Stochastic dynamics of time correlation in complex systems with discrete time

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In this paper we present the concept of description of random processes in complex systems with discrete time. It involves the description of kinetics of discrete processes by means of the chain of finite-difference non-Markov equations for time correlation functions (TCFs). We have introduced the dynamic (time dependent) information Shannon entropy $S_i(t)$ where $i=0,1,2,3,\ldots$, as an information measure of stochastic dynamics of time correlation ($i=0$) and time memory ($i=1,2,3,\ldots$). The set of functions $S_i(t)$ constitute the quantitative measure of time correlation disorder ($i=0$) and time memory disorder ($i=1,2,3,\ldots$) in complex system. The theory developed started from the careful analysis of time correlation involving dynamics of vectors set of various chaotic states. We examine two stochastic processes involving the creation and annihilation of time correlation (or time memory) in details. We carry out the analysis of vectors' dynamics employing finite-difference equations for random variables and the evolution operator describing their natural motion. The existence of TCF results in the construction of the set of projection operators by the usage of scalar product operation. Harnessing the infinite set of orthogonal dynamic random variables on a basis of Gram-Shmidt orthogonalization procedure tends to creation of infinite chain of finite-difference non-Markov kinetic equations for discrete TCFs and memory functions (MFs). The solution of the equations above thereof brings to the recurrence relations between the TCF and MF of senior and junior orders. This offers new opportunities for detecting the frequency spectra of power of entropy function $S_i(t)$ for time correlation ($i=0$) and time memory ($i=1,2,3,\ldots$). The results obtained offer considerable scope for attack on stochastic dynamics of discrete random processes in a complex systems. Application of this technique on the analysis of stochastic dynamics of RR intervals from human ECG's shows convincing evidence for a non-Markovian phenomemena associated with a peculiarities in short- and long-range scaling. This method may be of use in distinguishing healthy from pathologic data sets based in differences in these non-Markovian properties.

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I. INTRODUCTION

Manifold methods are successfully used in statistical physics for the description of distinctive characteristics of chaotic dynamics of complex systems [1–26]. Nevertheless, three vexing features which are difficult to yield a detailed and strict analysis are available in complex systems. Among them: nonstationarity, nonlinearity, and nonequlibrium phenomena. Furthermore, the significant peculiarities of complex systems are directly related to the discretness of time of object-subject registration response [9,14,15,17,24]. Non-Markov and long-range statistical memory effects also play the leading part in the complex systems behavior [7–9,17,27–33].

However, the discretness of time while considering the complex systems has not been taken into account until now, although it is discretness that is the most commonly encountered feature of real objects/subjects. On the other hand, the memory and time long-ranging effects are paramount. As a rule the state developed is complicated by the fact that the real complex systems are of nonphysical nature. Therefore, the direct methods of statistical physics derived from Hamiltonian formalism, exact equations of motion and Liouville's equation are not applicable in this case to its theoretical analysis. Meanwhile the real existence of complex systems in time and space generates a reliable evristic basis for the modeling in terms of the time discretness, memory and time long-range effects.

The present article is dedicated to statistical consideration of a discretization in temporary changes of complex systems of a substantial nature on the basis of the first principles. In Sec. II we briefly outline general definitions and proposals used to form the stochastic dynamics of discrete time sequences, and in Sec. III we suggest the geometrical presentation of stochastic dynamics of time correlation. Introduction of projection operators, splitting of equation of states vectors and matrix presentation of Liouville's quasioperator for the statistical description of random processes with discrete time are reported in Sec. IV, and introduction of the set of orthogonal random variables as well as construction of infinite chain of finite-difference non-Markov kinetic equations for discrete TCF are framed in Sec. V. A pseudohydrodynamic description of random processes is provided in Sec. VI, where the relative merits of this approach are set forth. In Sec. VII we define Shannon dynamical (time dependent) entropy for time correlation and time memory in complex systems. Aplication of technique on the analysis of stochastic dynamics of RR intervals from human ECG's are discussed in Sec. VIII. In Sec. IX we present the discussion and conclusions of the results obtained and possible opportunities for the experimental data processing.

II. BASIC ASSUMPTIONS AND DEFINITIONS

Following Gaspard [15] we consider a random process such as a sequence of random variables defined at successive

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times. We shall denote the random variable by

$$X=\{x(T),x(T+\tau),x(T+2\tau),\ldots,x(T+k\tau),\ldots,x(T+\tau N-\tau)\},\tag{1}$$

which corresponds to signal during the time period $t=(N-1)\tau$ where τ is time interval of signal discretization. The mean value $\langle X\rangle$, fluctuations δx_j, absolute (σ^2) and relative (δ^2) dispersion for a set of random variables (1) can be easily found by

$$\langle X\rangle=\frac{1}{N}\sum_{j=0}^{N-1}x(T+j\tau),\tag{2}$$

$$x_j=x(T+j\tau),\quad \delta x_j=x_j-\langle X\rangle,\tag{3}$$

$$\sigma^2=\frac{1}{N}\sum_{j=0}^{N-1}\delta x_j^2,\tag{4}$$

$$\delta^2=\frac{\sigma^2}{\langle X\rangle^2}=\frac{\frac{1}{N}\sum_{j=0}^{N-1}\delta x_j^2}{\left\{\frac{1}{N}\sum_{j=0}^{N-1}x(T+j\tau)\right\}^2}.\tag{5}$$

The abovementioned values determine the statical (independent from time) properties of the system considered. The normalized time correlation function (TCF) [1–3,7–9] depending on current time $t=m\tau, N-1\geqslant m\geqslant 1$ can be conveniently used for the analysis of dynamic properties of complex systems

$$a(t)=\frac{1}{(N-m)\sigma^2}\sum_{j=0}^{N-1-m}\delta x(T+j\tau)\,\delta x(T+(j+m\tau)).\tag{6}$$

TCF usage means that developed method is just for complex systems, when correlation function exist. In forthcoming papers we intend to apply developed method for discrete random processes analysis in complex systems in practical psychology, cardiology (for the development of diagnosis method of cardiovascular diseases), financial and ecological systems, seismic phenomena, etc. The properties of TCFs $a(t)$ are easily determined by Eq. (6)

$$\lim_{t\to 0}a(t)=1,\ \lim_{t\to\infty}a(t)=0.\tag{7}$$

We have to recognize that the second property in Eq. (7) is not always satistifed for the real systems even with arbitrary big values of time t or number $(N-1)=t/\tau$. Taken into account fact that the process is discrete, we must rearrange all standard operation of differentiation and integration [34,35]

$$\frac{dx}{dt}\to\frac{\Delta x(t)}{\Delta t}=\frac{x(t+\tau)-x(t)}{\tau},$$

$$\int_a^b x(t)dt=\sum_{j=0}^{n-1}x(T_a+j\tau)\Delta t=\tau\sum_{j=0}^{n-1}x(T_a+j\tau)=n\tau\langle X\rangle,$$

$$b-a=c,\quad c=\tau n.\tag{8}$$

The first derivative on the right is recorded in Eq. (8). The second derivative on the right is also derived easily,

$$\frac{d^2x(t)}{dt^2}\to\frac{\Delta x}{\Delta t}\left(\frac{\Delta x}{\Delta t}\right)=\tau^{-2}\{[x(t+2\tau)-x(t+\tau)]-[x(t+\tau)-x(t)]\}=\tau^{-2}\{x(t+2\tau)-2x(t+\tau)+x(t)\}.\tag{9}$$

Now let us proceed to the description of the dynamics of the process. For real systems values $x_j=x(T+j\tau)$ and $\delta x_j=\delta x(T+j\tau)$ result from the experimental data. Thus we can introduce in Shannon's manner [17] the evolution operator $U(T+t_2,T+t_1)$ in as follows ($t_2\geqslant t_1$):

$$x(T+t_2)=U(T+t_2,T+t_1)x(T+t_1).\tag{10}$$

For brevity let us present Eq. (10) in the form

$$x(j)=U(j,k)x(k),\quad j\geqslant k,\quad j,k=0,1,2,\ldots,N-1.\tag{11}$$

The time operator of one step shift τ along a discrete trajectory is conviniently considered by means of two nearest values $x(t+\tau)$ and $x(t)$

$$x(t+\tau)=U(t+\tau,t)x(t).\tag{12}$$

Owing to Eqs. (10)–(12) a formal equation of motion is derivable for any $x\in(x_0,x_1,x_2,\ldots,x_{N-1})$

$$\frac{dx}{dt}\to\frac{\Delta x(t)}{\Delta t}=\tau^{-1}\{x(t+\tau)-x(t)\}=\tau^{-1}\{U(t+\tau,t)-1\}x(t).\tag{13}$$

Let us consider Eq. (13) in terms of x_j

$$\frac{\Delta x_j(t)}{\Delta t}=\frac{x_{j+1}(t+\tau)-x_j(t)}{\tau}=\tau^{-1}\{U(t+\tau,t)-1\}x_j(t)$$

and then introduce a Liouville's quasioperator $\hat{L}$ as follows:

$$\frac{dx}{dt}=\frac{\Delta x(t)}{\Delta t}=i\hat{L}(t,\tau)x(t),$$

$$\hat{L}(t,\tau)=(i\tau)^{-1}[U(t+\tau,t)-1].\tag{14}$$

Now in accordance with Refs. [15,16] let us present a set of values of random variables $\delta x_j=\delta x(T+j\tau), j=0,1,\ldots,N-1$ as a k-component vector of system state

$$\mathbf{A}_k^0(0)=(\delta x_0,\delta x_1,\delta x_2,\ldots,\delta x_{k-1})=[\delta x(T),\delta x(T+\tau),\ldots,\delta x(T+(k-1)\tau)].\tag{15}$$

Now we can introduce the scalar product operation

$$\langle \mathbf{A}\cdot\mathbf{B}\rangle=\sum_{j=0}^{k-1} A_j B_j \tag{16}$$

with or without indication of obvious time dependence of vectors **A** and **B**, respectively, in the set of vectors $\mathbf{A}_k^0(0)$ and $\mathbf{A}_{m+k}^m(t)$ where $t=m\tau$ and

$$\begin{aligned}\mathbf{A}_{m+k}^m(t)&=\{\delta x_m,\delta x_{m+1},\delta x_{m+2},\ldots,\delta x_{m+k-1}\}\\&=\{\delta x(T+m\tau),\delta x(T+(m+1)\tau),\delta x(T\\&\quad+(m+2)\tau),\ldots,\delta x(T+(m+k-1))\tau\}.\end{aligned} \tag{17}$$

A number $k<N-1$ determines the vectors' dimension. The functions (4),(5) can be expressed in terms of scalar product (16)

$$\sigma^2=\frac{1}{N}\langle \mathbf{A}_N^0\cdot\mathbf{A}_N^0\rangle=N^{-1}\{\mathbf{A}_N^0\}^2,$$

$$\delta^2=\frac{N^{-1}\langle \mathbf{A}_N^0\cdot\mathbf{A}_N^0\rangle}{\langle X\rangle^2}.$$

A k- component vector $\mathbf{A}_{m+k}^m(t)$ displaced to the distance $t=m\tau$ on the discrete time scale can be formally presented by the time evolution operator $U(t+\tau,t)$ as follows:

$$\begin{aligned}\mathbf{A}_{m+k}^m(t)&=U(T+m\tau,T)\mathbf{A}_k^0(0)\\&=\{U[T+m\tau,T+(m-1)\tau]U[T+(m-1)\tau,T\\&\quad+(m-2)\tau]\cdots U(T+\tau,T\}\mathbf{A}_k^0(0).\end{aligned} \tag{18}$$

The normalized TCF in Eq. (6) can be rewritten in a more compact form by means of Eqs. (17),(18) ($t=m\tau$ is discrete time here)

$$a(t)=\frac{\langle \mathbf{A}_k^0\cdot\mathbf{A}_{m+k}^m\rangle}{\langle \mathbf{A}_k^0\cdot\mathbf{A}_k^0\rangle}=\frac{\langle \mathbf{A}_k^0(0)\cdot\mathbf{A}_{m+k}^m(t)\rangle}{\langle \mathbf{A}_k^0(0)^2\rangle}. \tag{19}$$

Replacement of Eq. (6) by Eq. (19) is true if the numbers $k<N-1$ satisties the condition

$$\sigma^2\cong k^{-1}\sum_{j=1}^{k-1}\delta x_j^2 \quad\text{or}\quad \sigma^2=\lim_{k\to\infty}k^{-1}\sum_{j=0}^{k-1}\delta x_j^2. \tag{20}$$

The condition of quasistationarity of processes under consideration

$$\left|\frac{da(T,t)}{dT}\right|\ll\left|\frac{da(t)}{dt}\right|, \tag{21}$$

serves the other criterion of validity for such replacement. The TCF $a(T,t)$ in Eq. (21) is viewed on a time scale (point T) at the distance t from the zero point. Such vector notion is very helpful for the analysis of dynamics of random processes by means of finite-difference kinetic equations of non-Markov type.

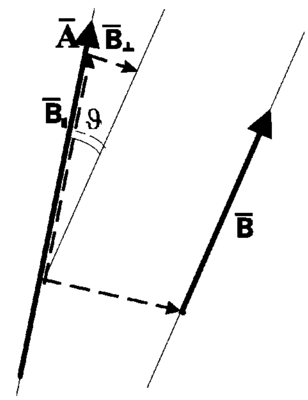

FIG. 1. Simple geometrical notion on vectors, their scalar product and normalized TCF of random variables.

III. GEOMETRICAL NOTION OF STOCHASTIC DYNAMICS OF TIME CORRELATION

First of all let us consider the projection operation in the set of vectors for different system states. It is easy to introduce it employing the above scalar product (16). Then it is necessary to introduce vectors $\mathbf{A}=\mathbf{A}_k^0(0)$ and $\mathbf{B}=\mathbf{A}_{m+k}^m(m\tau)$. Using Fig. 1 and simple geometrical notions we can demonstrate the following relations in terms of these symbols:

$$(1)\ \ \langle \mathbf{A}\cdot\mathbf{B}\rangle=|\mathbf{A}|\cdot|\mathbf{B}|\cos\vartheta,\quad \cos\vartheta=a(t),$$

$$(2)\ \ \mathbf{B}=\mathbf{B}_\parallel+\mathbf{B}_\perp,$$

$$(3)\ \ \mathbf{B}_\parallel=|\mathbf{B}|\cos\vartheta\frac{\mathbf{A}}{|\mathbf{A}|}=\frac{\mathbf{A}}{|\mathbf{A}|}|\mathbf{B}|a(t),\quad |\mathbf{B}_\parallel|^2=|\mathbf{A}|^2\{a(t)\}^2,$$

$$(4)\ \ |\mathbf{B}_\perp|=|\mathbf{B}|\sin\vartheta=|\mathbf{B}|\{1-[a(t)]\}^{1/2}, \tag{22}$$

where symbol $|\mathbf{A}|$ denotes the vector **A** length. Geometrical distance $R(\mathbf{A},\mathbf{B})$ between two vectors **A** and **B** can also be found:

$$R(\mathbf{A},\mathbf{B})=\{|\mathbf{A}-\mathbf{B}|^2\}^{1/2}=\left\{\sum_{j=0}^{k-1}(\mathbf{A}_j-\mathbf{B}_j)^2\right\}^{1/2}.$$

Using the latter and taking into account Eqs. (6), (19) we can find

$$\begin{aligned}R[\mathbf{A}_k^0(0),\mathbf{A}_{m+k}^m(t)]&=\{|\mathbf{A}_{m+k,\perp}^m(t)|^2\}^{1/2}\\&=\sqrt{2}|\mathbf{A}_{m+k}^m(t)|\{1-a(t)\}^{1/2}.\end{aligned}$$

The equation above immediately shows that the distance is determined by the dynamics of evolution of correlation process. Owing to the property (7) the following relation $\lim_{t\to\infty}R(\mathbf{A}_k^0(0),\mathbf{A}_{m+k}^m(t))=\sqrt{2k\sigma^2}$, where σ^2 is the variance can be developed. With regard to Eq. (22) the correlation decay in limit $t\to\infty$ may result in complete annihilation of parallel component of state $\mathbf{A}_{m+k}^m(t)$ vector. Then the state of the system at the moment $t\to\infty$ is entirely determined by the perpendicular component $\mathbf{A}_{m+k,\perp}^m(t)$ of the full vector $\mathbf{A}_{m+k}^m(t)$.

It follows from Eqs. (22) that in the set of state $\{\mathbf{A}_k^0(0),\mathbf{A}_{m+k}^m(t)\}$ vectors at different values of t, m, and k, TCF of random processes $a(t)$ plays a crucial role as an indicator of two interrelated states of a complex system. One of them deals with the creation of correlation and is specified by the $\mathbf{B}_{\parallel}$ component, whereas the second one is related to the annihilation of correlation and determined by the component $\mathbf{B}_{\perp}$. It results in the fact that in the limit of great $t\to\infty$ the following relation:

$$\lim_{t\to\infty} \mathbf{A}_{m+k,\parallel}^m(t)=\mathbf{0}, \quad \lim_{t\to\infty} \mathbf{A}_{m+k,\perp}^m(t)=\mathbf{A}_{m+k}^m(t) \tag{23}$$

is immediately fulfilled in correspondence with to Bogolubov's [36] principle of correlation attenuation.

From the physical point of view this fact means that TCF $a(t)$ represents two interrelated states determined by creation and annihilation of correlation. Hence it follows that such consideration must be given to both processes in an explicit form for stochastic dynamics of random processes' correlation.

IV. SPLITTING OF EQUATION OF VECTORS MOTION AND LIOUVILLIAN'S MATRIX PRESENTATION

It is obvious from Eq. (22) that TCF $a(t)$ is originated by projection of vector $\mathbf{A}_{m+k}^m(t)$ (18), where time $t=m\tau$ on the initial vector of state $\mathbf{A}_k^0(0)$ [see, for example, formula (19)]. The following construction of projection operator:

$$\Pi\mathbf{A}_{m+k}^m(t)=\mathbf{A}_k^0(0)\frac{\langle\mathbf{A}_k^0(0)\mathbf{A}_{m+k}^m(t)\rangle}{\langle|\mathbf{A}_k^0(0)|^2\rangle}=\mathbf{A}_k^0(0)a(t) \tag{24}$$

results from here. It is turn projection operator Π from Eq. (24) has the following properties

$$\Pi=\frac{|\mathbf{A}_k^0(0)\rangle\langle\mathbf{A}_k^0(0)|}{\langle|\mathbf{A}_k^0(0)|^2\rangle}, \quad \Pi^2=\Pi, \quad P=1-\Pi,$$

$$P^2=P, \quad \Pi P=0, \; P\Pi=0. \tag{25}$$

A pair of projection operators Π and P are idempotent and mutually supplementary. Figure 1 shows that projector Π projects on the direction $\mathbf{A}_k^0(0)$, whereas the orthogonal operator P transfers all vectors to the orthogonal direction.

Let us consider quasidynamic finite-difference Liouville's Eq. (14) for the vector of fluctuations

$$\frac{\Delta}{\Delta t}\mathbf{A}_{m+k}^m(t)=i\hat{L}(t,\tau)\mathbf{A}_{m+k}^m(t). \tag{26}$$

The vectors $\mathbf{A}_{m+k}^m(t)$ generate the vector finite-dimensional space $A(k)$ with scalar product in which [according to Eqs. (24),(25)] the orthogonal projection operation [37,38] is expressed by

$$A(k)=A'(k)+A''(k), \quad \mathbf{A}_{m+k}^m(t)\in A(k),$$

$$A'(k)=\Pi A(k), \quad A''(k)=PA(k)=(1-\Pi)A(k). \tag{27}$$

Operators Π and P split Euclidean space $A(k)$ into two mutually-orthogonal subspaces. This permits to split dynamical equation (26) into two equations within two mutually supplementary subspaces [38–40] as follows:

$$\frac{\Delta A'(t)}{\Delta t}=i\hat{L}_{11}A'(t)+i\hat{L}_{12}A''(t), \tag{28}$$

$$\frac{\Delta A''(t)}{\Delta t}=i\hat{L}_{21}A'(t)+i\hat{L}_{22}A''(t). \tag{29}$$

In the equations above we crossout for short space elements indices A,A' and A'' and matrix elements arguments $\hat{L}_{ij}$, $\hat{L}_{ij}=\Pi_i\hat{L}\Pi_j$, $\Pi_1=\Pi$, $\Pi_2=P=1-\Pi$, $i=1,2$. In line with Refs. [39,40] we write down Liouville's operator in matrix form

$$\hat{L}=\begin{pmatrix}\hat{L}_{11} & \hat{L}_{12}\\ \hat{L}_{21} & \hat{L}_{22}\end{pmatrix},$$

$$\hat{L}_{11}=\Pi\hat{L}\Pi, \quad \hat{L}_{12}=\Pi\hat{L}P,$$

$$\hat{L}_{21}=P\hat{L}\Pi, \quad \hat{L}_{22}=P\hat{L}P. \tag{30}$$

Operators $\hat{L}_{ij}$ act in the following way: $\hat{L}_{11}$ from A' to A', $\hat{L}_{22}$ from A'' to A'', $\hat{L}_{21}$ - from A' to A'', and $\hat{L}_{12}$ operates from A'' to A'.

To simplify Liouville Eqs. (28),(29) we exclude the irrelevant part $A''(t)$ and construct closed equation for relevant part $A'(t)$. For this purpose let us solve Eq. (29) step by step:

$$\frac{\Delta}{\Delta t}\{\mathbf{A}_{m+k}^m(t)\}''=i\hat{L}_{21}\{\mathbf{A}_{m+k}^m(t)\}'+i\hat{L}_{22}\{\mathbf{A}_{m+k}^m(t)\}''. \tag{31}$$

Considering Eq. (8) we arrive at finite-difference solution of this equation in the following form:

$$\frac{\Delta A''(t)}{\tau}=\tau^{-1}[A''(t+\tau)-A''(t)]=i\hat{L}_{21}A'(t)+i\hat{L}_{22}A''(t), \tag{32}$$

$$A''(t+\tau)=\{1+i\tau\hat{L}_{22}\}A''(t)+i\tau\hat{L}_{21}A'(t). \tag{33}$$

Applying Eqs. (32), (33) we find

$$A''(t+2\tau)=(1+i\tau\hat{L}_{22})^2A''(t)+(1+i\tau\hat{L}_{22})\{i\tau\hat{L}_{21}A'(t)\} + \{i\tau\hat{L}_{21}\}A'(t+\tau), \tag{34}$$

for $m=2$ and

$$A''(t+3\tau)=(1+i\tau\hat{L}_{22})^3A''(t)+(1+i\tau\hat{L}_{22})^2\{i\tau\hat{L}_{21}A'(t)\} + (1+i\tau\hat{L}_{22})\{i\tau\hat{L}_{21}A'(t+\tau)\} + \{i\tau\hat{L}_{21}A'(t+2\tau\}, \dots, \tag{35}$$

for $m=3$, respectively. In general case we find

$$A''(t+m\tau)=\{1+i\tau\hat{L}_{22}\}^m A''(t)+\sum_{j=0}^{m-1}\{1+i\tau\hat{L}_{22}\}^j \times\{i\hat{L}_{21}A'(t+(m-1-j)\tau)\} \tag{36}$$

for the arbitrary number of m steps. Then after the substitution of right side of Eq. (36) for Eq. (28) we obtain the closed finite-difference kinetic equation for the relevant parts of vectors

$$\frac{\Delta}{\Delta t}A'(t+m\tau)=i\hat{L}_{11}A'(t+m\tau)+i\hat{L}_{12}\{1+i\tau\hat{L}_{22}\}^m A''(t) -\hat{L}_{12}\sum_{j=0}^{m+1}\{1+i\tau\hat{L}_{22}\}^j\tau\hat{L}_{21} \times A'[t+(m-1-j)\tau]. \tag{37}$$

To simplify this equation, let us consider the idempotentity property, and then determine $(0\leqslant k\leqslant m-1)$

$$A''(t)=0,\quad \{1+i\tau\hat{L}_{22}\}^k A''(t)=0. \tag{38}$$

Transfering from vectors $\mathbf{A}^m_{m+k}$ in Eq. (37) to a scalar value of TCF $a(t)$ by means of suitable projection we come to the closed finite-difference discrete equation for the initial TCF

$$\frac{\Delta a(t)}{\Delta t}=i\omega_0^{(0)}a(t)-\tau\Omega_0^2\sum_{j=0}^{m-1}M_1(j\tau)a(t-j\tau). \tag{39}$$

Here Ω_0 is the general relaxation frequency whereas frequency $\omega_0^{(0)}$ describes the eigenspectrum of the Liouville's quasioperator $\hat{L}$

$$\omega_0^{(0)}=\frac{\langle \mathbf{A}_k^0(0)\hat{L}\mathbf{A}_k^0(0)\rangle}{\langle|\mathbf{A}_k^0(0)|^2\rangle},\quad \Omega_0^2=\frac{\langle \mathbf{A}_k^0\hat{L}_{12}\hat{L}_{21}\mathbf{A}_k^0(0)\rangle}{\langle|\mathbf{A}_k^0(0)|^2\rangle}. \tag{40}$$

Function $M_1(j\tau)$ in the right side of Eq. (39) is the first order memory function

$$M_1(j\tau)=\frac{\langle \mathbf{A}_k^0(0)\hat{L}_{12}\{1+i\tau\hat{L}_{22}\}^j\hat{L}_{21}\mathbf{A}_k^0(0)\rangle}{\langle \mathbf{A}_k^0(0)\hat{L}_{12}\hat{L}_{21}\mathbf{A}_k^0(0)\rangle},\quad M_1(0)=1. \tag{41}$$

Equation (39) alongside with Eqs. (40),(41) present first order discrete non-Markov kinetic equation for the discrete time correlation function $a(t)$. However, our consequent step will be to perform a further generalization of discrete TCF analysis and to obtain finite-difference equation for the first order memory function $M_1(j\tau)$ and so on.

V. INTRODUCTION OF THE SET OF ORTHOGONAL RANDOM VARIABLES AND CONSTRUCTION OF INFINITE CHAIN OF FINITE-DIFFERENCE NON-MARKOV KINETIC EQUATIONS FOR DISCRETE MEMORY FUNCTIONS

The discrete memory function $M_1(j\tau)$ (41) in Eq. (29) is in its turn the normalized TCF, evolution of which is defined by the deformed (compressed) Liouvillian $(\hat{L}^{(0)}=\hat{L})$

$$\hat{L}^{(1)}=\hat{L}_{22}^{(0)}=\hat{L}_{22}=(1-\Pi)\hat{L}(1-\Pi) \tag{42}$$

for a new dynamical variable $B^{(1)}=i\hat{L}_{21}\mathbf{A}_k^0(0)$. Thus, we can completely repeat for $M_1(j\tau)$ the whole procedure within Eqs. (24)–(41), and obtain the following non-Markov kinetic equation for the normalized TCF. The infinite chain of equations for the initial TCF and memory functions of increasing order results from multiple repetition of similar procedure.

However, this chain of equations can be obtained differently, i.e. much shorter and less costly. For this purpos let us employ the method developed earlier for the physical Hamilton systems with the continuous time in Refs. [40,41]. Moreover the lack of Hamiltonian and the time discretness must be taken into account.

Let us remember that natural equation of motion (14) is the finite-difference Liouville's equation

$$\frac{\Delta}{\Delta t}x(t)=i\hat{L}x(t), \tag{43}$$

where the Liouville quasioperator is

$$\hat{L}=\hat{L}(t,\tau)=(i\tau)^{-1}\{U(t+\tau,t)-1\}. \tag{44}$$

Succesively applying the quasioperator $\hat{L}$ to the dynamic variables $\mathbf{A}^m_{m+k}(t)$ ($t=m\tau$, where τ is a discrete time step) we obtain the infinite set of dynamic functions

$$\mathbf{B}_n(0)=\{\hat{L}\}^n\mathbf{A}_k^0(0),\quad n\geqslant 1. \tag{45}$$

Using variables $\mathbf{B}_n(0)$ one can find the formal solution of evolution Eq. (43) in the form

$$\mathbf{A}^m_{m+k}(m\tau)=\{1+i\tau\hat{L}\}^m\mathbf{A}_k^0(0)=\sum_{j=0}^{m}\frac{m!(i\tau)^{m-j}}{j!(m-j)!}\mathbf{B}^0_{m-j}(0). \tag{46}$$

However, a similar form of dynamic variables is deficient. That is why we prefer the use the orthogonal variables as vectors $\mathbf{W}_n$ given below. Employing Gram-Schmidt orthogonalization procedure [42] for the set of variables $\mathbf{B}_n(0)$ one can obtain the new infinite set of dynamical orthogonal variables, i.e., vectors $\mathbf{W}_n$

$$\langle \mathbf{W}_n^*(0),\mathbf{W}_m(0)\rangle=\delta_{n,m}\langle|\mathbf{W}_n(0)|^2\rangle, \tag{47}$$

where the mean $\langle\cdots\rangle$ should be read in terms of Eqs. (16)–(18) and $\delta_{n,m}$ is Kronecker's symbol. Now we may easily introduce the recurrence formula in which the senior values $\mathbf{W}_n=\mathbf{W}_n(t)$ are connected with the juniour values

$$\mathbf{W}_0=\mathbf{A}_k^0(0),\quad \mathbf{W}_1=\{\hat{L}-\omega_0^{(0)}\}\mathbf{W}_0, \quad \mathbf{W}_n=\{\hat{L}-\omega_0^{(n-1)}\}\mathbf{W}_{n-1}-\Omega_{n-1}^2W_{n-2},\quad n>1. \tag{48}$$

Here we used the equation, given earlier in Eq. (40) for number $n=0$

$$\omega_0^{(n)}=\frac{\langle \mathbf{W}_n\hat{L}\mathbf{W}_n\rangle}{\langle|\mathbf{W}_n|^2\rangle},\quad \Omega_n^2=\frac{\langle|\mathbf{W}_n|^2\rangle}{\langle|\mathbf{W}_{n-1}|^2\rangle}, \tag{49}$$

where Ω_n is the general relaxation frequency, and frequency $\omega_0^{(n)}$ completely describes the eigenspectrum of Liouville's quasioperator $\hat{L}$ (44). Now the arbitrary variables $\mathbf{W}_n$ may be expressed directly through the inital variable $\mathbf{W}_0=\mathbf{A}_k^0(0)$ by means of Eq. (48)

$$\mathbf{W}_n=\begin{vmatrix} \hat{L}-\omega_0^{(0)} & \Omega_1 & 0 & \cdots & 0 \\ \Omega_1 & \hat{L}-\omega_0^{(1)} & \Omega_2 & \cdots & 0 \\ 0 & \Omega_2 & \hat{L}-\omega_0^{(2)} & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \hat{L}-\omega_0^{(n-1)} \end{vmatrix} \times \mathbf{W}_0 . \tag{50}$$

The physical sense of $\mathbf{W}_n$ variables (vectors of state) can be cleared up in the following way. For example, in the continuous matter physics, the local density fluctuations may be considered as initial variables. So the local flow density, energy density and energy flow density fluctuations are the dynamic variables $\mathbf{W}_n$ where numbers $n \geqslant 1$. The careful usage of the abovementioned variables within the long-wave limits creates the basis for the condensed matter theory in hydrodynamic approximation. The set of the orthogonal variables (48) [see also Eq. (47)] can be connected with the set of projection operators. The later projects the arbitrary dynamic variable (i.e., vector of state) Y on the corresponding vector of the set

$$\Pi_n=\frac{|\mathbf{W}_n\rangle\langle\mathbf{W}_n^*|}{\langle|\mathbf{W}_n|^2\rangle}, \quad \Pi_n^2=\Pi_n , \quad P_n=1-\Pi_n ,$$
$$P_n^2=P_n , \quad \Pi_n P_n=0,$$
$$\Pi_n\Pi_m=\delta_{n,m}\Pi_n , \quad P_nP_m=\delta_{n,m}P_n , \quad P_n\Pi_n=0. \tag{51}$$

Let us take into consideration the fact that both sets (45) and (50) are infinite. If we execute the operations in the Euclidean space of dynamic variables then the formal expressions (51) must be understood as follows:

$$\Pi_n\mathbf{Y}=\mathbf{W}_n\frac{\langle\mathbf{W}_n^*\mathbf{Y}\rangle}{\langle|\mathbf{W}_n|^2\rangle}, \quad \mathbf{Y}\Pi_n=\mathbf{W}_n^*\frac{\langle\mathbf{Y}W_n\rangle}{\langle|\mathbf{W}_n|^2\rangle}. \tag{52}$$

Now according to Eqs. (28)–(30),(51),(52) we can introduce the following notation for the splitting of the Liouville's quasioperator into the diagonal ($\hat{L}_{ii}^{(n)}$) and nondiagonal ($\hat{L}_{ij}^{(n)}$) matrix elements with $i\neq j$, $n\geqslant 1$:

$$\hat{L}^{(n)}=P_{n-1}\hat{L}^{(n-1)}P_{n-1},$$
$$\hat{L}_0=\hat{L},\hat{L}_{ij}^{(n)}=\Pi_i^{(n-1)}\hat{L}\Pi_j^{(n-1)}, \quad i,j=1,2,$$
$$\Pi_1^{(n)}=\Pi_n , \quad \Pi_2^{(n)}=P_n=1-\Pi_n . \tag{53}$$

For example, we come to the following equations:

$$\hat{L}_{22}^{(0)}=\hat{L}_0=\hat{L}, \quad \hat{L}_{22}^{(n)}=P_{n-1}P_{n-2}\cdots P_0\hat{L}P_0\cdots P_{n-2}P_{n-1}. \tag{54}$$

for the second diagonal matrix elements. Successively applying projection operators Π_n and P_n for the discrete equation (43) in the set of normalized TCF ($t=m\tau$)

$$M_n(t)=\frac{\langle\mathbf{W}_n[1+i\tau\hat{L}_{22}^{(n)}]^m\mathbf{W}_n\rangle}{\langle|\mathbf{W}_n(0)|^2\rangle} \tag{55}$$

we obtain the infinite hierarchy of connected non-Markov finite-difference kinetic equations ($t=m\tau$)

$$\frac{\Delta M_n(t)}{\Delta t}=i\omega_0^{(n)}M_n(t)-\tau\Omega_{n+1}^2\sum_{j=0}^{m-1}M_{n+1}(j\tau)M_n(t-j\tau), \tag{56}$$

where $\omega_0^{(n)}$ is the eigen and Ω_n is the general relaxation frequency as follows:

$$\omega_0^{(n)}=\frac{\langle W_n^* L_n W_n\rangle}{\langle|W_n|^2\rangle}, \quad L_n=L_{22}^{(n)}, \quad \Omega_n^2=\frac{\langle|W_n|^2\rangle}{\langle|W_{n-1}|^2\rangle}.$$

A set of functions $M_n(t)$ (55),(56) except $n=0$

$$M_0(t)=a(t)=\frac{\langle\mathbf{A}_k^0(0)\cdot\mathbf{A}_{m+k}^m(t)\rangle}{\langle|\mathbf{A}_k^0(0)|^2\rangle}, \quad t=m\tau$$

can be considered as functions characterizing the statistical memory of time correlation in the complex systems with discrete time. The initial TCF a(t) and the set of discrete memory functions $M_n(t)$ in Eq. (56) are of crucial role for the further consideration. It is convenient to rewrite the set of discrete kinetic Eqs. (56) as the infinite chain of coupled non-Markov discrete equations of nonlinear type for the initial discrete TCF a(t) (discrete time $t=m\tau$ everywhere)

$$\frac{\Delta a(t)}{\Delta t}=-\tau\Omega_1^2\sum_{j=0}^{m-1}M_1(j\tau)a(t-j\tau)+i\omega_0^{(0)}a(t),$$
$$\frac{\Delta M_1(t)}{\Delta t}=-\tau\Omega_2^2\sum_{j=0}^{m-1}M_2(j\tau)M_1(t-j\tau)+i\omega_0^{(1)}M_1(t),$$
$$\frac{\Delta M_2(t)}{\Delta t}=-\tau\Omega_3^2\sum_{j=0}^{m-1}M_3(j\tau)M_2(t-j\tau)+i\omega_0^{(2)}M_2(t). \tag{57}$$

These finite-difference Eqs. (56) and (57) are very similar to famous Zwanzig'-Mori's chain (ZMC) of kinetic equations [43–45], which plays the fundamental role in modern statistical physics of nonequilibrium phenomena with the smooth time. It should be noted that ZMC's are true only for the physical quantum and classical systems with smooth time governed by Hamiltonian. Our finite-difference kinetic equations (56), (57) are valid for complex systems lacking Hamiltonian, the time being discrete and the exact equations of motion being absent. However, the ''dynamics'' and ''motion'' in the real complex systems are undoubtedly abundant and are immediately registered during the experiment. The first three of those Eqs. (57) in the whole infinite chain (56) form the basis for the quasihydrodynamic description of random processes in complex systems.

VI. A PSEUDOHYDRODYNAMIC DESCRIPTION OF RANDOM PROCESSES IN COMPLEX SYSTEMS

At first let us find the matrix elements $\hat{L}_{ij}$ of complex systems Liouvillian's quasioperator. Employing Eqs. (24), (25),(30),(53),(54) we successively found

$$i\hat{L}_{11}^{(0)}=\Pi\frac{a(\tau)-a(0)}{\tau}=a'(0)\Pi, \tag{58}$$

$$i\hat{L}_{21}^{(0)}=\{\tau^{-1}[U(t+\tau,t)-1]-a'(0)\}\Pi, \tag{59}$$

$$i\hat{L}_{12}^{(0)}=\Pi\{\tau^{-1}[U(t+\tau,t)-1]-a'(0)\}, \tag{60}$$

$$\begin{aligned} i\hat{L}_{22}^{(0)}&=i\hat{L}-i\{\hat{L}_{11}^{(0)}+\hat{L}_{12}^{(0)}+\hat{L}_{21}^{(0)}\}\\ &=\tau^{-1}[U(t+\tau,t)-1]-\tau^{-1}\Pi\{U(t+\tau,t)-1\}\\ &\quad-\tau^{-1}1\{U(t+\tau,t)-1\}\Pi+a'(0)\Pi. \end{aligned} \tag{61}$$

A diagonal matrix element $\hat{L}_{22}^{(0)}$ is the part of ''compressed'' evolution quasioperator, which in its turn is equal to

$$1+i\tau\hat{L}_{22}=U(t+\tau,t)+\tau a'(0)\Pi-\{\Pi,U(t+\tau,t)-1\}_+, \tag{62}$$

where the anticommutator of appropriate operator is designate by the brackets $\{A,B\}_+=AB+BA$. One can see from Eq. (62) that the ''compressed'' evolution operator differs from the natural operator $U(t+\tau,t)$ because of the presence of contributions, associated with the first and the following derivatives of TCF the initial TCF $a(t)$.

The large-scale presentation of the memory function $M_1(t)$ is suitable mostly for practical applications. Using Eqs. (58)–(61),(41),(54) we also find the succession of the first five points of discrete functions $M_1(j\tau)$ where $j=1,2,3,4$ and

$$M_1(0)=1,\ M_1(\tau)=\{1/a''(0)\}\{a''(\tau)-2\tau a'(0)a''(0)\}. \tag{63}$$

The ''Gaussian'' behavior of TCF at the zero point $t=0$

$$a'(0)=\frac{\langle \mathbf{A}_k^{(0)}(0)\{U_\tau-1\}\mathbf{A}_k^{(0)}(0)\rangle}{\langle|\mathbf{A}_k^0(0)|^2\rangle}=0$$

should be taken into account in the subsequent discussion. It is proved accurately and connected directly with the orthogonality property of dynamical variables (47)–(50). It gives us an opportinity to simplify the memory function formula as follows:

$$M_1(0)=1,\quad M_1(\tau)=\left\{\frac{a''\tau}{a''(0)}\right\},$$

$$M_1(2\tau)=\left\{\frac{1}{a''(0)}\right\}\{a''(2\tau)-2\tau a''(0)a'(\tau)+\tau[a''(0)]^2\},$$

$$\begin{aligned} M_1(3\tau)=&\left\{\frac{1}{a''(0)}\right\}\{a''(3\tau)-\tau a'(2\tau)a''(0)\\ &-2\tau a''(\tau)a'(\tau)+\tau^2a''(\tau)a''(0)+\tau a'(\tau)a''(0)\}, \end{aligned}$$

$$\begin{aligned} M_1(4\tau)=&\left\{\frac{1}{a''(0)}\right\}\{a''(4\tau)-\tau a'(3\tau)a''(0)\\ &-\tau a'(2\tau)a''(\tau)+\tau a'(2\tau)a''(0)-\tau a''(\tau)a'(2\tau)\\ &+\tau^2[a''(\tau)]^2-\tau a''(\tau)a'(\tau)+\tau^2a''(\tau)a(\tau)a''(0)\\ &-\tau^2a(\tau)[a''(0)]^2\}. \end{aligned} \tag{64}$$

Further presentation the following values $M_i(j\tau)$ (numbers $j\geq 5$) constitute the extremely complicated combinatory problem. As analysis of Eqs. (64) shows second derivative's behavior

$$M_1(j\tau)\cong\left\{\frac{1}{a''(0)}\right\}a''(j\tau) \tag{65}$$

contributes mainly into functions $M_1(j\tau)$.

Now let us move to practical realization of Eqs. (57), forming a basis of pseudohydrodynamic description of correlation dynamics. Thus using orthogonal dynamic variables (47), (48),(50), we immediately obtain

$$\begin{aligned} \hat{W}_0&=\mathbf{A}_k^0,\\ \hat{W}_1&=\{\hat{L}-\omega_0^{(0)}\}\hat{W}_0=\hat{L}\hat{W}_0=(i\tau)^{-1}(U_\tau-1)\mathbf{A}_k^0(0),\\ \hat{W}_2&=\hat{L}\hat{W}_1-\Omega_1^2\hat{W}_0\\ &=\{\hat{L}^2-\Omega_1^2\}\hat{W}_0\\ &=(i\tau)^{-2}\{U_\tau-1\}^2\mathbf{A}_k-\Omega_1^2\mathbf{A}_k^0,\\ \hat{W}_3&=\hat{L}\hat{W}_2-\Omega_2^2\hat{W}_1\\ &=\hat{L}(\hat{L}^2-\Omega_1^2)\hat{W}_0-\Omega_2^2\hat{L}\hat{W}_0\\ &=\{\hat{L}^3-(\Omega_1^2+\Omega_2^2)\hat{L}\}\hat{W}_0\\ &=\{(i\tau)^3[U_\tau-1]^3-(i\tau)^{-1}(\Omega_1^2+\Omega_2^2)(U_\tau-1)\}\mathbf{A}_k^0. \end{aligned} \tag{66}$$

Simple relation for the eigenfrequencies and general relaxation frequencies

$$\begin{aligned} &\omega_0^{(n)}=\frac{\langle\hat{W}_n\hat{L}\hat{W}_n\rangle}{\langle|\hat{W}_n|^2\rangle}=0,\quad \Omega_n^2=\frac{\langle|\hat{W}_n|^2\rangle}{\langle|\hat{W}_{n-1}|^2\rangle},\quad \Omega_1^2=|a^{(2)}(0)|,\\ &\Omega_2^2=\frac{a^{(4)}(0)-[a^{(2)}(0)]^2}{|a^2(0)|},\\ &\Omega_3^2=\frac{a^{(6)}(0)-2a^{(4)}(0)(\Omega_1^2+\Omega_2^2)-(\Omega_1^2+\Omega_2^2)^2a^{(2)}(0)}{a^{(4)}(0)-[a^{(2)}(0)]^2} \end{aligned} \tag{67}$$

should be taken into consideration here. The orthogonal variables $\hat{W}_n$ in Eq. (66) can be easily rearranged as follows:

$$\hat{W}_0=\mathbf{A}_k^0, \quad \hat{W}_1=-i\frac{d}{dt}\mathbf{A}_k^0=-i\frac{\Delta}{\Delta t}\mathbf{A}_k,$$

$$\hat{W}_2=\left\{\frac{d^2}{dt^2}+\Omega_1^2\right\}\mathbf{A}_k^{(0)}=\left\{\left(\frac{\Delta}{\Delta t}\right)^2+\Omega_1^2\right\}\mathbf{A}_k^0,$$

$$\hat{W}_3=i\left\{\frac{d^3}{dt^3}+(\Omega_1^2+\Omega_2^2)\frac{d}{dt}\right\}\mathbf{A}_k^0 =i\left\{\left(\frac{\Delta}{\Delta t}\right)^3+(\Omega_1^2+\Omega_2^2)\frac{\Delta}{\Delta t}\right\}\mathbf{A}_k^0. \tag{68}$$

Those formulas (68) have considerable utility inasmuch as they permit to see the structure of formation of orthogonal variables and junior orders memory functions for the numbers $n=1,2,3$. Equations (64),(67),(68) open up new fields of construction of quasikinetic description of random processes $\{\mathbf{A}_k^0(0),\mathbf{A}_{m+k}^m(m\tau)\}$. By analogy with hydrodynamics the variables $\hat{W}_0$, $\hat{W}_1$, $\hat{W}_2$, and $\hat{W}_3$ in Eq. (68) play the role similar to that of the local density, local flow, local energy density, and energy flow. It is clear that this is only formal analogy and the variables $\hat{W}_n$ do not possess any physical sense. However, such analogies can be helpful in revealing of the real sense of orthogonal variables.

To describe pseudohydrodynamics we have to use the set of first three discrete kinetic Eqs. (57) with frequences Ω_i^2 $(i=1,2,3)$ derived from Eqs. (67). It is essential that all frequences Ω_i^2 are connected straightly with the properties of the initial TCF $a(t)$ only. The latter can be easily derived directly from the experimental data [45–47]. Thus the system of Eqs. (57) has considerable utility for the experimental investigations of statistical memory effects and non-Markov processes in complex systems.

Among them it seems to us that one could propose more physical interpretation of the different terms in the right side of the three Eqs. (68). For example, term $-i\Delta A/\Delta t$ is similar to a dissipation, $\Delta^2 A/\Delta t^2$ is similar to an inertia, and $\Omega^2 A(t)$ is similar to a restoring force. Third derivative $\Delta^3 A/\Delta t^3$ is the finite-difference generic form of the Abraham-Lorenz force corresponding to dissipation feedback due to radiative losses [see, for instance, formula (3) in Ref. [48] for a recent experimental evidence in frictional systems].

VII. SHANNON ENTROPY FOR THE TIME CORRELATION AND TIME MEMORY IN COMPLEX SYSTEMS

According to the results in Sec. VI, the information measure for the description of random processes in complex systems can be expressed not only via TCF, but also by means of the certain set of time memory functions. To accomplish that let us return to Sec. III in which we presented the geometrical picture of stochastic dynamics of correlation. In a line with Shannon [17] in case of discrete source of information we were able to determine a definite rate of generating information, namely, the entropy of the underlying stochastic information by introduction fidelity evaluation function $\nu[P(x,y)]$. Here the function $P(x,y)$ is the two-dimensional distribution of random variables (x,y) and

$$\nu[P(x,y)]=\int\int dx\,dy\,P(x,y)\rho(x,y), \tag{69}$$

where the function $\rho(x,y)$ has the general nature of the ''distance'' between x and y. As pointed by Shannon [17] the function $\rho(x,y)$ is not a ''metric'' in the strict sense, however, since in general it does not satisfy either $\rho(x,y)=\rho(y,x)$ or $\rho(x,y)+\rho(y,z)\geqslant\rho(x,z)$. It measures how undesirable it is according to our fidelity criterion (69) to receive y when x transmitted. According to Shannon [17] any evolution of fidelity must correspond mathematically to the operation of a simple ordering of systems by the transmission of a signals within the certain tolerance. According to Shannon [17] the following is simple example of fidelity evaluation function

$$\nu(P(x,y))=\langle(x(t)-y(t))^2\rangle. \tag{70}$$

In our case it is convenient to consider the initial vector $A_k^0(0)$ as a variable x and the final vector $A_{m+k}^m(t)$ at time $t=m\tau$ for a variable y. The distance function $\rho(x,y)$ [17]

$$\rho(x,y)=\frac{1}{T}\int_0^T dt\{x(t)-y(t)\}^2 \tag{71}$$

is the most commonly used measure of fidelity.

Taking into accout Eqs. (69),(71) and the results in Sec. III as the fidelity function one can use the following function of geometrical distance:

$$\nu(P(\mathbf{A}_k^0(0),\mathbf{A}_{m+k}^m(t)))=2k\sigma^2\{1-a(t)\}, \tag{72}$$

where distance function is

$$\rho(\mathbf{A}_k^0(0),\mathbf{A}_{m+k}^m(t))=R^2(\mathbf{A}_k^0(0),\mathbf{A}_{m+k}^m(t)). \tag{73}$$

According to Ref. [17] partial solution of the general maximizing problem for determining the rate of generating information of a source can be given using Lagrange's method and considering the following functional:

$$\int\int\left\{P(x,y)\ln\frac{P(x,y)}{P(x)P(y)}+\mu P(x,y)\rho(x,y) +\nu(x)P(x,y)\right\}dx\,dy, \tag{74}$$

where the function $\nu(x)$ and μ are unknown. The following equation for the conditional probability can be obtained by variation on $P(x,y)$

$$P_y(x)=\frac{P(x,y)}{P(y)}=B(x)\exp\{-\lambda\rho(x,y)\}. \tag{75}$$

This shows that with best encoding the conditional probability of a certain cause for various received y, $P_y(x)$ will decline exponentially with the distance function $\rho(x,y)$ between values the x and y in problem. Unknown constant λ is

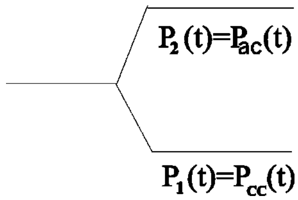

FIG. 2. Scheme of simplified two-lewel description of a complex systems state. Two probability $P_{cc}(t)$ and $P_{ac}(t)$ decsribes a stochastic processes of creation (existence) and annihilation (decay) of time correlation.

defined by the required fidelity, and function $B(x)$ in the case of continuous variables obeys the normalization condition

$$\int B(x)\exp\{-\lambda\rho(x,y)\}dx=1. \tag{76}$$

Since the distance function $\rho(x,y)$ (71) is dependent only on the vectors difference $\rho(x,y)=\rho(x-y)$, we get a simple solution for the special case $B(x)=\alpha$

$$P_y(x)=\alpha\exp\{-\lambda\rho(x-y)\}=\alpha\exp\{-c[1-a(t)]\} \tag{77}$$

instead of Eq. (75). Constants α and λ result from the corresponding normalizing condition and in accordance with the required fidelity. From the physical point of view the basic value of solution (77) is directly related to the occurence of the TCF $a(t)$. Therefore,the solution (77) describes the state of the system with certain level and scale of correlation.

Now let us employ Shannon's solution for continuous variables (75), (77) and pass to simplified discrete two-level description of the system. Then let us consider the conditional probability (77) which describes the state on time axis at the moment $t=m\tau$ as corresponding to the creation of correlation. Whereas the other state at the fixed moment $t=m\tau$ which accounts for the state with the absence (annihilation) of correlation will exist. Let us introduce two probabilities (see Fig. 2), which will fit normalizing condition

$$P_1(t)+P_2(t)=1,\quad P_1(t)=P_{cc}(t),\quad P_2(t)=P_{ac}(t),$$

$$P_{cc}(t)+P_{ac}(t)=1. \tag{78}$$

In the case of two levels Shannon entropy

$$S=-\sum_{i=1}^{2} P_i\ln P_i \tag{79}$$

increases at full disorder and takes its limiting value

$$\lim_{t\to\infty}S=\lim_{t\to\infty}S(t)=\ln 2. \tag{80}$$

To find unknown parameters α and c in two-level description (creation and annihilation of correlation) in Eq. (75) we should take into account normalization condition, principle of entropy increase (80) at $t\to\infty$ and of entropy extremality (presence of minimum) at full order when the following relationship: $\lim_{t\to o}a(t)=1$ is true for the TCF. We obtained the following equation:

$$\lim_{t\to 0}S(t)=-\{\alpha\ln\alpha+(1-\alpha)\ln(1-\alpha)\}=0$$

for the parameters α and c ($c\geqslant 0$, $0\leqslant\alpha\leqslant 1$) having regard to these requirements. Among two solutions ($\alpha_1=1$, $\alpha_2=0$) only the first one ($\alpha_1=1$) has physical sense. Two probabilities calculated by means of Eq. (77) will satisfy conditions (78), (80)

$$P_1(t)=P_{cc}(t)=\exp\{-(\ln 2)[1-a(t)]\}, \tag{81}$$

$$P_2(t)=P_{ac}(t)=1-\exp\{-(\ln 2)[1-a(t)]\}. \tag{82}$$

respectively. In accordance with two-level description it would be convenient to deal with two dynamic channels of entropy [creation (cc) and annihilation (ac)] of correlation (see Fig. 2)

$$S_{cc}(t)=(\ln 2)\{1-a(t)\}\exp\{-(\ln 2)[1-a(t)]\}, \tag{83}$$

$$S_{ac}(t)=-\{1-\exp[-(\ln 2)(1-a(t))]\}\times\ln\{1-\exp[-(\ln 2)(1-a(t))]\}. \tag{84}$$

The probabilities obtained are in the line with full dynamic (time dependent) information Shannon entropy

$$\begin{aligned}S_0(t)&=S_{cc}(t)+S_{ac}(t)\\&=\ln 2\{1-a(t)\}\exp\{-(\ln 2)[1-a(t)]\}\\&\quad-(1-\exp\{-(\ln 2)[1-a(t)]\})\\&\quad\times\ln(1-\exp\{-(\ln 2)[1-a(t)]\}).\end{aligned} \tag{85}$$

The entropy introduced in to Eqs. (83)–(85) characterized a quantitative measure of disorder in the system related to creation and annihilation of dynamic correlation. The probabilistic and entropy channels (81)–(84) possess the following asymptotic behavior:

if $a(t)\to 1$,

$$P_1(t)=P_{cc}(t)\cong 1+(\ln 2)[a(t)-1],$$

$$P_2(t)=P_{ac}(t)\cong(\ln 2)[1-a(t)],$$

$$S_{cc}(t)\cong-\{1+(\ln 2)[a(t)-1]\}\ln\{1+(\ln 2)[a(t)-1]\},$$

$$S_{ac}(t)\cong-(\ln 2)[1-a(t)]\ln\{\ln 2[1-a(t)]\},$$

$$S_0(t)\cong-\{1+(\ln 2)[a(t)-1]\}\ln\{1+(\ln 2)[a(t)-1]\}-\ln 2[1-a(t)]\ln\{(\ln 2)[1-a(t)]\}$$

and if $a(t)\to 0$,

$$P_1(t)=P_{cc}(t)\cong\tfrac{1}{2}\{1+(\ln 2)a(t)\}, \tag{86}$$

$$P_2(t)=P_{ac}(t)\cong\tfrac{1}{2}\{1-(\ln 2)a(t)\},$$

$$S_{cc}(t)\cong-\tfrac{1}{2}\{1+(\ln 2)a(t)\}\ln\{\tfrac{1}{2}[1+(\ln 2)a(t)]\},$$

$$S_{ac}(t)\cong-\tfrac{1}{2}\{1-(\ln 2)a(t)\}\ln\{\tfrac{1}{2}[1-(\ln 2)a(t)]\},$$

$$S_0(t)\cong -\tfrac{1}{2}\{1+(\ln 2)a(t)\}\ln\{\tfrac{1}{2}[1+(\ln 2)a(t)]\}$$
$$-\tfrac{1}{2}\{1-(\ln 2)a(t)\}\ln\{\tfrac{1}{2}[1-(\ln 2)a(t)]\}.$$

Subsequent boundary conditions result from the equations above in terms of Bogolubov's principle of attenuation of correlation (7) as follows:

$$\lim_{t\to 0} P_{cc}=1,\quad \lim_{t\to 0} P_{ac}(t)=0, \tag{87}$$

$$\lim_{t\to 0} S_{cc}(t)=0,\quad \lim_{t\to 0} S_{ac}(t)=0,\quad \lim_{t\to 0} S_0(t)=0, \tag{88}$$

$$\lim_{t\to\infty} P_{cc}(t)=\frac{1}{2},\quad \lim_{t\to\infty} P_{ac}=\frac{1}{2},$$

$$\lim_{t\to\infty} S_{cc}(t)=\lim_{t\to\infty} S_{ac}(t)=\frac{\ln 2}{2},\quad \lim_{t\to\infty} S_0(t)=\ln 2. \tag{89}$$

It is conditions (87) that give us an opportunity to present two different states associated with the creation (cc) (in the time moment $t=0$, $P_{cc}=1$) and annihilation (ac) [at the point $t=0$, $P_{ac}(0)=0$] of correlation. Owing to discretness of the TCF $a(t)$ all functions $P_{\alpha\beta}$, $S_{\alpha\beta}$ as well as $S_0(t)$ ($\alpha=a,c$; $\beta=c$) are discrete in the real complex systems.

The results obtained in Sec. VI permit us to present the set of entropies for the states connected with the set of orthogonal variables W_i and set of memory functions $M_i(t)=\{M_1(t),M_2(t),M_3(t),\dots\}$. In analogy with Eqs. (81)–(89) these functions describe non-Markov and memory effects in the system under discussion

$$P_1^{M_i}(t)=P_{cM_i}(t)=\exp\{-\ln 2[1-M_i(t)]\}, \tag{90}$$

$$P_2^{M_i}(t)=P_{am_i}(t)=1-\exp\{-(\ln 2)[1-M_i(t)]\}, \tag{91}$$

$$S_i(t)=(\ln 2)[1-M_i(t)]\exp\{(\ln 2)[1-M_i(t)|\}$$
$$-\{1-\exp[-(\ln 2)(1-M_i)]\}$$
$$\times\ln\{1-\exp[-(\ln 2)(1-M_i)]\}, \tag{92}$$

where $i=1,2,3$. Four corresponding entropies $S_0(t)$, $S_1(t),S_2(t)$, and $S_3(t)$ and their power frequency spectra are available from the set of four time functions [TCF $a(t)$ and three memory functions $M_1(t)$, $M_2(t)$, $M_3(t)$]. Equations (81)–(92) are of great value because they allow us to estimate stochastic dynamics of the real complex systems with discrete time. As a matter of principle the first three memory functions $M_i(t)(i=1,2,3)$ are easy to find via Eq. (57). Using dimensionless parameter $\varepsilon_1=\tau^2\Omega_1^2$ and solution of the first finite-difference Eq. (57) we can calculate the discrete function $M_1(j\tau)$ at the points $j=0,1,2,\dots$, as follows:

$$M_1(0)=1,\quad M_1(\tau)=-a(2\tau)+\varepsilon_1^{-1}\{a(2\tau)-a(3\tau)\},$$

$$M_1(2\tau)=-\{a(2\tau)M_1(\tau)+a(3\tau)\}+\varepsilon_1^{-1}\{a(3\tau)-a(4\tau)\},$$

$$M_1(3\tau)=-\{a(2\tau)M_1(2\tau)+a(3\tau)M_1(\tau)+a(4\tau)M_1(0)\}$$
$$+\varepsilon_1^{-1}\{a(4\tau)-a(5\tau)\},$$

$$\cdots M_1(m\tau)=-\sum_{j=0}^{m-1} M_1(j\tau)a\{(m+1-j)\tau\}$$
$$+\varepsilon_1^{-1}[a\{(m+1)\tau\}-a\{(m+2)\tau\}]. \tag{93}$$

In the general case solving the chain of Eqs. (55),(57) we can find the recurrence relations between the memory functions of junior and higher orders in the following form:

$$M_s(m\tau)=-\sum_{j=0}^{m-1} M_s(j\tau)M_{s-1}[(m+1-j)\tau]$$
$$+\varepsilon_s^{-1}\{M_{s-1}[(m+1)\tau]-M_{s-1}[(m+2)\tau]\},$$
$$\varepsilon_s=\tau^2\Omega_s^2,\quad s=1,2,3,\dots\ . \tag{94}$$

The relations obtained allow us to derive straightly the necessary memory functions $M_s(t)$ of any order $s=1,2,\dots$ from experimental data using the registered TCF $a(m\tau)$ [46,47]. Relaxation frequencies Ω_i^2, $i=1,2,3,\dots$, given in Eq. (94) are available to experimental registration. Thus, it is fair to say that the applications of Eq. (94) will open up fresh opportunities for detailed study of statistical properties of correlations in the complex systems. The very fact of existence of finite-difference Eqs. (55),(57) enables us to develop any functions directly from the experiment. Therefore, the availability of discretness permits to enhance substantially the capability to get information for the complex systems' state.

In conclusion let us show the equations, which characterize the rate of entropy production. It is obvious from conditions (87)–(89) as well as Eqs. (81)–(85) that the rate of entropy growth $\partial S/\partial t$ within the interval $(0,\infty)$ takes different sign values and is determined by the entropy behavior in the channels of creation and annihilation of correlation

$$\frac{\partial S_0}{\partial t}=\left(\frac{\partial S_1^{(0)}}{\partial t}\right)+\left(\frac{\partial S_2^{(0)}}{\partial t}\right)=\left(\frac{\partial S_{cc}(t)}{\partial t}\right)+\left(\frac{\partial S_{ac}(t)}{\partial t}\right), \tag{95}$$

$$\frac{\partial S_1^{(0)}(t)}{\partial t}=-(\ln 2)a'(t)\exp\{-(\ln 2)[1-a(t)]\}$$
$$\times\{1-(\ln 2)[1-a(t)]\}, \tag{96}$$

$$\frac{\partial S_2^{(0)}(t)}{\partial t}=-(\ln 2)a'(t)\exp\{-(\ln 2)[1-a(t)]\}$$
$$\times\{1+\ln[1-\exp[-(\ln 2)(1-a(t)]\}, \tag{97}$$

$$\frac{\partial S_0(t)}{\partial t}=(\ln 2)a'(t)\exp\{-(\ln 2)[1-a(t)]\}$$
$$\times\{\ln\{1-\exp[-(\ln 2)(1-a(t))]\}+\ln 2[1-a(t)]. \tag{98}$$

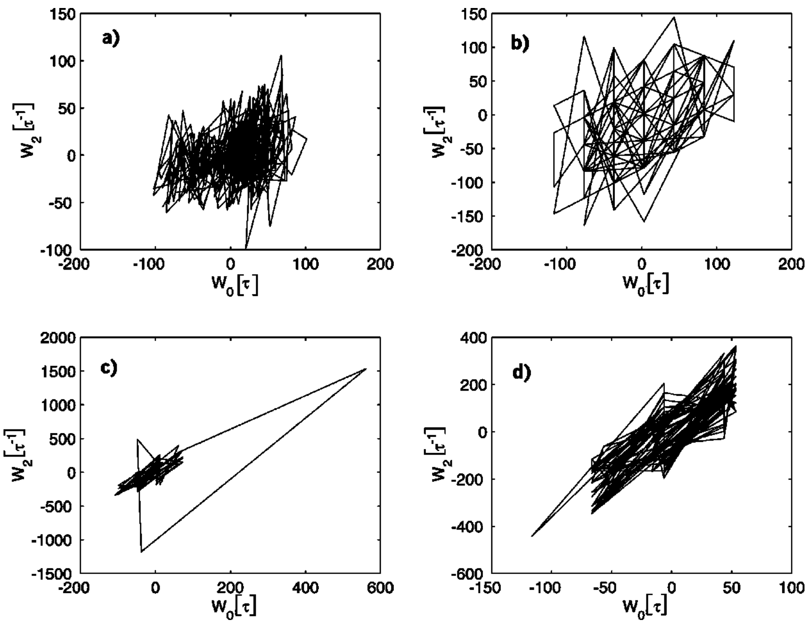

FIG. 3. Phase-time portrait in orthogonal variables (W_0, W_2) plain [see formulas (66),(68) for fourth group of patients]: healthy (a), patient with rhythm driver migration (b), patient after myocardial infarction (c), and patient after MI with subsequent SCD (d). As a matter of fact we utilized dimensionless variables W_0/τ and W_2/τ^{-1}.

The derivatives $a'(t)$ and $S_0'(t)$ here should be read in terms of Eqs. (8),(13). Since the derivative $a'(t)$ is finite within the whole time interval $(0,\infty)$: $|a'(t)|<c$, (where c is positive constant) the rate of entropy growth obeys the following boundary conditions:

$$\lim_{t\to 0}\left(\frac{\partial S_0}{\partial t}\right)=0, \quad \lim_{t\to\infty}\left(\frac{\partial S_0}{\partial t}\right)=0. \tag{99}$$

Formulas (95)–(99) are useful for the discussion of the experimental data. Close inspection of these equations shows that the behavior of derivative $(\partial S_0/\partial t)$ is described in many respects by the function $a'(t)=\tau^{-1}[a(t+\tau)-a(t)]$, which is in its turn can be obtained from the time series observed. Relations analogous to Eqs. (95)–(99) are easily available for the sequence of memory functions $M_i(t)$ (55) as well.

VIII. APPLICATION ON ANALYSIS OF STOCHASTIC DYNAMICS OF *RR* INTERVALS IN HUMAN ECG'S

Let us use the stochastic dynamics of RR intervals from human ECG's to illustrate some practical value of the approach developed. It is well known [24,26,49–60] that the statistical analysis of related dynamics allows the reliable quantitative characteristics of the human cardiovascular system states and trusty diagnostics of the various heart diseases [61–65].

Most investigators into heart rate dynamics have emphasized continuous functions, whereas the heart beat itself is in a crucial respect a discrete event. We present here experimental evidence that by considering this quality, the behavior of RR intervals may be appreciated as a result of discrete dynamics. To demonstrate effectiveness of non-Markovian approach we only take four typical particular cases from the whole the set of experimental data [66], which are available at our disposal. They are related to the case of healthy man (a), patient with a rhythm driver migration (b), patient after myocardial infarction (MI) (c), and patient after myocardial infarction (MI) with subsequent sudden cardial death (SCD) (d). Following standard medical practice, each from 112 person had an age, sex, and disease status matched pair serving as the control.

Results of our calculations, based on formulas of the theory and presented in previous sections, are shown on Figs. 3–8. It is necessary to mark that as a matter of convenience all variables and functions in a Figs. 3–8 are submitted in dimensionless form. Frequency ω everywhere is indicated in terms of units of $2\pi/\tau$. The orthogonal variables W_0 and W_2 in a Fig. 3 are written in units of τ and τ^{-1}, respectively. Frequency spectra $\mu_0(\omega)$, $\mu_1(\omega)$, and $\mu_2(\omega)$ in Figs. 4–6 are figured in terms of units of τ^2. Values $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ in Figs. 7, 8 are dimensionless values. Figure 3 shows phase trajectories, obtained for four different groups of patients in the orthogonal variables (W_0,W_2) plane. Let us remind ourselves, that in correspondence with formulas (64), (68) the variable W_0 presents RR intervals fluctuations, and W_2 is the second orthogonal variable and due to Eq. (68) is combination of an inertia force minus a restoring force. These variables have dimensions τ and τ^{-1}, respectively, where $\tau=\langle l_{RR}\rangle$ is the average value of the RR interval in time sequence. The set of characteristic parameters is collected in Table I. Let us mention the strong difference of numerical value of the first general relaxation frequency Ω_1 frequency for four different groups of patients. Figures 4 –6 show power frequency spectra for three different time functions for typical patients from four different groups. Figure 3(a) corresponds to a strange attractor, Fig. 3(b) corresponds to quasi-periodic motion, Fig. 3(c) 3(d) demonstrate the obviously expressed correlation of phase variables W_0 and W_2. Although the frequency ω is measured in units of $2\pi/\tau$ and power in τ^2, respectively. Figure 4 shows the power spectrum of TCF fluctuations of RR intervals. The data, shown in Figs. 5, 6 are correspondingly related to power spectra of first and second memory functions. The functions themselves are calculated from formulas (57), (68), and (94).

Figures 7, 8 require special explanation. They show frequency spectra of first two points $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ of statistical spectra of non-Markovity parameter (NMP) ϵ_i, where $i=1,2,\ldots$. A presentation of the NMP spectrum was intro-

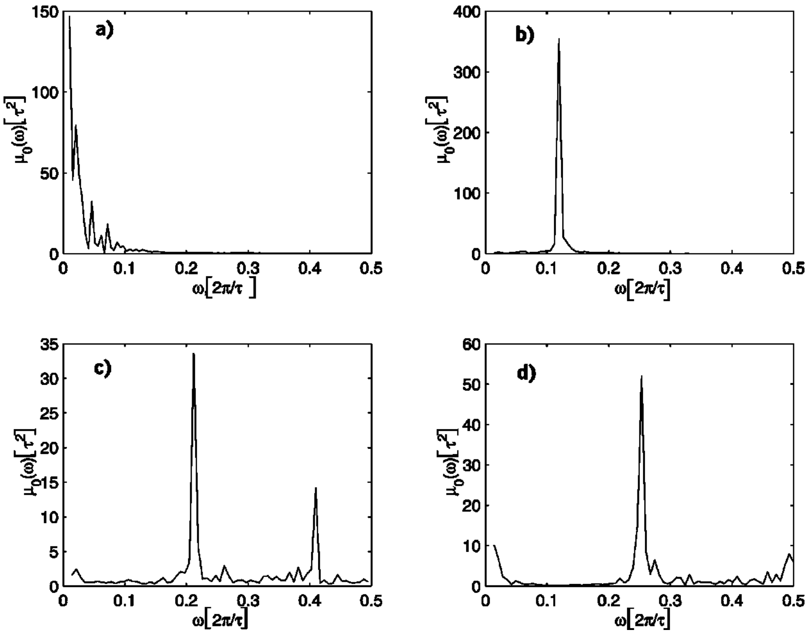

FIG. 4. Frequency spectrum of power $\mu_0(\omega)$ for TCF of fluctuation of RR intervals for fourth patient groups: healthy (a), patient with rhythm driver migration (b), patient after myocardial infarction (c), and patient after MI with subsequent SCD (d). The schedule is submitted in dimensionless units. The frequency is marked in terms of units of $(2\pi/\tau)$, the function $\mu_0(\omega)$ is figured in units of τ^2.

duced earlier in Refs. [67,68] and was then used in statistical physics of liquids [69,70]. Close to that given in Refs. [67,68] definitions of non-Markovity were developed later in Refs. [71–74]. In comparision with Refs. [67–69] here we generalize NMP conception for frequency dependent case

$$\epsilon_i(\omega)=\left\{\frac{\mu_{i-1}(\omega)}{\mu_i(\omega)}\right\}^{1/2},$$

where $i=1,2,\ldots$, and $\mu_i(\omega)$ is power frequency spectrum of ith level.

As is shown by Yulmetyev *et al.* in articles [67–70] NMP value of ϵ_i allows us to obtain a quantitative estimate of non-Markovity effects and statistical collective memory in random changes of experimentally measured data. Parameter ϵ_i allows us to divide all processes in three important cases [67–70]. Markovian processes correspond to $\epsilon\gg 1$, while quasi-Markovian processes correspond to situation with $\epsilon > 1$. The limit case $\epsilon\sim 1$ describes non-Markovian processes. In this case the time scale of memory processes and correlations (or junior and senior memory functions) coincide with each other.

From Figs. 3–8 one can easily obtain sharp differences between four groups of patients for all types of frequency spectra. For instance, frequency spectrum of TCF power for healthy [Fig. 4(a)] is almost reproduced in NMP $\epsilon_1(\omega)$ spectrum given in Fig. 7(a). Also it is slightly deformed in the spectra of first [Fig. 5(a)] and second [Fig. 6(a)] memory functions and is strongly transformed in NMP $\epsilon_2(\omega)$ spectrum [Fig. 8(a)]. Sharp peak in the vicinity of the point with $\omega\sim 0.125$ f.u., being characteristical for the patient (b), is seen in the power spectrum of first and second MF's [Fig. 5(b), 6(b)]. However, for other spectra of type b [for example, Figs. 6(b), 7(b), 8(b)] quite complicated structure ap-

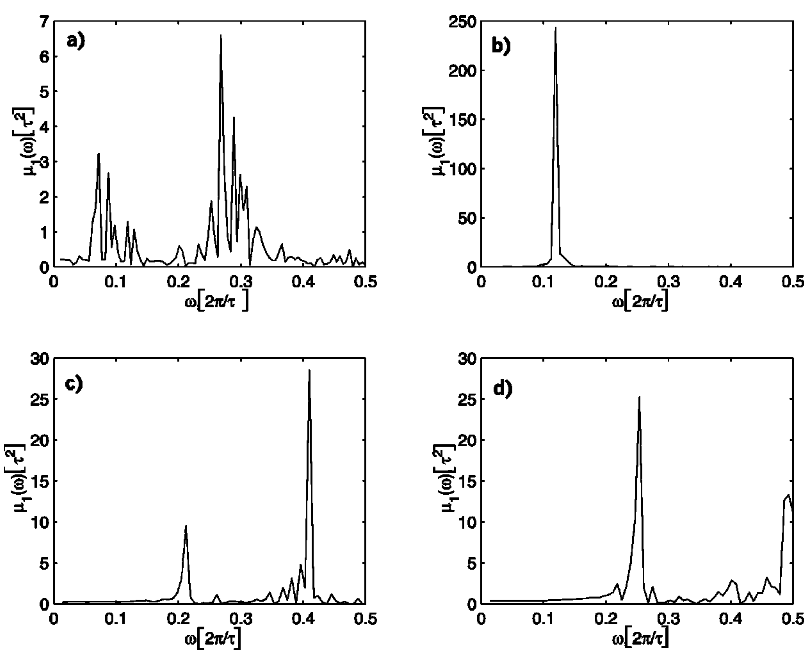

FIG. 5. Frequency spectrum of power $\mu_1(\omega)$ for the first MF $M_1(t)$ for fourth patient groups: healthy (a), patient with rhythm driver migration (b), patient after myocardial infarction (c), and patient after MI with subsequent SCD (d). The schedule is submitted in dimensionless units. The frequency is marked in terms of units of $(2\pi/\tau)$, the function $\mu_1(\omega)$ is figured in units of τ^2.

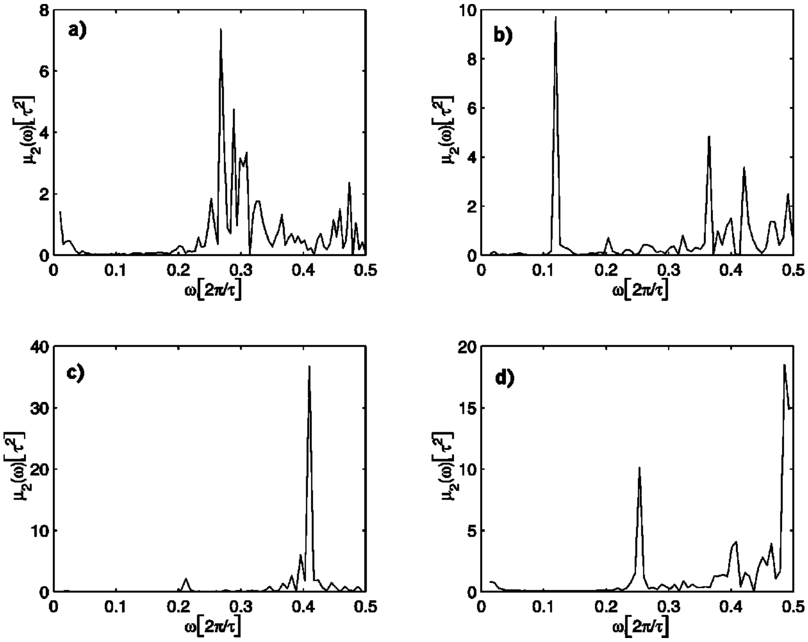

FIG. 6. Frequency spectrum of power $\mu_2(\omega)$ for the second MF $M_2(t)$ for fourth patient groups: healthy (a), patient with rhythm driver migration (b), patient after myocardial infarction (c), and patient after MI with subsequent SCD (d). The schedule is submitted in dimensionless units. The frequency is marked in terms of units of $(2\pi/\tau)$, the function $\mu_2(\omega)$ is figured in units of τ^2.

pears. Frequency spectrum of type (c), which is characteristic for IM, contains two sharply expressed spectral peaks nearly the frequencies, approximately 0.2 and 0.4 f.u. on the background of low intensity white noise. These peaks are conserved in the spectra of first [Fig. 5(c)] and second [Fig. 6(c)] MF. In NMP spectra $\epsilon_i(\omega)$, $\epsilon_2(\omega)$, complicated structure of spectral lines also appears. In characteristic case of patient with SCD frequency spectra of type (d) everywhere contain sharp peaks close to frequency 0.25 f.u. We would like to mention that all frequency spectra (5, 6, 7, and 8) are persuasive for strongly expressed non-Markovity for time change of RR intervals.

Figures 7(a)-7(d) and 8(a)–8(d) shows, that all values of NMP $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ lie in small interval of values (0–30). This fact convincingly tell us about characteristic statistic memory and noticeable non-Markovity effects in statistical dynamics of RR intervals from human ECG's. Obtained results on non-Markovian properties of temporal behavior of RR intervals justify significant and characteristic differences in data for all four groups of patients. We hope that the use of non-Markovian dynamics in the spirit of developed theory will incorporate development of more precise estimate of the state of cardiovascular systems for healthy as well as for more careful diagnostics of different patients.

IX. DISCUSSION

The present paper deals with two interrelated important results. The first one is connected with the establishment of the chain of finite-difference non-Markov kinetic equations for the discrete TCF. In this case the state of complex systems at the definite level of correlation is described by two vectors constructed over the strict determined rules. It is natural finite-difference equation of motion, being the pecu-

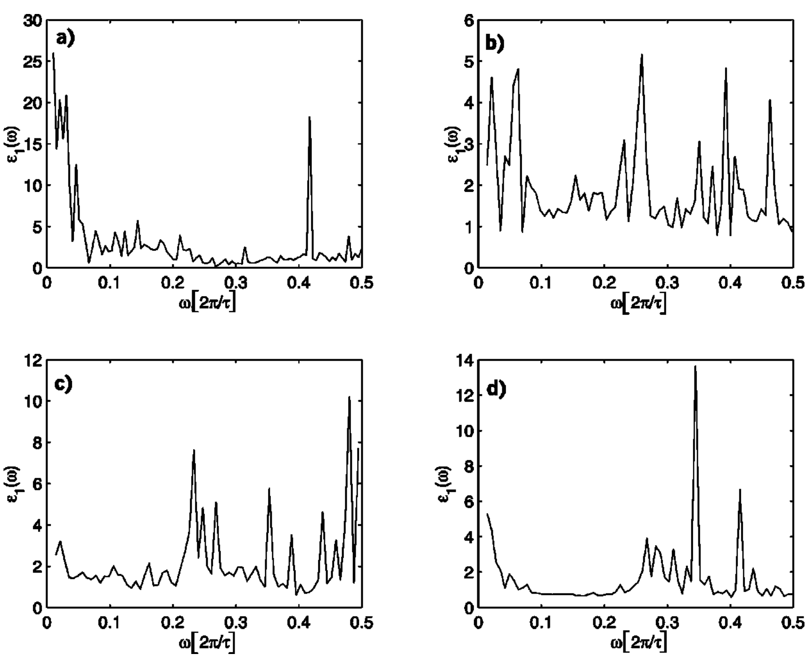

FIG. 7. Frequency spectrum of the first point in the statistical spectrum on non-Markovity parameter $\epsilon_1(\omega)$ for fourth patient groups: healthy (a), patient with rhythm driver migration (b), patient after myocardial infarction (c), and patient after MI with subsequent SCD (d). The schedule is submitted in dimensionless units. The frequency is marked in terms of units of τ^2.

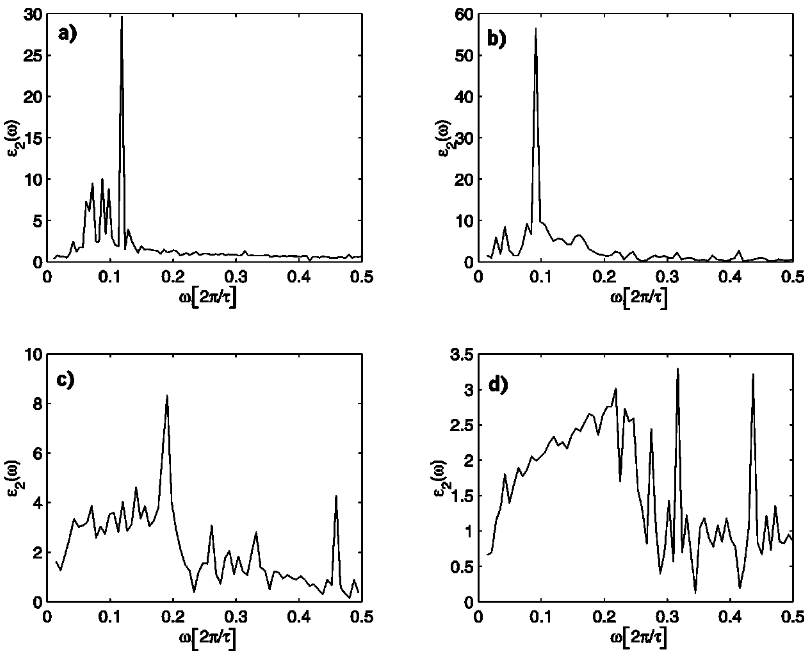

FIG. 8. Frequency spectrum of the second point in the statistical spectrum on non-Markovity parameter $\epsilon_2(\omega)$ for fourth patient groups: healthy (a), patient with rhythm driver migration (b), patient after myocardial infarction (c), and patient after MI with subsequent SCD (d). The schedule is submitted in dimensionless units. The frequency is marked in terms of units of τ^2.

liar analog of Liouville equations for the initial dynamic variables which are of particular interest for our analysis. In the subsequent discussion we employ the strict deduced mathematical fact of the existence of the normalized TCF. Due to the operation of scalar product the availability of TCF makes it possible to introduce the projection operators in the space of vectors of states. Those projection operations and matrix elements of Liouville's quasioperator ensure the splitting of natural equations of motion and then they are solved in the closed finite-difference form. Using Gram-Schmidt orthogonalization procedure we find an infinite set of the orthogonal dynamic random variables. This allows us to obtain the whole infinite chain of finite-difference kinetic equations for the initial discrete TCF. These equations contain the set of all memory functions characterizing the complete spectrum of non-Markov processes and statistical memory effects in the complex system. The presence of discretness and the very fact of the existence of finite-difference structure enable, in principle, to find all memory functions solving successively kinetic equations for the TCF. Parameters of these equations can be easily obtained from the experimentally registered TCF. In chaotic dynamics of complex systems the TCF above plays the role similar to that of the statistical integral in equilibrium statistical physics.

Another important result of our work is the dynamic (time dependent) information Shannon entropy given in terms of the TCF. This allows us to use the information measure for the quantitative characteristic of two interrelated correlation channels. One of them corresponds to the creation of time correlation and the other to the annihilation of correlation.

For that as we employ one of the classical Shannon's results [17], related to the introduction of fidelity evolution function and distance function between two vectors of state. The existence of a new information measure opens up new fields for exploration of information characteristics of complex systems. In particular, some interesting data arise from calculations frequency spectra of power of information entropy.

The important consequence of the results obtained is the usage of power spectra of memory functions $M_j(m\tau)$, where $m=0,1,2,3,\ldots$ and $j=1,2,3,\ldots$. The set of three junior memory functions with numbers $j=1,2,3$ provides the basis for the pseudohydrodynamical description of the complex system. In practice, any memory function can be extracted from the experimental time sets and experimentally recorded TCF. These criteria provide the possibility to get reliable information about non-Markov processes and memory effects in natural evolution of complex systems. In principle,

TABLE I. Set of ECG's data for the various group of patients.

Patient	Mean of RR intervals $\tau=\langle l_{RR}\rangle$ (ms)	Absolute variance σ (ms)	Relative variance δ (%)	A first general relaxation frequency Ω_1^2 [units of $(2\pi/\tau)^2$]
Healthy man	781	40.9	5.2	0.24
Rhythm driver migration	756	55.9	7.4	0.57
After myocardial Infarction	647	45.8	7	2.04
After myocardial infarction With subsequent sudden Cardiac death	776	32.3	4.8	2.34

the new point in the analysis of complex systems arises from the opportunity to construct the dynamical information Shannon entropy for the experimental memory functions. Undoubtedly, detection of the frequency spectra of power of entropy for memory functions gives us new unique information about the statistical non-Markov properties as well as memory effects in complex systems of various nature.

Application of the theory developed on the analysis of dynamics of RR intervals from human ECG's strongly suggest the substantially non-Markovian properties of the this dynamics. Here we have obtained non-Markovian quantitative characteristics for the fourth various groups of patients. One might expect this method may be use in distinguishing healthy from pathalogic data sets based in differences in its non-Markovian properties.

In conclusion it may be said that this paper describes a first-principle derivation of a hierarchy of finite-difference equations for time correlation function of out-of-equilibrium systems without Hamiltonian. The approach developed seems to have potentials and offer few advantages over the usual Hamiltonian point of view. A similar situation is true apparently with regard to turbulence, aging, for instance, as in spin glasses and glasses as well as experimental time series for living, social, and natural complex systems (physiology, cardiology, finance, psychology, and seismology, etc.).

By way of illustration it is significant that the anomalous scaling of simultaneous correlation function in turbulence is intimately related to the breaking of temporal scale invariance, which is equivalent to the appearence of infinitely many time times scales in the time dependence of time-correlation functions. In Refs. [75] temporal multiscaling on the basis of the continued fraction representation of turbulent correlation function [76] was addressed within the framework the Zwanzig-Mori formalism [43,44] which was applied to the time correlation function in turbulence. It has been shown by Grossman and Thomas [76] that the Zwanzig-Mori formalism applied to turbulent systems described by Navier- Stokes-like equations.

Mode coupling equatins have been considered in various areas of many particle physics for an approximate treatment of the dynamics of particles in glasses [77,78]. These equations are obtained if one represent within the Zwanzig-Mori formalism [43,44] correlation functions in terms of memory kernels and then expressed the latter via a factorization approximation in terms of the former for the glass transition of molecular liquids [79]. It has been found by Heuer *et al.* [80] that a model-free interpretation of higer-order correlation function determined by NMR reveals important information about the complex dynamics close to glass transition of polymers. This has been demonstrated with spin glasses [81] to show how a hierarchical model of spin glasses relaxation can display aging behavior in the time scale, similar to what is found in spin glasses and other complex systems out of thermodynamical equilibrium. The application of the approach developed on the analysis of the temporal behavior of complex systems of various natures will be available in our forthcoming papers.

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