Nonadiabatic Quantum Brownian Rectifiers

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We study both *dissipative* and *nondissipative* quantum transport in discrete Brownian rectifiers being driven by *nonthermal* noise that is *unbiased* on average. In the absence of dissipation the current is always zero and the ballistic diffusion changes into normal diffusion. The dissipative quantum dynamics exhibits current (with distinctive reversals) as a result of the cooperative interplay between dissipative forces and external fluctuations. Considering the *nonlinear* current response to aperiodic, noisy forces we predict aperiodic quantum stochastic resonance. The nonthermal fluctuations can considerably enhance, as well as suppress, the thermal quantum diffusion. [S0031-9007(98)06616-2]

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The constructive role of nonthermal (deterministic or stochastic) forces and equilibrium fluctuations for driven transport in one-dimensional periodic structures can generate unexpected novel phenomena. In particular, new concepts such as fluctuation driven transport in periodic structures that lack a reflection symmetry (ratchets) [1], or the phenomenon of stochastic resonance [2] in threshold systems, boost the importance of nonthermal fluctuations to a level, where it must be viewed as a source for order and complexity in its own right. Directed current in *adiabatically* rocked *quantum* ratchets has been discussed only recently [3]. Other examples of anomalous transport properties which do not exploit the ratchet mechanism have recently been investigated in driven periodic tight-binding (TB) lattices. In particular, an absolute negative conductance near zero dc bias can be induced by the combined effects of dc and ac fields [4], or dc field and external noise [5]. In this Letter, our focus will be on *control* of quantum transport in periodic TB lattices, being solely driven by *unbiased* nonthermal noise. No current appears in the absence of quantum dissipation. However, as a result of a ratchetlike mechanism, a finite current occurs when quantum equilibrium fluctuations interplay with (classical) nonthermal noise. We shall consider the resulting current in the whole regime of adiabatic-to-nonadiabatic fluctuation time scales of nonthermal unbiased forces. In addition, we show that the current is maximal for an optimal value T of the environmental temperature, this being a signature of *aperiodic quantum stochastic resonance*.

Because of the generality of the model, our results can find application in a variety of physical and chemical systems. For example, new lithography and low temperature techniques allow for fabrication and measurement of the current-voltage characteristics in small Josephson junctions [6], or in suitably designed semiconductor superlattices [7]. Another potential class for applications is optical lattices formed by interfering beams of light [8].

To start, we consider a single-band tight-binding Hamiltonian $H(t) = H_{\mathrm{TB}} + H_{\mathrm{ext}}(t) + H_{\mathrm{B}}$ which accounts both for quantum dissipation and external, generally time-dependent, unbiased forces. The first term H_{TB} describes the Hamiltonian of the bare multistate system

$$H_{\mathrm{TB}} = -\frac{\hbar\Delta}{2}\sum_{n=-\infty}^{\infty}(|n\rangle\langle n+1| + |n+1\rangle\langle n|), \quad (1)$$

where $|n\rangle$ denotes the localized (Wannier) states, and $\hbar\Delta$ is the tunneling coupling energy between neighboring states. The driving influence is characterized by

$$H_{\mathrm{ext}}(t) = -e\mathcal{E}(t)\hat{q}, \qquad \hat{q} = a\sum_n n|n\rangle\langle n|, \quad (2)$$

where a is the lattice period, $\hat{q}$ is the position operator for the particle on the lattice, and $\mathcal{E}(t)$ is the external time-dependent (generally random) electric field. Quantum dissipation is modeled by an ensemble of harmonic oscillators bilinearly coupled to the driven system [9]; i.e.,

$$H_{\mathrm{B}} = \frac{1}{2}\sum_i\left[\frac{p_i^2}{m_i} + m_i\omega_i^2\left(x_i - \frac{c_i}{m_i\omega_i^2}\hat{q}\right)^2\right]. \quad (3)$$

The environmental influences are fully captured by the spectral density $J(\omega) = (\pi/2)\sum_i c_i^2/(m_i\omega_i)\delta(\omega - \omega_i)$. The model (1)–(3) can be considered as a one-band truncation for the problem of a quantum Brownian particle motion in a driven periodic potential. This situation holds in the limit of high potential barriers when the particle dynamics entails only tunneling between neighboring ground states. It is assumed that neither the temperature nor nonthermal fluctuations can cause any essential population of higher levels in the wells.

The statistical transport quantities of interest are the mean particle position $\langle q(t)\rangle = a\sum_n np_n(t)$ and the mean squared position $\langle q^2(t)\rangle = a^2\sum_n n^2p_n(t)$, with $p_n(t)$ being the site populations. In turn, the relevant transport quantities in the presence of unbiased ($\overline{\eta(t)} = 0$) time-dependent random force $\eta(t) = ea\mathcal{E}_\eta(t)/\hbar$ are the noise-averaged stationary quantum current

$$J_{\mathrm{st}} = e\lim_{t\to\infty}\frac{d}{dt}\overline{\langle q(t)\rangle}, \quad (4)$$

and quantum diffusion coefficient

$$D = \frac{1}{2} \lim_{t\to\infty} \frac{d}{dt} \overline{\langle [q(t) - \overline{\langle q(t) \rangle}]^2 \rangle}, \tag{5}$$

respectively. To be definite, we consider nonthermal fluctuations described by a kangaroo process. Such a stochastic process randomly assumes discrete values $\eta(t) = \varepsilon_i$, $i = 1, \ldots, N$ with probabilities $w_i = w(\varepsilon_i)$, $\sum_i w_i = 1$, and average sojourn time $\tau_i = 1/\nu_i$ in state ε_i [10]. In particular, an asymmetric dichotomic Markov process (DMP) is the simplest example of a kangaroo process switching between two states $\eta(t) = \{\varepsilon_1, \varepsilon_2\}$ with flip rates $\nu_{1,2}$, yielding the stationary probabilities $w_{1,2} = \nu_{2,1}/(\nu_1 + \nu_2)$. From $\overline{\eta(t)} = 0$ it follows that $\varepsilon_1 \nu_2 = -\varepsilon_2 \nu_1$. The DMP is completely characterized by three independent parameters: The noise variance $\sigma^2 = \overline{\eta^2(t)} = \varepsilon_1 |\varepsilon_2|$, the inverse autocorrelation time $\nu = (\nu_1 + \nu_2)/2$, and the asymmetry measure $\xi \equiv \ln(|\varepsilon_2|/\varepsilon_1)$. Note that DMPs with equal σ and ν but with different ξ possess the same autocorrelation function $\overline{\eta(t)\eta(t')} = \sigma^2 e^{-\nu|t-t'|}$.

In order to clarify the role of quantum dissipation, let us first consider the nondissipative dynamics.

Coherent dynamics.—We characterize the particle by the density matrix $\rho_{n,m}(t)$, with $p_n(t) \equiv \rho_{n,n}(t)$ being the site populations. Following the approach in [11], the characteristic function $F(k,t) = \sum_n e^{ikn} p_n(t)$, with $-\pi \leq k < \pi$, can be evaluated *exactly* for arbitrary driving $\mathcal{E}(t)$. The mean position $\langle q(t) \rangle = -iaF'_k(0,t)$ is given by

$$\langle q(t) \rangle = \langle q(0) \rangle + a|K|\Delta \int_0^t d\tau \sin[\phi(\tau, 0) + \varphi], \tag{6}$$

with $K = \sum_n \rho_{n-1,n}(0)$, $\tan\varphi = \mathrm{Im}\,K/\mathrm{Re}\,K$, $\phi(t,\tau) = \int_\tau^t \varepsilon(t')\,dt'$, and $\varepsilon(t) = ea\mathcal{E}(t)/\hbar$. This *exact* relation has some intriguing consequences.

(i) Because of the boundness of (6) for *any* field, the stationary dc current is always zero. Consider a particle prepared in a pure initial state described by the Gaussian wave packet $|\psi(0)\rangle = \sum_n c_n |n\rangle$, with $c_n = c_0 e^{-\delta n^2}$. Then, in the limit $\delta \ll 1$, the coherence parameter K in (6) can be related to the wave packet width $1/\delta$; i.e., $K \sim e^{-\delta/2}$. Next, switch on a dc-field $\mathcal{E}(t) = \mathcal{E}_{dc}$. In this case $\langle q(t) \rangle$ exhibits Bloch oscillations [12] with frequency $\varepsilon_{dc} = ea\mathcal{E}_{dc}/\hbar$, and with spatial amplitude $\Delta q = \hbar\Delta|K|/e\mathcal{E}_{dc}$. For mixed states described with a diagonal density matrix ($K = 0$), (6) yields $\langle q(t) \rangle = \langle q(0) \rangle$.

(ii) Next consider the influence of additional unbiased fluctuations $\eta(t)$ on the Bloch oscillations; i.e., $\varepsilon(t) = \varepsilon_{dc} + \eta(t)$. The average of (6) over the external noise reduces to the evaluation of the noise-averaged Kubo propagator $\Phi(t - \tau) = \overline{\exp(i \int_\tau^t \eta(t')\,dt')}$. For a general kangaroo process this latter problem can be solved exactly [10]. For a DMP the complex-valued propagator is

$$\Phi_{\mathrm{DMP}}(t) = e^{-\gamma t/2}[\cos(\zeta t) + (\chi/2\zeta)\sin(\zeta t)], \tag{7}$$

with $\zeta = \sqrt{\sigma^2 - \chi^2/4}$, and $\chi = \nu - 2i\sigma\sinh(\xi/2)$. Because the propagator (7) decays, the Bloch oscillations are exponentially suppressed.

Next we focus on the quantum diffusion behavior by assuming a diagonal initial density matrix. From $\langle q^2(t) \rangle = -a^2 F''_{kk}(0,t)$ we find

$$\frac{d}{dt}\langle q^2(t) \rangle = a^2\Delta^2 \int_0^t d\tau \cos[\phi(t,\tau)]. \tag{8}$$

With $\varepsilon(t) = 0$, Eq. (8) yields $\langle q^2(t) \rangle = \langle q^2(0) \rangle + a^2\Delta^2 t^2/2$; i.e., quantum diffusion is ballistic. In the case of a dc field, $\langle q^2(t) \rangle$ oscillates with Bloch frequency ε_{dc}. However, the combined action of a dc field and nonthermal noise asymptotically leads to $\overline{\langle q^2(t) \rangle} \to 2Dt$ with diffusion coefficient $D = \frac{1}{2} a^2\Delta^2 \,\mathrm{Re}\,\tilde{\Phi}(i\varepsilon_{dc})$, with $\tilde{\Phi}$ being the Laplace-transformed Kubo propagator. For a DMP we obtain

$$D = \frac{1}{2} \frac{\Delta^2\sigma^2\nu a^2}{(\varepsilon_{dc} - \sigma e^{-\xi/2})^2(\varepsilon_{dc} + \sigma e^{\xi/2})^2 + \nu^2\varepsilon_{dc}^2}, \tag{9}$$

which approaches $D = \Delta^2 a^2 \nu / 2\sigma^2$ for $\varepsilon_{dc} \to 0$.

Dissipative dynamics.—As just established, an initially localized particle does not support a finite stationary current in the absence of dissipation. We study here the effects of quantum dissipation within the so-termed noninteracting blip approximation (NIBA) leading to a generalized master equation (GME) for the occupation probabilities p_n. It has been recently derived for an arbitrary driving field in [4] within a real-time path-integral approach, and in [5] by use of the small polaron picture up to second order perturbation theory in the bath-renormalized intersite coupling. As discussed in [9,13], and confirmed by real-time quantum Monte Carlo calculations in [14], the NIBA is a good approximation for a TB particle at high enough temperatures and/or strong dissipation when transport proceeds by sequential tunneling. The GME reads

$$\dot{p}_n(t) = \int_0^t \{W^{(+)}(t,\tau)p_{n-1}(\tau) + W^{(-)}(t,\tau)p_{n+1}(\tau) - [W^{(+)}(t,\tau) + W^{(-)}(t,\tau)]p_n(\tau)\}\,d\tau, \tag{10}$$

with forward (+) and backward (−) integral kernel

$$W^{(\pm)}(t,\tau) = \frac{1}{2}\Delta^2 e^{-Q'(t-\tau)} \cos[Q''(t-\tau) \mp \phi(t,\tau)]. \tag{11}$$

In (11), $Q'(t)$ and $Q''(t)$ are the real and imaginary parts, respectively, of the bath correlation function [9]

$$Q(t) = \frac{a^2}{\hbar\pi} \int_0^\infty d\omega \frac{J(\omega)}{\omega^2} \times \frac{\cosh(\beta\hbar\omega/2) - \cosh[\omega(\frac{\beta}{2} - it)]}{\sinh(\beta\hbar\omega/2)},$$

where $T = 1/k_B\beta$ is the temperature. The integrodifferential equation for the characteristic function $F(k,t)$ stemming from (10) yields

$$\frac{d}{dt}\langle q(t)\rangle = a\int_0^t \Gamma^-(t,\tau)\,d\tau\,, \tag{12}$$

$$\frac{d}{dt}\langle q^2(t)\rangle = a^2\int_0^t d\tau\,[\Gamma^+(t,\tau) + 2\Gamma^-(t,\tau)\langle q(\tau)\rangle/a]. \tag{13}$$

with $\Gamma^{\pm} = W^{(+)} \pm W^{(-)}$. Again, for vanishing dissipation (12) predicts no current, in agreement with the above exact discussion, and (13) reduces to (8). Equation (12) can be averaged *exactly* over the external noise for any kangaroo process. This yields the main result

$$J_{\rm st} = ea\Delta^2\int_0^\infty dt\,\exp[-Q'(t)]\sin[Q''(t)]\,{\rm Im}[\Phi(t)]\,. \tag{14}$$

Although no static bias is present, the stationary current is found to be *nonzero* for *unbiased* noise $\overline{\eta(t)} = 0$ (rectification effect) whenever higher order odd moments are nonzero (asymmetric noise). Consider the case of asymmetric DMP. Then $\Phi(t)$ in (14) is given by (7). For a symmetric DMP (asymmetry $\xi = 0$) the propagator $\Phi_{\rm DMP}(t)$ is real, and there is *no* current. If $\xi \neq 0$, a finite current appears. In the adiabatic limit, $\nu \ll \omega_c$, $\nu \ll \sigma$, we can approximate $\Phi(t) \approx \Phi_{\rm ad}(t) = \sum_i w_i e^{i\varepsilon_i t}$. From (14) this yields the adiabatic current $J_{\rm ad} = \sum_i w_i J_i$, where J_i is the dc current which corresponds to the ith discrete value ε_i of the nonthermal noise. Then $J_{\rm ad} \propto \eta^3(t) \approx \xi\sigma^3$ for small σ and ξ. Hence, the appearance of directed transport is a highly nonlinear cooperative effect, involving the interplay between the asymmetric driving and quantum dissipation. Until now the above results are valid for any dissipation mechanism. We consider next Ohmic friction with a spectral density $J(\omega) = (2\pi\hbar/a^2)\alpha\omega e^{-\omega/\omega_c}$, cutoff $\omega_c \gg \Delta$, and dimensionless friction strength α [9]. This allows for an exact analytical evaluation of the bath correlation function $Q(t)$. Hence, (14) with (7) allows for a numerical calculation (within the NIBA) of the stationary current $J_{\rm st}$ induced by a DMP over the whole nonadiabatic regime. $J_{\rm st}$ is depicted vs fluctuation strength $\sigma = \sqrt{\varepsilon_1|\varepsilon_2|}$ in Fig. 1. We note that distinct deviations occur for nonadiabatic, nonthermal DMP—driving from its adiabatic limit (full line in Fig. 1). The absolute value $|J_{\rm st}|$ is maximal at some optimal value of σ. Moreover, as depicted in the inset, the *sign* of the current can change as the damping strength α is increased. The temperature dependence of the current is plotted in Fig. 2. At certain values of the parameters, $|J_{\rm st}|$ exhibits a maximum. Because the current appears as the nonlinear response to the *aperiodic* external signal, the existence of this maximum can be interpreted as a signature of aperiodic quantum stochastic resonance [15]. In order to elucidate these results we apply a short-time approximation valid for large friction [5], yielding $Q'(t) \approx 2\alpha\omega_c^2 t^2\kappa_{\rm eff}$ and $Q''(t) \approx 2\alpha\omega_c t$, where $\kappa_{\rm eff} = 1/2 + \kappa^2\Psi'(1+\kappa)$ with $\Psi'(z)$ being the derivative of the digamma function, and $\kappa = k_BT/\hbar\omega_c$. To lowest order in σ and ξ the adiabatic current reads explicitly

$$J_{\rm ad} \approx -\frac{\sqrt{2\pi}\,ea\Delta^2}{64\omega_c^4}\,\frac{e^{-\alpha/2\kappa_{\rm eff}}}{\sqrt{\alpha}\,\kappa_{\rm eff}^{7/2}}\left(\frac{1}{3} - \frac{\kappa_{\rm eff}}{\alpha}\right)\xi\sigma^3. \tag{15}$$

Thus, (15) predicts a *change of sign* of the current, as well as a maximum of $|J_{\rm ad}|$, which depends both on temperature and on dissipation strength α.

The evaluation of the noise average for the quantum diffusion in (5) presents a difficult task: Here, we restrict

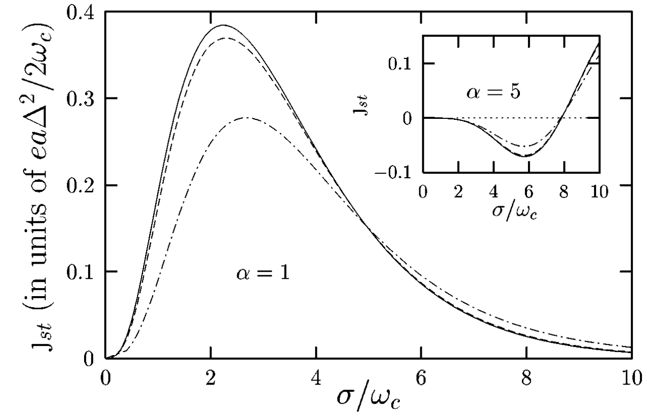

FIG. 1. Stationary current $J_{\rm st}$ vs noise strength σ for a dichotomic Markov process with zero mean and asymmetry parameter $\xi = 1/2$. The solid line depicts the adiabatic approximation. The dashed and dot-dashed lines correspond to inverse autocorrelation times $\nu = 0.1\omega_c$ and $\nu = \omega_c$, respectively. The chosen temperature is $k_BT = 0.2\hbar\omega_c$ and the Ohmic friction value is $\alpha = 1$. The inset depicts a current reversal at strong friction $\alpha = 5$.

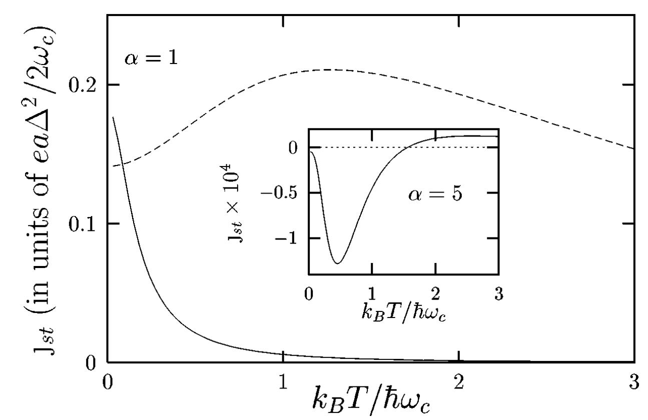

FIG. 2. Aperiodic quantum stochastic resonance (AQSR). The solid line depicts the stationary current $J_{\rm st}$ vs temperature T for $\sigma = 0.5\omega_c$ and $\sigma = 7\omega_c$ (dashed line). Other parameters are $\nu = 0.1\omega_c$, $\xi = 1$, and $\alpha = 1$. The occurrence of the maximum of $|J_{\rm st}|$ is a signature of AQSR. The inset for $\sigma = 0.5\omega_c$ depicts a current reversal vs temperature at fixed friction strength $\alpha = 5$.

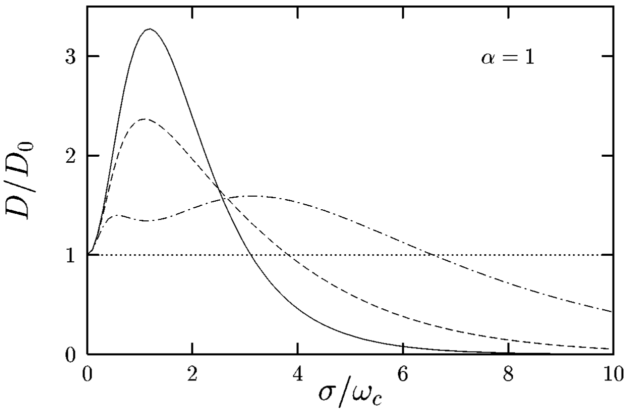

FIG. 3. The scaled fluctuation driven quantum diffusion D/D_0 vs noise strength σ for various asymmetry parameters $\xi = 0$ (solid line), $\xi = 1$ (dashed line), and $\xi = 2$ (dash-dotted line). $D_0 \equiv D(\sigma = 0)$ denotes the diffusion coefficient in the absence of nonthermal noise. For small σ the diffusion is strongly enhanced. This effect is maximal for a symmetric (i.e., $\xi = 0$) dichotomic noise. Other parameters are $\nu = 0.01\omega_c$, $\Delta = 0.1\omega_c$, $k_B T = 0.2\hbar\omega_c$, and $\alpha = 1$.

ourselves to the adiabatic approximation within the Markov limit of (10). This enables us to evaluate analytically the adiabatic diffusion coefficient $D_{\rm ad}$ for an arbitrary kangaroo process. For a DMP we obtain

$$D_{\rm ad} = \frac{a^2}{4\cosh(\xi/2)}[e^{\xi/2}\gamma_1^+ + e^{-\xi/2}\gamma_2^+] + \frac{a^2}{4\nu\cosh^2(\xi/2)}[\gamma_1^- + \gamma_2^-]^2, \qquad (16)$$

where $\gamma_i^\pm = \int_0^{t\to\infty}\Gamma^\pm(t,\tau)\,d\tau$ with $\phi(t,\tau) = \varepsilon_i(t-\tau)$. We note that, in the limit of vanishing dissipation, (16) does not reduce to the exact limit $D = \Delta^2 a^2\nu/2\sigma^2$ derived below (9). This flaw originates from the breakdown of the Markov approximation for weak dissipation. As depicted with Fig. 3, diffusion can be either increased or decreased, as compared to the case of zero nonthermal noise ($\sigma = 0$) with diffusion coefficient

$$D_0 = \frac{1}{2}a^2\Delta^2\int_0^\infty dt\,\exp[-Q'(t)]\cos[Q''(t)]. \qquad (17)$$

Strong asymmetry ξ tends to smear out this effect.

In summary, we have found several new phenomena for nonthermally driven periodic quantum systems as represented by a TB Hamiltonian. For zero quantum dissipation we observe a characteristic suppression of Bloch oscillations, as well as identically vanishing current. Moreover, we find a nonthermal noise induced crossover from ballistic (at zero dc bias) to normal diffusion. With noise strength σ approaching zero the corresponding diffusion coefficient in (9) tends to infinity. Note also that the switch-on of finite nonthermal noise overcomes the zero diffusion that characterizes Wannier-Stark localization. In contrast, a noise-directed current always occurs in a *dissipative* TB lattice in the presence of unbiased, asymmetric forcing. This rectification effect is *robust*; e.g., it is manifest also for harmonic mixing signals, $\varepsilon(t) = c_1\cos(\Omega t) + c_2\cos(2\Omega t)$ [16]. Current reversals do occur both vs noise strength σ (Fig. 1) and vs temperature T (Fig. 2). This driven system also exhibits signatures of aperiodic quantum stochastic resonance, as well as a suppressed diffusion as compared to that of the undriven quantum Brownian system. These novel surprising features regarding rectification, current reversal, and control of diffusion are expected to be observable in already fabricated superlattices [7] and/or in optical lattices [8].

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[1] R. D. Astumian, Science **276**, 917 (1997); F. Jülicher, A. Ajdari, and J. Prost, Rev. Mod. Phys. **69**, 1269 (1997).
[2] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, Rev. Mod. Phys. **70**, 233 (1998); K. Wiesenfeld and F. Moss, Nature (London) **373**, 33 (1995).
[3] P. Reimann, M. Grifoni, and P. Hänggi, Phys. Rev. Lett. **79**, 10 (1997).
[4] L. Hartmann, M. Grifoni, and P. Hänggi, Europhys. Lett. **38**, 497 (1997).
[5] I. A. Goychuk, E. G. Petrov, and V. May, Phys. Lett. A **238**, 59 (1998).
[6] *Single Charge Tunneling,* edited by H. Grabert and M. H. Devoret, NATO ASI, Ser. B, Vol. 294 (Plenum, New York, 1992).
[7] B. J. Keay *et al.,* Phys. Rev. Lett. **75**, 4098 (1995); **75**, 4102 (1995).
[8] M. G. Prentiss, Science **260**, 1078 (1993); A. Hemmerich *et al.,* Phys. Rev. Lett. **75**, 37 (1995); S. R. Wilkinson *et al.,* Phys. Rev. Lett. **76**, 4512 (1996).
[9] U. Weiss, *Quantum Dissipative Systems,* Series in Modern Condensed Matter Physics Vol. 2 (World Scientific, Singapore, 1993).
[10] A. Brissaud and U. Frish, J. Math. Phys. (N.Y.) **15**, 524 (1974).
[11] D. H. Dunlap and V. M. Kenkre, Phys. Rev. B **37**, 6622 (1988).
[12] M. Holthaus and D. W. Hone, Philos. Mag. B **74**, 105 (1996).
[13] U. Weiss and H. Grabert, Phys. Lett. **108A**, 63 (1985); W. Zwerger, Phys. Rev. B **35**, 4737 (1987).
[14] C. H. Mak and R. Egger, Phys. Rev. E **49**, 1997 (1994).
[15] A. Fulinski, Phys. Rev. Lett. **79**, 4926 (1997); C. Heneghan *et al.,* Phys. Rev. E **54**, R2228 (1996); J. J. Collins *et al.,* Nature (London) **376**, 236 (1995).
[16] I. Goychuk and P. Hänggi (to be published).