that the last two terms in upper Eq. (6) cancel for certain values of x_1.

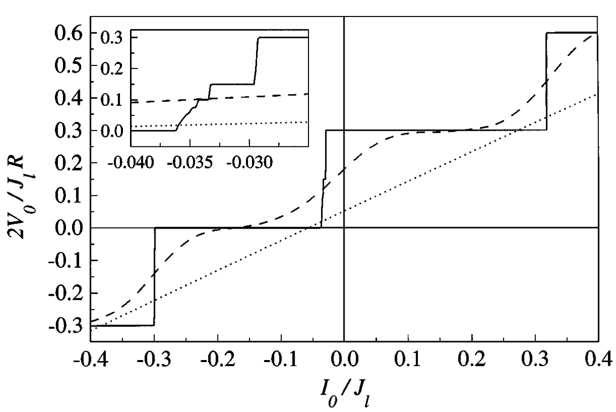

FIG. 4. Same as in Fig. 3 for frequency $\omega = 0.3$ and $A = 1.7$, for $D = 0$ (solid line), 0.01 (dashed line), and 0.5 (dotted line). Inset: magnified picture showing steps at fractional values of ω in the $D = 0$ curve.

Going back to Figs. 3 and 4, we notice the remarkable property that there is a finite range of I_0 values in which *the sign of the average voltage is independent of the average external current* [18]. For a narrower range of parameters (see, e.g., in Fig. 4), and in the absence of noise, it is possible to obtain, not only the same sign, but also *the same value* of V_0, regardless of the value and sign of I_0. Therefore, we conclude that the asymmetric SQUID we propose here can be used as a device for *voltage rectification*. From the curves presented here, we note that this mechanism of voltage rectification will operate more efficiently at low frequencies and for not too small ac amplitudes [19]. On the other hand, the analysis given in the preceding paragraph indicates that, under the effect of an external ac voltage source, the SQUID of Fig. 1 could not yield current rectification.

In our analysis, we have assumed for convenience that certain ideal relations between the parameters of the different junctions are satisfied. One may wonder whether the physical effects we have discussed may be affected by minor deviations from those specific values, especially when the two junctions in series are not identical and the simple relation $\varphi_1(t) = \varphi_2(t)$ cannot always be valid. For the case of zero noise, analytical considerations suggest that a weakened ratchet effect and a structure of shorter steps will remain. We have performed a numerical check by treating φ_1 and φ_2 as independent variables. For $\omega = 0.01$ and $A = 1$ and 1.7, and assuming differences of order between 1% and 10% (namely, $R_2/R_1 = J_2/J_1 = R_r/2R_1 = 1.01$ and 1.1), we find that the dc voltage at zero current bias decreases within 5% to 30% and that the voltage plateaus are shortened by about one-half. These results underline the robustness of the predicted physical behavior (in particular, the ratchet effect) against small deviations from the ideal structure.

For a typical SQUID, the inductance can be $L \sim 10^{-10}$ H [6]. Thus, currents $\lesssim 10^{-6}$ A are required for the condition $\Phi \simeq \Phi_{ext}$ to be satisfied. For typical tunnel junctions, the overdamped limit is not easily achieved unless one operates very close to the critical temperature [6]. Operation in a wider range of temperatures could, however, be achieved by adding shunts of sufficiently low resistance. For $J_l = 10^{-6}$ A, $R = 1\ \Omega$, the "units" of temperature, frequency, and voltage are 48 K, 125 MHz, and 0.5 μV. From our numerical results, we conclude, for instance, that for $T = 0.48$ K and $\Omega = 37.5$ MHz the dc voltage is $V_0 \sim 0.1\ \mu$V at zero dc current.

In conclusion, we have demonstrated the feasibility of a novel effect in the dynamics of the phase across an asymmetric SQUID threaded by a magnetic flux. The ratchet structure of the effective potential experienced by the phase through the ring favors one sign of its time derivative. Under an oscillating current source, the dc current-voltage characteristics present striking properties such as displaced Shapiro steps and the possibility of having a finite dc voltage with a zero dc current, and vice versa. Within a certain range of parameters, the same sign, and even the same value, of the dc voltage can be obtained regardless of the sign of the external dc current. This mechanism of voltage rectification has been shown to be robust in the presence of moderate noise and of small deviations of the junction parameters from the proposed ideal behavior. Estimates for a single SQUID suggest that the predicted ratchet-induced voltage shift is indeed measurable. The effect could be conveniently amplified by placing many similar devices in series.

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[18] It is important to note, however, that, although the phase velocity has the right sign in a majority of cases, there are particular combinations of parameters (typically, for moderate-to-large ω and small-to-moderate values of D and A) in which $\langle\dot{\varphi}\rangle$ has the "wrong" sign, as already noted for the mechanical analog of the asymmetric SQUID (cf. Fig. 1 of Ref. [3]).

[19] In the limit $D \rightarrow 0$ and $\omega \rightarrow 0$, a minimum value of A is required to obtain a nonzero dc voltage.