Tunneling Center as a Source of Voltage Rectification in Josephson Junctions

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(Received 19 June 1997)

A novel mechanism of fluctuation-induced voltage rectification in a Josephson junction is analyzed. A point contact with a defect tunneling incoherently between two states is proposed as a source of nonthermal fluctuations which produce asymmetric dichotomic noise. The dc current-voltage characteristics are calculated, and several transport regimes are identified according to the frequency, amplitude, and asymmetry of the noise. The limiting cases of fast and slow fluctuations are investigated analytically. The loaded dc curves exhibit voltage rectification (nonzero voltage at zero current bias) and pronounced singularities in the differential resistance. The system can be used to measure selected characteristics of the tunneling dynamics. [S0031-9007(97)05019-9]

PACS numbers: 74.40.+k, 74.50.+r, 85.25.Dq

The effect of thermal noise on the current-voltage (I-V) characteristics of Josephson junctions for a long time has been the object of considerable research activity. A great deal of the theoretical work performed in this context has been based on the capacitively and resistively shunted junction (CRSJ) model, in which the Josephson junction is viewed as a superconducting link of critical current I_c placed in parallel with an Ohmic resistance R and a capacitance C [1]. The parallel capacitance C can be neglected when $(2e/\hbar)I_c R^2 C \ll 1$ (overdamped limit). Within this approximation, the authors in [2] found that the main effect of thermal noise is that of smoothing the cusplike features in the I-V curves and bending them towards the linear behavior of an Ohmic resistor. More recently, there has been a growing interest in the study of directed transport induced by nonequilibrium fluctuations in spatially periodic structures without reflection symmetry (ratchets) [3,4]. It has been demonstrated that the phase across an asymmetric dc superconducting quantum interference device (SQUID) threaded by a magnetic flux is subject to an effective ratchet potential [5], and in the presence of ac current sources this feature has been shown to yield remarkable transport properties such as displaced Shapiro steps and, in particular, the existence of a nonzero dc voltage in the presence of a zero dc current.

Privileged particle motion in one direction can also be induced in systems with *symmetric* periodic (i.e., not ratchetlike) potentials but subject to *asymmetric* two-state noise of zero average [6]. Such dynamical systems have received theoretical attention in the recent past [7–9]. However, these prior studies have been restricted to the case of *zero bias force*, and no consideration has been given to specific situations that might be of interest for applications in Josephson devices. Another limitation of the existing literature is that only diffusive transport regimes have been considered in which the particle slides (experiences no barriers) for both values of the dichotomic force. In this Letter, we aim to fill these gaps and study the full I-V characteristics of a Josephson junction subject to an external current which fluctuates asymmetrically between two possible values. Most importantly, we also propose a specific realization of this device, consisting of coupling a Josephson junction to a point contact whose resistance fluctuates randomly due to the presence of an active asymmetric tunneling center such as observed in [10]. Unlike in the prominent case of thermal fluctuations [2], the presence of dichotomic noise does not smooth the current-voltage characteristics. On the contrary, it gives rise to a nondifferential structure caused by the emergence of novel transport regimes.

We wish to analyze the dynamic behavior of the circuit schematically depicted in Fig. 1. All its elements are conventional except for a resistance that fluctuates between two values r_a and r_b with mean waiting times t_a and t_b, respectively. This can be achieved by inserting into the circuit a point contact whose conductance is controlled by an asymmetric two-level system tunneling *incoherently* between the two states with rates $1/t_a$ and $1/t_b$ [10,11]. Accordingly, the fluctuations in the point contact resistance can be modeled as a stationary Markovian dichotomic process (telegraphic noise).

In the *overdamped* regime, the dynamics of the circuit is governed by the stochastic equations for current $I(t)$ and the Josephson phase $\varphi(t)$, i.e.,

$$I(t) = I_l(t) + I_r(t) = I_c \sin(\varphi) + \frac{\hbar}{2eR}\dot{\varphi} - \Xi(t), \quad (1)$$

$$\frac{\hbar}{2e}\dot{\varphi} = V_l + [r(t) + R_l]I_l(t) = -V_r + R_r I_r(t), \quad (2)$$

where $r(t) \in \{r_a, r_b\}$ is telegraphic noise and $\Xi(t)$ describes thermal fluctuations. If $R \ll R_l, R_r$, (1) and (2)

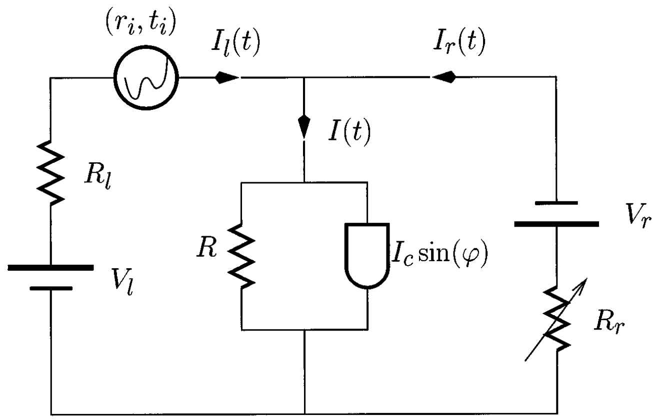

FIG. 1. Plot of the circuit proposed to test the influence of random telegraphic noise on a Josephson junction. The novel element is a normal point contact whose resistance randomly takes values $t_i (i = 1, b)$, with mean waiting times t_i, due to the presence of a near defect tunneling incoherently between two sites.

lead to the equation

$$I_0 + \Lambda(t) + \Xi(t) = \frac{\hbar}{2eR}\dot{\varphi} + I_c \sin(\varphi), \qquad (3)$$

$$I_0 \equiv \frac{V_r}{R_r} - \frac{V_l}{t_a + t_b}\left(\frac{t_a}{R_l + r_a} + \frac{t_b}{R_l + r_b}\right) \qquad (4)$$

being an average load current. The variable $\Lambda(t) \in \{-I_a, I_b\}$ behaves as dichotomic noise, with $I_{a,b} = t_{b,a} V_l (t_a + t_b)^{-1}[(R_l + r_a)^{-1} - (R_l + r_b)^{-1}]$, and has a zero mean, i.e., $\langle \Lambda(t) \rangle = 0$. The combined effect of the voltage sources $V_{r,l}$ and large resistances $R_{r,l}$ shown in Fig. 1 is that of providing a fluctuating current source whose dc value I_0 can be adjusted—for a given telegraphic noise pattern—by varying R_r.

At this point we introduce dimensionless variables $\tau = (2eRI_c/\hbar)t$ and $x(\tau) = \phi(t)$, and rewrite (3) as

$$F + \xi(\tau) + \lambda(\tau) = \frac{dx}{d\tau} + \sin(x), \qquad (5)$$

where $F = I_0/I_c$. The rescaled Gaussian thermal noise $\xi(\tau) = \Xi(t)/I_c$ has zero average $\langle \xi(\tau) \rangle = 0$ and correlation $\langle \xi(\tau)\xi(0) \rangle = 2D_T \delta(\tau)$, with $D_T = 2ek_BT/\hbar I_c$. The dimensionless dichotomic noise $\lambda(\tau) = \Lambda(t)/I_c \in \{-I_a/I_c, I_b/I_c\} \equiv \{-a, b\}$ is uncorrelated with $\xi(t)$, again has zero mean, and is exponentially correlated, i.e., $\langle \lambda(\tau)\lambda(0) \rangle = (Q/\tau_0)\exp(-|\tau|/\tau_0)$, where $\tau_0^{-1} = \tau_a^{-1} + \tau_b^{-1}$ [with $\tau_{a,b} = (2eRI_c/\hbar)t_{a,b}$] is the inverse correlation time and $Q = ab\tau_0$ is the noise intensity. Hereafter, we take $r_a < r_b$, which implies $a > 0, b > 0$.

The dc voltage V_0 is given by the Josephson relation $V_0 = RI_c \langle dx/d\tau \rangle$, where the brackets denote an average over noise. We will focus mostly—but not exclusively—on the effect of dichotomic noise. Thermal effects are indeed negligible if $D_T \ll 1$. Since $D_T \simeq 4.4 \times 10^{-5} T(\mathrm{K})/I_c(\mathrm{mA})$, we expect that temperature effects can be neglected over a wide region of parameters.

The process $x(\tau)$ defined in (5) is a non-Markovian stochastic process. From the continuity equation for its probability distribution $P(x, \tau)$ one obtains an expression for the probability current $J(x, \tau)$ [6]. The stationary probability current J is obtained in the long time limit $t \to \infty$ by imposing periodic boundary conditions $P(x) = P(x + 2\pi)$ on the corresponding stationary distribution $P(x)$ and normalizing it over the period 2π as $\int_0^{2\pi} P(x)\,dx = 1$. For vanishing temperature, i.e., when $\xi(\tau)$ is set to zero, the probability current J is determined by the equation [see Eq. (11) in [7]]

$$-[D(x)P(x)]' + f(x)P(x) = [1 + \tau_0 f'(x)]J, \qquad (6)$$

where $D(x) = Q[1 - f(x)/a][1 + f(x)/b]$ is the effective diffusion function and $f(x) = F - \sin(x)$ is a deterministic "force." The stationary mean phase velocity is given by $\langle dx/d\tau \rangle = 2\pi J$, which leads to a dc voltage $V_0 = 2\pi RI_c J$. Equation (6) is an ordinary differential equation that can be solved in closed analytical form. Three cases must be distinguished according to the structure of the roots of $D(x)$. In the *first case*, when $D(x) \neq 0$ for all x, the probability current J assumes the form [7]

$$J = \frac{1 - e^{\Psi(2\pi)}}{\int_0^{2\pi} dz\, D^{-1}(x) e^{-\Psi(x)} \int_x^{x+2\pi} dy [1 + \tau_0 f'(y)] e^{\Psi(y)}}, \qquad (7)$$

where $\Psi(x) = -\int_0^x dy\, f(y)/D(y)$ is a nonequilibrium potential. Equation (7) can in turn be applied to two physically different situations: (i) When $a > 1 + F$ and $b > 1 - F$, then barriers are suppressed for the two values of the dichotomic noise. Hence, both forward and backward transitions are possible [this corresponds to a diffusive regime with $D(x) > 0$]. (ii) When $F > 1 + a$ or $F < -1 - b$, then "particles" can move only to the right or to the left, respectively [$D(x) < 0$]. The *second case* occurs when both $1 - f(x)/a$ and $1 + f(x)/b$ have real roots. Now, the probability current J vanishes because the particle cannot overcome the barriers posed by the effective potentials $V_\lambda(x) = -(F + \lambda)x - \cos(x)$, with $\lambda = -a, b$; hence the voltage is zero (cf. dashed curve for $Q = 0.2$ in Fig. 4 below). In the *third case*, the roots of $D(x)$ come from only one of the two brackets. If, for instance, $1 - f(x)/a$ has two real roots in the period interval and $1 + f(x)/b$ has none, then only forward transitions take place. Let x_1 be a root of $1 - f(x)/a$ with $f'(x_1) < 0$ [the potential $V_{-a}(x)$ has a local minimum at x_1], and let $x_2 < x_1$ be the root in the interval $(x_1 - 2\pi, x_1)$. From (6), we arrive at the novel result

$$J^{-1} = \int_{x_1 - 2\pi}^{x_1} dx\, D^{-1}(x) e^{-\Psi(x)} \times \int_x^{x_2} d[1 + \tau_0 f'(y)] e^{\Psi(y)}. \qquad (8)$$

Since a variation of F may drive the system to a different transport regime, we expect some type of cusps in the dc I-V curves. While these curves remain continuous, the *differential structure exhibits jumps*. These sharp

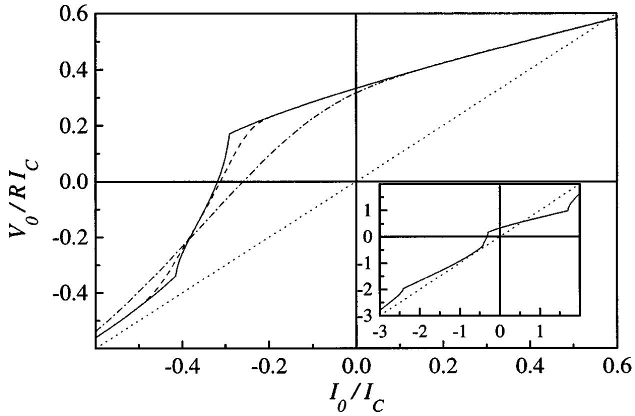

FIG. 2. dc V-I curves in the adiabatic limit, at temperatures $D_T \equiv (2e/\hbar I_c)k_B T = 0$ (solid), 0.01 (dashed), and 0.1 (dash-dotted). The linear resistor behavior ($V_0 = RI_0$) is shown for comparison (dotted). Inset: enlarged plot for the case of zero temperature (solid) and for a linear resistor (dotted). Here, $\alpha = 1\sqrt{2}$ and $b = \sqrt{2}$.

transitions in the differential resistance are caused by the appearance and disappearance of barriers in the effective potentials $V_\lambda(x)$, and they occur at values $F = \pm 1 + a, \pm 1 - b$. This nondifferential structure can be observed in Figs. 2–4, and all four cusps can be clearly seen in the inset of Fig. 2. We note that for zero current bias the voltage does not drop to zero. This effect can be clearly seen in Figs. 2–4. We conclude that a single Josephson junction subject to a current source with asymmetric dichotomic noise can yield voltage rectification.

Let us consider several limiting cases. In the adiabatic limit ($\tau_0 \to \infty$) with intensity $Q = ab\tau_0$ and asymmetry a/b of the noise kept fixed (i.e., $a \to 0, b \to 0$), the deterministic load behavior [i.e., $\xi(\tau) = \lambda(\tau) = 0$] is recovered. On the other hand, if $\tau_0 \to \infty$ with a and b fixed, we obtain

$$\left\langle \frac{dx}{d\tau} \right\rangle = \frac{\tau_a}{\tau_a + \tau_b} \langle v_{-a} \rangle + \frac{\tau_b}{\tau_a + \tau_b} \langle v_b \rangle, \qquad (9)$$

where $\langle v_a \rangle = \mathrm{sgn}(F + \alpha)\sqrt{(F + \alpha)^2 - 1}$, if $|F + \alpha| > 1\,(\alpha = -a, b)$, and zero otherwise. Clearly, $\langle v_\alpha \rangle$ is the steady-state average velocity of a particle whose dynamics is governed by the equation of motion $dx/d\tau = F + \alpha - \sin(x)$. The structure which follows from (9) can be observed in Fig. 2. The main physical features of these dynamical systems are present already in this simple, well-defined adiabatic limit: namely, rectification and a *discontinuous* structure in the differential resistance.

In the opposite limit of fast fluctuations ($\tau_0 \to 0$) with fixed a and b, the noise has practically no time to act, and the deterministic load curve $\langle v_{\alpha=0} \rangle$ is recovered. An alternative realization of the fast limit occurs when both $a, b \to \infty$, and Q is fixed. Then, the dichotomic noise tends to Gaussian white noise with the noise strength Q playing the role of an effective dimensionless temperature. We can use Eq. (7) to obtain $J_0(Q, F) = Q[1 - e^{\phi(2\pi)}][\int_0^{2\pi} dx\, e^{-\phi(x)} \int_x^{x+2\pi} dy\, e^{\phi(y)}]^{-1}$, where $\phi(x) = Q^{-1}(1 - Fx - \cos x)$. The semiclassical behavior of a Josephson junction in the presence of thermal noise has been studied in [2], where it was shown that the main effect of temperature is that of bending the dc curves toward the linear resistor limit, $V_0 = RI_0$. Inspection of Fig. 3 reveals that, close to this fast limit, a high degree of asymmetry is needed to obtain significant rectification. The pronounced discontinuities in the differential resistance can be explained in the same physical terms as given above for the general case. An important point is that the magnitude of the voltage at zero current bias depends nonmonotonically on the asymmetry I_b/I_a and the fluctuation intensity Q, exhibiting a bell-shaped behavior. With fixed asymmetry, there is an *optimum* value of the intensity (and

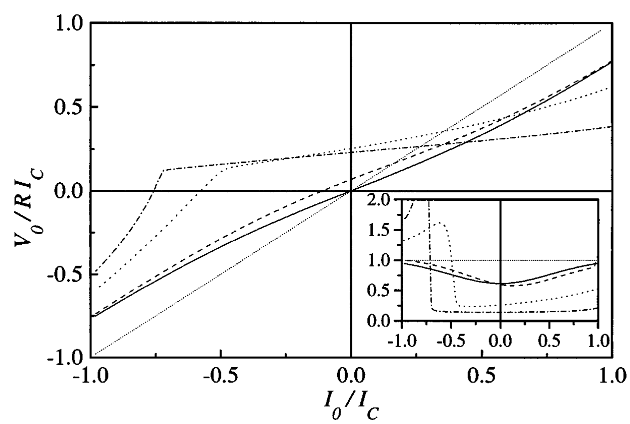

FIG. 3. dc V-I curves for $\tau_0 \equiv (2eRI_c/\hbar)t_0 = 0.1$, $Q \equiv ab\tau_0 = 1$, and for several values of the asymmetry parameter: $I_b/I_a = 1$ (solid), 2 (dashed), 32 (dotted), and 128 (dash-dotted). The linear resistor dc curve $V_0 = RI_0$ (thin, short dotted) is also shown. Inset: the corresponding dimensionless differential resistances (dV_0/RdI_0).

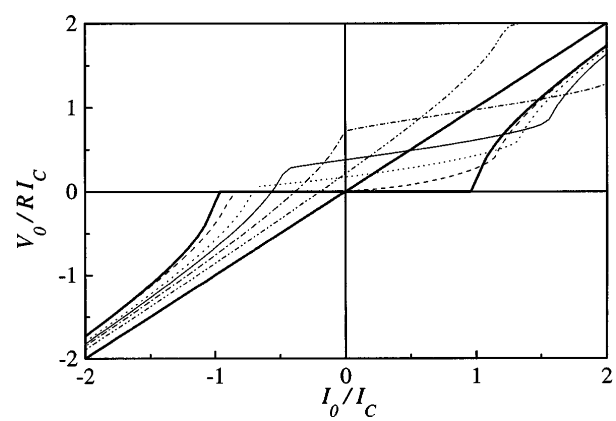

FIG. 4. dc V-I curves for $\tau_0 \equiv (2eRI_c/\hbar)t_0 = 1$, asymmetry $I_b/I_a = 10$, and for several values of the dichotomic noise intensity: $Q \equiv ab\tau_0 = 0.2$ (dashed), 1 (dotted), 3 (solid), 10 (dash-dotted), and 50 (dash–double-dotted). The thick lines correspond to an Ohmic resistor ($V_0 = RI_o$) and to deterministic RSJ behavior ($V_0 = R\sqrt{I_0^2 - I_c^2}$).

vice versa) maximizing the voltage (see Figs. 4 and 3, respectively).

Figure 4, with $\tau_0 = 1$, depicts the behavior when intense noise drives the system towards the Ohmic limit. There are remarkable differences with Ref. [2], where the crossover from zero to strong thermal noise was studied. While equilibrium noise induces a smooth transition to a linear I-V characteristic, asymmetric dichotomic noise drives the system to the same limit but through a rich, nondifferential intermediate structure.

The expression for $J_0(Q, F)$ that has been derived in the fast limit reappears in the adiabatic limit ($\tau_0 \gg 1$) in the presence of finite thermal noise with $D_T \neq 0$. The leading effect of the dichotomic force in this limit is that of weighting the two possible values of the voltage, so that

$$J_{\text{ad}} = \frac{1}{\tau_a + \tau_b} (\tau_a J_a + \tau_b J_b), \tag{10}$$

where $J_a = J_0(D_T, F - a)$ and $J_b = J_0(D_T, F + b)$. The maximum value of J_{ad} at zero current bias is $1/2\pi$, and it is achieved when $a = 1$ and $b \to \infty$. The effect of temperature has been analyzed in Fig. 2. As expected, it rounds off singularities while maintaining the rectification of voltage. Higher temperatures drive the system towards the Ohmic limit. Actually, for $D_T \gtrsim 1$ (not shown), the junction already behaves essentially like an Ohmic resistor.

The unit of inverse time is $t^{-1} = 2eRI_c/\hbar$, so that $t^{-1}(\text{MHz}) \sim 3R(\text{m}\Omega)I_c(\mu\text{A})$. To work in the overdamped limit at low effective temperatures and not very high frequencies, sufficiently small shunt resistances are needed. For example, if $R \sim 1$ mΩ and $I_c \sim$ μA, the maximum zero bias voltages obtained will be of the order of 1 nV, which is experimentally accessible. Compared with the voltage rectifier detailed in [5], the device proposed here seems simpler to build, because no fine adjustment of several Josephson junctions is needed. The magnitude of the rectified voltages in the two cases are comparable. The device described here can be used to test the tunneling process occurring at the point contact. For instance, by measuring the positions of the four sharp transitions in the differential resistance (which theoretically occur at $F = \pm 1 + a$ and $F = \pm 1 - b$), one can determine the values of a and b, and, in particular, the asymmetry of the double well, $t_a/t_b = b/a$. The value of τ_0 can be determined with greatest accuracy when the system is away from both the adiabatic and the fast limits. The real times t_a and t_b can be obtained from the knowledge of the critical voltage RI_c of the junction. A potential major advantage of this device is that it could be used to test the properties of a fluctuating two-level system whose dynamics is too fast to be resolved in real time. In this context we note that $\tau_0 \to 0$ should not necessarily be identified with nonresolvable fast fluctuations of $r(t)$. Actually, by adjusting the junction relaxation time $\hbar/2eRI_c$ such a "fast dynamics" can be made to correspond to a correlation time $\tau_0 \sim 1$.

In summary, we have proposed a novel mechanism to induce voltage rectification in a Josephson junction based on the coupling to a quantum point with a fluctuating two-level system in its vicinity (see Fig. 1). Such a device provides a very adjustable system where the influence of asymmetric nonequilibrium noise on biased I-V characteristics can be experimentally studied. We have identified conditions for maximum voltage rectification. A rich structure in the differential resistance is shown to appear due to the existence of various transport regimes. This structure is absent in systems subject to thermal noise only. The characteristic jumplike structure of the differential resistance may lead to the devising of novel control elements. The system analyzed could be used to measure the dynamics of a tunneling center close to a point contact in situations where observation of the conductance in real time is impractical.

F. S. and I. Z. would like to thank Jan van Ruitenbeek for valuable discussions. This work has been supported by DGICyT (PB93-1248), KBN (Grant No. 2P03B07911), the Deutsche Forschungsgemeinschaft, the EC-HCM Programme, the Spain-Germany Programme of Acciones Integradas, and the Comunidad Autónoma de Madrid.

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