Quantum stochastic resonance in symmetric systems

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We investigate the low-temperature quantum stochastic resonance (QSR) phenomenon in a two-level system (TLS) which is coupled to an Ohmic heat bath. In contrast to common belief we find that QSR occurs also for *symmetric* (i.e., unbiased) TLS's if the viscous friction parameter α exceeds a critical value: We demonstrate that with respect to the spectral power amplification measure QSR always occurs for $\alpha > 1$; in contrast, the output signal-to-noise ratio exhibits an amplification only for $\alpha > 3/2$. [S1063-651X(99)12305-5]

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I. INTRODUCTION

The discovery of the phenomenon of stochastic resonance (SR) [1] in nonlinear systems, whereby an ambient noise source can optimally enhance the detection of a weak information carrying signal, has triggered a large body of research to this—at first glance—paradoxical effect. By now, stochastic resonance, which is utterly a nonlinear phenomenon, is well understood and it seems to be even more common than originally suspected; see Ref. [2] for a comprehensive review and further references, or Refs. [3,4] for introductory surveys into this exciting field. In this context, we note that most of the research thus far, which mainly addressed physical, chemical and biological systems, has predominantly been based on classical stochastic dynamics. The SR phenomenon has only recently been taken into the quantum world with a few contributions [5–10]. As such, QSR is still in its infancy, but attracts increasing attention [2,4]. Some prominent quantum results are the established increase of quasiclassical SR by several orders of magnitude. This boost emerges due to finite temperature tunneling contributions [8]. Another point raised in the literature refers to the very low-temperature behavior of QSR [2,4]. It is commonly assumed that QSR in the deep cold can only emerge in biased systems [5–7]. Then, the degradation of the response with increasing (quantum) noise intensity can be offset by a noise intensity increasing Arrhenius factor (provided by the detailed balance factor of the bias). In contrast, in a symmetric TLS this helping role of an exponential Arrhenius factor is missing. It has thus been tacitly, but incorrectly suspected that the weak algebraic dependence on temperature of the corresponding low-temperature quantum rate is generically not sufficient to counterbalance the degradation of the response caused by increasing the temperature. Recently, it has been shown by use of *numerical* path integral calculations, however, that QSR at moderately low temperatures does indeed occur in *unbiased*, i.e., symmetric quantum double-well systems [10]. Our purpose with this work is to resolve such an apparent contradiction, and to obtain decisive and clear *analytical* insight into the conventional QSR phenomenon at very low temperatures.

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II. MODEL

Let us consider a two-level system (TLS), being bilinearly coupled to the heat bath, and which is subjected to a weak periodic driving force

$$f(t) = A_0 \cos(\Omega t). \tag{1}$$

The total Hamiltonian of the considered driven system reads

$$\hat{H}(t) = -\frac{1}{2}[\epsilon + 2x_0 f(t)]\hat{\sigma}_z + \frac{1}{2}\hbar\Delta\hat{\sigma}_x - x_0\hat{\sigma}_z\sum_\lambda \kappa_\lambda(b_\lambda^+ + b_\lambda) + \sum_\lambda \hbar\omega_\lambda\left(b_\lambda^+ b_\lambda + \frac{1}{2}\right). \tag{2}$$

This driven spin-boson Hamiltonian describes the reduced quantum tunneling dynamics in an asymmetric double-well potential with minima located at $x_{\min} = \pm x_0$, and with the energy bias ϵ [12–14]. The boson operators b_λ^+, b_λ correspond to heat bath oscillators with frequencies ω_λ, and $\hat{\sigma}_{z,x}$ are the usual Pauli matrices. The tunneling dynamics can be characterized by the time-dependent position operator $\hat{x}(t) = x_0\hat{\sigma}_z(t)$. Furthermore, $\hbar\Delta$ in Eq. (2) is the tunneling coupling energy between the two lowest energy levels. The bath influence on the TLS dynamics is captured by an *operator* random force $\hat{\xi}(t) = \sum_\lambda \kappa_\lambda (b_\lambda^+ e^{i\omega_\lambda t} + b_\lambda e^{-i\omega_\lambda t})$. Due to the inherent Gaussian statistics of the bath, its statistical properties are defined by the autocorrelation function [12–14]

$$\langle \hat{\xi}(t)\hat{\xi}(0)\rangle_\beta = \frac{\hbar}{\pi}\int_0^\infty J(\omega)[\coth(\beta\hbar\omega/2)\cos(\omega t) - i\sin(\omega t)]d\omega. \tag{3}$$

Here, the bath spectral density $J(\omega) = (\pi/\hbar)\sum_\lambda \kappa_\lambda^2 \delta(\omega - \omega_\lambda)$ has been introduced, $\langle \cdots \rangle_\beta$ denotes the thermal average, and $\beta = 1/k_B T$ is the inverse temperature. We assume that $J(\omega)$ acquires the Ohmic form $J(\omega) = (2\pi\hbar/4x_0^2)\alpha\omega e^{-\omega/\omega_c}$. Here, α quantifies the dimensionless viscous friction strength and ω_c is the cutoff frequency of the bath spectrum. As customary used [2], the driving force $f(t)$ plays the role of an *input signal*, and the thermal noise averaged asymptotic, time-periodic ($t \to \infty$) deviation $\langle \delta\hat{x}(t)\rangle_\beta$ from the *equilibrium* mean position is considered as the averaged *output signal*. For instance, in the case of superconducting quantum interference devices (SQUID's) the input signal corresponds to a periodically applied magnetic flux modulation, and the output relates to the total time-periodic magnetic flux [15]. The used two level approximation is well justified at low temperatures $k_B T \ll \hbar\omega_0$ and for a small time-dependent bias $|\epsilon + 2x_0 f(t)| \ll \hbar\omega_0$, where $\hbar\omega_0$ measures the energy splitting between the lowest tunnel doublet and the first higher lying excited state in the bistable double well.

III. LINEAR RESPONSE THEORY

Within the framework of linear response theory (LRT), the input signal and the averaged output response are related by

$$\langle \delta\hat{x}(t)\rangle_\beta = \int_{-\infty}^{t} \chi(t-t')f(t')dt', \tag{4}$$

where $\chi(t)$ denotes the response function. The linear susceptibility of TLS is defined as the one-sided Fourier transform $\tilde{\chi}(\omega) = \int_0^\infty e^{i\omega t}\chi(t)dt$. Furthermore, the spectral power of the fluctuations reads $S_{xx}(\omega) = \int_{-\infty}^{\infty} e^{i\omega\tau}\overline{C}_{xx}(\tau)d\tau$, where

$$\overline{C}_{xx}(\tau) = \lim_{\mathcal{T}\to\infty} \frac{1}{2\mathcal{T}} \int_0^{\mathcal{T}} \langle \delta\hat{x}(t)\delta\hat{x}(t+\tau) + \delta\hat{x}(t+\tau)\delta\hat{x}(t)\rangle_\beta dt \tag{5}$$

is the time-averaged symmetrized autocorrelation function of TLS fluctuations. Note that the spectral power $S_{xx}(\omega)$ in the considered weak driving limit can be decomposed as

$$S_{xx}(\omega) = |\tilde{\chi}(\omega)|^2 S_{ff}(\omega) + S_{xx}^{(0)}(\omega). \tag{6}$$

Here, $S_{xx}^{(0)}(\omega)$ stands for the spectral power of spontaneous fluctuations in the absence of driving, and $S_{ff}(\omega)$ is the spectral power of the signal, which is defined similarly to $S_{xx}(\omega)$. Moreover, $S_{xx}^{(0)}(\omega)$ is related to the linear susceptibility $\tilde{\chi}(\omega)$ by the thermal *fluctuation-dissipation theorem* (FDT) [16]

$$S_{xx}^{(0)}(\omega) = \hbar \coth\left(\frac{\hbar\omega}{2k_B T}\right) \mathrm{Im}\,\tilde{\chi}(\omega). \tag{7}$$

An evaluation of either $S_{xx}^{(0)}(\omega)$, or $\tilde{\chi}(\omega)$ for the spin-boson model (2) presents a difficult task which—apart from the case $\alpha = 1/2$, [6,7]—can be solved approximately only. To this end, let us consider the driven TLS dynamics subjected to the weak harmonic signal (1). In the regime, where *incoherent* transitions dominate and the tunneling coupling is small, i.e., $\Delta \ll \omega_c$, the TLS dynamics can be described within the so-termed noninteracting blip approximation (NIBA) [12–14]. The corresponding generalized master equation within NIBA for the evolution of $\langle\hat{\sigma}_z\rangle_\beta$ in arbitrary fields is well known [17–19]. An analysis of the asymptotic solution of this equation for the case of weak *adiabatic* driving (1) yields [6,7]

$$\tilde{\chi}(\omega) = \frac{1}{k_B T} \frac{x_0^2}{\cosh^2(\epsilon/2k_B T)} \frac{W}{W - i\omega}. \tag{8}$$

Here, W denotes the quantum rate of *relaxation* of the average level populations. This rate is the sum, $W = W_+ + W_-$, of forward (W_+) and backward (W_-) rates, respectively, which satisfy the detailed balance condition in the form $W_+/W_- = e^{\epsilon/k_B T}$. The expression (8) is valid in the incoherent tunneling regime for $x_0 A_0, \hbar\Omega \ll \hbar\omega_c, \alpha k_B T$ [7]. For Ohmic friction, this incoherent regime occurs whenever $\alpha > 1/2$ for any temperature [12,13]. Moreover, we assume the condition $W \ll k_B T/\hbar$, which is readily obeyed in practice. In this case, the *quantum* FDT (7) can safely be substituted by its *classical analog*, yielding for the unperturbed spectral density of the TLS

$$S_{xx}^{(0)}(\omega) = \frac{x_0^2}{\cosh^2(\epsilon/2k_B T)} \frac{2W}{W^2 + \omega^2}. \tag{9}$$

The spectral power density (9) contemplates the random transitions between levels of the TLS with the *switching rates* $W_\pm$ determined by relaxation of the *mean* populations; it thus reflects the quasiclassical Onsager regression hypothesis. The incoherent quantum rate W coincides within NIBA with the golden rule expression, i.e. [12,13],

$$W = \Delta^2 \int_0^\infty dt \exp[-Q'(t)]\cos[Q''(t)]\cos[\epsilon t/\hbar]. \tag{10}$$

The functions $Q'(t)$ and $Q''(t)$ in Eq. (10) denote the real and imaginary parts of the bath correlation function, respectively, i.e. [13],

$$Q'(t) + iQ''(t) = \frac{4x_0^2}{\hbar^2} \int_0^t dt_1 \int_0^{t_1} \langle\hat{\xi}(t_2)\hat{\xi}(0)\rangle_\beta dt_2 + i\lambda t,$$

wherein, $\hbar\lambda = 4x_0^2 \int_0^\infty d\omega\, J(\omega)/\pi\omega$ denotes the bath reorganization energy [19]. For the considered case herein, the function $Q(t)$ can be evaluated in the closed analytical form to yield (see, e.g., Ref. [20])

$$Q''(t) = 2\alpha \arctan(\omega_c t),$$

$$Q'(t) = 2\alpha \ln\left\{ \sqrt{1+\omega_c^2 t^2} \frac{\Gamma^2(1+\kappa)}{|\Gamma(1+\kappa+i\omega_M t)|^2} \right\}. \tag{11}$$

In Eq. (11), $\Gamma(z)$ denotes the complex gamma-function, and we used the abbreviations $\omega_M = k_B T/\hbar$, and $\kappa = \omega_M/\omega_c$. These expressions allow for a numerical evaluation of the quantum rate (10) with good accuracy. Moreover, in the low-temperature domain $\pi k_B T \ll \hbar\omega_c$ and for small bias $\epsilon \ll \hbar\omega_c$ one arrives at the well-known analytical approximation [21]

$$W = \frac{\Delta^2}{2\omega_c \Gamma(2\alpha)} \left(\frac{2\pi k_B T}{\hbar\omega_c}\right)^{2\alpha-1} \times \Gamma\left|\left(\alpha + i\frac{\epsilon}{2\pi k_B T}\right)\right|^2 \cosh\left(\frac{\epsilon}{2k_B T}\right). \tag{12}$$

It is worth pointing out here that the regime of validity of the two-level approximation is not restricted by the temperature domain $k_B T \ll \hbar\omega_c$. For example, in proton transferring molecular complexes in nonpolar media the energy gap between the lowest tunneling doublet and the next one is about $\omega_0 \sim 400\ \mathrm{cm}^{-1}$; which exceeds the cutoff frequency $\omega_c \sim 80\ \mathrm{cm}^{-1}$ [22]. For such cases, $k_B T/\hbar\omega_c$ can take on rather arbitrary values within the validity of a well-founded

TLS approximation, but the physics is no longer described within the low-temperature approximation used inherently in Eq. (12).

IV. QSR: SPECTRAL POWER AMPLIFICATION VERSUS SIGNAL-TO-NOISE RATIO

Let us next consider in detail the case of periodic forcing (1). The spectral power density of the signal is immediately found to read

$$S_{ff}(\omega)=\frac{\pi}{2}A_0^2[\delta(\omega+\Omega)+\delta(\omega-\Omega)]. \tag{13}$$

Combining Eqs. (13) and (9) in Eq. (6) one recognizes that the spectral power density of the output (within LRT) consists of two δ-spikes which are superimposed on a broadband Lorentzian ''background.'' This situation characterizes conventional stochastic resonance for a weak input signal. To quantify it, we use two different measures, namely, the spectral power amplification (SPA) of the signal and the signal-to-noise ratio (SNR) at the output [2]. The SPA η is related to the integral intensity of both spikes, and is defined by [23]

$$\eta(\Omega)=\pi A_0^2|\tilde{\chi}(\Omega)|^2. \tag{14}$$

Note that η has the dimensionality of $[x_0^2]$. For the present case it is convenient to use a dimensionless measure given by $\tilde{\eta}=(\hbar\omega_c/A_0x_0)^2(2x_0)^{-2}\eta$. The signal-to-noise ratio R is defined by the ratio between the spectral power amplification (14) and the spectral power intensity of the spontaneous fluctuations at the driving frequency Ω [2] in absence of driving. It has the dimension of a frequency and reads within the LRT

$$R(\Omega)=\frac{\pi A_0^2|\tilde{\chi}(\Omega)|^2}{S_{xx}^{(0)}(\Omega)}. \tag{15}$$

Upon combining Eqs. (8) and (9) one obtains for the spectral power amplification

$$\eta(\Omega)=\frac{1}{(k_BT)^2}\frac{\pi A_0^2x_0^4}{\cosh^4(\epsilon/2k_BT)}\frac{W^2}{W^2+\Omega^2}, \tag{16}$$

and likewise for the SNR

$$R=\frac{\pi A_0^2x_0^2}{2(k_BT)^2}\frac{W}{\cosh^2(\epsilon/2k_BT)}. \tag{17}$$

Note that within the considered adiabatic approximation the SNR measure R does not depend on the angular driving frequency Ω.

We focus now on the unbiased, symmetric TLS dynamics (i.e., $\epsilon=0$) in the low-temperature domain, where the approximation (12) is fully valid. For the scaled SPA $\tilde{\eta}$ we derive the main result

$$\tilde{\eta}=\frac{\pi^3w_0^2\tilde{T}^{4(\alpha-1)}}{w_0^2\tilde{T}^{4(\alpha-1/2)}+\Omega^2}, \tag{18}$$

where we introduced the scaled temperature given by $\tilde{T}=2\pi k_BT/\hbar\omega_c$ and $w_0\equiv\Delta^2\Gamma^2(\alpha)/2\omega_c\Gamma(2\alpha)$. As is clearly seen from Eq. (18), the spectral power amplification η exhibits a *monotonic* decrease vs increasing temperature *if* $\alpha\leqslant 1$. Thus, no QSR phenomenon occurs for a symmetric TLS in this parameter region, which is in full agreement with the previous findings in this regime [5–7]. The behavior changes, however, in the strong dissipation regime $\alpha>1$: Then—although not of exponential strong form—the algebraic increase of the incoherent rate W, being proportional to $T^{(2\alpha-1)}$ with increasing friction strength is, in fact, *sufficient* to counterbalance the algebraic decrease (proportional T^{-2}) of the output response with increasing temperature. This finding thus extends prior research studies [5–7] to the whole regime of viscous dissipation strength $0<\alpha<\infty$. In particular, we find that for the SPA measure QSR occurs for all $\alpha>1$. For given Ω, the maximum

$$\tilde{\eta}_{\max}=\pi^3\frac{[2(\alpha-1)]^{2(\alpha-1)/(2\alpha-1)}}{2\alpha-1}(w_0/\Omega)^{2/(2\alpha-1)} \tag{19}$$

in the signal power amplification takes place at

$$\tilde{T}_{\max}=[2(\alpha-1)]^{1/2(2\alpha-1)}(\Omega/w_0)^{1/(2\alpha-1)}. \tag{20}$$

Note that the SPA maximum $\eta_{\max}$ and its position $T_{\max}$ are related in the considered low-temperature approximation by a scaling low $\eta_{\max}\sim T_{\max}^{-2}$ independently of α. Moreover, substituting Eq. (20) into Eq. (12) (at $\epsilon=0$) we obtain the condition

$$W(T_{\max})=\sqrt{2(\alpha-1)}\Omega \tag{21}$$

for *approximate* matching of the time scales between incoherent tunneling dynamics and external driving at the SPA maximum. This time-scale matching underpins the interpretation of stochastic resonance as a synchronization phenomenon [2,23]. For very low Ω [such that the corresponding $T_{\max}(\Omega)<10^{-2}\hbar\omega_c/k_B$] the full numerical results for the synchronization scaling function $f(\alpha,\Omega)=W(T_{\max})/\Omega$ are consistent with its low-temperature approximation $f_{LT}(\alpha)=\sqrt{2(\alpha-1)}$ in Eq. (21). In particular, $f_{LT}(\alpha)$ actually provides an upper bound for the true synchronization scaling function, i.e., $f(\alpha,\Omega)\leqslant f_{LT}(\alpha)$. With increasing angular frequency Ω a maximum for $f(\alpha,\Omega)$ vs α does appear (not shown).

The corresponding bell-shaped QSR behavior is depicted in Fig. 1(a) for the case $\alpha=1.44$; this specific value is of relevance for the experimental SQUID dynamics as investigated in Ref. [11] in absence of a periodic driving. The authors do look forward, of course, to having this QSR result verified by experimental practitioners.

With respect to the SNR measure the situation is more delicate. We note that the signal-to-noise ratio R is actually a monotonic decreasing function of temperature T for this particular value of friction strength [see Fig. 1(b)]. This finding portrays the principal difference between the two QSR measures in use, particularly if applied in the quantum domain. For SNR it follows from Eqs. (17) and (12) that $R\propto T^{2\alpha-3}$. Thus, quantum stochastic resonance, as quantified by SNR—i.e., an increase of SNR with increasing noise strength—*also* occurs in a symmetric ($\epsilon=0$), TLS, *if* $\alpha>3/2$. We emphasize that the prediction of strictly monotonic increase of R versus T for $\alpha>3/2$ is a flaw of the low-temperature approxi-

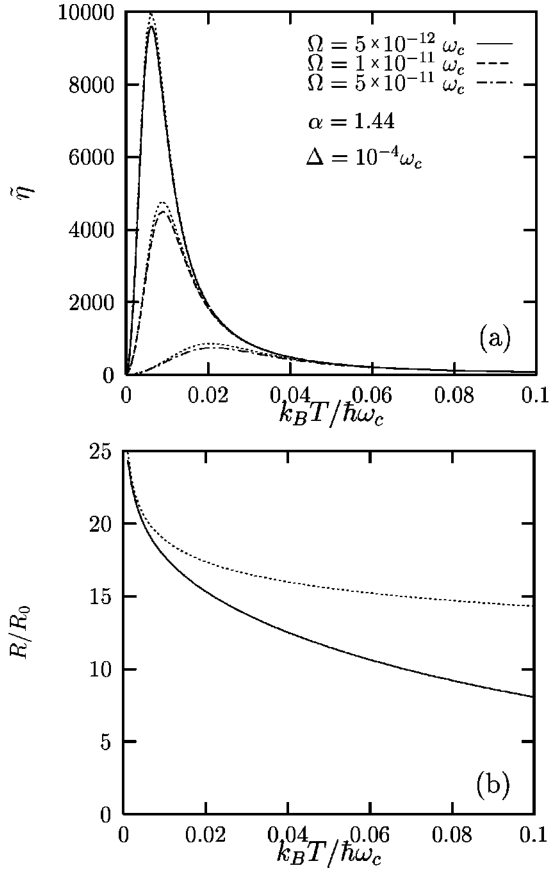

FIG. 1. (a) Scaled spectral power amplification of the signal $\tilde{\eta}$ vs scaled temperature for a symmetric TLS with Ohmic coupling strength $\alpha = 1.44$. Calculations were evaluated according to Eq. (16), using the exact NIBA relaxation rate as defined in Eqs. (10), (11), for different angular driving frequencies Ω. The scaling cutoff frequency ω_c is typically of order $10^{12}\ \mathrm{s}^{-1}$ which corresponds approximately to 7.6 K. The dotted lines in (a) and (b) nearby the corresponding curves depict the result based on the low-temperature approximation in Eq. (12), cf. Eq. (18). The QSR enhancement of the signal power amplification is quite striking. (b) Signal-to-noise ratio plotted in the scaled unit of $R_0 = (A_0 x_0/\hbar\omega_c)^2(\Delta^2/\omega_c)$ vs scaled temperature for the same system parameters. Here, no QSR occurs with respect to the SNR measure.

mation used for Eq. (12). By use of the full expression in Eq. (10) and for k_BT still *essentially* less than $\hbar\omega_c$, SNR exhibits a true maximum. This maximum is shifted towards higher temperatures with increasing frictional strength α (see Fig. 2).

V. SUMMARY

In summary, we revisited quantum stochastic resonance in the deep cold within the linear response theory approach. Our detailed analysis revealed [see Eq. (18)] that QSR, as quantified by the SPA measure, occurs also for symmetric, *unbiased* dissipative TLS systems whenever the frictional strength α exceeds the critical value $\alpha = 1$. This finding is contrary to prior assertions in Refs. [5–7] where this strong friction regime has not been addressed in detail. In particular, due to the absence of an exponential detailed balance factor for a symmetric TLS, is has been anticipated incorrectly that the degradation of the output response with increasing noise strength (the temperature) cannot be sufficiently offset by the algebraic temperature dependence of the incoherent, symmetric TLS rate.

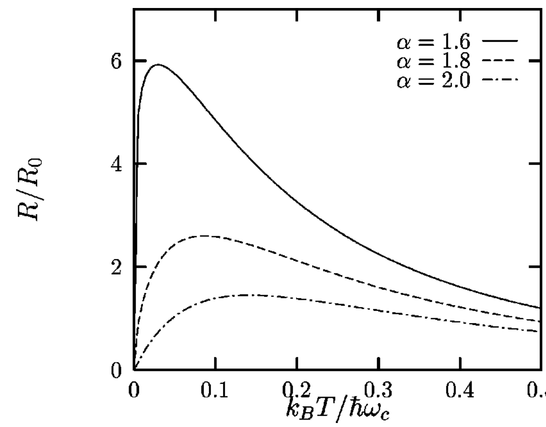

FIG. 2. Enhancement of SNR in unbiased TLS. Scaled signal-to-noise ratio vs scaled temperature for differing dissipation strengths α.

With regard to the SNR measure, the related quantum SR occurs only for $\alpha > 3/2$ [see Figs. 1(b),2]. Quantum stochastic resonance can take place also in a parameter region $k_BT < \hbar\omega_c$, for $3/2 < \alpha < 2$, beyond the validity of the low temperature approximation $\pi k_BT \ll \hbar\omega_c$ inherent in Eq. (12). With increasing α the position of SNR maximum for SNR is shifted towards $k_BT \sim \hbar\omega_c$ (at $\alpha \sim 5$) (not shown). Moreover, because the SNR approximation does not involve a dependence on angular driving frequency, its maximum behavior clearly cannot properly typify the inherent stochastic synchronization mechanism between noise-assisted (tunneling) transport through the barrier region and external periodic signal modulation which lies at the roots of the SR phenomenon [2]. In contrast, the SPA measure exhibits a peak behavior that with increasing angular frequency driving Ω shifts towards higher temperature (and thus higher total rates W). This is in qualitative agreement with such a rough, approximate matching of time scales for stochastic (QSR) synchronization [see Eq. (21)]. Finally, we note that these results for QSR in the strong friction regime are expected to become observable for a periodically modulated magnetic flux dynamics in rf SQUID's that are operating in the mK temperature region [11].

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