Modeling share returns - an empirical study on the Variance Gamma model

Andreas W. Rathgeber · Johannes Stadler · Stefan Stöckl

Abstract Due to the fact that there has been only little research on some essential issues of the Variance Gamma (VG) process, we have recognized a gap in literature as to the performance of the various estimation methods for modeling empirical share returns. While some papers present only few estimated parameters for a very small, selected empirical database, Finlay and Seneta (Int Stat Rev 76:167–186, 2008) compare most of the possible estimation methods using simulated data. In contrast to Finlay and Seneta (2008) we utilize a broad, daily, and empirical data set consisting of the stocks of each company listed on the DOW JONES over the period from 1991 to 2011. We also apply a regime switching model in order to identify normal and turbulent times within our data set and fit the VG process to the data in the respective period. We find out that the VG process parameters vary over time, and in accordance with the regime switching model, we recognize significantly increasing fitting rates which are due to the chosen periods.

Keywords Variance Gamma model · Parameter estimation methods · Regime switching model · Empirical data

JEL Classification G15

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1 Introduction

Distribution assumptions and jump discontinuities play an important role in modeling share returns. Jumps can generate fat tails, and in doing so they can influence the excess skewness and kurtosis. Fama (1965) studies the distribution of daily returns on the Dow Jones Industrial Average Index (DOW JONES) over the period from 1957 to 1962. In his analysis, he includes the stocks of the thirty companies listed on the DOW JONES, rejects the normality assumption and suggests that non-normal distribution assumptions would probably fit more accurately. However, he is not able to find a Paretian stable distribution (at least one finite mean and one finite variance according to Mandelbrot (1963)) for his non-normal distribution assumptions. Based on these results Officer (1972) or Hsu et al. (1974) - amongst others - demand better modeling approaches for non-stationary share returns as their attempts to estimate Paretian stable distributions failed. Praetz (1972) was the first trying to improve share return modeling by means of an inverse gamma distribution which implies a rescaled Student t-distribution for returns. His results are superior to Fama's (1965) and are regarded as the basis for all future stochastic variance models.

Since Madan and Seneta (1987) took the results of Praetz (1972) and published the first symmetric version of the Variance Gamma (VG) process with mean zero, there has been some progress in developing the VG process, a Lévy process (other known Lévy processes are for example the hyperbolic or normal inverse gaussian process by Barndorff-Nielsen (1977, 1995) or Eberlein and Keller (1995)), as alternatives to the common Brownian Motion model for stock market returns. There exist two major parts in literature about the VG process. First, the univariate case, in which Madan and Seneta (1990) extend the Black-Scholes model by applying the VG process within the pricing framework to the Variance Gamma option pricing model. Madan et al. (1998) conclude that Variance Gamma option pricing reduces the pricing bias - in contrast to the Black-Scholes model - as the VG process covers the excess kurtosis, which is a result of jumps. Daal and Madan (2005) use this new idea for an empirical examination of the Variance Gamma option pricing model, the traditional Black-Scholes model and Merton's (1976) jump diffusion model for foreign currency options. They reaffirm Madan and Seneta's (1990) findings that the Variance Gamma option pricing model performs better than the others do. For further applications of the VG process see for example Leicht and Rathgeber (2014). Furthermore, there exist several variations of the VG process, like the CMGY process by Carr et al. (2002). Second, the multivariate case, which cares about the integration of correlations and therefore of the dependence between the Lévy processes. For an overview see for example Luciano and Schoutens (2006), Luciano and Sameraro (2008), Semeraro (2008) or Luciano et al. (2014). Summed up, the VG process, like the other Lévy processes, offers lots of possibilities in asset pricing, risk modeling by reducing pricing errors or model miscalibrations. They help to incorporate jumps, map a more realistic market behaviour in comparison to traditional models and therefore are important instruments in the field of financial economics.

In contrast to the two-parametric Brownian Motion the VG process mentioned above is a four-parametric stochastic process. Therefore, these two methods used for stock market modeling differ widely as the VG process captures the skewness and kurtosis in addition to the mean and standard deviation. The VG process considers both, the symmetric increase in the left and right tail probabilities of the return distribution (kurtosis) and the asymmetry of the left and right tails of the return density (skewness). These properties allow for a more accurate representation of stock returns. The VG process parameters (for the univariate as well as most parts for the multivariate case) can be obtained by the application of several methods, such as the simplified method of moments, the method of moments, the maximum likelihood estimation, the empirical characteristic function, the Bayesian inference and Markov chain Monte Carlo method, or the minimum χ^2 method. For an overview see Madan and Seneta (1987), Madan and Seneta (1990), Seneta (2004) and particularly Finlay and Seneta (2008).

However, there has been only little research on some essential issues of the VG process, so far. We have recognized a gap in literature as to the performance of the various estimation methods for modeling empirical share returns. While some papers present only few estimated parameters for a very small, selected empirical database, Finlay and Seneta (2008) compare most of the possible estimation methods using simulated data. In contrast to Finlay and Seneta (2008), we utilize a broad, daily, and empirical data set consisting of the stocks of each of the companies listed on the DOW JONES over the period from 1991 to 2011. Additionally, the calibration quality as well as the parameters' range of the VG process are dependent on time. This means that parameters vary in different market phases. As market participants are exposed to varying situations, the selection of the correctly fitted model is an essential element of their work. This leads us to apply a regime switching model in order to identify normal and turbulent regimes within our data set and to fit the VG process to the data in the respective period. This approach has two major advantages. First, it results in a more accurate parameter estimation, which avoids over- or underestimation of the real VG process. Second, the fitting rate - the fact that the returns follow a VG process - increases significantly. Thus, the use of the regime switching model adds new knowledge to the framework of the VG process.

The remainder of this paper is structured as follows. The next section provides an overview of the theoretical background of the VG process itself, the several estimation methods for the VG process parameters, and our hypotheses. Section 3 introduces the research design including the data set and the methodology of differentiating between normal and turbulent times in financial markets. Section 4 presents the results of the VG process parameter estimation. Subsequent to the discussion of the results in Section 5, Section 6 concludes the paper.

2 Theoretical background

This section presents the framework for modeling risky assets by means of a VG process and provides a brief overview of the estimation methods for the VG process parameters. We define the price of a risky asset S_t at point in time t ($t \geq 0$) with the following traditional model

$$S_t = S_0 e^{r_t}, \tag{1}$$

where $r_t = \ln \frac{S_t}{S_0}$ is the compounded log return from $t = 0$ to t. r_t follows the stochastic process $r_t = ct + \theta T_t + \sigma W(T_t)$, where c, θ and $\sigma(>0)$ are real constants as defined by Seneta (2004). $W(\cdot)$ represents the traditional Brownian Motion. Therefore, this approach is also known as the VG process as a modified Brownian Motion. Besides, Schoutens (2003) and Cont and Tankov (2004) describe the VG process as the difference between two independent gamma processes. Luciano and Schoutens (2006) take these results and define a Lévy Triplet by means of the difference between two gamma processes. Tjetjep and Seneta (2006) model the VG process as a normal-variance-mean-mixture-model. Seneta (2004) models the market activity time $(T_t)_{t\geq 0}$ as a positive, monotonically non-decreasing random process with stationary increments $\tau_t = T_t - T_{t-1}$ $(t \geq 1)$ and $T_0 = 0$ (almost surely). It is assumed that the expected duration of an increment is $E(\tau_t) = 1 \; \forall \, t \geq 0$. This simplification allows normalizing the expected economically relevant measure to 1. Seneta (2004) and Finlay and Seneta (2008) - amongst others - use the corresponding increments X_t of S_t instead of the compounded return r_t for fitting the VG process. This means that $X_t = c + \theta(T_t - T_{t-1}) + \sigma(W(T_t) - W(T_{t-1}))$ and from the distribution properties of $W(\cdot)$, $\sigma(W(T_t) - W(T_{t-1})) \overset{D}{=} \sigma\sqrt{T_t - T_{t-1}}W(1)$, the relevant formula for X_t can be derived

$$X_t \overset{D}{=} c + \theta\tau_t + \sigma\sqrt{\tau_t}W(1). \tag{2}$$

Both, Seneta (2004) and Finlay and Seneta (2006) model τ_t with a gamma distribution[1] $\gamma\left(\frac{1}{\nu}, \frac{1}{\nu}\right)$ with $\nu > 0$ and a gamma function $\Gamma\left(\frac{1}{\nu}\right)$ embedded in the gamma probability density function (PDF)

$$f_\tau(w) = \begin{cases} \frac{1}{\nu^{\frac{1}{\nu}}} \frac{w^{\frac{1}{\nu}-1} e^{\frac{-w}{\nu}}}{\Gamma(\frac{1}{\nu})} & w, \nu > 0 \\ 0 & \text{else} \end{cases} \tag{3}$$

which fulfills the requirements that $E(\tau_t) = 1$ and $Var(\tau_t) = \nu$. Before Madan and Seneta (1990) introduced the VG process in 1990, there had not existed a closed form for the PDF of the VG process. They only identified a closed form for the characteristic function. By means of a Bessel function K it is possible to close this research gap (see Cont and Tankov 2004) and to define $f_X(x)$ as the PDF of the VG process

$$f_X(x) = \frac{2}{\sigma\sqrt{2\pi}\nu^{\frac{1}{\nu}}\Gamma\left(\frac{1}{\nu}\right)} e^{\theta\frac{x-c}{\sigma^2}} \left(\frac{|x-c|}{\sqrt{\frac{2\sigma^2}{\nu} + \theta^2}}\right)^{\frac{1}{\nu}-\frac{1}{2}} K_{\frac{1}{\nu}-\frac{1}{2}}\left(\frac{|x-c|\sqrt{\frac{2\sigma^2}{\nu}+\theta^2}}{\sigma^2}\right), \tag{4}$$

with θ, σ, ν and c representing the VG process parameters, according to Seneta (2004). As suggested for example by both, Madan and Seneta (1990) and Cont

[1] In general, a $\gamma(b, p)$ distribution has $f(x) = \frac{b^p}{\Gamma(p)} x^{p-1} e^{-bx} \; \forall \, b, p, x > 0$ as PDF.

and Tankov (2004), we apply the Bessel function of the second kind. Furthermore, the characteristic function $\phi_X(x) = E(e^{iux})$ for the VG process can be generated by means of the PDF (see formula (4)) and Madan et al. (1998) define it as

$$\phi_X(u) = e^{icu}\left(1 - i\theta\nu u + \frac{\sigma^2\nu u^2}{2}\right)^{-\frac{1}{\nu}}, \tag{5}$$

with $-\infty < u < \infty$ and $i = \sqrt{-1}$. On the basis of these properties the four moments of the VG process can be calculated. By using the moment generating function $m_\tau(x) = E(e^{ux})$ for the time increments τ_t, $m_\tau(u) = (1 - \nu u)^{-\frac{1}{\nu}}$, the i-th moment $E[(\tau_t - 0)^i]$, with $i = 1, \ldots, 4$, about zero, the i-th central moment $M_i = E[(\tau_t - 1)^i]$, with $i = 1, \ldots, 4$, we get $E[(X_t - E(X_t))^2] = c + \theta$, $E[(X_t - E(X_t))^2] = \sigma^2 + \theta^2 M_2$, $E[(X_t - E(X_t))^3] = 3\theta\sigma^2 M_2 + \theta^3$ and $E[(X_t - E(X_t))^4] = 3\sigma^4(1 + M_2) + 6\sigma^2\theta^2(M_2 + M_3) + \theta M_4$. Finally, we present the four moments $\tilde{M}_i$, with $i = 1, \ldots, 4$, of the VG process mean (μ), variance (σ^2), skewness (β) and kurtosis (κ) depending on the parameter set $\eta = (\sigma, \theta, \nu, c)$

$$\tilde{M}_1 := \hat{\mu} = c + \theta \tag{6}$$

$$\tilde{M}_2 := \hat{\sigma^2} = \sigma^2 + \theta^2\nu \tag{7}$$

$$\tilde{M}_3 := \hat{\beta} = \frac{2\theta^3\nu^2 + 3\sigma^2\theta\nu}{(\sigma^2 + \theta^2\nu)^{\frac{3}{2}}} \tag{8}$$

$$\tilde{M}_4 := \hat{\kappa} = 3 + \frac{3\sigma^4\nu + 12\sigma^2\theta^2\nu^2 + 6\theta^4\nu^2}{(\sigma^2 + \theta^2\nu)^2} \tag{9}$$

Tjetjep and Seneta (2006) calculate some upper and lower bounds for the skewness and the kurtosis. All the bounds depend on the subordinated VG process with its parameter ν. The skewness is embedded in the range $-3\sqrt{\nu} < \beta < 3\sqrt{\nu}$ and the kurtosis is embedded in the range $3\nu+3 < \kappa < 6\nu+3$. For a more detailed derivation of the respective formulas see, for example, Madan and Seneta (1987, 1990) and Seneta (2004).

For fitting the PDF of the VG process and the normalized moments mentioned above, the related literature suggests several approaches. In the subsequent Table 1 we provide a short overview of six possible different VG process estimation methods.

For testing the quality of the parameter estimation methods we use common applied methods such as the Kolomogorov-Smirnov (KS) test (see Massey (1951)), the χ^2 test (see Finlay and Seneta (2008)) and particularly the Anderson-Darling test (see Anderson and Darling (1952)) for taking a special look at the tails of the distributions.

Having described the theoretical background, we can develop some hypotheses. All hypotheses derive from the characteristic behavior of the stock markets and the estimation methods of the VG process parameters. So far, Finlay and Seneta (2008) have been the only authors who conducted a broad comparison of the estimation methods of the VG process. Their findings are mainly based on a simulated VG

Table 1 Overview estimation methods

Method	Formula	Description
method of moments (MOM)	$\begin{aligned}(\sigma^*, \theta^*, \nu^*)_{MOM} &= \underset{\sigma,\theta,\nu}{\operatorname{argmin}} \sum_{i=2}^{4} \left(\frac{M_i^E - \tilde{M}_i(\sigma,\theta,\nu)}{M_i^E} \right)^2 \\ c^* &= M_1^E - \theta^*\end{aligned}$	<u>Idea:</u> We use the modified least-square method and minimize the relative deviation between the i-th sample moment M_i^E calculated from the empirical data set and the i-th moment $\tilde{M}_i$, with $i = 2, \ldots, 4$. As the parameter c, only has an effect on $\tilde{M}_1$, it could be separately calculated. <u>Literature:</u> Tjetjep and Seneta (2006)
simplified method of moments (SMOM)	$\begin{aligned}\sigma^* &= \sqrt{M_2^E} \\ \nu^* &= \frac{M_4^E}{3} - 1 \\ \theta^* &= \frac{M_3^E \sigma^*}{3\nu*} \\ c^* &= M_1^E - \theta^*\end{aligned}$	<u>Idea:</u> We assume a symmetric case of the empirical distribution of the log returns and therefore we can approximate $\theta \approx 0$. This restriction implies $\theta^2 = \theta^3 = \theta^4 = 0$ and by means the empirical moments M_i^E, with $i = 1, \ldots, 4$ we obtain the SMOM. <u>Literature:</u> Seneta (2004), Finlay and Seneta (2006)
maximum likelihood estimation (MLE)	$\begin{aligned}\mathcal{LL}(\eta) &= \underset{\sigma,\theta,\nu}{\operatorname{argmax}} \sum_{i=1}^{N} \ln\left(f_X(z \mid \sigma, \theta, \nu)\right) \\ f_X(z) &= \frac{2}{\sigma\sqrt{2\pi}\,\nu^{\frac{1}{\nu}}\,\Gamma(\frac{1}{\nu})} e^{\theta\frac{z-\theta}{\sigma^2}} \left(\frac{\lvert z-\theta\rvert}{\sqrt{\frac{2\sigma^2}{\nu}+\theta^2}} \right)^{\frac{1}{\nu}-\frac{1}{2}} K_{\frac{1}{\nu}-\frac{1}{2}} \left(\frac{\lvert z-\theta\rvert\sqrt{\frac{2\sigma^2}{\nu}+\theta^2}}{\sigma^2} \right) \\ \text{with} \quad z &= X_t - M_1^E \quad \text{and} \quad c^* + \theta^* = \frac{1}{N}\sum_{i=1}^{N} X_i\end{aligned}$	<u>Idea:</u> We use the log-likelihood function $\mathcal{LL}(\eta) = \ln(\mathcal{L}(\eta))$ for the maximization. All the observations of the random VG process are adjusted by the first empirical moment M_1^E in order to get a VG process of the three parameters σ, θ and ν with $f_X(z)$ as PDF. <u>Literature:</u> Efron (1982), Aldrich (1997), Seneta 2004
empirical characteristic function (ECF)	$\begin{aligned}\Phi_{ECF}(\omega) &= \frac{1}{N}\sum_{j=1}^{N} e^{i\omega X_j} \\ \eta^* &= \underset{\eta}{\operatorname{argmin}} \int_{-\infty}^{+\infty} \mid \Phi_{X,\eta}(\omega) - \Phi_{ECF}(\omega) \mid^2 e^{-\omega^2} d\omega\end{aligned}$	<u>Idea:</u> This method matches the characteristic function derived from the VG process model $\Phi_{X,\eta}(\omega)$ with the empirical characteristic function obtained from empirical data. We first calculate an estimator for the empirical characteristic function $\Phi_{ECF}(\omega)$ using the N observed log-returns $X = (X_1, \ldots, X_N)$ where $i = \sqrt{-1}$ and ω are the evaluation points and then set up the optimization problem. <u>Literature:</u> Yu (2004)

Table 1 (continued)

Method	Formula	Description
Bayesian inference and Markov chain Monte Carlo (BI)	$f_X(\eta \mid x) = \frac{\mathcal{L}(\eta) f_X(\eta)}{\int \mathcal{L}(\eta) f_X(\eta) d\eta}$ $\eta^* = \mu(f_X(\eta \mid x))$	Idea: The posterior probability is the probability of the parameter set η which is based on the evidence of $X : f_X(\eta \mid x)$, where $X = (X_1, \ldots, X_N)$ represents the N observed log returns. The likelihood function $\mathcal{L}(\eta)$ describes the probability of the evidence given $f_X(x \mid \eta)$. By assuming a prior probability $f_X(\eta)$, we can use the Bayes theorem and calculate the posterior probability $f_X(\eta \mid x)$. On the basis of this function the mean of the posterior probability η^* is defined. We use WinBUGS for the MCMC simulation as described by Finlay and Seneta (2008) in detail. Literature: Roberts (1965), Finlay and Seneta (2008)
minimum $x^2 (x^2)$	$\tilde{O}_i(\eta) = N \int_{B_{i-1}}^{B_i} f_{X,\eta}(x) dx$ $\eta^* = \underset{\eta}{\operatorname{argmin}} \sum_{i=1}^{I} \frac{\left(O_i - \tilde{O}_i(\eta)\right)^2}{\tilde{O}_i(\eta)}$	Idea: The χ^2 method minimizes the relative difference between the observed and expected numbers of log returns in the determined intervals. The basis are N observed log returns $X = (X_1, \ldots, X_N)$ and the determination of I intervals with a vector $B := (B_i)_{i=0,\ldots,I}$ as right borders of the intervals. The number of the expected observations $\tilde{O}_i(\eta)$ in any interval is dependent on the parameter set η. Finlay and Seneta (2008) suggest that the log returns are divided into 1 % sample quantile bands. Literature: Berkson (1980), Finlay and Seneta (2008)

Note: This table presents the six estimation methods used in order to calibrate the VG model process data set and could be summarized, that in most instances, the MLE method is the superior method and that the χ^2 estimation is - on average - the next best method. In accordance with Madan and Seneta (1987, 1990) and Daal and Madan (2005), who use only one or two estimation methods, there is no evidence for a large empirical data set, that stock markets returns can be modeled by a VG process, although it contains the typical features of returns like drift, volatility and jumps. For the purpose of closing this gap, we assimilate the properties of the estimation methods, the VG process and the typical features of empirical asset returns and define our first hypothesis (H1a).

Hypothesis 1a (H1a)*. The VG process is able to describe the behavior of empirical share returns.*

The SMOM can be obtained from the empirical data set more or less directly. The empirical log returns often include outliers due to extreme events at the stock markets. These events affect the kurtosis directly and thus, produce some bias. Therefore, the minimization problem of the MOM leads to an overestimation of the parameter ν and thus, to a higher market activity as to the gamma function itself. The ECF method is closely related to the MOM. We have derived the four moments of the VG process from the moment generating function, which is similar to the characteristic function. For that reason, the ECF method is frequently confronted with the same overestimation of ν. The BI method is independent of a particular optimization approach and relies on a simulation. As it is true for all simulations, the results are affected by the assumptions made within the simulation framework, such as the prior distribution of the parameter set η or the intervals for the parameter set η. Nevertheless, this approach should provide more stable and better results compared to the SMOM, MOM and ECF method. The MLE and the χ^2 methods are two approaches, which directly use the PDF for fitting the VG process. While the MLE method modifies the PDF to a three-parametric PDF for numerical reasons, the χ^2 method utilizes the original PDF. The use of the PDF enables a close fitting between the empirical PDF and the PDF of the VG process. Particularly, the problem of extreme events can be captured more easily as the χ^2 method employs intervals being able to handle these effects. Therefore, it can be stated that the χ^2 method could be superior to that of the MLE. We build the next hypothesis (H1b) on these fundamental aspects.

Hypothesis 1b (H1b)*. If the empirical share returns follow a VG process, the χ^2 method provides the best approximation of the VG process for an empirical data set.*

According to Hamilton (1989), Jeanne and Masson (2000) or Cerra and Saxena (2003) - amongst others - the stock markets' behavior can change from normal (= non-volatile) times to turbulent (= volatile) times over a longer period as a result of structural breaks. This market attitude can also affect the VG process' fitting procedure and the quality of the estimated parameter set η. Therefore, the fact whether the VG process is fitted to a data set taken from a normal time, or a turbulent time, or a mixture of both can be important. For example, the use of a badly selected data set can imply that the variance influencing parameter σ under- or overestimates the real VG process or that the asymmetry component θ is biased. With this knowledge, we apply a regime switching model (for an overview of this model see Hamilton (1989) or Hamilton (2005)) for the purpose of identifying normal and turbulent times at the stock markets and calibrate the VG process using this selected database. Therefore, we formulate the next hypothesis (H2).

Hypothesis 2 (H2). *If a stock faces different market phases, the parameter set η of the VG process must vary over time.*

Previous literature on the VG process' fitting models - e.g. Seneta (2004), Tjetjep and Seneta (2006), and Finlay and Seneta (2008) - use simulated data or only an empirical data set taken from a short period of time for the whole sample. Due to insufficient empirical testing of the above mentioned six fitting methods for large data sets, we use the DOW JONES as well as all index stocks over the period from 1991 to 2011. In accordance with hypothesis H2 and the assumption that the inclusion of a regime switching model contributes to a parameter improvement we state our last hypothesis (H3).

Hypothesis 3 (H3). *The inclusion of a regime switching model in a VG process' framework increases the fitting rate of each parameter estimation method.*

We can state that H1a and H1b provide a hint for the results of the other two hypotheses. Furthermore, H2 is closely connected to H3. Therefore, the tests of these hypotheses will also be joined in Section 4.

3 Research design

3.1 Data

We distinguish between the data set needed for fitting the VG process and the data set needed for the regime switching model. For analyzing the VG process we use daily returns of DOW JONES stocks and of the index itself over the period from 01.01.1991 to 31.12.2011. The stocks represent the actual composition of the DOW JONES at the end of the year 2011. With this framework we calibrate the VG process for 29 stocks and the index itself. We only exclude Kraft Foods Inc. from our research, because they went public only in June 2001. Therefore only limited daily returns are available. Furthermore, we calibrate the regime switching model with weekly closing prices of the DOW JONES. For back-testing the results of this model with the safe haven theory we take the yields of daily United States of America (US) Treasuries with a maturity of 6 months (m), 12 months, 5 years (y) and 10 years as well as the closing prices of gold. All our data are provided by Thomson Reuters Datastream. Table 2 summarizes the data set and Table 12 in Appendix A1 provides some descriptive information about the data set.

Table 2 Empirical data overview

Underlying	Timeframe	Interval	Observations	Usage
Dow Jones industrial average index	01.01.1991 to 31.12.2011	daily	5298	Fitting VG process
Dow Jones industrial average index	01.01.1991 to 31.12.2011	weekly	1094	Regime switching model
Dow Jones industrial average index stocks	01.01.1991 to 31.12.2011	daily	5298	Fitting VG process
US treasury 6 months middle rate	01.01.1991 to 31.12.2011	daily	5298	Robustness
US treasury 12 months middle rate	01.01.1991 to 31.12.2011	daily	5298	Robustness
US treasury 5 years middle	01.01.1991 to 31.12.2011	daily	5298	Robustness
US treasury 10 years middle	01.01.1991 to 31.12.2011	daily	5298	Robustness
Gold	01.01.1991 to 31.12.2011	daily	5298	Robustness

Note: This table presents an overview about the data set provided by Thomson Reuters Datastream and used in our empirical analysis. Hereby, it gives some information about the timeframe, the interval and the number of observations and the usage in the respective model

Our data set includes the different phases of the past 21 years. It covers bullish markets which could be observed in the periods from 1991 to 1998 and from 2003 to 2007, the dot-com hype and the crisis subsequent to it, the events of 9/11, the financial crisis as well as the effects of the European sovereign debt crisis on US stocks. Additionally, it includes extreme jump events, like for example the over 30 % drop of Protector & Gamble on 07.03.2000. This broad range of data allows us to provide a profound analysis of the VG process in the next sections.

3.2 Sector classification

In some cases, it can be useful to group shares into sectors in order to identify common behavior and trends in different market phases. We use the North American Industry Classification System (NAICS) for classifying the companies listed on the DOW JONES into sectors. As to that, the classification system of the NAICS employs a six-digit code. In our case, it often suffices to rely on the first two digits. In some cases, however, such as in the manufacturing sector, which a lot of companies listed on the DOW JONES operate in, the third digit has to be employed. We identify the following six sectors, which sometimes cover more than one industry. For an overview see Table 3.

3.3 Identification of normal and turbulent timeframes

For identifying normal and turbulent timeframes in our economic time series we use a regime switching model. Our focus is simply to distinguish normal and

Table 3 Sectors covered by the DOW JONES according to NAICS

Sector	Coverage	Companies
Information & entertainment	information sector entertainment sector	ATT, MICROSOFT VERIZON, WALT DISNEY
Finance & insurance	financial sector insurance sector	AMEX, BOA, JP MORGAN TRAVELERS
Engineering	transportation equipment manufacturing machinery manufacturing primary metal manufacturing	ALCOA, CAT, UTC
Trade & food	retail trade wholesale food industry	COCA COLA, HOME DEPOT, MCDONALDS, WAL MART, JNJ, PG
Chemistry & oil	chemical manufacturing petroleum and coal products manufacturing	CHEVRON, DU PONT, EXXON MERCK, PFIZER
Electrical & component	electrical sector electrical component sector	3M, CISCO, GE, HP INTEL, IBM

Note: This table presents the sectors, we have identified and clustered in the DOW JONES according to NAICS. Furthermore, it shows the sector classification of the companies of the DOW JONES. The abbreviations of the companies can be found in Table 12 in Appendix A1

turbulent timeframes for the forthcoming parameter estimations. Therefore, in our point of view a common regime switching model like Hamilton (1989) is appropriate and more complex regime switching models for example with time varying transition probabilities by Diebold et al. (1994) are not necessary. We follow the approach by Hamilton (1988, 1989, 1994, 2005) and the implementation and calculations by Perlin (2007) as proposed by Alexander (2008). For calibrating the regime switching model we use a database of weekly closing prices from January 1991 to December 2011. We prefer the weekly database to the daily database as the results are more stable. Figure 1 shows the development of the DOW JONES as well as the smoothed probability that the share index is in a turbulent regime.

By means of these smoothed probabilities, we divide the total timeframe into four sub-periods - two normal states (from 01.01.1991 to 08.06.1998 and from 14.04.2003 to 15.10.2007) and two turbulent states (from 15.06.1998 to 07.04.2003 and from 22.10.2007 to 31.12.2011). Hamilton (1989) concludes that his model makes very clear decisions about the probability of being in a certain state. In Hamilton's context this means that only few smoothed probabilities should lie between 0.3 and 0.7 and the algorithm usually identifies fairly clear decisions about the states. With this knowledge he suggests using the decision criterion smoothed probability ≥ 0.5 in a two state model. In the first sub-period we treat the two short periods with a high smoothed probability as outliers and therefore, consider them to be within a normal

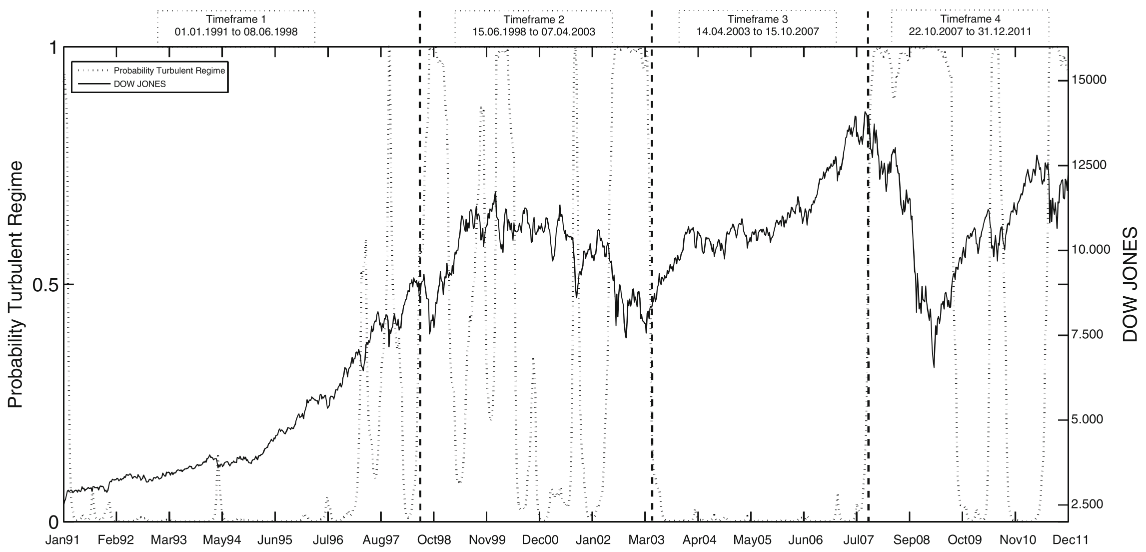

Fig. 1 Regime switching model - DOW JONES. This figure shows the development of the DOW JONES and the corresponding probability that the DOW JONES is in a turbulent regime from 1991 to 2011. All calculations are based on the implementation of the regime switching model by Perlin (2007) and on a weekly database

time. In the second and fourth sub-period we face times switching from turbulent to normal and back to turbulent with high frequency. We apply Hamilton's (1989) decision criteria and demand an expected smoothed probability $\mu_{sm} \geq 0.5$ for a turbulent time. Therefore, we perform an approximate Gauss test with a 95 % significance level and the null hypothesis $H_0 : \mu \geq 0.5$ and $H_1 : \mu < 0.5$ as the alternative hypothesis. For more information about the regime switching model and the applied approximate Gauss-test see Appendix A2.

To sum up, we classify our four sub-periods as follows: from 01.01.1991 to 08.06.1998 as a normal period; from 15.06.1998 to 07.04.2003 as a turbulent period; from 14.04.2003 to 15.10.2007 as a normal period; from 22.10.2007 to 31.12.2011 as a turbulent period. We use this division in the subsequent sections. We also do a robustness test by using the safe haven theory. The results are also shown in Appendix A2.

4 Empirical results

At first, we try to fit a VG process over the total period of 21 years by means of the six presented estimation methods. Table 4 demonstrates that the quality of the fitting rates (= percentages of shares where we could not reject the null hypothesis that the log returns follow a VG process) tested by the KS test is not acceptable except for the MLE method and the χ^2 method. In contrast to the simulated VG process data, we even notice a strong decrease of the fitting rates with increasing

Table 4 KS test: fitting rates of all estimation methods - total timeframe

	SMOM	MOM	MLE	BI	ECF	χ^2
μ_d	0.0862	0.0919	0.0205	0.0467	0.0604	0.0189
1 %	0	0	73.33	10	3.33	83.33
5 %	0	0	36.67	0	0	50
10 %	0	0	23.33	0	0	33.33
15 %	0	0	20	0	0	26.67
20 %	0	0	20	0	0	23.33

Note: This table presents the fitting rates by the KS test of all estimation methods over the total timeframe. All fitting rates are indicated in %. Furthermore μ_d shows the mean maximum absolute deviation between the estimated and the empirical CDF

levels of significance. Furthermore, μ_d indicates the mean maximum deviation of the 30 empirical samples. As is the case for the fitting rates, μ_d is too large and therefore does not reveal a high-quality approximation for all methods. Only the MLE method and the χ^2 method positively stands out from these methods. For a robustness test of the fitting rates, have a look at the χ^2 test's results in Appendix A3 (see Table 15).

These results make clear that it is hardly possible to estimate a VG process over a very long period. Problems result from data covering too many different market phases in which the stocks change their behavior. This means that the increments' PDF has various symmetries, fat tails on the left or right end of the distribution or a time varying volatility as most stocks behave differently during the different periods. Table 11 presents - among other things - the parameter estimation results via the χ^2 method and the corresponding significance levels over the period from 1991 to 2011. It becomes apparent, that the fitting is not sector depended and there are problems with low and kurtosis distributions. It seems that the change of the market behavior of stocks in the last two decades has also affected

Table 5 KS test: fitting rates of all estimation methods - timeframe 1

	SMOM	MOM	MLE	BI	ECF	χ^2
μ_d	0.0687	0.0686	0.0403	0.0428	0.0532	0.0433
1 %	10	10	53.33	53.33	23.33	36.67
5 %	3.33	3.33	20	23.33	6.67	26.67
10 %	0	0	10	16.67	0	20
15 %	0	0	10	13.33	0	16.67
20 %	0	0	3.33	3.33	0	13.33

Note: This table presents the fitting rates by the KS test of all estimation methods over timeframe 1. All fitting rates are indicated in %. Furthermore μ_d shows the mean maximum absolute deviation between the estimated and the empirical CDF

Table 6 KS test: fitting rates of all estimation methods - timeframe 2

	SMOM	MOM	MLE	BI	ECF	χ^2
μ_d	0.0545	0.0541	0.0183	0.0182	0.0342	0.0204
1 %	56.67	56.67	100	100	76.67	100
5 %	43.33	43.33	100	100	70	100
10 %	26.67	26.67	100	100	70	96.67
15 %	23.33	23.33	100	100	66.67	96.67
20 %	23.33	23.33	100	100	63.33	96.67

Note: This table presents the fitting rates by the KS test of all estimation methods over timeframe 2. All fitting rates are indicated in %. Furthermore μ_d shows the mean maximum absolute deviation between the estimated and the empirical CDF

the fitting characteristics of the stochastic processes and therefore emphasizes our hypotheses H2 and H3. From the results we learn that a randomly selected timeframe or fitting method used for calibrating a stochastic process does not ensure significant results. With this background knowledge we run the same parameter estimations for the four timeframes identified by the regime switching model and the safe haven theory. Tables 5, 6, 7 and 8 clearly show increasing fitting rates for all estimation methods compared to the total timeframe estimation (see Table 4). For a robustness check see Tables 16, 17, 18 and 19 in Appendix A3. While, for example, the fitting rate for the χ^2 method is 83.33 % (1 % significance level) over the total timeframe, the fitting rates vary over the four identified timeframes. In the less volatile and calm timeframe 1, where we consequently obverse less jumps, it decrease to 36.67 % while in all the other three timeframes all samples could be fitted successfully. This fact highlights that the data used to fit a model plays an important role and a well-defined period of the data set increases the probability of obtaining high-quality estimations. Besides, we also consider the mean maximum absolute deviation μ_d between the estimated

Table 7 KS test: fitting rates of all estimation methods - timeframe 3

	SMOM	MOM	MLE	BI	ECF	χ^2
μ_d	0.0736	0.0724	0.0184	0.0568	0.0542	0.0186
1 %	53.33	53.33	100	43.33	53.33	100
5 %	46.67	46.67	100	30	46.67	100
10 %	43.33	43.33	100	16.67	36.67	100
15 %	43.33	43.33	100	16.67	33.33	100
20 %	43.33	43.33	100	16.67	33.33	100

Note: This table presents the fitting rates by the KS test of all estimation methods over timeframe 3. All fitting rates are indicated in %. Furthermore μ_d shows the mean maximum absolute deviation between the estimated and the empirical CDF

Table 8 KS test: fitting rates of all estimation methods - timeframe 4

	SMOM	MOM	MLE	BI	ECF	χ^2
μ_d	0.0816	0.0811	0.0208	0.0618	0.0621	0.0208
1 %	23.33	23.33	100	50	30	100
5 %	16.67	16.67	100	46.67	13.33	100
10 %	10	13.33	100	33.33	3.33	100
15 %	10	10	100	23.33	3.33	93.33
20 %	6.67	6.67	100	20	3.33	93.33

Note: This table presents the fitting rates by the KS test of all estimation methods over timeframe 4. All fitting rates are indicated in %. Furthermore μ_d shows the mean maximum absolute deviation between the estimated and the empirical CDF

and the empirical CDF. Particularly, we do not find a huge difference concerning the well fitting quality between the last three timeframes. Further, there is a trend that the deviations themselves decrease significantly by comparing timeframe 1 with the other three timeframes (as to the χ^2 method from 0.0433 to round about to 0.02). Again, these trends reaffirm that stock markets are more likely to follow a VG process. We cannot reject our hypothesis H3 as we obtain strongly improved fitting rates from the application of the regime switching model. As the fitting quotes and μ_d by the KS test do not give a clear answer about the best fitting method (H1b) and the KS test is often criticized only focusing on one point of the distribution, we also use the Anderson Darling test. This test especially helps to capture the extreme events in the return distributions. Table 9 shows that in general the χ^2 method is preferable to the MLE method. Table 10 gives a more detailed view at the single fitting results. During timeframe 2 to timeframe 4, it is clearly visible that for the heavily tailed stocks, the χ^2 method works better than the MLE method. Timeframe 1 again highlights the problems for fitting jump processes to less tailed returns, while the results for the overall period emphasizes that for a very large number of observations with a high kurtosis the χ^2 method works definitively best. All in all, we find an outperformance of the χ^2 method for all sub-periods

Table 9 Summary Anderson Darling test statistics

	SMOM	MOM	MLE	BI	ECF	χ^2
Overall	75.3799	86.5248	2.477	2.9055	36.2853	1.9605
Timeframe 1	11.2572	11.1686	1.7332	4.0998	4.9172	3.8414
Timeframe 2	8.2616	7.9506	0.4601	0.4625	3.581	0.475
Timeframe 3	17.8564	16.9886	0.4981	0.6885	8.2424	0.4545
Timeframe 4	12.4214	12.2695	0.617	0.6305	8.6627	0.6117

Note: This table presents the mean of the Anderson Darling test statistics for all estimated timeframes and the total period for heavy tailed data. The MLE method is the second best approach and particularly outperforms the moment based estimation methods. The BI method seems to work in a more or less acceptable way, but does not justify the efforts undertaken. These results lead to a non-rejection of H1b and confirm the simulated VG process' results by Finlay and Seneta (2008) with the only difference that the χ^2 method is slightly preferred for empirical returns.

So far we have focused on the fitting rates and the quality of the estimations. Now we start a more detailed analysis of the stocks' parameters of the VG process (see Table 11). At first, we take the parameters of the total timeframe and the four sub-periods into account. We see that the total timeframe tends to average over the four sub-timeframes. This means, for example, that we overestimate σ, θ or ν in a normal period and underestimate them in a turbulent period. With respect to simulation and forecasting applications of asset prices by means of a VG process model this can lead to enormous biases. While σ and ν errors particularly lead to a mismatch of the actual market behavior, a θ error results in a wrong assumption of the log returns' symmetry. In normal timeframes there is a trend towards a right symmetric stochastic process, whereas in turbulent timeframes there is a trend towards a left symmetric VG process' behavior. All in all, we can not reject H1a and H2 as the share distributions follow a VG process and their parameters vary over time. Having looked at the VG process parameters in a more general manner, we continue with some detailed information about the sectors:

Information & entertainment and electrical & component:
We notice a different behavior in the two turbulent periods. During the dot-com crisis, these two sectors tend to have a higher σ and ν in relation to the other sectors, while these sectors are less volatile during the financial crisis (see Table 12 in Appendix A1). Particularly, enterprises in the telecommunication and technology sector reveal a stable performance and offer a good opportunity for diversification.

Finance & insurance:
This sector has an extremely high σ and ν during the financial crisis. The VG process is able to capture this behavior according to the KS test and the respective significance levels. Therefore, the VG process and the jumps generated by a gamma function using the parameter ν are helpful approaches to cover this difficult market behavior. These findings are in contrast to the first timeframe where the finance & insurance stocks do not directly follow a VG process. This change in market behavior highlights the transition from a stable to a volatile and nervous sector as a consequence of a financial crisis and political and economic changes.

Engineering:
This sector seems to be strongly dependent on the current economic situation. In turbulent phases σ and ν increase. For that reason, the VG process is a good procedure for modeling stocks of cyclical industries.

Chemistry & oil and trade & food:

The VG process parameters σ and ν show that these sectors decouple from the rest of the market in turbulent phases. The parameters indicating nervous markets are lower than those of other sectors. From a risk perspective, we can - again - find some opportunities for diversification.

To sum up, the parameters analysis emphasizes the importance of a clearly defined separation of normal and turbulent market phases and verifies our second hypothesis H2. Additionally, the examination of the four parameters allows a more detailed analysis of stocks from a risk perspective. In some cases, there is only a concentration on the mean and the variance. The VG process allows an extended consideration of risk by also taking the symmetry of stocks' log return PDF, the kurtosis or the market activity with the parameter ν into account.

5 Discussion

After presenting the empirical results, we put them in relation to each other. The quality of a VG process depends on the estimation method used. Unlike comparable cases, where there often exists a trade off between estimation quality and estimation time, the χ^2 method allows a high quality estimation by keeping the estimation effort acceptable. Furthermore, we notice that we even get high fitting rates for the χ^2 method when we apply the very restrictive KS test. The detailed analysis with the Anderson Darling test reveals, that for heavy tailed data the χ^2 method particularly works well. This fact verifies the good VG process approximation by means of the χ^2 method.

We contribute to existing literature, such as Seneta (2004) or Finlay and Seneta (2008), by comparing all estimation methods with a large empirical data set of the DOW JONES for a period of more than 21 years. In contrast to Seneta (2004) and Finlay and Seneta (2008), we do not choose the timeframes for fitting the VG process randomly, we apply a regime switching model for identifying normal and turbulent timeframes. This additional aspect offers a good opportunity for identifying time varying VG process parameters and is also a progress in avoiding the over- or underestimation of the actual parameters. However, we are aware that the differentiation between normal and turbulent timeframes is only based on the DOW JONES and not on a single asset decision. Therefore, our results and the behavior of an asset are always compared to the market behavior of the DOW JONES. Nevertheless, the regime switching model increases the fitting rates of the six estimation methods significantly. We also notice that markets are more likely to follow jump processes, such as the VG process, as we identified increasing fitting rates over the four timeframes. Besides improved fitting rates, the regime switching model highlights that VG process parameters vary over time and the sensitivity of

Table 10 Comparison Anderson Darling values for MLE method and χ^2 method

	Overall			01.01.91 to 08.06.98			09.06.98 to 07.04.03			08.04.03 to 15.10.07			16.10.07 to 31.12.11		
Underlying	MLE	χ^2	kurt.	MLE	χ^2	kurt.	MLE	χ^2	kurt.	MLE	χ^2	kurt.	MLE	χ^2	kurt.
DOW JONES	1.0942	0.7726	11.0595	0.3936	0.378	9.2529	0.2227	0.1951	5.2473	0.3225	0.3439	4.0028	0.6379	0.8762	9.0548
3M	1.75	1.4904	7.0189	1.389	3.0285	7.4763	0.1953	0.2246	4.6444	0.7703	0.4768	11.227	0.2667	0.2356	6.2279
ATT	2.4414	2.4434	7.3472	2.0751	7.1134	4.3469	0.1903	0.2703	4.2529	0.3005	0.3384	6.0431	0.737	0.6193	11.5428
ALCOA	2.9219	2.038	10.192	1.1357	4.3072	4.3911	0.4028	0.8488	4.5605	0.2052	0.2371	4.4476	0.5685	0.6396	7.7191
AMEX	2.6575	1.9298	9.7163	3.5354	5.5275	4.3852	0.4306	0.4614	4.337	0.191	0.1932	5.6372	0.6234	0.7089	8.0594
BOA	9.4922	6.6208	28.5908	1.8355	5.1061	4.4691	0.1784	0.229	4.0228	0.3317	0.2522	17.8379	2.0956	2.6152	11.9545
BOEING	2.7748	2.4104	9.1874	2.3325	4.5955	9.4352	0.554	0.4093	8.3657	0.255	0.1862	4.4527	0.446	0.4609	5.9979
CAT	1.492	1.2336	6.6702	1.1568	1.8156	5.2941	0.2472	0.4111	4.6392	0.5797	0.4501	13.2132	0.1951	0.2336	5.7833
CHEVRON	2.432	1.7952	11.7333	1.3438	3.7473	3.6426	0.2431	0.2946	4.3684	0.3819	0.4661	3.3189	0.7361	0.5055	13.9167
CISCO	1.1385	0.798	8.0608	1.2622	1.1935	6.7491	0.8211	0.8966	5.3794	0.6145	0.405	9.0828	0.3431	0.2977	10.7532
COCA COLA	1.9393	1.5589	8.16	2.4356	5.3775	4.3616	0.4124	0.5764	5.4724	1.241	0.7535	12.9903	0.7411	0.3218	13.0496
DU PONT	2.861	2.364	6.6685	1.8846	3.2743	4.7178	0.481	1.3383	4.6154	0.6686	0.5859	5.3784	0.4561	0.4556	6.2158
EXXON	2.7721	2.4703	11.2628	1.6926	14.208	4.4718	0.3715	0.325	5.0946	0.4014	0.5726	4.1442	1.0316	0.6393	14.3472
GE	4.4202	4.1069	10.4269	1.6862	3.2753	4.1155	0.5292	0.5088	4.8392	0.2813	0.5064	3.976	0.7615	0.641	8.416
HP	1.8261	1.2166	9.9101	1.5465	1.262	9.328	0.2806	0.2809	6.0694	0.506	0.3454	11.8901	0.359	0.2941	13.4328
HOME DEPOT	1.6732	1.336	17.4898	1.3929	3.4428	4.299	0.5856	0.3889	17.1058	0.3496	0.3174	5.5017	0.137	0.1414	6.1274
INTEL	2.2528	1.7047	8.4734	1.9233	1.9476	6.1273	0.7551	0.5918	6.4593	0.6924	0.4562	7.2715	0.4177	0.3725	6.2704
IBM	2.1807	1.6502	9.7128	1.5757	1.4202	8.0503	0.5204	0.4069	7.4296	0.4159	0.3688	8.177	0.4742	0.4711	6.6812
JP MORGAN	2.9609	2.0521	12.8131	1.5539	4.0297	5.6341	0.2244	0.3852	5.8297	0.1838	0.1712	5.3276	0.8129	0.8459	9.8085
JNJ	0.7646	0.6623	9.9462	1.1851	2.4298	3.9766	0.4693	0.3699	10.8591	0.4525	0.5601	5.47	0.708	0.6705	14.8506
MCDONALDS	2.5159	2.1562	7.074	3.1158	6.1099	3.8388	0.5002	0.4269	6.3587	0.7861	0.8947	7.0631	0.2296	0.2798	7.1815
MERCK	3.2598	2.5043	22.1316	1.4448	2.2208	4.4364	0.5935	0.5046	4.8773	1.7973	1.0583	99.1117	1.3776	1.8954	10.9305

Table 10 (continued)

	Overall			01.01.91 to 08.06.98			09.06.98 to 07.04.03			08.04.03 to 15.10.07			16.10.07 to 31.12.11		
Underlying	MLE	χ^2	kurt.	MLE	χ^2	kurt.	MLE	χ^2	kurt.	MLE	χ^2	kurt.	MLE	χ^2	kurt.
MICROSOFT	0.8846	0.8179	8.2046	0.6064	0.8856	3.9835	0.5184	0.4299	6.6051	0.5993	1.1878	15.0967	0.5049	0.5731	10.0871
PFIZER	1.6643	1.5472	5.9685	0.9562	1.7017	3.7361	0.4047	0.5331	4.4018	0.9739	0.5392	15.1114	0.8681	0.6793	7.0399
PG	1.6546	0.899	71.6152	1.011	2.2293	3.9868	0.9603	0.5152	67.7332	0.3385	0.3503	5.3827	0.4402	0.3424	9.3729
TRAVELERS	4.1028	3.1738	15.6198	2.154	2.9149	5.4562	0.3698	0.4382	5.598	0.4148	0.4416	5.3153	0.9202	0.726	16.7754
UTC	1.512	1.1903	27.456	1.6707	3.0409	5.171	0.5596	0.1856	27.8778	0.143	0.2808	4.338	0.2959	0.6117	7.9252
VERIZON	1.6471	1.4991	7.6634	1.78	4.8192	5.392	0.4541	0.4452	5.122	0.1797	0.3735	4.4121	0.3323	0.2823	9.3772
WAL MART	2.5793	2.3276	6.8408	4.166	11.0746	6.7831	0.4116	0.4838	4.1411	0.2608	0.2745	4.2474	0.5679	0.4005	10.18
WALT DISNEY	2.6449	2.0449	9.9639	1.7541	2.7655	5.0917	0.917	0.8748	7.5465	0.3042	0.2489	12.6829	0.4255	0.5158	7.955

NOTE: This table presents the Anderson Darling test statistic values for the MLE method as well as for the χ^2 method. Furthermore, we additionally show the kurtosis for highlighting, that the χ^2 method is especially well working for fat tails. A smaller Anderson Darling value indicates a more accurate fitting

Table 11 VG process' parameter estimation results DOW JONES via χ^2 method

	Overall			01.01.91 to 08.06.98			09.06.98 to 07.04.03			08.04.03 to 15.10.07			16.10.07 to 31.12.11		
Underlying	σ	θ	ν	σ	θ	ν	σ	θ	ν	σ	θ	ν	σ	θ	ν
DOW JONES	0.01068*	−0.00065*	0.91136*	0.00764*	−0.0004*	0.50316*	0.01348*	0.00064*	0.41424*	0.00721*	−0.00061*	0.54423*	0.01546*	−0.00112*	1.2322*
3M	0.01502*	0.00016*	0.70178*	0.01239	0.00083	0.57019	0.01892*	0.0036*	0.42644*	0.01013*	0.00125*	0.43289*	0.01805*	−0.00187*	0.82186*
ATT	0.01736	0.00039	0.66999	0.01371	0.00208	0.40887	0.02525*	0.00074*	0.36485*	0.01208*	0.00038*	0.49449*	0.01696*	−0.00112*	0.88967*
ALCOA	0.02339*	0.00136*	0.66688*	0.016	0.00443	0.38942	0.02717*	0.00419*	0.28615*	0.01739*	−9e-05*	0.40153*	0.03623*	−0.00584*	0.88105*
AMEX	0.02278*	0.00048*	0.74004*	0.01791	0.00432	0.36145	0.02766*	−0.00077*	0.17433*	0.01161*	0.00214*	0.68545*	0.0346*	−0.00204*	1.2102*
BOA	0.02401	−0.00067	1.141	0.01592	0.00229	0.45226	0.02524*	0.00163*	0.40153*	0.00878*	−0.00115*	0.34375*	0.05*	−0.0007*	1.4312*
BOEING	0.01932*	0.0004*	0.5619*	0.01526	0.00314	0.41967	0.02475*	−0.00014*	0.40191*	0.01379*	0.00215*	0.20471*	0.02305*	−0.0022*	0.64927*
CAT	0.02082*	0.00104*	0.54958*	0.01725*	0.0043*	0.35738*	0.02449*	0.00408*	0.37858*	0.0151*	−0.00304*	0.29712*	0.02694*	−0.00183*	0.75005*
CHEVRON	0.01541*	−0.00084*	0.42505*	0.01298	0.00095	0.35069	0.01733*	0.00161*	0.40601*	0.01252*	−0.00294*	0.10564*	0.01991*	−0.00295*	0.83117*
CISCO	0.02782*	−0.00164*	0.61395*	0.02774*	−0.00163*	0.2286*	0.0397*	−0.0001*	0.28691*	0.01606*	0.00125*	0.31948*	0.02228*	−0.00171*	0.82701*
COCA COLA	0.01449*	0.00066*	0.58709*	0.01348	0.00295	0.23918	0.02056*	0.00174*	0.48246*	0.00818*	0.00171*	0.3737*	0.01373*	−0.00199*	0.92383*
DU PONT	0.01805*	0.00117*	0.52545*	0.01511	0.00127	0.2677	0.02278*	0.0036*	0.44394*	0.01112*	−0.00013*	0.23274*	0.02331*	−0.00351*	0.76545*
EXXON	0.01466	−0.00116	0.47743	0.01131	0.00147	0.55824	0.01777*	−0.00034*	0.23858*	0.01221*	−0.00319*	0.30796*	0.01779*	−0.00094*	0.79338*
GE	0.01805*	0.00013*	0.75656*	0.01232	0.00424	0.17968	0.02324*	0.00422*	0.31785*	0.00993*	0.00252*	0.35245*	0.02586*	−0.00279*	1.0416*
HP	0.0236*	7e−05*	0.6505*	0.02102*	0.00366*	0.28985*	0.03388*	0.00274*	0.45485*	0.0157*	0.00066*	0.48653*	0.0221*	−0.00251*	0.88144*
HOME DEPOT	0.02037*	0.00151*	0.57188*	0.01713	0.00192	0.31271	0.02859*	0.00295*	0.37731*	0.01306*	0.0031*	0.31573*	0.02215*	0.00093*	0.86554*
INTEL	0.02525*	0.00033*	0.51306*	0.02327*	−0.00019*	0.27718*	0.03639*	0.00203*	0.25643*	0.01655*	0.0003*	0.24601*	0.0228*	−0.00203*	0.68664*
IBM	0.01797*	0.0003*	0.71629*	0.01729*	0.00195*	0.41764*	0.02484*	0.00186*	0.47136*	0.01026*	0.0007*	0.4302*	0.01644*	−0.00155*	0.77415*
JP MORGAN	0.02413*	0.00132*	0.88943*	0.01871	0.00187	0.46183	0.02959*	0.00348*	0.38619*	0.01173*	0.00045*	0.58657*	0.03685*	0.00201*	1.1487*
JNJ	0.01415*	0.00094*	0.62183*	0.01516*	0.00198*	0.23322*	0.01757*	0.00291*	0.32244*	0.00898*	0.00074*	0.66016*	0.01143*	9e−05*	0.97293*
MCDONALDS	0.0159	0.00119	0.43935	0.01424	0.00467	0.26262	0.02099*	0.00277*	0.44337*	0.01337*	0.00102*	0.45521*	0.01442*	−0.00131*	0.84997*
MERCK	0.01733*	−0.0002*	0.48267*	0.01592*	0.00384*	0.33771*	0.02031*	−0.00153*	0.23514*	0.01349*	−0.00164*	0.64434*	0.02024*	−0.00119*	0.85032*

Table 11 (continued)

Underlying	Overall			01.01.91 to 08.06.98			09.06.98 to 07.04.03			08.04.03 to 15.10.07			16.10.07 to 31.12.11		
	σ	θ	ν	σ	θ	ν	σ	θ	ν	σ	θ	ν	σ	θ	ν
MICROSOFT	0.02087*	0.00157*	0.69379*	0.01998*	0.0054*	0.24609*	0.02737*	0.00299*	0.27872*	0.01136*	0.00096*	0.8062*	0.02059*	−0.00137*	0.82284*
PFIZER	0.01796*	0.00162*	0.48926*	0.01713*	0.00528*	0.20305*	0.02279*	0*	0.35977*	0.01217*	0.00194*	0.40508*	0.01749*	0.00142*	0.65317*
PG	0.01399*	0.00077*	0.63754*	0.01354*	0.00412*	0.21926*	0.01934*	−0.00037*	0.58344*	0.00846*	0.00101*	0.43616*	0.01284*	−0.00084*	0.9814*
TRAVELERS	0.01747	−0.00019	0.74601	0.01214	0.00098	0.23533	0.02362*	0.00385*	0.50624*	0.01269*	0.00046*	0.55678*	0.02302*	−0.00062*	1.1753*
UTC	0.0171*	0.00082*	0.6381*	0.01391	0.00272	0.51858	0.02321*	0.00023*	0.13847*	0.01117*	0.00152*	0.30131*	0.01954*	−0.00137*	0.93603*
VERIZON	0.01637*	−0.00052*	0.56822*	0.01351	0.00127	0.35481	0.0234*	0.00376*	0.30733*	0.0112*	0.00063*	0.39625*	0.0167*	−0.00156*	0.93192*
WAL MART	0.01732*	0.00128*	0.56255*	0.01614	0.00384	0.30099	0.02465*	0.00445*	0.39113*	0.0108*	0.0011*	0.30405*	0.01339*	−0.00078*	0.76847*
WALT DISNEY	0.01928*	0.00078*	0.58492*	0.01532	0.00351	0.2908	0.02658*	0.00338*	0.34363*	0.01268*	−0.00055*	0.29718*	0.02209*	−0.00045*	0.86583*

Note: This table presents the estimated parameters of the VG process by the χ^2 method for the total timeframe as well as for all for sub timeframes. As c only affects the midpoint of the PDF we neglect it in this analysis and focus on the more important parameters σ, θ and ν. * indicates a 1 % significance level for the KS test the VG process itself becomes apparent. As a consequence, we can suggest two main points in the context of modeling share returns and calibrating risk models. First, it is essential to use an accurate estimation method and second, we highly recommend to take care about the timeframes used for calibrating the models for avoiding over- or underestimation.

6 Conclusion

We have taken the common knowledge about the VG process (see Madan and Seneta 1987, 1990; Seneta 2004; Finlay and Seneta 2008, amongst others) and the knowledge about the estimation methods of the VG process estimation methods by means of a simulated VG process data set (see Finlay and Seneta 2008) and applied it on a large empirical data set, the DOW JONES. Based on the theoretical background we develop four hypotheses, each of which includes an aspect of practical usage of the VG process. We reach our aim to improve the quality of a VG process by integrating a regime switching model developed by Hamilton (1989) and Hamilton (2005) into the VG process framework, and in this way, identify VG process parameters which depend on timeframes (hypothesis H2). This approach results in the improvement of fitting rates (hypothesis H3), which are determined by the conservatively adjusted KS test for CDF. Finlay and Seneta (2008) suggest using a MLE method or χ^2 method in order to fit a VG process by means of a simulated data set. We can verify this knowledge for empirical data, but in contrast to Finlay and Seneta (2008), we find a superiority of the χ^2 method for empirical data with a high kurtosis (hypothesis H1b). Finally, we can verify that empirical share returns can be modeled by a VG process (H1a). All these findings are representative of empirical stock return time series, as we apply a very large data set. For other studies and time series simulated and/or empirical data is necessary.

The regime switching model allows us to have a closer look at the parameters of the VG process over the various normal and turbulent regimes over our daily data set comprising 21 years. Furthermore, we can learn that such a differentiation helps to avoid an over- or underestimation of the real VG process stock market behavior.

However, we state that our results only cover the DOW JONES and the new knowledge about the VG process is limited to this data set. The scope of the analysis could be broadened to cover other markets such as the commodities, currency or bond markets by extending the data set. Furthermore, this paper focuses on the identification of the estimation method fitting the VG process best. On the basis of these results we could compare the VG process with other stochastic models.

To sum up, our findings should ensure a better understanding of the VG process in connection with an empirical dataset. This empirical study on the VG process should provide a useful basis for further research on the permanent consideration of the combination of the regime switching model and the VG process.

Acknowledgments We thank the participants of the European Financial Management Association (EFMA) conference in Reading 2013 and of the Eighth World Congress of the Bachelier Finance Society in Brussels 2014 for the helpful comments.

Appendix A1

Table 12 Descriptive statistics: DOW JONES, US Treasuries and gold

Underlying	Abbreviation	Overall N=5298				01.01.91' to 08.06.98 N=1922				09.06.98 to 07.04.03 N=1174				08.04.03 to 15.10.07 N=1140				16.10.07 to 31.12.11 N=1062			
		mean	std. dev.	skew.	kurt.	mean	std. dev.	skew.	kurt.	mean	std. dev.	skew.	kurt.	mean	std. dev.	skew.	kurt.	mean	std. dev.	skew.	kurt.
Dow Jones Industrial Average	DOW JONES	0.0003	0.0113	−0.1467	11.0595	0.0006	0.008	−0.4564	9.2529	0	0.0137	−0.0717	5.2473	0.0005	0.0072	−0.1696	4.0028	−0.0001	0.0162	0.0018	9.0548
3M Comp.	3M	0.0003	0.0157	−0.0338	7.0189	0.0004	0.0133	−0.3922	7.4763	0.0005	0.0194	0.3621	4.6444	0.0003	0.0112	−0.7609	11.227	−0.0001	0.0186	−0.1144	6.2279
AT&T Inc.	ATT	0.0002	0.018	0.064	7.3472	0.0006	0.0147	−0.0537	4.3469	−0.0005	0.0255	−0.0537	4.2529	0.0006	0.0124	0.2434	6.0431	−0.0003	0.0183	0.6125	11.5428
Alcoa Inc.	ALCOA	0	0.0254	−0.0629	10.192	0.0004	0.0171	0.2132	4.3911	0.0002	0.0279	0.2823	4.5605	0.0005	0.0176	−0.0591	4.4476	−0.0014	0.0382	−0.154	7.7191
American Express Comp.	AMEX	0.0004	0.0243	0.0256	9.7163	0.001	0.019	0.1474	4.3852	0.0001	0.028	−0.0824	4.337	0.0006	0.012	0.2561	5.6372	−0.0002	0.0353	0.0913	8.0594
Bank of America Corp.	BOA	0	0.0291	−0.3287	28.5908	0.0011	0.0168	0.0136	4.4691	−0.0001	0.0253	−0.0105	4.0228	0.0003	0.0095	−1.3257	17.8379	−0.0021	0.0535	−0.1436	11.9545
Boeing Comp.	BOEING	0.0002	0.0203	−0.3125	9.1874	0.0003	0.0171	−0.2996	9.4352	−0.0003	0.0259	−0.6019	8.3657	0.0011	0.0141	0.2761	4.4527	−0.0003	0.0235	0.1303	5.9979
Caterpillar Inc.	CAT	0.0005	0.0216	−0.0649	6.6702	0.0008	0.0185	0.0922	5.2941	0.0001	0.0248	0.0306	4.6392	0.001	0.016	−1.0962	13.2132	0.0001	0.0272	0.0532	5.7833
Chevron Corp.	CHEVRON	0.0003	0.0164	0.0887	11.7333	0.0004	0.0135	0.0689	3.6426	−0.0002	0.0175	0.1131	4.3684	0.0009	0.0126	−0.2596	3.3189	0.0001	0.0223	0.1791	13.9167
Cisco Systems Inc.	CISCO	0.0009	0.0291	−0.0699	8.0608	0.0025	0.0287	−0.3802	6.7491	−0.0002	0.0405	0.2095	5.3794	0.0008	0.0174	0.0256	9.0828	−0.0006	0.0242	−0.4648	10.7532
Coca-Cola Comp.	COCA COLA	0.0003	0.0153	0.0537	8.16	0.0011	0.0145	0.1571	4.3616	−0.0006	0.0208	−0.072	5.4724	0.0003	0.0092	−0.6785	12.9903	0.0002	0.0149	0.6126	13.0496
E. I. du Pont de Nemours and Comp.	DU PONT	0.0002	0.0189	−0.1104	6.6685	0.0007	0.016	−0.0427	4.7178	−0.0004	0.0233	0.1329	4.6154	0.0002	0.0114	−0.3331	5.3784	−0.0001	0.024	−0.2842	6.2158
Exxon Mobil Corp.	EXXON	0.0004	0.0159	0.0579	11.2628	0.0005	0.0126	0.0459	4.4718	0	0.0184	0.1696	5.0946	0.0009	0.0123	−0.4133	4.1442	−0.0001	0.0205	0.14	14.3472
General Electric Comp.	GE	0.0003	0.0192	0.0052	10.4269	0.001	0.0131	0.1345	4.1155	0	0.0237	0.0648	4.8392	0.0003	0.0103	0.1903	3.976	−0.0008	0.0277	0.0353	8.416
Hewlett Packard Comp.	HP	0.0004	0.0253	−0.1961	9.9101	0.001	0.0228	−0.1898	9.328	−0.0002	0.0352	−0.0032	6.0694	0.001	0.0172	−0.2008	11.8901	−0.0007	0.0235	−0.6504	13.4328
Home Depot Inc.	HOME DEPOT	0.0005	0.0217	−0.6915	17.4898	0.0013	0.0181	−0.0316	4.299	−0.0001	0.0305	−1.2396	17.1058	0.0002	0.0137	0.2021	5.5017	0.0002	0.0226	0.4347	6.1274
Intel Corp.	INTEL	0.0006	0.0265	−0.3828	8.4734	0.0016	0.0241	−0.3443	6.1273	−0.0002	0.0376	−0.3469	6.4593	0.0004	0.0173	−0.5716	7.2715	0	0.0235	−0.093	6.2704

Table 12 (continued)

Underlying	Abbreviation	Overall N=5298 mean	std. dev.	skew.	kurt.	01.01.91' to 08.06.98 N=1922 mean	std. dev.	skew.	kurt.	09.06.98 to 07.04.03 N=1174 mean	std. dev.	skew.	kurt.	08.04.03 to 15.10.07 N=1140 mean	std. dev.	skew.	kurt.	16.10.07 to 31.12.11 N=1062 mean	std. dev.	skew.	kurt.
International Business Machines Corp.	IBM	0.0004	0.0191	0.0306	9.7128	0.0005	0.0189	0.2308	8.0503	0.0002	0.0263	−0.0748	7.4296	0.0003	0.0107	−0.4182	8.177	0.0004	0.017	0.1138	6.6812
Johnson & Johnson	JP MORGAN	0.0004	0.0264	0.2754	12.8131	0.0015	0.02	0.4357	5.6341	−0.0005	0.0305	0.1342	5.8297	0.0005	0.012	0.0908	5.3276	−0.0003	0.0392	0.3086	9.8085
JP Morgan Chase & Co.	JNJ	0.0004	0.0147	−0.1719	9.9462	0.0008	0.0159	0.08	3.9766	0.0004	0.0187	−0.6881	10.8591	0.0001	0.009	0.2529	5.47	0	0.0126	0.6197	14.8506
McDonald's Corp.	MCDONALDS	0.0005	0.0167	−0.0251	7.074	0.0008	0.0152	0.0765	3.8388	−0.0006	0.0222	−0.1262	6.3587	0.0011	0.0139	0.5489	7.0631	0.0005	0.0147	−0.0208	7.1815
Merck & Co. Inc.	MERCK	0.0002	0.0188	−1.0759	22.1316	0.0008	0.0166	0.008	4.4364	−0.0001	0.0209	−0.0289	4.8773	0	0.0174	−5.2355	99.1117	−0.0003	0.0214	−0.5128	10.9305
Microsoft Corp.	MICROSOFT	0.0006	0.0216	−0.0017	8.2046	0.0017	0.0207	0.1293	3.9835	0	0.0286	−0.1019	6.6051	0.0001	0.0126	−1.0883	15.0967	−0.0001	0.0218	0.3271	10.0871
Pfizer Inc.	PFIZER	0.0004	0.0186	−0.1731	5.9685	0.0013	0.0178	0.0822	3.7361	−0.0001	0.0233	−0.2222	4.4018	−0.0002	0.0137	−0.9535	15.1114	−0.0001	0.0186	−0.0829	7.0399
Procter & Gamble Comp.	PG	0.0004	0.0156	−2.8588	71.6152	0.0007	0.0143	0.1289	3.9868	0.0001	0.0229	−4.0805	67.7332	0.0004	0.0087	0.0179	5.3827	−0.0001	0.0138	−0.1937	9.3729
The Travelers Companies Inc.	TRAVELERS	0.0003	0.0192	0.3251	15.6198	0.0005	0.0129	0.0622	5.4562	−0.0001	0.0246	0.3512	5.598	0.0004	0.0129	0.1417	5.3153	0.0001	0.0261	0.321	16.7754
United Techn-ologies Corp.	UTC	0.0005	0.0182	−1.1176	27.456	0.0007	0.0148	0.2348	5.171	0.0003	0.0255	−1.9704	27.8778	0.0009	0.0112	0.2266	4.338	−0.0001	0.0199	0.2657	7.9252
Verizon Commun-ications Inc.	VERIZON	0.0001	0.0173	0.1543	7.6634	0.0003	0.0145	0.1843	5.392	−0.0002	0.0241	0.0871	5.122	0.0003	0.0115	0.0763	4.4121	0	0.0178	0.331	9.3772
Wal-Mart Stores, Inc.	WAL MART	0.0004	0.0181	0.0466	6.8408	0.0008	0.0183	−0.1906	6.7831	0.0005	0.025	0.12	4.1411	−0.0001	0.011	−0.006	4.2474	0.0003	0.0144	0.2162	10.18
Walt Disney Comp.	WALT DISNEY	0.0003	0.0205	−0.0146	9.9639	0.0008	0.0162	0.3018	5.0917	−0.0006	0.0282	−0.287	7.5465	0.0007	0.0135	0.8861	12.6829	0.0001	0.0231	0.2961	7.955
US Treasury 6 Months Middle Rate	Yield US T-Bills 6M	−0.0009	0.0479	1.9063	72.2473	−0.0001	0.0094	−0.4021	14.0125	−0.0012	0.0148	−1.9668	21.1502	0.0011	0.014	0.6377	14.6526	−0.0039	0.104	1.0214	16.3243
US Treasury 12	Yield US	−0.0007	0.0296	0.1421	21.9554	−0.0001	0.0106	−0.3149	10.6034	−0.0012	0.0179	−0.4974	13.6807	0.0011	0.0178	0.5029	12.6729	−0.0033	0.0588	0.2148	6.7941

Table 12 (continued)

Underlying	Abbreviation	Overall N=5298 mean	std. dev.	skew.	kurt.	01.01.91' to 08.06.98 N=1922 mean	std. dev.	skew.	kurt.	09.06.98 to 07.04.03 N=1174 mean	std. dev.	skew.	kurt.	08.04.03 to 15.10.07 N=1140 mean	std. dev.	skew.	kurt.	16.10.07 to 31.12.11 N=1062 mean	std. dev.	skew.	kurt.
Months Middle Rate	T-Bills 12M																				
US Treasury 5 Years Middle Rate	Yield US T-Bills 5Y	−0.0004	0.0212	−0.3591	14.7355	−0.0002	0.0099	0.3221	6.1617	−0.0005	0.0158	0.505	7.5961	0.0004	0.0157	0.149	6.8479	−0.0015	0.039	−0.2734	5.9453
US Treasury 10 Years Middle Rate	Yield US T-Bills 10Y	−0.0003	0.0147	−0.3183	12.0457	−0.0002	0.0085	0.4658	6.204	−0.0003	0.0125	0.4756	5.3934	0.0001	0.0117	0.1882	5.1336	−0.0008	0.0248	−0.399	6.4587
gold	gold	0.0003	0.0098	−0.2008	10.1476	−0.0001	0.0061	−0.8394	13.9815	0.0001	0.0087	1.0436	14.6729	0.0007	0.0104	−0.6007	5.7193	0.0007	0.0145	−0.3211	6.0371

Note: This table presents the results of the approximate Gauss test for the smoothed probabilities of the regimeswitching model. It shows the the observations as well as the test statistics for each timeframe based on a 5 % significance level

Appendix A2

As we have no idea about the distribution of the smoothed probabilities, we do not assume any distribution for calculating the test statistic value V. Table 13 demonstrates that we can reject H_o in the first and third sub-period, but not in other periods.

Table 13 Descriptive statistics: approximate Gauss test for smoothed probabilities

Timeframe	Observations	μ_{sm}	σ_{sm}	V	Rejection interval
01.01.1991 to 08.06.1998	387	0.067651	0.16408	−51.8376	$(-\infty, -1.6449)$
15.06.1998 to 07.04.2003	252	0.48732	0.40017	−0.50313	$(-\infty, -1.6449)$
14.04.2003 to 15.10.2007	236	0.013478	0.036818	−202.9983	$(-\infty, -1.6449)$
22.10.2007 to 31.12.2011	219	0.60208	0.44601	3.3869	$(-\infty, -1.6449)$

NOTE: This table presents the results of the approximate Gauss test for the smoothed probabilities of the regime switching model. It shows the the observations as well as the test statistics for each timeframe based on a 5 % significance level

We also present the estimated transition matrix P^* for the regime switching model for the DOW JONES

$$P^* = \begin{pmatrix} 0.99 & 0.01 \\ (0.00^{***}) & (0.01^{***}) \\ 0.04 & 0.96 \\ (0.00^{***}) & (0.00^{***}) \end{pmatrix}, \tag{10}$$

which provides the probabilities of switching from one state to another. The figures in brackets denoted are the test values and the significances levels, with *** indicating a 1 % significance level.

Table 14 Correlation between DOW JONES and US Treasuries and gold

Timeframe	Yield US T-Bills 6m	Yield US T-Bills 12m	Yield US T-Notes 5y	Yield US T-Notes 10y	gold
01.01.1991 to 08.06.1998	0.32654	0.29208	−0.25329	−0.51722	−0.45986
15.06.1998 to 07.04.2003	0.59068	0.61947	0.76689	0.80246	−0.67852
14.04.2003 to 15.10.2007	0.761	0.75848	0.73629	0.63986	0.90282
22.10.2007 to 31.12.2011	0.45577	0.41158	0.24329	0.23187	0.31944

Note: This table presents the correlation between DOW JONES and US Treasuries and gold. We apply this as a robustness test and therefore use different maturities for the US Treasuries

For running a robustness test which is in accordance with the safe haven theory, we have a look at the correlations between the DOW JONES, US Treasury yields and gold. In turbulent times, the correlation between the DOW JONES and the US Treasury yields increases. This means that while stock markets decrease, US Treasury prices increase (yields =decrease). Furthermore, gold decouples from the DOW JONES in turbulent times.

Appendix A3

The following six tables show the fitting rates based on the χ^2 test. As mentioned above we use these results as a robustness check for the KS test. Each time a robustness test is needed, a reference can be found in the main part of this paper. The results of the χ^2 test are in accordance with the KS test and verify the χ^2 method as the best estimation method of the VG process. All in all, the results only differ in that they reveal lower fitting rates.

Table 15 χ^2 test: fitting rates of all estimation methods - total timeframe

	SMOM	MOM	MLE	BI	ECF	χ^2
μ_{χ^2}	764.4	826.68	93.8	95.7	406.44	84.46
1 %	0	0	3.33	3.33	0	3.33
5 %	0	0	0	0	0	3.33
10 %	0	0	0	0	0	3.33
15 %	0	0	0	0	0	0
20 %	0	0	0	0	0	0

Note: This table presents the fitting rates by the χ^2 test as a robustness test for all estimation methods over the total timeframe. All fitting rates are indicated in %. μ_{χ^2} is the mean of the χ^2 test statistic values

Table 16 χ^2 test: fitting rates of all estimation methods - timeframe 1

	SMOM	MOM	MLE	BI	ECF	χ^2
μ_{χ^2}	203.76	201.87	116.28	132.95	150.97	118.87
1 %	0	0	3.33	3.33	0	3.33
5 %	0	0	3.33	3.33	0	3.33
10 %	0	0	3.33	3.33	0	3.33
15 %	0	0	3.33	3.33	0	3.33
20 %	0	0	3.33	3.33	0	3.33

Note: This table presents the fitting rates by the χ^2 test as a robustness test for all estimation methods over timeframe 1. All fitting rates are indicated in %. μ_{χ^2} is the mean of the χ^2 test statistic values

Table 17 χ^2 test: fitting rates of all estimation methods - timeframe 2

	SMOM	MOM	MLE	BI	ECF	χ^2
μ_{χ^2}	70.79	69.34	22.26	21.91	54.41	21.18
1 %	20	20	86.67	90	50	90
5 %	13.33	13.33	56.67	63.33	36.67	66.67
10 %	10	10	46.67	50	36.67	56.67
15 %	10	10	46.67	43.33	33.33	46.67
20 %	10	10	43.33	43.33	26.67	46.67

Note: This table presents the fitting rates by the χ^2 test as a robustness test for all estimation methods over timeframe 2. All fitting rates are indicated in %. μ_{χ^2} is the mean of the χ^2 test statistic values

Table 18 χ^2 test: fitting rates of all estimation methods - timeframe 3

	SMOM	MOM	MLE	BI	ECF	χ^2
μ_{χ^2}	169.48	160.51	24.36	24.97	97.55	21.65
1 %	40	36.67	76.67	80	33.33	93.33
5 %	23.33	26.67	56.67	60	16.67	66.67
10 %	16.67	13.33	43.33	43.33	10	56.67
15 %	13.33	10	30	33.33	10	50
20 %	13.33	10	26.67	26.67	10	40

Note: This table presents the fitting rates by the χ^2 test as a robustness test for all estimation methods over timeframe 3. All fitting rates are indicated in %. μ_{χ^2} is the mean of the χ^2 test statistic values

Table 19 χ^2 test: fitting rates of all estimation methods - timeframe 4

	SMOM	MOM	MLE	BI	ECF	χ^2
μ_{χ^2}	149.99	137.74	21.36	20.99	113.56	20.08
1 %	10	10	86.67	86.67	3.33	86.67
5 %	6.67	10	76.67	76.67	0	80
10 %	6.67	6.67	63.33	66.67	0	76.67
15 %	0	0	60	60	0	66.67
20 %	0	0	50	53.33	0	60

Note: This table presents the fitting rates by the χ^2 test as a robustness test for all estimation methods over timeframe 4. All fitting rates are indicated in %. μ_{χ^2} is the mean of the χ^2 test statistic values

References

Aldrich J (1997) R. A. Fisher and the making of maximum likelihood 1912-1922. Stat Sci 3:162–176

Alexander C (2008) Market risk analysis - practical financial econometrics, 1st edn., vol 2. The Wiley Finance Series, New Jersey

Anderson TW, Darling DA (1952) Asymptotic theory of certain "goodness of fit" criteria based on stochastic processes. Ann Math Stat 23:193–212

Barndorff-Nielsen OE (1977) Exponentially decreasing distributions for the logarithm of particle size. Proc R Soc Lond A Math Phys Sci 353:401–409

Barndorff-Nielsen OE (1995) Normal inverse gaussian distributions and the modeling of stock returns, Research Report no. 300, Department of Theoretical Statistics, Aarhus University

Berkson J (1980) Minimum chi-square, not maximum likelihood. Ann Stat 8:457–487

Carr PP, Geman H, Madan DB, Yor M (2002) The fine structure of asset returns - an emprical investigation. J Bus 32:75–305

Cerra V, Saxena SW (2003) Did output recover from the Asian crisis. International Monetary Fund, Working Paper

Cont R, Tankov P (2004) Financial modelling with jump processes, 1st edn. CRC Press, New York

Daal EA, Madan DB (2005) An empirical examination of the Variance Gamma model for foreign currency options. J Bus 78:2121–2152

Diebold F, Lee J-H, Weinbach G (1994) Regime switching with time-varying transition probabilities. In: Hargreaves CP (ed) Nonstationary time series analysis and cointegration. Oxford University Press, Oxford and New York, pp 283–302

Eberlein E, Keller U (1995) Hyperbolic distributions in finance. Bernoulli 1:281–299

Efron B (1982) The 1981 Wald memorial lectures - maximum likelihood and decision theory. Ann Stat 10:340–356

Fama EF (1965) The behavior of stock-market prices. J Bus 38:34–105

Finlay R, Seneta E (2006) Stationary-increment student and Variance Gamma processes. J Appl Prob 43:441–453

Finlay R, Seneta E (2008) Stationary-increment Variance Gamma and t models - simulation and parameter estimation. Int Stat Rev 76:167–186

Hamilton JD (1988) Rational expectations econometric analysis of changes in regime - an investigation of the term structure of interest rates. J Econ Dyn Control 12:385–423

Hamilton JD (1989) A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57:357–384

Hamilton JD (1994) Time series analysis, 1st edn. Princeton University Press, New Jersey

Hamilton JD (2005) Regime switching models, 1st edn. Palgrave Dictionary of Economics, New Jersey

Hsu DA, Miller RB, Winchern DW (1974) On the stable Paretian behavior of stock-market prices. J Am Stat Assoc 69:108–113

Jeanne O, Masson P (2000) Currency crises, sunspots and markov-switching regimes. J Int Econ 50:327–350

Luciano E, Marena M, Sameraro P (2014) Dependence calibration and portfolio fit with factor based time changes. Carlo Alberto Notebooks, Working Paper

Luciano E, Sameraro P (2008) Multivariate Variance Gamma and gaussian dependence: a study with copulas. Carlo Alberto Notebooks, Working Paper

Luciano E, Schoutens W (2006) A multivariate jump-driven financial asset model. Quant Finan 6:385–402

Leicht JJ, Rathgeber AW (2014) Guaranteed stop orders as portfolio insurance - An analysis for the German stock market. Journal of Derivatives & Hedge Funds, forthcoming

Madan DB, Seneta E (1987) Simulation of estimates using the empirical characteristic function. Int Stat Rev 55:153–161

Madan DB, Seneta E (1990) The Variance Gamma VG model for share market returns. J Bus 63:511–524

Madan DP, Carr PP, Chang EC (1998) The Variance Gamma process and option pricing. Eur Finan Rev 2:79–105

Mandelbrot B (1963) The variation of certain speculative prices. J Bus 36:394–419

Massey FJ (1951) The Kolmogorov-Smirnov test for goodness of fit. J Am Stat Assoc 46:68–78

Officer RR (1972) The distribution of stock returns. J Am Stat Assoc 67:807–812

Perlin M (2007) MS regress - the MATLAB package for Markov regime switching models, unpublished working paper, UFRGS - Escola de Administracao (Porto Alegre)

Praetz PD (1972) The distribution of share price changes. J Bus 45:49–55

Roberts HV (1965) Probabilistic prediction. J Am Stat Assoc 60:50–62

Schoutens W (2003) Lévy processes in finance: pricing financial derivatives, 1st edn. The Wiley Series in Probability and Statistics, New Jersey

Semeraro P (2008) A multivariate Variance Gamma model for financial applications. Int J Theor Appl Finan 11:1–18

Seneta E (2004) Fitting the Variance Gamma model to financial data. J Appl Probab 41:177–187

Tjetjep A, Seneta E (2006) Skewed normal-variance-mean models for asset pricing and the method of moments. Int Stat Rev 74:109–126

Yu J (2004) Empirical characteristic function estimation and its applications. Econ Rev 23:93–123