VOLTAGE RECTIFICATION IN A DRIVEN ASYMMETRIC SQUID

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We argue that the phase across an asymmetric dc SQUID threaded by a magnetic flux can experience a ratchet (periodic and asymmetric) potential. Under an external ac current, a rocking ratchet mechanism operates whereby one sign of the time derivative of the phase is favored. We show that there exists a range of parameters in which a fixed sign of the average voltage across the ring occurs, regardless of the sign of the external current dc component.

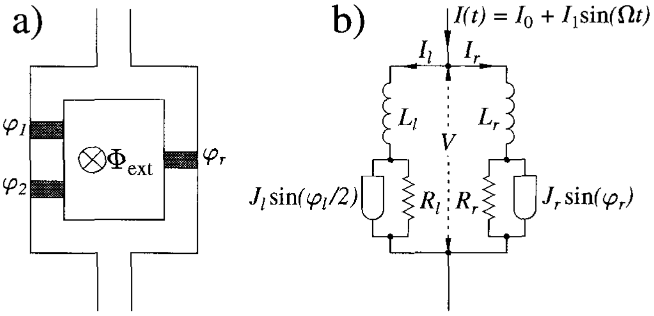

Fig. 1. (a) Asymmetric SQUID with three junctions. (b) Equivalent circuit.

A rocking ratchet is formed by a particle subject to a periodic and asymmetric potential (ratchet) and to a time periodic, additive (state-independent) force of zero average. The combination of spatial asymmetry and nonequilibrium, unbiased forces favors motion in one direction [Magnasco, 1993; Bartussek *et al.*, 1994; Ajdari *et al.*, 1994]. Here we propose a realization of the rocking ratchet mechanism in an asymmetric dc SQUID with two Josephson junctions in one of the arms and one junction in the other arm (see Fig. 1). When the ring is threaded by a magnetic flux $\Phi_{\rm ext}$, the potential $\tilde{U}(\varphi)$ for the phase $\varphi(t) \equiv 2\varphi_1(t) = 2\varphi_2(t) \equiv \varphi_l(t)$ across the ring is a ratchet (see Fig. 2). When the potential $\tilde{U}(\varphi)$ is "rocked" by an external ac current, one sign of $\dot{\varphi}$ is favored. Thus, there must be a range of parameters for which a fixed sign of the average dc voltage across the SQUID, $V_0 = \langle V(t)\rangle \equiv \hbar\langle\dot{\varphi}\rangle/2e$ (with the brackets denoting the long time limit $t \to \infty$ of the time average over one period of the driving), occurs regardless of the sign of the external dc current component I_0. Neglecting inductance effects, the phases across the left and right arms differ by $\varphi_l - \varphi_r = -\varphi_{\rm ext} + 2\pi n$, with $\varphi_{\rm ext} \equiv 2\pi\Phi_{\rm ext}/\Phi_0$. Two identical junctions in series on the left arm act like one junction with a $\sin(\varphi_l/2)$ current-phase relation [Zapata & Sols, 1996]. In the overdamped limit (neglecting capacitances), the phase $\varphi \equiv \varphi_l$ through the system satisfies the equation

$$\frac{\hbar}{eR}\dot{\varphi} = -J_l\sin\left(\frac{\varphi}{2}\right) - J_r\sin(\varphi + \varphi_{\rm ext}) + I(t)\,, \quad (1)$$

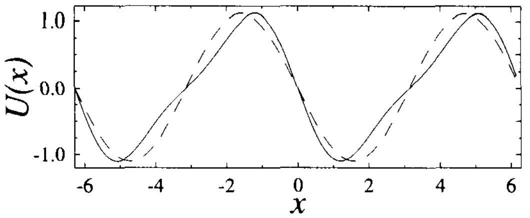

Fig. 2. Ratchet potential (solid) given after Eq. (2) of the text, compared to the sine potential (dashed) $U(x) = 1.1\ \sin(x)$.

where $R_l = R_r \equiv R$ and $I(t) = I_0 + I_1 \sin(\Omega t)$ is an external current. If $J_r/J_l = 1/2$ and $\Phi_{\text{ext}} = \Phi_0/4$, the phase dynamics is given by

$$\frac{dx}{d\tau} = -\frac{\partial}{\partial x}U(x) + F + A\ \sin(\omega\tau) + \xi(\tau)\,, \qquad (2)$$

where $x \equiv (\varphi + \pi)/2$, $\tau \equiv (eRJ_l/2\hbar)t$, $F \equiv I_0/J_l$, $A \equiv I_1/J_l$, $\omega \equiv 2\hbar\Omega/eRJ_l$, $D \equiv ek_BT/\hbar J_l$, $\xi(\tau)$ is Gaussian noise with $\langle\xi(\tau)\xi(0)\rangle = 2D\delta(\tau)$, and $U(x) = -[\sin(x) + (1/4)\sin(2x)]$ is the ratchet-shaped effective potential (see Fig. 2). Hence, the average voltage across the SQUID is $V_0 = (J_lR/2)\,\langle dx/d\tau\rangle$.

Figure 3 shows the dc current-voltage characteristics for $\omega = 0.01$, $A = 1$, and different noise strengths D (temperatures). The result is compared to that obtained for a symmetric potential with the same barrier height. The main effect of the ratchet shape of the potential is that the dc current-voltage characteristics is globally shifted towards more positive values of V_0. In the $D = 0$ case with $U(x)$ being the sine potential, the velocity $\langle dx/d\tau\rangle$ is quantized at values $n\omega$, $n = 0, \pm 1, \ldots$ ("Shapiro Steps"), while for the ratchet potential substeps may also appear [Zapata *et al.*, 1996]. A small amount of noise ($D = 0.01$) wipes out the structure of steps but does not destroy the ratchet-induced shift in the dc I–V characteristic. The inset of Fig. 3 shows, that Ohmic behavior ($V_0 \propto I_0$) is achieved with stronger noise or for large enough I_0 [Zapata *et al.*, 1996]. Here, we show results for low frequencies. For a given amplitude, the ratchet behavior tends to disappear as the frequency grows. On the other hand, for a given frequency, there is an optimum amplitude that maximizes the ratchet effect [Bartussek *et al.*, 1994].

We remark that there is a finite range of I_0 values in which *the sign of the average voltage is independent of the sign of the average external current.* For a narrower range of parameters, and in the absence of noise, it is possible to obtain, not only the same sign, but also *the same value* of V_0, regardless of the value and sign of I_0. Thus, the asymmetric SQUID of Fig. 1 can be used as a device for *voltage rectification.*

One may wonder whether the physical effects discussed here might disappear if the two junctions in series are not exactly identical. For $D = 0$ and differences in the parameter values of one to ten

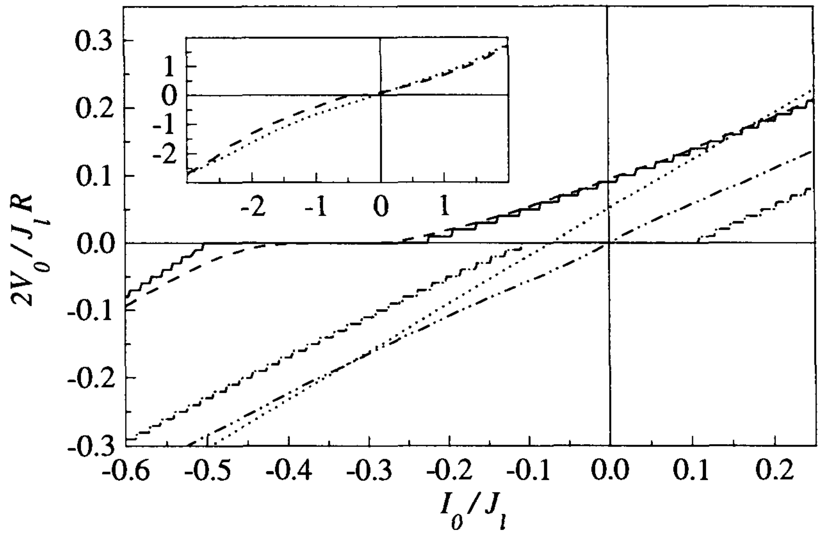

Fig. 3. For amplitude $A = 1$ and frequency $\omega = 0.01$ the dc current-voltage characteristics which arise in a ratchet and in a sine potential, cf. Fig. 2, are compared: Results are given for the ratchet potential with $D = 0$ (solid), $D = 0.01$ (dashed) and for the sine potential with $D = 0$ (dashed-dotted), $D = 0.01$ (dashed-double dotted). The inset gives I_0–V_0 curves for the ratchet potential with $D = 0.01$ (dashed) and $D = 0.5$ (dotted).

percent, we have checked that the ratchet effect survives, more precisely the dc voltage at $I_0 = 0$ only decreases by ten to thirty percent. These results underline the robustness of the predicted physical behavior.

For a typical SQUID, the inductance can be $L \sim 10^{-10}$ Henrys. Thus, currents $\lesssim 10^{-6}$ A are required for the condition $\Phi \simeq \Phi_{\rm ext}$ to be satisfied. For typical tunnel junctions, the overdamped limit is difficult to achieve unless one operates very close to the critical temperature. Operation in a wider range of temperatures could, however, be achieved by adding shunts of sufficiently low resistance. For $J_l = 10^{-6}$ A, $R = 1$ Ohm, the "units" of temperature, frequency and voltage are 48 K, 125 MHz, and 0.5 μV. From our numerical results, we may conclude, for instance, that for $T = 0.48$ K and $\Omega = 37.5$ MHz the dc voltage is $V_0 \sim 0.1$ μV at zero dc current.

In summary, we demonstrate the feasibility of a novel effect in the dynamics of the phase across an asymmetric SQUID threaded by a magnetic flux. The ratchet structure of the effective potential experienced by the phase through the ring favors one sign of its time derivative. Under an oscillating current source, the dc current-voltage characteristics present striking properties such as displaced Shapiro steps and the possibility of having a dynamically induced finite dc voltage with a zero dc current. Within a certain range of parameters, the same sign, and even the same value, of the dc voltage can be obtained regardless of the sign of the external dc current. This mechanism of voltage rectification is robust against moderate noise and small deviations of the junction parameters from the proposed ideal behavior. Estimates for a single SQUID suggest that the predicted ratchet-induced voltage shift is indeed measurable. The effect could be conveniently amplified by placing many similar devices in series.

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