Quantification of heart rate variability by discrete nonstationary non-Markov stochastic processes

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We develop the statistical theory of discrete nonstationary non-Markov random processes in complex systems. The objective of this paper is to find the chain of finite-difference non-Markov kinetic equations for time correlation functions (TCF) in terms of nonstationary effects. The developed theory starts from careful analysis of time correlation through nonstationary dynamics of vectors of initial and final states and nonstationary normalized TCF. Using the projection operators technique we find the chain of finite-difference non-Markov kinetic equations for discrete nonstationary TCF and for the set of nonstationary discrete memory functions (MF's). The last one contains supplementary information about nonstationary properties of the complex system on the whole. Another relevant result of our theory is the construction of the set of dynamic parameters of nonstationarity, which contains some information of the nonstationarity effects. The full set of dynamic, spectral and kinetic parameters, and kinetic functions (TCF, short MF's statistical spectra of non-Markovity parameter, and statistical spectra of nonstationarity parameter) has made it possible to acquire the in-depth information about discreteness, non-Markov effects, long-range memory, and nonstationarity of the underlying processes. The developed theory is applied to analyze the long-time (Holter) series of *RR* intervals of human ECG's. We had two groups of patients: the healthy ones and the patients after myocardial infarction. In both groups we observed effects of fractality, standard and restricted self-organized criticality, and also a certain specific arrangement of spectral lines. The received results demonstrate that the power spectra of all orders ($n=1,2,\dots$) MF $m_n(t)$ exhibit the neatly expressed fractal features. We have found out that the full sets of non-Markov, discrete and nonstationary parameters can serve as reliable and powerful means of diagnosis of the cardiovascular system states and can be used to distinguish healthy data from pathologic data.

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I. INTRODUCTION

The study of information processing in life systems is one of the central problems in modern science. It is now well known that in many natural sequences the elements are not arranged randomly, but exhibit long-range correlations. For a long period of time it was suggested that many complex systems observed in nature should be described only by some of low-dimensional nonlinear dynamic models. The properties of these systems were supposed to be expressed by Lyapunov exponents, unique fractal dimensions or Kolmogorov-Sinai entropy. However, such low dimensionality can be expected for rather coherent phenomena such as observed in laser systems. Alive data seems to have a more complicated structure largely due to high-dimensional and many-factor processes and due to the pronounced effects of random fluctuations and long-time memory effects.

Since the time of Refs. [1–6] heart rate variability (HRV) serves as one of the most reliable and authentic methods of testing the state of a human heart in the norm and in the pathology [7]. In particular, the analysis of HRV has promoted the establishment of reliable connections between the functioning of a vegetative nervous system and a sudden heart death [4,8–12]. At present there are many diverse approaches by theoretical physics to the problems of nonlinear properties of HRV description. The following things should be mentioned: the fractal approach based on scaling of a frequency spectrum on power law $1/\omega^{\alpha}$ [13–15], the calculation of correlation dimension [16], the simulation by nonlinear oscillators [16,17], the calculation of the Kolmogorov entropy [16], usual [18] and dynamic [19] Shannon entropy, the use of dynamics of lattice spins as a model of arrhythmia [20], Fano-factor and Allan-factor [14], the wavelet analysis [21], and the detrended fluctuation analysis [22,23]. The following methods are also employed here: the multifractal analysis [24], the multiscaled randomness [25], the Markov formalization of dynamics [26], and the terminal dynamics model of heart beat [27]. In a recent paper, Teich *et al.* [28] demonstrated the manner in which various measures of fluctuation of the sequence of interbeat intervals could be used to assess the presence or likelihood of cardiovascular disease.

The profound analysis of the dynamics of heart beats dynamics reveals that the fundamental methods of the statistical physics based on the Hamilton formalism and exact equations of motion are directly inapplicable for its quantitative description. On the other hand, the discretization of events and long-time event-event correlation are very relevant in similar dynamics. Recently, a non-Markov theory of discrete stochastic processes was developed in Ref. [29]. The approach advanced in [29] makes the calculation of the wide set of non-Markov characteristics of an arbitrary complex system from experimental database possible.

In the present paper we develop a non-Markov approach [29] for the study of long-time correlations in chaotic long-time dynamics of *RR* intervals from human electrocardio-

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gram's (ECG's). *RR* interval is defined as the time distance between nearest *R* peaks in human electrocardiogram. The generalization will consist in taking into account the nonstationarity of stochastic processes and its further applications to the analysis of HRV.

We should bear in mind that one of the key moments of the spectral approach in the analysis of stochastic processes consists in the use of normalized time correlation function (TCF)

$$a_0(t) = \frac{\langle\langle \mathbf{A}(T)\mathbf{A}(T+t)\rangle\rangle}{\langle \mathbf{A}(T)^2\rangle}, \tag{1.1}$$

where the time T is the beginning of a time serial, $\mathbf{A}(t)$ is a state vector of a complex system, $|\mathbf{A}(t)|$ is the length of vector $\mathbf{A}(t)$, and the double angular brackets indicate a scalar product of vectors and ensemble averaging. The ensemble averaging is, of course, needed in Eq. (1.1) when correlation and other characteristic functions are constructed. The average and scalar product becomes equivalent when a vector is composed of elements from a discrete-time sampling, as done later in the paper. Here a continuous formalism is discussed for convenience. However, further, from Sec. II we shall consider only a case of discrete processes.

The above-stated designation is true only for stationary systems. In a nonstationary case Eq. (1.1) is not true and should be changed. The concept of TCF can be generalized in case of discrete nonstationary sequence of signals. For this purpose the standard definition of the correlation coefficient in probability theory for the two random signals X and Y must be taken into account:

$$\rho = \frac{\langle\langle \mathbf{XY}\rangle\rangle}{\sigma_X\sigma_Y}, \quad \sigma_X = \langle|\mathbf{X}|\rangle, \quad \sigma_Y = \langle|\mathbf{Y}|\rangle. \tag{1.2}$$

In Eq. (1.2) the multicomponent vectors $\mathbf{X}$, $\mathbf{Y}$ are determined by fluctuations of signals x and y accordingly, σ_X^2, σ_Y^2, represent the variances of signals $\mathbf{x}$ and $\mathbf{y}$, and values $|\mathbf{X}|$, $|\mathbf{Y}|$ represent the lengths of vectors $\mathbf{X}$, $\mathbf{Y}$, correspondingly. Therefore, the function

$$a(T,t) = \frac{\langle\langle \mathbf{A}(T)\mathbf{A}(T+t)\rangle\rangle}{\langle|\mathbf{A}(T)|\rangle\langle|\mathbf{A}(T+t)|\rangle} \tag{1.3}$$

can serve as the generalization of the concept of TCF (1.1) for nonstationary processes $\mathbf{A}(T+t)$. Nonstationary TCF (1.3) obeys the conditions of the normalization and attenuation of correlation

$$a(T,0) = 1, \quad \lim_{t\to\infty} a(T,t) = 0.$$

According to the Eqs. (1.1) and (1.3) for the quantitative description of nonstationarity it is convenient to introduce a function of nonstationarity

$$\gamma(T,t) = \frac{\langle|\mathbf{A}(T+t)|\rangle}{\langle|\mathbf{A}(T)|\rangle} = \left\{\frac{\sigma^2(T+t)}{\sigma^2(T)}\right\}^{1/2}. \tag{1.4}$$

One can see that this function equals the ratio of the lengths of vectors of final and initial states. In case of the stationary process the dispersion does not vary with the time (or its variation is very weak). Therefore the following relations

$$\sigma(T+t) = \sigma(T), \quad \gamma(T,t) = 1 \tag{1.5}$$

are true for the stationary process.

Due to the condition (1.5) the following function

$$\Gamma(T,t) = 1 - \gamma(T,t), \tag{1.6}$$

is convenient to consider as a dynamic parameter of nonstationarity. This dynamic parameter can serve as a quantitative measure of nonstationarity of the process under investigation. According to Eqs. (1.4)–(1.6) it is reasonable to suggest the existence of three different classes of nonstationarity

$$|\Gamma(T,t)| = |1-\gamma(T,t)| \begin{cases} \ll 1, & \text{weak nonstationarity} \\ \sim 1, & \text{intermediate nonstationarity} \\ \gg 1, & \text{strong nonstationarity.} \end{cases} \tag{1.7}$$

The existence of a dynamic parameter of nonstationarity makes it possible to determine, on principle, the type of nonstationarity of the investigated process and to find its spectral characteristics from the experimental data base. We intend to use Eqs. (1.4), (1.6), and (1.7) for the quantitative description of effects of nonstationarity in the investigated temporary series of *RR* intervals of human ECG's for healthy people and patients after myocardial infarction (MI).

Here we shall show that the complex dynamics of heart rate fluctuation can be described in detail by the set of nonstationary non-Markov properties on the whole. There are two problems, which we would like to decide. One of them is, how important is the discretization and long-range memory effects in the behavior of cardiovascular systems? The second problem is defined by the following: whether it is possible to use nonstationary and non-Markov properties to diagnose the state of a human heart. We demonstrate that the solution to these problems is possible to be found.

The paper is organized in the following way. In Sec. II, we present the nonstationary generalization of our previous paper [29], used here for the analysis of HRV. In Sec. III we describe the data and standard technique of ECG in the time and frequency domains. This section contains technical details of obtaining experimental data on Holter monitoring of ECG's for healthy people and patients after MI. The results of quantitative calculations of the phase portraits, the memory functions, their power frequency spectra, the frequency dependence of the first three points in statistical spectrum of non-Markovity parameter, and in statistical spectrum of nonstationarity parameter will be shown in Sec. IV. The last, Sec. V, contains the discussion of the results obtained and the conclusion.

II. STATISTICAL THEORY OF NONSTATIONARY DISCRETE NON-MARKOV PROCESSES IN COMPLEX SYSTEMS

Here we shall extend original results of the statistical theory of discrete non-Markov processes in complex systems, developed recently by us in Ref. [29], for the case of nonstationary processes. The theory [29] is developed on the basis of the first principles and represents a discrete finite-difference analogy for complex systems of well-known Zwanzig'-Mori's kinetic equations [30,31] in the statistical physics of condensed matter.

We examine a discrete stochastic process $X(T+t)$, where $t=m\tau$,

$$X=\{x(T),x(T+\tau),x(T+2\tau),\ldots,x(T+k\tau),\ldots,\times x(T+(N-1)\tau)\}, \tag{2.1}$$

where $\mathbf{T}$ is the beginning of the time and τ is a discretization time. The normalized TCF

$$a(t)=\frac{1}{(N-m)\sigma^2}\sum_{j=0}^{N-t-m}\delta x(T+j\tau)\delta(T+[j+m]\tau) \tag{2.2}$$

is a convenient means to analyze dynamic properties of complex systems. Here is entered variance σ^2, fluctuation $\delta x(T+j\tau)$, and mean value $\langle x\rangle$,

$$\delta x_j=\delta x(T+j\tau)=x(T+j\tau)-\langle x\rangle,$$

$$\sigma^2=\frac{1}{(N-m)}\sum_{j=0}^{N-1-m}\{\delta x(T+j\tau)\}^2, \tag{2.3}$$

$$\langle x\rangle=\frac{1}{(N-m)}\sum_{j=0}^{N-1-m}x(T+j\tau), \tag{2.4}$$

and discrete time t is equal $t=m\tau$.

In general, the mean value, the variance and TCF in Eqs. (2.2), (2.3), and (2.4) is dependent on numbers m and N. The similar situation is especially typical for the case of nonstationary processes. All indicated values cease to depend on numbers m and N for stationary processes at $m\ll N$. The definition of TCF in Eq. (2.2) is true only for stationary processes.

Now we shall try to take into account this important dependence. With this purpose we shall form two k-dimensional vectors of state by the process (2.1)

$$\mathbf{A}_k^0=(\delta x_0,\delta x_1,\delta x,\ldots,\delta x_{k-1}),$$

$$\mathbf{A}_{m+k}^m=(\delta x_m,\delta x_{m+1},\delta x_{m+2},\ldots,\delta x_{m+k-1}). \tag{2.5}$$

When a vector of a state is composed of elements from a discrete-time sampling, the average and scalar product in Eq. (1.1) becomes equivalent. In a Euclidean space of vectors of state (2.5), TCF $a(t)$,

$$a(t)=\frac{\langle \mathbf{A}_{N-1-m}^0\mathbf{A}_{N-1}^m\rangle}{(N-m)\{\sigma(N-m)\}^2}=\frac{\langle \mathbf{A}_{N-1-m}^0\mathbf{A}_{N-1}^m\rangle}{|\mathbf{A}_{N-1-m}^0|^2} \tag{2.6}$$

describes the correlation of two different states of the system $(t=m\tau)$. Here the brackets $\langle\cdots\rangle$ indicate the scalar product of the two vectors. The dimension dependence of the corresponding vectors is also taken into account in the variance $\sigma=\sigma(N-m)$. As a matter of fact TCF $a(t)$ represents $\cos\vartheta$, where ϑ is the angle between the two vectors from Eq. (2.5). Let us introduce a unit vector of dimension $(N-m)$ as follows:

$$\mathbf{n}=\frac{\mathbf{A}_{N-1-m}^0}{\sqrt{(N-m)\sigma^2}}. \tag{2.7}$$

Then it is possible to present TCF $a(t)$ (2.2) as follows:

$$a(t)=\langle \mathbf{n}(0)\mathbf{n}(t)\rangle. \tag{2.8}$$

From the above discussion it is clear that Eqs. (2.6)–(2.8) are true for stationary processes only. In case of nonstationary processes it is necessary to redefine TCF to take into account the nonstationarity in the variance σ^2 in a line with Eqs. (1.2)–(1.7). For this purpose we shall redefine a unit vector of the final state as follows:

$$\mathbf{n}(t)=\frac{\mathbf{A}_{N-1}^m(t)}{|\mathbf{A}_{N-1}^m(t)|}. \tag{2.9}$$

Then for nonstationary processes it is convenient to write TCF as the scalar product of the two unit vectors of the initial and final states

$$a(t)=\langle \mathbf{n}(0)\mathbf{n}(t)\rangle=\frac{\langle \mathbf{A}_{N-1-m}^0(0)\mathbf{A}_{N-1}^m(t)\rangle}{|\mathbf{A}_{N-1-m}^0(0)||\mathbf{A}_{N-1}^m(t)|}. \tag{2.10}$$

Now we shall consider dynamics of nonstationary stochastic process. The equation of motion of a random variable x_j can be written in a finite-difference form for $0\leqslant j\leqslant N-1$ [29] as follows:

$$\frac{dx_j}{dt}\Rightarrow\frac{\Delta\,\delta x_j}{\Delta t}=\frac{\delta x_j(t+\tau)-\delta_j(t)}{\tau}. \tag{2.11}$$

Then it is convenient to express the discrete evolution of single step operator as follows:

$$x[T+(j+1)\tau]=\hat{U}(T+(j+1)\tau,T+j\tau)x(T+j\tau). \tag{2.12}$$

In the case of stationary process we can rewrite the equation of motion (2.11) in a more simple form

$$\frac{\Delta\,\delta x_j}{\Delta t}=\tau^{-1}\{\hat{U}(\tau)-1\}\delta x_j. \tag{2.13}$$

The invariance of the mean value $\langle x\rangle$ is taken into account in an equation, Eq. (2.13),

$$\langle x\rangle=\hat{U}(\tau)\langle x\rangle,\quad \{\hat{U}(\tau)-1\}\langle x\rangle=0. \tag{2.14}$$

In case of nonstationary process it is necessary to turn to the equation of motion for vector of the final state $\mathbf{A}_{m+k}^{m}(t)$ $(k=N-1-m)$

$$\frac{\Delta \mathbf{A}_{m+k}^{m}(t)}{\Delta t}=i\hat{L}(t,\tau)\mathbf{A}_{m+k}^{m}(t), \tag{2.15}$$

where the Liouville's quasioperator is

$$\hat{L}(t,\tau)=(i\tau)^{-1}\{\hat{U}(t+\tau,t)-1\}. \tag{2.16}$$

It is well known that, in general, a stochastic trajectory does not obey a linear equation, so the general evolution operator and Liouville's quasioperator should probably be nonlinear. Furthermore, in statistical physics Liouville's operator acts upon the probability densities of dynamical variables, as well as upon the variables themselves such as in the Mori paper [31]. The evolution of density would indeed be linear. But Mori, in Ref. [31], used Liouville's operator in the quantum equation of motion. In line with Mori [31] Eqs. (2.13) and (2.15) can be considered as the formal and exact equation of motion of a complex system.

As a matter of fact, the discrete set of values of the discrete evolution operator is considered in Eqs. (2.15) and (2.16). This set is convenient for presenting as a $(N-1)$-dimensional diagonal matrix

$$\hat{U}(t+\tau,t)=\begin{pmatrix} \hat{U}(T+\tau,T) & 0 & 0 & \cdots & 0 \\ 0 & \hat{U}(T+2\tau,T+\tau) & 0 & \cdots & 0 \\ 0 & 0 & \hat{U}(T+3\tau,T+2\tau) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \hat{U}(T+[N-1]\tau,T+[N-2]\tau) \end{pmatrix}. \tag{2.17}$$

Here each diagonal matrix element acts on the corresponding component of the state vector. Then it is convenient to rewrite the Liouville's quasioperator in Eq. (2.15) in the form of $(N-1)$-dimensional diagonal matrix as follows:

$$\hat{L}(t,\tau)=(i\tau)^{-1}\times\begin{pmatrix} \hat{U}(T+\tau,T)-1 & 0 & 0 & \cdots & 0 \\ 0 & \hat{U}(T+2\tau,T+\tau)-1 & 0 & \cdots & 0 \\ 0 & 0 & \hat{U}(T+3\tau,T+2\tau)-1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \hat{U}(T+[N-1]\tau,T+[N-2]\tau)-1 \end{pmatrix}. \tag{2.18}$$

It is obvious from Eqs. (2.17) and (2.18) that the diagonal elements of matrices $\hat{U}$ and $\hat{L}$ are operators themselves. Here each diagonal matrix element acts on the corresponding component of the state vector. The matrix representation of the Liouville's quasioperator (2.18) and the evolution operator (2.17) allow to take into account the nonstationary peculiarities of the dynamics of the multidimensional vector of the final state of the system.

We shall use the formulas (2.15) and (2.18) further only for compactness.

So, due to the Eqs. (2.10) and (2.15)–(2.18) it is possible to take into account the nonstationarity of the stochastic process. Now let us introduce the linear projection operator in Euclidean space of the state vectors

$$\Pi \mathbf{A}(t)=\frac{\mathbf{A}(0)\rangle\langle \mathbf{A}(0)\mathbf{A}(t)\rangle}{|\mathbf{A}(0)|^2}, \quad \Pi=\frac{\mathbf{A}(0)\rangle\langle \mathbf{A}(0)}{\langle \mathbf{A}(0)\mathbf{A}(0)\rangle}, \tag{2.19}$$

where angular brackets in the numerator represent the boundaries of action for the scalar product.

For the analysis of the dynamics of stochastic process $\mathbf{A}(t)$, the vector $\mathbf{A}_k^0(0)$ from Eq. (2.5) can be considered as the vector of the initial state $\mathbf{A}(0)$, and vector $\mathbf{A}_{m+k}^{m}(t)$ from Eq. (2.5) at value $m+k=N-1$ can be considered as the vector of the final state $\mathbf{A}(t)$.

It is necessary to note that the projection operator (2.19) has the necessary property of idempotentity $\Pi^2=\Pi$. The presence of operator Π allows to introduce the mutually supplementary projection operator P as follows:

$$P=1-\Pi, \quad P^2=P, \quad \Pi P=P\Pi=0. \tag{2.20}$$

It is necessary to mark that both projectors Π and P are linear and can be recorded for the fulfillment of operations in the particular Euclidean space. Due to the property (2.10) and Eq. (1.4) it is easy to receive the required TCF as follows:

$$\Pi\mathbf{A}(t)=\Pi\mathbf{A}^{m}_{m+k}(t)=\mathbf{A}^{0}_{k}(0)\langle\mathbf{n}^{0}_{k}(0)\mathbf{n}^{m}_{k+m}(t)\rangle\gamma_1(t)$$
$$=\mathbf{A}^{0}_{k}(0)a(t)\gamma_1(t),$$

$$\gamma_1(t)=\frac{|\mathbf{A}^{m}_{m+k}(t)|}{|\mathbf{A}^{0}_{m}(0)|}. \tag{2.21}$$

Therefore the projector Π generates a unit vector along the vector of the final state $\mathbf{A}(t)$ and creates its projection on the initial state vector $\mathbf{A}(0)$.

The existence of a pair of two mutually supplementary projection operators Π and P allows to carry out the splitting of Euclidean space of vectors $A[\mathbf{A}(0),\mathbf{A}(t)\in A]$ into a straight sum of the two mutually supplementary subspaces as follows:

$$A=A'\dotplus A'',\quad A'=\Pi A,\quad A''=PA. \tag{2.22}$$

Substituting Eq. (2.22) in Eq. (2.16) we find Liouville's quasioperator $\hat{L}$ in a matrix form

$$\hat{L}=\hat{L}_{11}+\hat{L}_{12}+\hat{L}_{21}+\hat{L}_{22}, \tag{2.23}$$

where the matrix elements are introduced

$$\hat{L}_{11}=\Pi\hat{L}\Pi,\quad \hat{L}_{12}=\Pi\hat{L}P,\quad \dot{L}_{21}=P\hat{L}\Pi,\quad \hat{L}_{22}=P\hat{L}P. \tag{2.24}$$

Due to the properties (2.17) and (2.18) Euclidean space of values of Liouville's quasioperator $W=\hat{L}A$ will be generated by vectors $\mathbf{W}$ of dimension $k-1$

$$(\mathbf{W}(0)\in W,\mathbf{W}(t)\in W)$$

$$W=W'\dotplus W'',\quad W'=\Pi W,\quad W''=PW. \tag{2.25}$$

Matrix elements $\hat{L}_{ij}$ of the contracted description

$$\hat{L}=\begin{pmatrix}\hat{L}_{11} & \hat{L}_{12}\\ \hat{L}_{21} & \hat{L}_{22}\end{pmatrix} \tag{2.26}$$

act as follows: $\hat{L}_{11}$, from a subspace A' to subspace W'; $\hat{L}_{12}$, from A'' to W'; $\hat{L}_{21}$, from W' to W''; and $\hat{L}_{22}$, from A'' to W''

The projection operators Π and P allow to execute the contracted description of stochastic process. Splitting the dynamic equation (2.15) into two equations in the two mutually supplementary Euclidean subspaces (see, for example, [29]), we find

$$\frac{\Delta\mathbf{A}'(t)}{\Delta t}=i\hat{L}_{11}\mathbf{A}'(t)+i\hat{L}_{12}\mathbf{A}''(t), \tag{2.27}$$

$$\frac{\Delta\mathbf{A}''(t)}{\Delta t}=i\hat{L}_{21}\mathbf{A}'(t)+i\hat{L}_{22}\mathbf{A}''(t). \tag{2.28}$$

Following [29] it is necessary to eliminate first irrelevant part $\mathbf{A}''(t)$ to simplify Liouville's equation (2.15) and then to write a closed equation for relevant part $\mathbf{A}'(t)$. According to [29] this can be realized by the series of successive steps (for example, see Eqs. (32)–(36) in Ref. [29]). At first a solution to Eqs. (2.28) for the first step can be obtained in a form

$$\frac{\Delta\mathbf{A}''(t)}{\Delta t}=\frac{\mathbf{A}''(t+\tau)-\mathbf{A}''(t)}{\tau}=i\hat{L}_{21}\mathbf{A}'(t)+iL_{22}\mathbf{A}''(t),$$

$$\begin{aligned}\mathbf{A}''(t+\tau)&=\mathbf{A}''(t)+i\tau\hat{L}_{21}\mathbf{A}'(t)+i\tau\hat{L}_{22}\mathbf{A}''(t)\\&=\{1+i\tau\hat{L}_{22}\}\mathbf{A}''(t)+i\tau\hat{L}_{21}\mathbf{A}'(t)\\&=U_2(t+\tau,t)\mathbf{A}''(t)+i\tau\hat{L}_{21}(t+\tau,t)\mathbf{A}'(t).\end{aligned} \tag{2.29}$$

Here we considered the obvious ratio for operators of the particular step and introduced designations

$$\hat{U}_{22}(t+\tau,t)=1+i\tau\hat{L}_{22}(t+\tau,t). \tag{2.30}$$

Using Eqs. (2.29) and (2.30), we derive successively for the next step

$$\begin{aligned}\mathbf{A}''(t+2\tau)&=\hat{U}_{22}(t+2\tau,t+\tau)\mathbf{A}''(t+\tau)+i\tau\hat{L}_{21}(t+2\tau,t+\tau)\mathbf{A}'(t+\tau)\\&=\hat{U}_{22}(t+\tau,t+\tau)\{\hat{U}_{22}(t+\tau,t)\mathbf{A}''(t)+i\tau\hat{L}_{21}(t+\tau,t)\mathbf{A}'(t)\}+i\tau\hat{L}_{21}(t+2\tau,t+\tau)\mathbf{A}'(t+\tau)\\&=\hat{U}_{22}(t+2\tau,t+\tau)\hat{U}_{22}(t+\tau,t)\mathbf{A}''(t)+i\tau\{\hat{U}_{22}(t+2\tau,t+\tau)\hat{L}_{21}(t+\tau,t)\mathbf{A}'(t)+\hat{L}_{21}(t+2\tau,t+\tau)\mathbf{A}'(t+\tau)\}.\end{aligned} \tag{2.31}$$

Consequently, for the third step of evolution of the final state vector we obtain

$$\begin{aligned}\mathbf{A}''(t+3\tau)=&\hat{U}_{22}(t+3\tau,t+\tau)\hat{U}_{22}(t+2\tau,t+\tau)\hat{U}_{22}(t+\tau,t)\mathbf{A}''(t)+i\tau\{\hat{U}_{22}(t+3\tau,t+2\tau)\hat{U}_{22}(t+2\tau,t+\tau)\hat{L}_{21}(t+\tau,t)\mathbf{A}'(t)\\&+\hat{U}_{22}(t+3\tau,t+2\tau)\hat{L}_{21}(t+2\tau,t+\tau)\mathbf{A}'(t+\tau)+\hat{L}_{21}(t+3\tau,t+2\tau)\mathbf{A}'(t+2\tau)\}.\end{aligned} \tag{2.32}$$

Generally, after a series of mth successive discrete steps, the final result represents, respectively, the following:

$$\mathbf{A}''(t+m\tau)=\left\{\hat{T}\prod_{j=0}^{m-1}\hat{U}_{22}(t+[j+1]\tau,t+j\tau)\right\}\mathbf{A}''(t)+i\tau\sum_{j=0}^{m-1}\left\{\hat{T}\prod_{j'=j}^{m-2}\hat{U}_{22}(t+[j'+2]\tau,t+[j'+1]\tau)\right\}\hat{L}_{21}(t+[j+1]\tau,t+j\tau)\times\mathbf{A}'(t+j\tau). \tag{2.33}$$

Here $\hat{T}$ denotes the Dyson operator of chronological ordering. Substituting the irrelevant part in Eq. (2.27) to the right side of Eq. (2.33), we obtain the closed finite-difference equation for the relevant part of the state vector

$$\frac{\Delta}{\Delta t}\mathbf{A}'(t+m\tau)=i\hat{L}_{11}(t+[m+1]\tau,t+m\tau)\mathbf{A}'(t+m\tau)+i\hat{L}_{12}(t+[m+1]\tau,t+m\tau)\times\left[\left\{\hat{T}\prod_{j=0}^{m-1}\hat{U}_{22}(t+(j+1)\tau,t+j\tau)\right\}\mathbf{A}''(t)-\tau\sum_{j=0}^{m-1}\left\{\hat{T}\prod_{j'=j}^{m-2}\hat{U}_{22}(t+(j'+2)\tau,t+[j'+1]\tau)\right\}\hat{L}_{21}(t+[j+1]\tau,t+j\tau)\mathbf{A}'\times(t+j\tau)\right]. \tag{2.34}$$

Introducing the modified evolution operator by the formula

$$\hat{V}(t+m\tau,t+s\tau)=\hat{T}\prod_{j=s}^{m-1}\hat{U}_{22}(t+[i+1]\tau,t+j\tau), \tag{2.35}$$

where the integers m and s fit the condition $m>s$, we can rewrite Eq. (2.34) in the following form:

$$\frac{\Delta}{\Delta t}\mathbf{A}'(t+m\tau)=i\hat{L}_{11}(t+[m+1]\tau,t+m\tau)\mathbf{A}'(t+m\tau)+i\hat{L}_{12}(t+[m+1])\tau,t+m\tau)\times\hat{V}(t+m\tau,t)\mathbf{A}''(t)-\tau\sum_{j=0}^{m-1}\hat{L}_{12}\times(t+[m+1)\tau,t+m\tau)\times\hat{V}(t+m\tau,t+[j+1]\tau)\times\hat{L}_{21}(t+[j+1]\tau,t+j\tau)\mathbf{A}'(t+j\tau). \tag{2.36}$$

This vector equation can be simplified as well. For this purpose we shall take into account the idempotentity property of the projection operators. Substituting time arguments in Eqs. (2.36) $t-T$, $m\tau-t$ we receive ratio

$$\mathbf{A}''(T)=0,\quad \hat{V}(T+t,T)\mathbf{A}''(T)=0. \tag{2.37}$$

Substituting Eqs. (2.21) and (2.22) in Eq. (2.36), we derive a finite-difference kinetic equation of a non-Markov type for TCF $a(t=m\tau)$

$$\frac{\Delta a(t)}{\Delta t}=\lambda_1 a(t)-\tau\Lambda_1\sum_{j=0}^{m-1}M_1(t-j\tau)a(j\tau). \tag{2.38}$$

Here λ_1 is an eigenvalue and Λ_1 is the relaxation parameter of Liouville's quasioperator $\hat{L}$

$$\lambda_1=i\frac{\langle\mathbf{A}_k^0(0)\hat{L}\mathbf{A}_k^0(0)\rangle}{|\mathbf{A}_k^0(0)|^2},$$

$$\Lambda_1=\frac{\langle\mathbf{A}_k^0(0)\hat{L}_{12}\hat{L}_{21}\mathbf{A}_k^0(0)\rangle}{|\mathbf{A}_k^0(0)|^2}=\frac{\langle\mathbf{A}_k^0(0)\hat{L}^2\mathbf{A}_k^0(0)\rangle}{|\mathbf{A}_k^0(0)|^2}, \tag{2.39}$$

and angular brackets indicate a scalar product of the new vectors of state. Function $M_1(t-j\tau)$ in the right side of Eq. (2.38) represents the modified memory function (MF) of the first order

$$M_1(t-j\tau)=\frac{\gamma_1(t-j\tau)}{\gamma_1(t)}m_1(t-j\tau). \tag{2.40}$$

For stationary processes the function $\gamma_1(t)$ turns to unit then the memory functions $M_1(t)$ and $m_1(t)$ coincide with each other. The latter equation is the first kinetic finite-difference equation for TCF. It is remarkable that the non-Markovity, discretization, and nonstationarity of stochastic process can be considered here explicitly. Due to the account of nonstationarity both in TCF and in the first memory function this equation generalizes our results obtained recently in Ref. [29]. We introduced the following designations for functions in Eqs. (2.38) and (2.40):

$$m_1(t-j\tau)=\frac{\langle\mathbf{W}_1(0)\hat{V}(T+m\tau,T+j)\tau\mathbf{W}_1(j\tau)\rangle}{|\mathbf{W}_1(0)||\mathbf{W}_1(t-j\tau)|}, \tag{2.41}$$

$$\gamma_1(t-j\tau)=\frac{|\mathbf{W}_1(t-j\tau)|}{|\mathbf{W}_1(0)|}, \qquad (2.42)$$

$$\mathbf{W}_1(t-j\tau)=\hat{V}(T+t,T+j\tau)\hat{L}_{21}\mathbf{A}_k^0(0). \qquad (2.43)$$

The modified evolution operator in Eq. (2.42) has property $\hat{V}(T+t,T+t)=1$. It should be mentioned that a new dynamic parameter of nonstationarity $\gamma_1(t-j\tau)$, Eq. (2.42) appears in the first MF $M_1(t-j\tau)$. On the one hand, as indicated by Eq. (2.41), short MF $m_1(t-j\tau)$ represents an ordinary memory function of the first order, normalized with the view of the property of nonstationarity.

It is important to mark that the memory function $m_1(j\tau)$ is a normalized TCF for a new random variable $\mathbf{W}_1$,

$$\mathbf{W}_1(t)=\hat{V}(T+t,T)\hat{L}_{21}\mathbf{A}_k^0(0). \qquad (2.44)$$

Now contracted Liouville's quasioperator

$$L^{(1)}=L_{22}=P\hat{L}P \qquad (2.45)$$

determines the time evolution. Owing to the discreteness of time series and Eqs. (2.17) and (2.18) the dimension $(k-1)$ of new state vector $\mathbf{W}_1$ is a unit less than dimension (k) of initial vector $\mathbf{W}_0=\mathbf{A}_k^0$. It should be taken into account in numerical calculations. Now let us write in an obvious form an equation of motion for $\mathbf{W}_1(j\tau)$ with regard to Eqs. (2.15)–(2.18), (2.29), and (2.30)

$$\frac{\Delta\mathbf{W}_1(t)}{\Delta t}=\frac{\mathbf{W}_1(t+\tau)-\mathbf{W}_1(t)}{\tau}=i\hat{L}_{22}\mathbf{W}_1(t)=i\hat{L}^1\mathbf{W}_1(t). \qquad (2.46)$$

For a new random dynamic variable $\mathbf{W}_1(t)$ it is possible to repeat all the above-mentioned arguments, which we have used at finding the kinetic equation (2.38). Then it is possible to find the second equation for the short normalized memory function $m_1(j\tau)$.

However, it is more convenient to use the Gram-Schmidt orthogonalization procedure [32,33] for the set of new dynamical orthogonal variables

$$\langle\mathbf{W}_n,\mathbf{W}_m\rangle=\delta_{n,m}\langle|\mathbf{W}_n|^2\rangle, \qquad (2.47)$$

where $\delta_{n,m}$ is Kronecker's symbol. It is easy to find the recurrence formula for the orthogonal variable of different orders $(n\geqslant 2)$

$$\mathbf{W}_0=\mathbf{A}_k^0(0),\quad \mathbf{W}_1=\{i\hat{L}-\lambda_1\}?\mathbf{W}_0, \qquad (2.48)$$

$$\mathbf{W}_2=\{i\hat{L}-\lambda_2\}\mathbf{W}_1-\Lambda_1\mathbf{W}_0,$$

$$\mathbf{W}_3=\{i\hat{L}-\lambda_3\}\mathbf{W}_2-\Lambda_2\mathbf{W}_1-\nu_1\mathbf{W}_0,$$

$$\mathbf{W}_4=\{i\hat{L}-\lambda_4\}\mathbf{W}_3-\Lambda_3\mathbf{W}_2-\nu_2\mathbf{W}_1-\mu_1\mathbf{W}_0,$$

$$\mathbf{W}_{n+1}=\{i\hat{L}-\lambda_{n+1}\}\mathbf{W}_n-\Lambda_n\mathbf{W}_{n-1}-\nu_{n-1}\mathbf{W}_{n-2}-\mu_{n-2}\mathbf{W}_{n-3}+\cdots,$$

with notations for eigenvalues λ_n and relaxation parameters $\Lambda_n,\nu_{n-1},\mu_{n-2},\ldots$ of Liouville's quasioperator

$$\lambda_{n+1}=i\frac{\langle\mathbf{W}_n\hat{L}\mathbf{W}_n\rangle}{|\mathbf{W}_n|^2};\quad \Lambda_n=i\frac{\langle\mathbf{W}_{n-1}\hat{L}\mathbf{W}_n\rangle}{|\mathbf{W}_{n-1}|^2}, \qquad (2.49)$$

$$\nu_{n-1}=i\frac{\langle\mathbf{W}_{n-2}\hat{L}\mathbf{W}_n\rangle}{|\mathbf{W}_{n-2}|^2},$$

$$\mu_{n-2}=i\frac{\langle\mathbf{W}_{n-3}\hat{L}\mathbf{W}_n\rangle}{|\mathbf{W}_{n-3}|^2},\ldots.$$

From Eqs. (2.48) and (2.49) it will be obvious that in the cited Gram-Schmidt procedure from each new vector of state one should subtract the projection on to all previous vectors. Thereafter the orthogonalization (2.47) is complete. In the present form the new vector $\mathbf{W}_n$ would necessarily be perpendicular to the preceding vectors $\mathbf{W}_k$ with $k<n-2$. This problem affects the coupled finite-discrete equations for the memory functions, a central result in this paper.

As the initial stochastic process $\mathbf{W}_0(t)=\mathbf{A}(t)$ is nonstationary, all subsequent orthogonal dynamic variables $\mathbf{W}_n(t)$ [see, Eq. (2.44)] also describe nonstationary process. It is necessary to take into account that in this case all eigenspectrum values do not disappear, $\lambda_n\neq 0$. It is important to note that relaxation parameters Λ_n, ν_n, $\mu_n,\ldots$ will reflect nonequilibrium and nonstationary properties of the system considered. It is interesting to note that in the statistical theory Zwanzig and Mori a relaxation parameter Λ_n can be only positive. Therefore, unlike the case of physical systems, numerical values of parameters $\Lambda_n,\nu_{n-1},\mu_{n-2},\ldots$ can be both positive and negative in case of complex systems.

As noted above, by the simple, but cumbersome calculations it is possible to show that the first short memory function $m_1(t)$ represents a normalized TCF of the first dynamic variable W_1,

$$m_1(t)=\langle\mathbf{n}_w(0)\mathbf{n}_w(t)\rangle=\frac{\langle\mathbf{W}_1(0)\mathbf{W}_1(t)\rangle}{|\mathbf{W}_1(0)||\mathbf{W}_1(t)|},$$

$$\mathbf{W}_1(t)=\hat{V}(T+t,T)\mathbf{W}_1(0). \qquad (2.50)$$

Here $\mathbf{n}_w(0)$ and $\mathbf{n}_w(t)$ are the unit vectors in Euclidean space $W_1[\mathbf{W}_1(0),\mathbf{W}_1(t)\in W_1]$ of the new orthogonal vectors of the state with dimension equal $(k-1)$.

Following Eqs. (2.19)–(2.24) it is possible to introduce the sequence of projection operators Π_n in sequence of Euclidean subspaces $W_n(W_n(0),\mathbf{W}_n(t)\in\mathbf{W}_n)$ with $n\geqslant 1$

$$\Pi_n\mathbf{W}_n(t)=\frac{\mathbf{W}_n(0)\rangle\langle\mathbf{W}_n(0)\mathbf{W}_n(t)\rangle}{|\mathbf{W}_n(0)||\mathbf{W}_n|}=\mathbf{W}_n(0)m_n(t)\gamma_n(t),$$

$$\gamma_n(t)=\frac{|\mathbf{W}_n(t)|}{|\mathbf{W}_n(0)|}. \qquad (2.51)$$

Alongside a set of projectors Π_n it is possible to introduce a set of mutually supplementary projectors P_n,

$$P_n=1-\Pi_n,\quad P_n\Pi_n=\Pi_nP_n=0,$$
$$\Pi_n\Pi_m=\delta_{n,m}\Pi_n,\quad P_nP_m=\delta_{m,n}P_n. \tag{2.52}$$

Each pair of the projection operators Π_n, P_n splits the appropriate Euclidean space W_n of vectors of state $\mathbf{W}_n$, $\mathbf{W}_n(t)\in W_n$ into the two mutually supplementary subspaces

$$W_n=W_n'\dot{+}W_n'',\quad W_n'=\Pi_nW_{n,},\quad W_n''=P_nW_n. \tag{2.53}$$

Now we derive the discrete equation of motion of variable $\mathbf{W}_n(t)$,

$$\frac{\Delta\mathbf{W}_n(t)}{\Delta t}=\frac{1}{\tau}\{\mathbf{W}_n(t+\tau)-\mathbf{W}_n(t)\}=\frac{1}{\tau}\{\hat{V}^{(n)}(\tau)-1\}\mathbf{W}_n(t)$$
$$=i\hat{L}^{(n)}\mathbf{W}_n(t). \tag{2.54}$$

Here $L^{(n)}$ is a new Liouville's quasioperator

$$\hat{L}^{(n)}=\hat{L}_{22}^{(n)}=(i\tau)^{-1}\{\hat{V}^{(n)}(\tau)-1\}=P_n\hat{L}_{22}^{(n-1)}P_n. \tag{2.55}$$

Following the projection technique described above, we receive a chain of connected kinetic finite-difference equations of a non-Markov type for normalized short memory functions $m_n(t)$ in Euclidean space of state vectors of dimension $(k-n)$ $(t=m\tau,n\geqslant 1)$

$$\frac{\Delta m_n(t)}{\Delta t}=\lambda_{n+1}m_n(t)-\tau\Lambda_{n+1}\sum_{j=0}^{m-1}m_{n+1}(j\tau)m_n(t-j\tau)$$
$$\times\left\{\frac{\gamma_{n+1}(j\tau)\gamma_{n+1}(t-j\tau)}{\gamma_n(t)}\right\}, \tag{2.56}$$

$$m_{n+1}(t)=\frac{\langle\mathbf{W}_{n+1}(0)\mathbf{W}_{n+1}(t)\rangle}{|\mathbf{W}_{n+1}(0)||\mathbf{W}_{n+1}(t)|},$$
$$\gamma_n(j\tau)=\left\{\frac{|\mathbf{W}_n(j\tau)|}{|\mathbf{W}_n(0)|}\right\}. \tag{2.57}$$

Here $\gamma_n(j\tau)$ is the nth order nonstationarity function.

The set of all memory functions $m_1(t)$, $m_2(t)$, $m_3(t)$,… allows to describe non-Markov processes and statistical memory effects in the considered nonstationary system. For the particular case we receive a more simple form for the set of equations (2.47) for the first three short memory functions $(t=m\tau)$

$$\frac{\Delta a(t)}{\Delta t}=-\tau\Lambda_1\sum_{j=0}^{m-1}m_1(j\tau)\left\{\frac{\gamma_1(j\tau)\gamma_1(t-j\tau)}{\gamma_1(t)}\right\}a(t-j\tau)$$
$$+\lambda_1a(t),$$

$$\frac{\Delta m_1(t)}{\Delta t}=-\tau\Lambda_2\sum_{j=0}^{m-1}m_2(j\tau)\left\{\frac{\gamma_2(j\tau)\gamma_2(t-j\tau)}{\gamma_2(t)}\right\}m_1(t-j\tau)$$
$$+\lambda_2m_1(t),$$

$$\frac{\Delta m_2(t)}{\Delta t}=-\tau\Lambda_3\sum_{j=0}^{m-1}m_3(j\tau)\left\{\frac{\gamma_3(j\tau)\gamma_3(t-j\tau)}{\gamma_3(t)}\right\}m_2(t-j\tau)$$
$$+\lambda_3m_2(t). \tag{2.58}$$

Here relaxation parameters Λ_1, Λ_2, and Λ_3 are determined by Eq. (2.49), and the nonstationarity functions $\gamma_n(t)$ are introduced in Eqs. (2.42) and (2.57). Now by analogy with Eq. (1.6) we can introduce a set of dynamic parameters of nonstationarity (PNS) for the arbitrary nth relaxation level

$$\Gamma_n(T,t)=1-\gamma_n(t)=1-\gamma_n(T,t). \tag{2.59}$$

The whole set of values of dynamic PNS $\gamma_n(t)$ determines the broad spectrum of nonstationarity effects of the considered process.

The obtained equations are very similar to the well-known Zwanzig'-Mori's kinetic equations [30,31] in the nonequilibrium statistical physics of condensed matter. Let us mark three essential distinctions of our Eqs. (2.56) and (2.58) from the results of Refs. [30,31]. In Zwanzig'-Mori's theory the key moment in the analysis of considered physical systems is the presence of a Hamiltonian and an operation of a statistical averaging carried out with the help of the quantum density operator or the classic Gibbs distribution function. In the examined case both Hamiltonian and distribution functions are absent. In physics exact classic or quantum equations of motion exist, therefore, Liouville's equation and Liouville's operator are useful in many applications. The motion of both individual particles and the whole statistic system is described by states with smooth time. Therefore, for physical systems it is possible to use effectively the methods of integrodifferential calculus, based on mathematically habitual (but from the physical point of view difficult for understanding) representation of infinitesimal variations of values of coordinates and time. By nature the majority of complex systems is discrete. As is well known, discretization is inherent in a wide variety of both classic and quantum complex systems. It compels us to reject the concept of infinite small values and continuity and turn to the discrete-difference schemes. And, at last, the third feature is connected with incorporating the nonstationary processes into our theory. The Zwanzig'-Mori's theory is true only for stationary processes. Due to the introduction of normalized vectors of states and the use of the appropriate projection technique our theory allows to take into account nonstationary processes. The last ones can be described by the non-Markov kinetic equations together with the introduction of the set of nonstationarity functions.

The nonstationary theory advanced here, essentially differs from stationary case [29]. The external structure of the kinetic equations remains constant. As well as earlier, they represent the kinetic equations with memory. However, the functions and the parameters, which are included in these equations, differ appreciably from each other. As we have already marked above, nonstationarity effects are shown both in functions $\gamma_n(t)$, and in spectral and kinetic parameters. Therefore, it appears possible to carry the careful account of nonstationarity effects in complex systems.

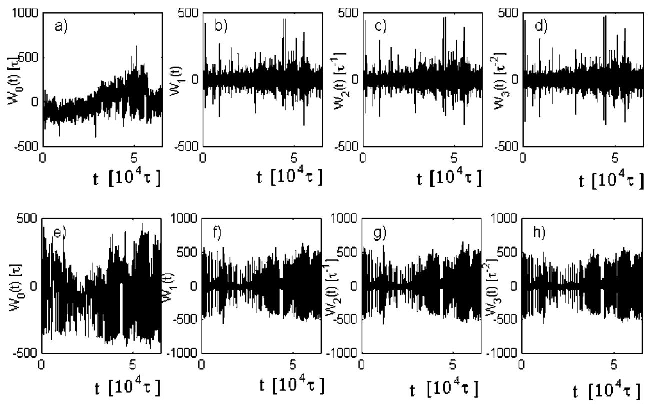

FIG. 1. The time record of the four first orthogonal variables W_0, W_1, W_2, and W_3 of healthy person, Kshf. (a)–(d), and a patient, Sibg., after MI (e)–(h) from the time dynamics of *RR* intervals of the ECG's. In both cases the disappearance of trends is appreciable. The scales of fluctuations of the orthogonal variables practically do not change.

III. EXPERIMENTAL DATA AND PREPROCESSING

A. Data

Dynamic ECG recording has been done on three channels. The bipolar orthogonal channel X is channel $N1$, Y is channel $N2$, and Z is channel $N3$. Our analysis is executed on the channel $N1$. The *RR* recordings were drawn from the Division of Cardiac Surgery of 6th Kazan city Hospital (Kazan, Tatarstan) congestive heart-failure database comprising 30 records from normal patients (age: 18–31 years; mean: 22 year) and 14 records from severe congestive heart-disease patients (age: 32–67 years, mean: 55 years). The recordings, which form a standard database for evaluating the merits of various measures for identification of heart disease, were made with a standard Holter Monitor (Astrocard Holter system, $2F$), digitized at a fixed value of 250 Hz. We use the long-time series to $2^{16}=65\,536$ beats to eliminate spurious effects due to variations in data to nonsinus beats associated with artifacts.

B. Patients

In this preliminary study, we have included a sample of patients subdivided into two groups. The first group consists of 30 healthy persons. In the second group there are 14 patients after MI with weak electrical risk (arrhythmias of low degree).

C. Traditional analysis

These techniques can be divided into time and frequency domains. In the time domain we have calculated the following standard functions: the phase portrait in plane projections of the multidimensional space of the dynamic orthogonal variables, the time correlation function throughout the observed time domain, and the set of the first three junior memories, and the first four nonstationarity functions. Also we have determined the following power spectra (PS): of the initial TCF's first, second, and third short MF's, PS of the first, second, and third points of the statistical spectrum of non-Markovity parameter and PS of the first four nonstationarity parameters.

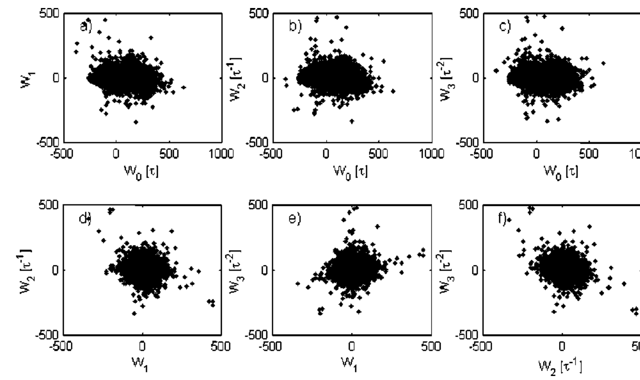

FIG. 2. Phase portraits of *RR* intervals dynamics from human ECG's in a plane of the two various orthogonal variables (W_i, W_j) for the healthy person (Kshf.), (a)–(f). In cases (a), (d), and (c) the phase clouds are stretched along a W_0 axis and they look similar to a pancake. In three other cases (d)–(f) they become more and more symmetrical and acquire spherical forms.

IV. THE QUANTITATIVE ANALYSIS OF LONG-RANGE MEMORY EFFECTS OF LONG-TIME DYNAMICS OF HUMAN ECG'S *RR* INTERVALS

In this section we shall present some results of the quantitative analysis of random dynamics of *RR* intervals of a healthy person and a patient after a MI ECG's within the framework of the theory developed in Sec. II. Figures 1–6 present typical examples of phase portraits, the power spectra of TCF and junior short memory functions and the frequency spectra of the first three points of a statistical spectrum of non-Markovity parameter. Figures 7 and 8 demonstrate the results of calculations of the statistical parameter of nonstationarity and its power spectrum for long-time series of *RR* intervals for the ECG's of healthy persons and patients after MI. In Figs. 1(a)–1(h) the representative time records of *RR* intervals of the ECG's of a healthy person

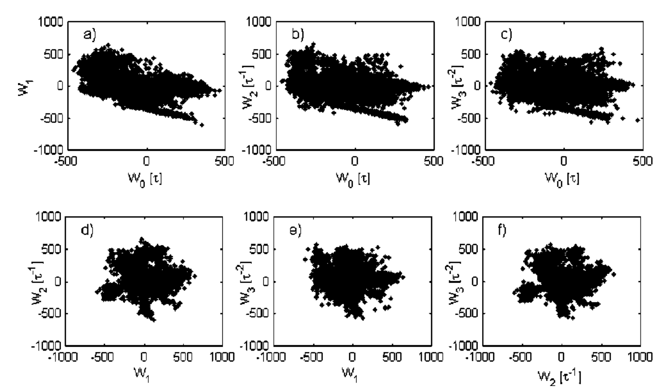

FIG. 3. Phase portraits of *RR* intervals dynamics from human ECG's in a plane of two various orthogonal variables (W_i, W_j) for patient Sibg. on the 20th day after M, (a)–(f). In all six plane projections there is a strong stratification of the phase clouds. Thus there are the shoots similar to the legs of an octopus. The similar stratification can serve as a serious indicator of MI.

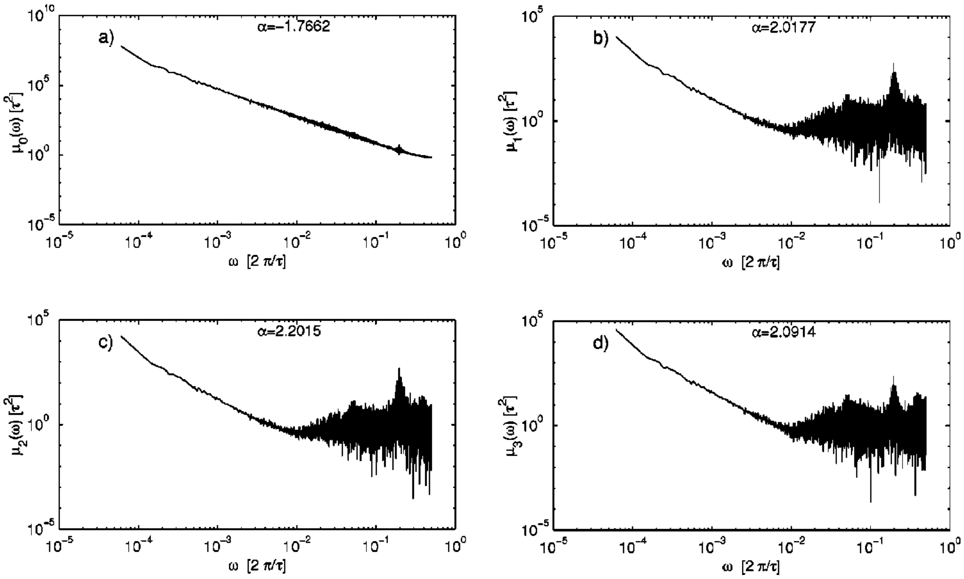

FIG. 4. (a)–(d) Power spectra $\mu_i(\omega)$, $i=0,1,2,3$ for the healthy person (Kshf.) from time dynamics of *RR* intervals of human ECG's in a double-log scale. In the spectrum of initial TCF self-organized criticality is observed. With the growth of order of the memory function there is a certain reduction of the linear (fractal) site. Besides, a high-frequency peak appears as a reflection of respiratory arrhythmia.

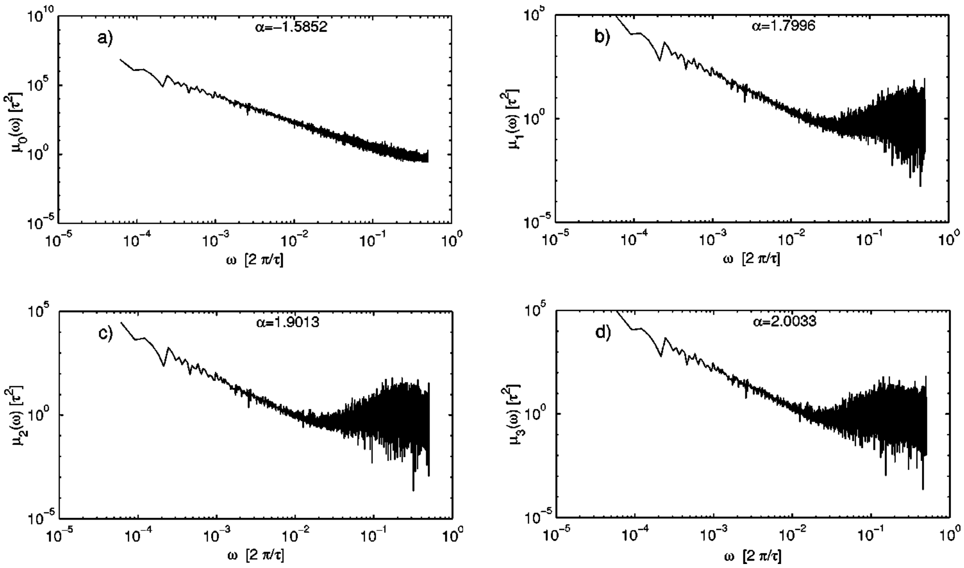

FIG. 5. (a)–(d) Power spectra $\mu_i(\omega)$, $i=0,1,2,3$ for the first four junior memory functions for the patient (Sibg.) after MI from the time dynamics of *RR* intervals of human ECG's in a double-log scale. All spectra have linear sites with fractal frequency dependence and there are no high-frequency peaks connected with respiratory arrhythmia. As a rule, fractal exponents are smaller here than in the case of a healthy person.

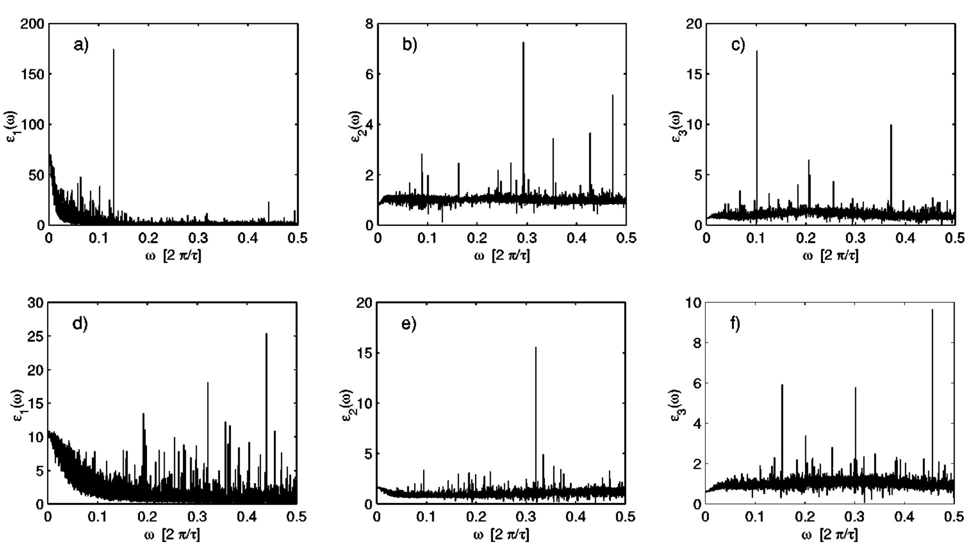

FIG. 6. The frequency dependence of the first three points of non-Markovity parameter for the healthy person (Kshf.) (a)–(c) and patient (Sibg.) after MI (d)–(f) from the time dynamics of *RR* intervals of human ECG's. In the spectrum of the first point of NMP $\epsilon_1(\omega)$ there is an appreciable low-frequency (long-time) component, which concerns the quasi-Markovian processes. Spectra NMP $\epsilon_2(\omega)$ and NMP $\epsilon_3(\omega)$ fully comply with non-Markovian processes within the whole range of frequencies.

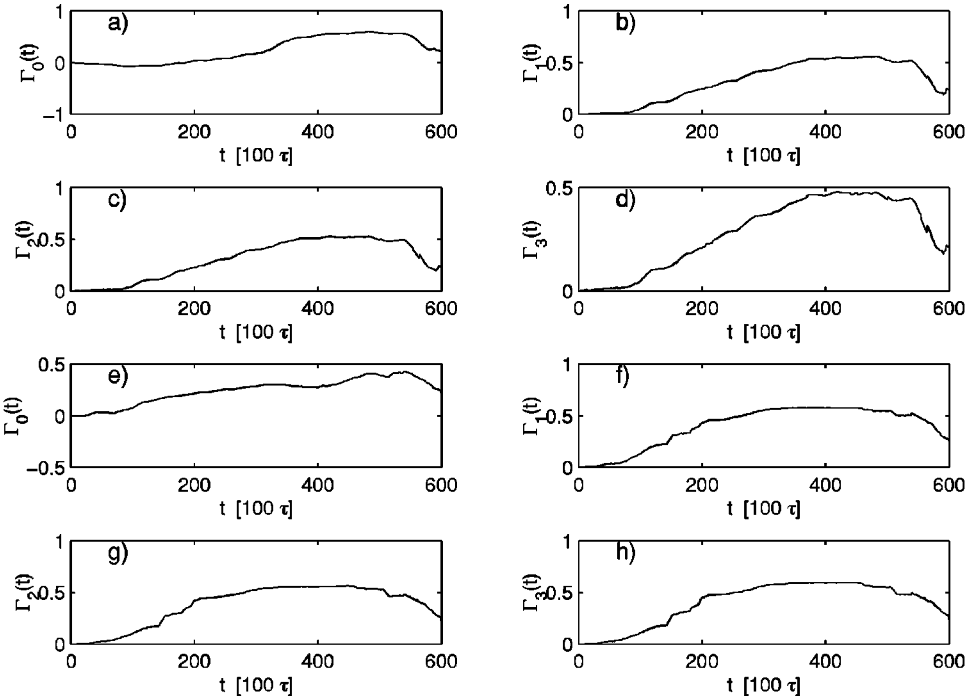

FIG. 7. The time dependence of the first four junior nonstationary dynamical parameters $\gamma_i(t)$, $i=0,1,2,3$ for the healthy person (Kshf.) (a)–(d) and the patient (Sibg.) after MI (e)–(h).

[Figs. 1(a)–1(d)] and a patient after MI [Figs. 1(e)–1(h)] are displayed for comparison. The records are given for the first four orthogonal variables W_0 [Figs. 1(a) and 1(e)], W_1 [Figs. 1(b) and 1(f)], W_2 [Figs. 1(c) and 1(d)], and W_3 [Figs. 1(d) and 1(h)]. Two circumstances have drawn our attention. The initial variable W_0 for the healthy person has a higher value than for the patient after MI. In comparison with the fluctuation W_0, the fluctuation scales W_2 and W_3 grow insignificantly both for the healthy person and for the patient after MI. However, the scale and amplitude of this fluctuation are much higher for the patient after MI than for the healthy person.

In Figs. 2(a)–2(f) and 3(a)–3(f) the phase clouds for the healthy person [Kshf, Figs. 2(a)–2(f)] and the patient after MI [Sibg., Figs. 3(a)–3(f)] in six plane projections (W_i, W_j) of first four orthogonal dynamic variables W_i, $i \neq j = 0,1,2,3$ are shown. In phase portraits of the healthy person [Figs. 2(a)–2(f)] in a plane (W_i, W_j) there is some asymmetry of a phase cloud along variable $W_i (i=1,2,3)$ at value $j=0$. But the projection of a phase cloud in planes (W_i, W_j) with i,j

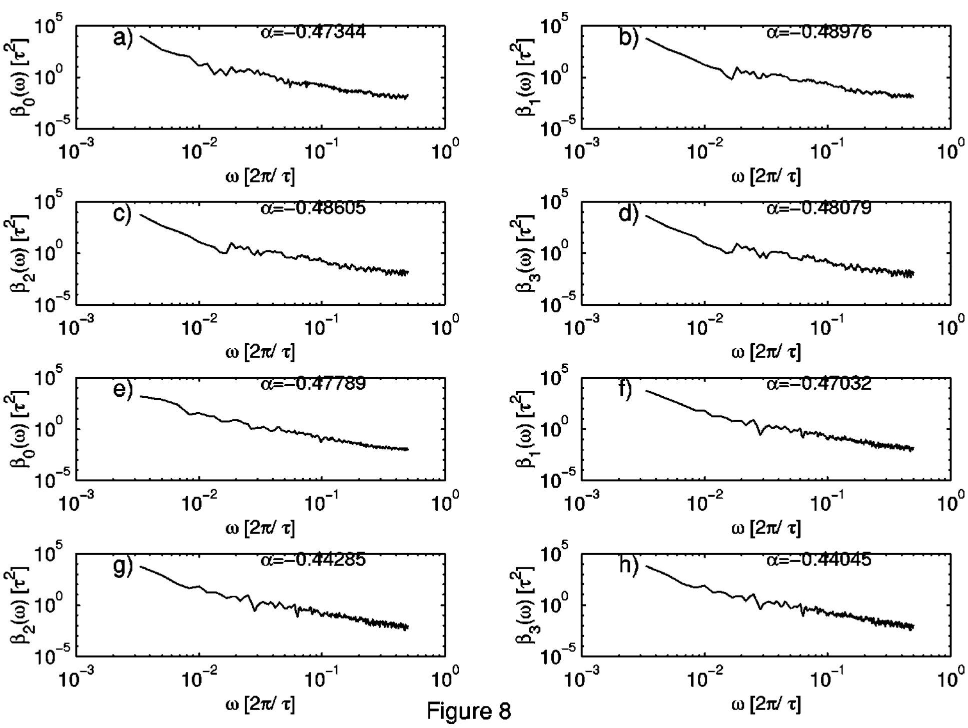

FIG. 8. The frequency dependence of the first four junior nonstationary dynamical parameters $\beta_i(\omega)$, $i=0,1,2,3$ for a healthy person (Kshf.) (a)–(d) and a patient after MI (Sibg) (e)–(h) in a double-log scale. The distinctions in fractal exponents are undistinguished. All spectra are characterized by a fractal frequency dependence. It is possible to notice some reduction of parameters α_2 and α_3 for the patient after MI.

TABLE I. Some kinetic and relaxation parameters in comparison for healthy and patient after MI.

	$\lambda_1(\tau^{-1})$	$\lambda_2(\tau^{-1})$	$\lambda_3(\tau^{-1})$	$\Lambda_1(\tau^{-2})$	$\Lambda_2(\tau^{-2})$
Healthy	−0.0267	−0.907	−0.974	0.005	−0.074
Patient after MI	−0.1676	−1.201	−1.048	0.254	0.156

$=1,2,3$ is characterized by the symmetrical distribution of the phase cloud. In the case of patients after MI some features are evident. The basic feature is a fingerlike scattering of the phase cloud in planes $W_{0,j}$ with numbers $j=1,2,3$. This scattering is so specific, that its occurrence represents the indicator of MI. The next feature is an octopuslike distribution of the phase clouds in the other three planes [see, Figs. 3(d)–3(f)].

In Figs. 4(a)–4(d) the power spectra of TCF $a(t) = m_0(t)$ [Fig. 4(a)], the first [Fig. 4(b)], the second [Fig. 4(c)], and the third [Fig. 4(d)] MF's of the dynamics of *RR* intervals of the ECG for the healthy person (Ksfh., y) are represented. The fractal peculiarities are found for the spectra of all memory functions (the zero (TCF) and the first, the second, and the third orders [Figs. 4(a) and 4(c)]). There appears a frequency dependence such as $\mu_i(\omega) \sim \omega^{-\alpha}$, $i = 0,2$. Fractal behavior exists in full frequency range only for the initial TCF [see, Fig. 4(a)]. The power spectra of the first three junior MF's $\mu_i(\omega)$, $i=1,2,3$, depict the nonfractal behavior in frequency domain $10^{-2} < \omega < 0.5$ f.u., 1 f.u. $=2\pi/\tau$, where the set of peaks is connected with the fast alteration of the first three orthogonal variables W_1, W_2, and W_3, which describe a human cardiovascular system (CVS) state.

Thus, the sudden emergence of a group of high-frequency peaks in the spectrum of the healthy person for functions μ_1, μ_2, $\mu_3(\omega)$ contradicts the standard point of view [7,17,21] and can serve as the proof of latent pathology in human CVS activity.

Let us return again to fractal behavior in Figs. 4(a) and 4(c). The self-similar behavior of spectra $\mu_i(\omega)$ and $\mu_2(\omega)$ for the healthy person is accompanied by a number of effects. The effects of respiratory arrhythmia (RA) are conspicuous in both Figs. 4(b) and 4(d). In a spectrum of the initial TCF [Fig. 4(a)] the influence of RA can be found on a frequency of 0.11 f.u. in the form of a weak spectral splash. In the spectrum of the next short MF's the same influence of RA is appreciably amplified owing to the Syuyumbike Tower effect [34]. The fractal behavior of all spectra is also associated with the phenomenon of the self-organized criticality (SOC) [35,36]. Nevertheless, the lengths of the linear segments in Figs. 4(a) and 4(d) are different. For example, for the initial TCF [see, Fig. 4(a)] criticality exists within the frequency range from 0.5 f.u. up to 5×10^{-4} f.u., and SOC is characteristic for the whole registered frequency area. Vice versa, SOC in the short MF's [see, Figs. 4(c)–4(d)] is seen only in the restricted frequency area from 10^{-2} f.u. up to frequency 5×10^{-4} f.u. As a result the restricted self-organized criticality (RSOC) is significant in the spectra of all short MF's.

The power spectra for patients after MI and for healthy persons differ a little. Fractality and criticality also exist in the spectra of patients after MI, but they have essentially limited character. Criticality is appreciable in the linear region for initial TCF [see, Fig. 5(a)] in the frequency interval from 0.4 f.u. up to frequency 0.9×10^{-4} f.u. and for all short MF's [Figs. 5(b) and 5(c)] in the frequency interval from $2\times10^{-2} < \omega < 6\times10^{-5}$ f.u. The packets of spectral lines appear in the power spectra of short MF's [Figs. 5(b) and 5(d)] in a high-frequency region from 2×10^{-2} f.u. up to 0.5 f.u.

Table I contains some kinetic and relaxation parameters of stochastic dynamics of RR intervals of human ECG's for healthy persons and patients after MI. It is possible to notice some similarity of our kinetic parameter λ_i with well-known Lyapunov's exponents. It is important to note that all λ_i are only negative numbers ($\lambda_i < 0$). Relaxation parameters Λ_i are both positive and negative. Numerical changes of these parameters can appear useful to diagnose CVS diseases. For example, it is visible from Table I that the transition from the healthy person to the patient after MI is accompanied by sharp change of parameter λ_1 (almost 6.28 times) and parameter Λ_1 (almost 50 times).

By analogy with Ref. [29] it is convenient to define the generalized non-Markov parameter for the frequency-dependent case as

$$\epsilon_i(\omega) = \left\{ \frac{\mu_{i-1}(\omega)}{\mu_i(\omega)} \right\}^{1/2}, \tag{4.1}$$

where $i=1,2,\ldots$, and $\mu_i(\omega)$ is the power spectrum of the ith memory function. It is convenient to use this parameter for quantitative description of long-range memory effects in the system considered together with memory functions defined above. The behavior of spectra of the first three points $\varepsilon_i(\omega)$ of the statistical spectrum of NMP for the healthy person [Figs. 6(a)–6(c)] and the patient after MI [Figs. 6(d)–6(f)] is rather informative. Careful analysis of these data shows that the dynamics of *RR* intervals is non-Markovian for the second and the third relaxation levels both for the healthy person and for the patient after MI. As seen from Figs. 6(b),6(c) and 6(e),6(f) the similar behavior arises for the first and the second non-Markovity parameters $\epsilon_2(\omega) \sim 1$ and $\epsilon_3(\omega) \sim 1$, everywhere within the whole frequency region. The behavior of $\epsilon_1(\omega)$ for the patient after MI within the whole frequency region [see, Fig. 6(d)] is typical for

TABLE II. Set of fractal exponents for power spectra of initial TCF and first three memory functions.

	α_0	α_1	α_2	α_3
Healthy	1.7662	2.0177	2.2015	2.0914
Patient after MI	1.5852	1.7996	1.9013	2.0033

non-Markov relaxation scenario. A sharp change of value of parameter $\epsilon_1(0)$ from the healthy person $[\epsilon_1(0)\sim 71.6]$ to the patient after MI $[\epsilon_1(0)\sim 11.0]$ (almost 6.5 times) is valuable for pathologic data sets based on the difference of these non-Markov properties. Careful analysis of Figs. 6 reveals a less prominent non-Markov behavior for the patient after MI rather than for the healthy person. CVS of the healthy person represents the more chaotic system whereas CVS of the patient after MI shows evidence of the more ordered system.

Table II contains a number of fractal exponents for the power spectra of the initial TCF and the first three junior memory functions for the healthy person and patient after MI. As may be seen from these tables crucial differences exist in fractal exponents α_0, α_1, α_2, and α_3 for the healthy person and the patient after MI. They are trustworthy means of distinguishing healthy cases from cardiac diseases.

In Figs. 7 and 8 numerical results of calculation of nonstationarity effects for the healthy person [Figs. 7(a)–7(d), and 8(a)–8(d)] and the patient after MI [Figs. 7(e)–7(h) and 8(e)–8(h)] are displayed. The time behavior of these effects is presented in Figs. 7(a)–7(h) through the time dependence of nonstationarity functions $\gamma_i(t)$. Frequency behavior is shown in Figs. 8(a)–8(h) by the frequency dependence of PS $\nu_i(\omega)$ of functions $\gamma_i(t)$. Figures 7(a)–7(f) convincingly display that according to our classification (1.7) the long-term dynamics of *RR* intervals of human ECG's, both for the healthy person and the patient with cardiac disease, concern the case of intermediate nonstationarity. From Figs. 8(a)–8(h) we can state with assurance that all PS $\nu_i(\omega)$ both for the healthy person and the patient after MI, $i=0,1,2,3$ demonstrate the similar fractal-like behavior with power law dependence $\omega^{-\alpha}$; exponents are in the range $0.47<\alpha<0.49$ as a rule. But the values of exponent α for the patient after MI for the case $i=2$ $(\alpha_2=0.442\,85)$ and $i=3$ $(\alpha_3=0.440\,45)$ are far outside of this range. These values of fractal exponent differ drastically from the similar values for PS for patients after MI and the first points of the statistical spectrum of NMP.

We emphasize especially that frequency spectra introduced above are characterized by a specific alternation of fractal spectra and spectra such as the color noise. In a certain sense the similar alternation reminds the peculiar alternation of effects of a Markov and non-Markov behavior for hydrodynamic systems in statistical physics of condensed systems detected for the first time in papers [37,38]. The fine specificity of such alternation appears essentially diverse for healthy persons and patients after MI. It is important to note that the similar alternation is completely absent in frequency spectrums of non-Markovity for the short-time series of *RR* intervals of human ECG's both for healthy persons and patients of various heart diseases [29].

As the research held by us demonstrates such alternation of non-Markov effects is typical only for long-time (Holter) series of RR intervals of human ECG's. It allows to use the fine points of this behavior for more comprehensive and detailed diagnosis of human CVS diseases.

V. DISCUSSION OF RESULTS AND CONCLUSION

In the present paper we have constructed the kinetic theory of discrete nonstationary non-Markov processes in complex systems of various nature. From the very beginning we developed the theory on the basis of nonstationary TCF. For finding the latter we have taken advantage of the general and exact definition of correlation coefficient of the stochastic processes in the probability theory. The construction of nonstationary TCF allows one to get the linear projection operator acting in a Euclidean space of nonstationary dynamic vectors of states. For the analysis of nonstationary dynamics of stochastic process we have constructed discrete-difference stochastic equation of motion, Liouville's quasioperator and evolution operator in the form of diagonal matrices. We have executed careful investigation of stochastic nonstationary dynamics of multidimensional vectors of initial and final chaotic states. To find the nonstationary TCF we have taken advantage of the technique of projection operators, developed in our previous paper [29]. We have especially updated it here to analyze nonstationary stochastic processes.

Due to splitting of a stochastic Liouville's equation into two mutually supplementary Euclidian subspaces we could receive the chain of connected finite-difference kinetic equations for discrete nonstationary TCF and MF's. Kinetic parameters and discrete functions (TCF and MF's of different orders) in this set of equations can be easily found from experimental time series. It makes possible to apply our theory in the study of the broad class of discrete nonstationary stochastic processes with a long-range memory. It is necessary to mark one more relevant feature of the developed theory. Our theory has certain analogy with the famous Zwanzig-Mori theory in statistical physics. But there are two key differences. First, our results are true for non-Hamilton systems, where there are no Hamiltonian and exact equations of motion. Second, our theory is specially adapted to account for the step-type behavior of the underlying process with discretization time τ. It is easy to notice that our theory contains Zwanzig'-Mori's results as the specific case. For this purpose it is necessary to proceed to a limit $\tau\rightarrow 0$ and to replace the stochastic Liouville's quasioperator on the physical quantum or classical Liouville's operator.

Another relevant result of this paper is the quantification of discrete nonstationary non-Markov stochastic processes in heart rate variability for healthy persons and for patients after MI by memory functions, non-Markovity and nonstationary parameters, and a long-range memory. We have established the existence of a large variety of interesting physical effects in different nonlinear spectra. Among them it is necessary to mark the fractal-like behavior of PS with power frequency law, the phenomena of SOC and RSOC, the spectral behavior of some frequency spectra in the form of white and color noises, the existence of the legibly expressed qualitative and quantitative differences in spectral and kinetic characteristics for healthy persons and patients after MI. Our preliminary study shows that the indicated differences can serve a trustworthy method of diagnosis of the state of cardiovascular systems for healthy persons and patients. The last circumstance is of special value for the results of theory developed here. An interesting feature of the advanced theory is that it justifies not only the change of absolute values, but also the sign of the relaxation parameters. Change of a sign of parameter is unattainable in the standard physical theory of stationary processes.

One of the most interesting experimental results of our study consists in the reliable registration of hydrodynamic effects of alternation of Markov and non-Markov effects in the behavior of power spectra for cardiac time series. The analysis of these terms provides insight into the nature of chaotic dynamics of HRV. All the abovementioned are in good agreement with the basic results of recent publications [39–43]. In particular, in Ref. [39] is offered an approach for analyzing signals with long-range correlations by decomposing the signal increment series into magnitude and sign series and analyzing their scaling properties. It is well known that many complex systems share statistical characteristics. For instance, in Ref. [40] a turbulence analogy is proposed for the long-term heart rate variability of healthy humans. In Ref. [41] it was revealed that when fluctuation in physical activity and other behavioral modifiers are minimized, a remarkable level of complexity of heart beat dynamics remains, while for neuroautonomic blockage the multifractal complexity decreases. Introducing a model of competitive population dynamics of biological species with clock dynamics incorporated, Daido [42] has shown that periods equal or close to that of the environment do not always guarantee overwhelming superiority and can even lead to extinction. Stanley and co-workers [43] have analyzed a complex rhythm of heart beats for patients at high risk for sudden cardiac death. They have shown that the rhythm can be described by a theoretical model consisting of two interacting oscillators with stochastic elements. Bunde *et al.* [44] have studied the heart rhythm in the different sleep stages [deep, light, and rapid eye movement sleep] that reflect different brain activities for both healthy subjects and patients with moderate sleep disorder. They have found a sleep phase finder that is based on the different heart rhythm in the different sleep stages, supplementing the quite tedious evaluation of the sleep phases by the standard electrophysiological procedures.

Thus, our observation suggests strongly that the fractal frequency behavior is one of the basic properties of the human cardiovascular system. From our standpoint the fundamental property of a human heart consists in the specific alternation of Markov and non-Markov memory effects. It is quite probable that the last conclusion is the key moment in understanding the physics of alive systems.

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