

Recovery Rate in the Event of an Issuer's Insolvency — Empirical Study on Implications for the Pricing of Credit Default Risks in German Corporate Bonds

Alexander Friesenegger

*Chair for Risk Management and Derivatives
University of Hohenheim, Germany
a.friesenegger@uni-hohenheim.de*

Andreas W. Rathgeber

*Institute of Materials Resource Management
Faculty of Mathematics and Natural Sciences
University of Augsburg, Germany
andreas.rathgeber@mrm.uni-augsburg.de*

Stefan Stöckl*

*ICN Business School Nancy-Metz, France
CEREFIGE (European Centre for Research in Financial Economics
and Business Management)
stefan.stoeckl@icn-groupe.fr*

According to the Jarrow–Turnbull model, coupon bonds are valued as a portfolio of zero-coupon bonds that, in the event of insolvency, pay a recovery rate at the end of their term. However, when it comes to valuations, the German insolvency law differs in certain respects. To find out whether a model adapted to the German insolvency law will prove to be more empirically robust, an empirical study of 103 corporate bonds was carried out over more than 800 trading days.

*Corresponding author.

1. Introduction

After insolvencies in the bond market had been in the shadow of stock market events for a long time, they have clearly shifted to the market participants' focus as a result of problems related to creditworthiness. This fact repeatedly brought about an increase in credit spreads and implicit default probabilities. Because of an increase in implicit default probabilities, the default event itself and thus the possible payment of a recovery rate in case of such an event is being brought back to the investors' attention. Accordingly, the amount of this recovery rate is considered in the valuation of these financial instruments; this gives rise to the question of whether the prevailing and potentially well-known framework for corporate bonds is sufficiently taken into account. For instance, the question about the extent to which the agreed-upon bond terms might affect the recovery rate has been hardly discussed so far.

As a result, standard models such as the one developed by Jarrow and Turnbull (1995) make simple assumptions. In these models, the recovery rate of zero-coupon bonds is always paid at the end of the original term (Jarrow and Turnbull, 1995; Jarrow *et al.*, 1997). The coupon bond's recovery rate results from the consideration that it can be regarded as a portfolio of zero-coupon bonds with the repayment of each coupon and redemption payment in the amount of the recovery rate being made at the bond's initial maturity. Overall, the recovery rate is a function of default, risk-free discounted payments. A similar approach can also be found in other models, such as Duffie (1998), who assumes the recovery rate to be dependent on the function of the amount to be paid back, or Bakshi *et al.* (2006), Duffie and Singleton (1999), and Jarrow and Turnbull (2000), who assume the recovery rate to be a function of the value before the default event. The latter, in particular, not only introduces the Jarrow–Turnbull (JT)¹ assumption for the recovery rate, but also introduces it — depending on the nominal amount — as a legally correct approach and shows that, under these conditions, a coupon bond's recovery rate is not identical to the recovery rate of a portfolio of corresponding zero-coupon bonds.

¹Jarrow and Turnbull (1995) are referred to as JT in the subsequent sections.

Both Delianedis and Lagnado (2002) and Hull and White (2000) compare these models as to the valuation of credit derivatives; particularly Delianedis and Lagnado (2002) tend to prefer the assumption of Jarrow and Turnbull (1995) that the recovery rate is a function of default, risk-free discounted payments. Bakshi *et al.* (2006) and Khuong-Huu *et al.* (2008) present similar results. Although they prove that the recovery rate as a function of the nominal amount is more stable, the pricing of the recovery rate according to Jarrow and Turnbull (1995) appears to be more realistic. Guha (2003), by contrast, shows that the recovery rate is paid relatively quickly and independently of the bond's coupon in the course of the U.S. Chapter 11 proceedings. The reason for this result is rooted in the American legal system, which, compared with Chapter 7, does not offer clear provisions for the Chapter 11 proceedings (Bris *et al.*, 2005).

In Germany, the handling of different types of bonds is predetermined by the German Insolvency Code (GIC).² In accordance with the GIC, the accrued interest and nominal amounts of coupon bonds are declared due at the time of the institution's bankruptcy proceedings, whereas interest accruing after that point in time is treated as non-priority interest during the proceedings.

The present paper aims at analyzing whether the simplified assumptions of the JT model regarding the recovery rate are also reflected by market prices or whether they can be better explained by a model that considers the German insolvency law.

The paper is organized as follows. Section 2 provides a brief description of the JT model and gives an overview of the German insolvency law's valuation-relevant provisions regarding the bond terms. Then, in Sec. 3, we integrate these provisions into the basic JT model and analyze their effect on prices theoretically. We empirically compare the presented model³ with the JT model in Sec. 4, to find out which of the two models is best at modeling the observed market prices. The paper concludes with a summary and an outlook on future research.

2. Description of the JT Model and Overview of the German Insolvency Law

2.1. *JT model*

The discrete version of the JT model determines a zero-coupon bond's value V_{ZCB}^0 at the point in time $t = 0$ with a term T as the payments' expected

²German insolvency code = "Insolvenzordnung".

³The model presented in this paper is referred to as the GIC model in figures and tables.

value, which is discounted by the default risk-free interest rate i_f , given that the default probability λ is risk-neutral. Accordingly, the JT model assumes the recovery rate to be generally paid out at the end of term T . This prevents a default from leading to premature repayment, which may result in a default risky zero-coupon bond having a higher value than a default, risk-free zero-coupon bond.

For a nominal amount V_{Nom} the following equation results:

$$V_{\text{ZCB}}^0 = \frac{(1 - \lambda_T^u) \cdot V_{\text{Nom}} + \lambda_T^u \cdot V_{\text{Nom}} \cdot \delta}{(1 + i_f)^T}, \quad (1)$$

where δ is the uniform recovery rate for priority and non-priority claims and λ_T^u is the independent default probability until the bond's maturity T , which is calculated as follows:

$$\lambda_T^u = \sum_{t=1}^T \lambda_t^d \cdot \prod_{c=1}^{t-1} (1 - \lambda_c^d), \quad (2)$$

where λ_T^u is derived from the subperiods' dependent probabilities λ_t^d , given that $\lambda_0^u = \lambda_0^d = 0$. The first factor represents the dependent default probability in the respective period, whereas the product of the remaining factors indicates the survival probability until that period. This default structure is illustrated in Fig. 1.

Evaluating coupon bonds, the JT model assumes that coupons are paid out at their maturity in case of default, and therefore coupon bonds can be treated like a portfolio of zero-coupon bonds. A coupon bond's value, which

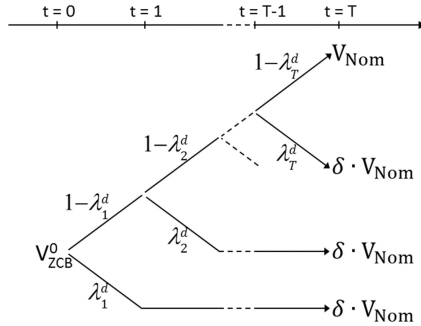


Fig. 1. Payments for a default, risk-free zero-coupon bond according to JT model (figure is analogous to Jarrow and Turnbull, 1995).

Legend: Bond's maturity T , nominal amount V_{Nom} , δ the recovery rate, λ_t^d the dependent default probability at time t , V_{ZCB}^0 the zero-coupon bond's value at time 0.

has a coupon rate c and is defined as

$$V_{\text{CB}}^0 = \sum_{t=1}^T \frac{(1 - \lambda_t^u) \cdot c \cdot V_{\text{Nom}} + \lambda_t^u \cdot V_{\text{Nom}} \cdot c \cdot \delta}{(1 + i_f)^t} + \frac{(1 - \lambda_T^u) \cdot V_{\text{Nom}} + \lambda_T^u \cdot V_{\text{Nom}} \cdot \delta}{(1 + i_f)^T}, \quad (3)$$

until point in time T , thus includes the expected discounted payments from both the coupons and the nominal amount.

2.2. *German insolvency law*

The JT model is generally based on the U.S. insolvency law, which differs from the German insolvency code in many ways. To determine the modifications that may be necessary for German bonds, the JT model is analyzed with regard to the German provisions.⁴

Accordingly, the GIC's provisions referring to creditor claims and their satisfaction from the assets in the insolvency (Eickmann, 2006, Section 39 RdNr. 13⁵) are of crucial significance — especially the provisions referring to the ranking of a claim. The ranking of a claim in relation to the other claims in the bankruptcy proceedings is determined by Section 39 GIC. Moreover, a non-priority ranking can be agreed upon explicitly by the creditor and debtor within the legal framework, whereas an agreed-upon super priority is not possible. Therefore, all claims that are not non-priority claims are regarded as priority claims in this paper.

The ranking of a claim affects the chronological payment structure and — because of the possibility of interim payments for priority claims during the bankruptcy proceedings — particularly the expectation as to how much of the outstanding claim will be paid back (Keenan, 2000), with priority claims being given absolute priority over non-priority claims.⁶

All claims that are not covered by Section 39 GIC are “proper” insolvency claims according to Section 38 GIC and they are generally of equal ranking. Particularly the “interest and late charges on the creditors’ claims [...] that

⁴As of December 2007. The Legal Services Act (“Gesetz zur Neuregelung des Rechtsberatungsrechts”) is the latest amendment (December 12, 2007).

⁵Commentaries on German laws are usually available in German. For this reason we do not translate the indication of the source. The term “Randnummer” (RdNr.) refers to subparagraphs.

⁶This is in contrast to the U.S. Chapter 11, in which the so-called “absolute priority violation” is quite common.

have accrued since the institution of the bankruptcy proceedings” are always non-priority claims according to the wording of Section 39.1 GIC.⁷

Summary of the first valuation-relevant provision:

Interest that has accrued over a period before the institution’s bankruptcy proceedings is of equal ranking as the claim itself (see also Section 39.3 GIC). Interest that has accrued since the institution’s bankruptcy proceedings, in contrast, is of subordinate or, at best, equal ranking compared with the claim itself.

Claims of equal ranking are paid back in proportion to their amount, i.e., in equal shares. Therefore, the amounts of the individual claims have to be determined.

In accordance with Section 41.1 GIC, also those claims that are not yet due are regarded as due when the bankruptcy proceedings are instituted. The amount of the due claim is determined by Section 41.2 GIC. The law differentiates between interest-bearing and non-interest-bearing claims; this results in a different handling of coupon bonds and zero-coupon bonds. The legally predefined procedure for determining the amounts of the claims follows a so-called “fair value view”; this may produce results that, at first, do not appear to make sense from an economic point of view.

With regard to zero-coupon bonds, Section 41.2 GIC determines that non-interest-bearing claims are to be discounted by a statutory interest rate, which is 5% according to Section 352 German Commercial Code⁸ (as of October 1, 2011), from their original maturity to the time of the institution’s bankruptcy proceedings. In the case of a non-bilateral commercial transaction, a statutory interest rate of 4% (according to Section 246 German Civil Code⁹) is to be applied. In this way, the zero-coupon bond’s implicit interest, which is considered in the bond price, is separated from the primary claim; this corresponds to the principle of subordination of interest accruing after the institution’s bankruptcy proceedings.¹⁰

⁷ German wording of the citation: “die seit der Eröffnung des Insolvenzverfahrens laufenden Zinsen und Säumniszuschläge auf Forderungen der Insolvenzgläubiger [...]”

⁸ German commercial code = “Handelsgesetzbuch”.

⁹ German Civil Code = “Bürgerliches Gesetzbuch”.

¹⁰ “It is aimed at equalization with the creditors of interest-bearing claims [...]” (Bitter, 2007, Section 41 RdNr. 17; German wording of this citation: es “soll eine Gleichstellung mit den Gläubigern verzinslicher Forderungen erreicht [...] werden”).

Summary of the second valuation-relevant provision:

The (discounted) primary claim of zero-coupon bonds is treated as an ordinary priority claim.

Because of the explicit reference to non-interest-bearing claims, it can also be derived from Section 41,2 GIC that interest-bearing claims have to be taken into account in the amount of their nominal amount (Bitter, 2007, Section 41 RdNr. 17, etc.). Therefore, the rate of the agreed interest is irrelevant. The primary claim of a coupon bond with an almost zero coupon clearly differs from that of a comparable zero-coupon bond. Apart from the primary claim, a (non-priority) interest claim is incurred to the creditors for the duration of the bankruptcy proceedings. Once again, it has to be differentiated between zero-coupon bonds and coupon bonds.

Summary of the third valuation-relevant provision:

With regard to coupon bonds, the interest that has accrued since the institution's bankruptcy proceedings is determined on the basis of the agreed coupon.¹¹

As to zero-coupon bonds, there is no explicit, contractual interest rate. However, because they are discounted by the statutory interest rate for the determination of a claim, it seems “only logical to grant the statutory interest rate [...] also to creditors of discounted claims” (Bitter, 2007, Section 41 RdNr. 19).¹² Summary of the fourth valuation-relevant provision:

With regard to zero-coupon bonds, the interest that has accrued since the institution's bankruptcy proceedings is determined on the basis of the statutory interest rate.

¹¹Ehricke (2007, Section 39 RdNr. 12 f) is of the same opinion. The application of a (relatively high) interest on arrears is out of the question because it is aimed at “preventing the creditor — right from the start — from receiving more than he is entitled to according to his claim” (German wording of this citation: es soll “im Ansatz verhindert werden, dass der Gläubiger mehr erhält als ihm nach dem Inhalt seiner Forderung zusteht”).

¹²German wording of the citation: “nur konsequent, auch den Gläubigern der abgezinsten Forderungen den gesetzlichen Zinssatz [...] zuzusprechen.”

The duration of the bankruptcy proceedings is not taken into account in the JT model because all payments are made at points in time that are known in advance. However, the time of the institution's bankruptcy proceedings in relation to the bond's maturity and the duration of the bankruptcy proceedings until the final payment are of crucial importance because of the GIC's provisions.

Summary of the last valuation-relevant provision:

The payment to creditors is additionally determined by the duration of the bankruptcy proceedings.

Apart from the final payment (Section 196 GIC), there is also the possibility of interim payments (Section 187.2) to satisfy a claim; however, this possibility is only for priority claims because “non-priority creditors [...] are not taken into account as to interim payments”¹³ (Section 187,2 GIC).

Analogous to the JT approach, the valuation-relevant consequences resulting from amounts and points in time of interim payments are not taken into account in the empirical analyses as it is hardly possible to anticipate them ex-ante because of a lack of universal practical experience (Füchsl and Weishäupl, 2007, RdNr. 1). Nevertheless, they are formally taken into account in the theoretical considerations.

Hence, the overall expected repayment in the case of an insolvency, which is based on the legal provisions presented and summarized previously, is dependent on the following factors of influence:

- Whether the bond is a coupon bond or a zero-coupon bond;
- The ranking of the primary claim;
- The recovery rate for priority and non-priority claims;
- The duration of the bankruptcy proceedings; and
- The bond's remaining term at the time of the institution's bankruptcy proceedings.

3. Theoretical Implications

3.1. *Zero-coupon bonds*

Next, the five valuation-relevant provisions will be implemented into the JT model. Similarly to Chapter 2, we use a discrete model. However, despite its

¹³German wording of the citation: “nachrangige Insolvenzgläubiger [...] bei Abschlagsverteilungen nicht berücksichtigt werden.”

simplicity, the model is suitable for considering real questions if calibrated adequately, as will become apparent later on.

A priority zero-coupon bond with a term T is considered. Therefore, the bond's remaining term is $T - t$ at each point in time $t = 0, \dots, T$. The nominal value V_{Nom} will be paid back if a company does not default during that term.

Let t_i be the point in time when a company declares bankruptcy. If the company defaults at $t_i < T$, then the bankruptcy proceedings will be instituted at t_i , and from that point in time all financial obligations will be met in accordance with the legal provisions only. The definite illiquidity is the consequence of a company's default; restructuring of the company is ruled out. Thus, a company remains insolvent at all subsequent points in time $t_i + 1, \dots, t_i + D$, with D indicating the duration of the bankruptcy proceedings (analog, Jarrow and Turnbull, 1995, p. 58). The assets in the insolvency are distributed for the last time and the company is liquidated after the completion of the bankruptcy proceedings at $t_i + D$.

The amount of the primary insolvency claim results from the discounting of V_{Nom} by the statutory interest rate i_g , which is divided into periods, from t_i to the bond's maturity T . In addition, there is a non-priority interest claim derived from the insolvency claim for the proceedings' duration D . The subsequent considerations of claims refer to the time of the bankruptcy proceedings' completion.

Until the end of the proceedings $t_i + D$, a total claim $C_{\text{ZCB},p}(t_i)$

$$C_{\text{ZCB},p}(t_i) = \frac{V_{\text{Nom}}}{1 + (T - t_i) \cdot i_g} + \left(\frac{V_{\text{Nom}}}{1 + (T - t_i) \cdot i_g} \right) \cdot i_g \cdot D \quad (4)$$

resulting from the primary and interest claims is incurred to the creditor.

Because of the prohibition against the application of compound interest, which is in accordance with Section 248.1 German Civil Code (exceptions from this prohibition are defined in Section 248.2 German Civil Code), the calculation differs from the commonly used economic form of discounting. Therefore, the primary claim and interest claim are determined by the application of Hofmann's formula (Bitter, 2007, Section 41 RdNr. 21 f).

Interim payments at a_t time t can be made at any discrete point in time between the bankruptcy proceedings' institution and the last period before the completion of the proceedings, and they can be made in any amount desired. For considering the payment structure, a (average) recovery rate δ of the total claim is required. In accordance with common practice, we assume that the interim payments' total is not larger than the total payment

that the creditor is entitled to in the course of distribution (Füchsl and Weishäupl, 2007, Section 187 RdNr. 9) so that

$$\sum_{t=t_i}^{t_i+D-1} a_t \leq C_{\text{ZCB},p}(t_i) \cdot \delta \quad (5)$$

applies. An interim payment made during the last period of the proceedings is regarded as part of the final payment. An interim payment a_t directly decreases the amount of the primary insolvency claim so that the outstanding primary claim is

$$C_{\text{ZCB},p}^{\text{Prim}}(t_i) = \frac{V_{\text{Nom}}}{1 + (T - t_i) \cdot i_g} - \sum_{t=t_i}^{t_i+D-1} a_t, \quad (6)$$

at the end of the proceedings at $t_i + D$. In addition, there is the non-priority interest claim without compound interest, which is based on the primary claim at the time of the bankruptcy proceedings' institution, so that the outstanding interest claim is

$$C_{\text{ZCB},p}^{\text{Interest}}(t_i) = \sum_{t=t_i}^{t_i+D-1} \left(\frac{V_{\text{Nom}}}{1 + (T - t_i) \cdot i_g} - \sum_{c=t_i}^{2t_i+D-1-t} a_c \right) \cdot i_g, \quad (7)$$

at $t_i + D$ after the application of Eq. (6). To comply with the legal provisions of the GIC, a compounding/discounting of interim payments has deliberately been omitted from this equation and the subsequent equations that include interim payments.

$$C_{\text{ZCB},p}^{\text{Interest}}(t_i) = i_g \cdot \sum_{t=t_i}^{t_i+D-1} \frac{V_{\text{Nom}}}{1 + (T - t_i) \cdot i_g} - (D - t + t_i) \cdot a_t \quad (8)$$

is obtained from transformation. According to this approach, the interest return related to the interim payment is deducted from the total interest claim. This deduction from the first interim payment after the declaration of bankruptcy is made for the proceedings' overall duration; the duration is shorter with regard to all further payments. Thus, the outstanding claim's total amount $C_{\text{ZCB},p}(t_i)$ is

$$\begin{aligned} C_{\text{ZCB},p}(t_i) = & \frac{V_{\text{Nom}}}{1 + (T - t_i) \cdot i_g} - \sum_{t=t_i}^{t_i+D-1} a_t + i_g \cdot \sum_{t=t_i}^{t_i+D-1} \frac{V_{\text{Nom}}}{1 + (T - t_i) \cdot i_g} \\ & - (D - t + t_i) \cdot a_t \end{aligned} \quad (9)$$

at the end of the bankruptcy proceedings, with interim payments being taken into account. Recovery rates for claims of different ranking differ in

that not all claims can be fully met. A considerable non-priority recovery rate can be identified in the United States, partially on the basis of the absolute priority rule violation within the context of Chapter 11, but the recovery rate's expected value is greater than zero even in cases according to Chapter 7 (Bris *et al.*, 2005).

Thus, the payment P_{ZCB} resulting from a claim C_{ZCB} is

$$P_{\text{ZCB},p}(t_i) = \frac{V_{\text{Nom}}}{1 + (T - t_i) \cdot i_g} \cdot \delta_V - \sum_{t=t_i}^{t_i+D-1} a_t + i_g \cdot \sum_{t=t_i}^{t_i+D-1} \frac{V_{\text{Nom}}}{1 + (T - t_i) \cdot i_g} - (D - t + t_i) \cdot a_t \cdot \delta_N \quad (10)$$

at the end of the proceedings, whereby δ_V is the recovery rate for the nominal value (priority) und δ_N is the recovery rate for the interest payments (non-priority).

All in all, the value of a priority zero-coupon bond $V_{\text{ZCB},p}^0$ with a remaining term of T periods is calculated using

$$V_{\text{ZCB},p}^0 = \sum_{t=t_i}^T \frac{(\lambda_{t_i}^u - \lambda_{t_i-1}^u)}{(1 + i_f)^{t_i}} \cdot \left[P_{\text{ZCB},p}(t_i) \cdot (1 + i_f)^{-D} + \sum_{c=t_i}^{t_i+D-1} a_c \cdot (1 + i_f)^{-(c-t_i)} \right] + \frac{(1 - \lambda_T^u)}{(1 + i_f)^T} \cdot V_{\text{Nom}}, \quad (11)$$

where $P_{\text{ZCB},p}(t_i)$ is determined according to Eq. (10). When comparing Eq. (11) with the corresponding equation of the JT model (1), differences in the payment structure in the event of a default can be recognized. Although the payment of the recovery rate in the JT model is made at the end of the bankruptcy proceedings (see also Fig. 2), the nominal amount and the payment of the recovery rate in our modified model are — in the first instance — discounted by the statutory interest rate to the time of default (see also Fig. 2). In contrast to the JT model, the (final) payment is not made at the end of the term but at the end of the proceedings. Moreover, the presented model even includes the possibility of interim payments during the bankruptcy proceedings. For the purpose of valuation, all payments are then discounted by the risk-free interest rate to the day of valuation according to (11).

The priority payment in the presented model is cumulatively lower than that of the JT model because the nominal amount is discounted before it is paid out. This disadvantage is weakened by the non-priority interest payment (see Fig. 2), which is made at the end of the proceedings.

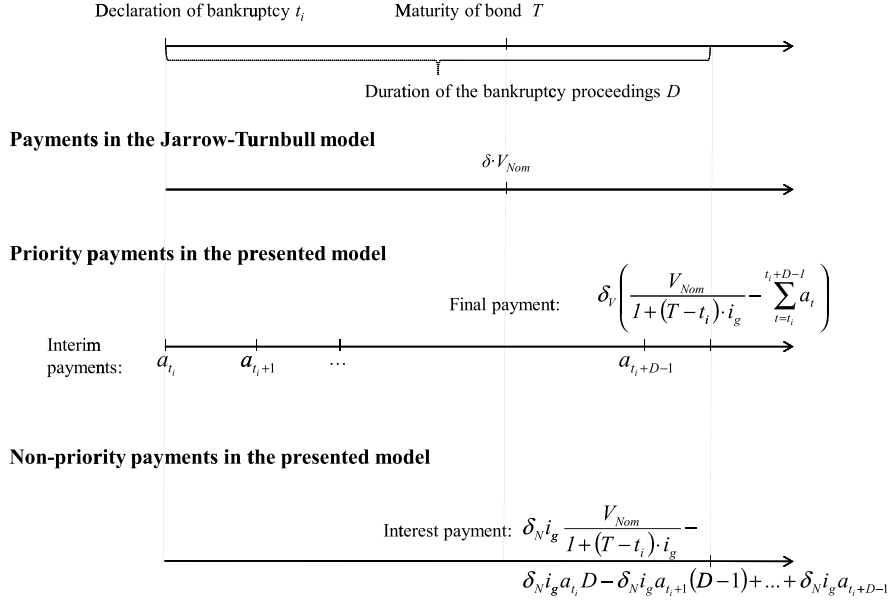


Fig. 2. Comparison of payments of zero-coupon bonds after declaring bankruptcy.

Legend: Bond's maturity T , nominal amount V_{Nom} , recovery rate (JT) δ , recovery rate (priority) δ_v , recovery rate (non-priority) δ_N , a_t interims payment at time t , statutory interest rate i_g , duration of bankruptcy proceedings D .

Because of the discounting of the nominal amount, the claim is in fact divided into a priority and non-priority claim. In the case of equal parameterization of the models, the values derived from the JT model are higher, particularly in case of a long remaining term and short duration of the proceedings, than those derived from the modified model, even if the recovery rates for priority and non-priority claims are assumed to be equal.

Apart from the comparison with the JT model, comparisons with the models of Duffie (1998) and Duffie and Singleton (1999) are also reasonable. A comparison of the case in which the recovery rate is a function dependent on the repayable amount at the time of declaring bankruptcy shows that the value derived from the presented model is considerably lower as a result of the discounting by the statutory interest rate. Duffie and Singleton (1999), in contrast, assume the recovery rate to be a function of the value at the time bankruptcy is declared, which produces results that are closer to those of the model presented in this paper. However, the rate of the statutory interest compared with that of the current market interest (risk-free interest) is crucial.

These considerations refer to priority zero-coupon bonds. If non-priority zero-coupon bonds are to be considered instead, there are two changes. First, interim payments during the bankruptcy proceedings can be ruled out regarding non-priority claims; and second, primary and interest claims are of the same ranking and therefore have the same recovery rate. Therefore, the payment at the end of the bankruptcy proceedings is

$$P_{\text{ZCB},s}(t_i) = V_{\text{Nom}} \cdot \frac{1 + D \cdot i_g}{1 + (T - t_i) \cdot i_g} \cdot \delta_N; \quad (12)$$

hence, the value is

$$V_{\text{ZCB},s}^0 = \sum_{t_i=1}^T \frac{(\lambda_{t_i}^u - \lambda_{t_i-1}^u)}{(1 + i_f)^{t_i+D}} \cdot V_{\text{Nom}} \cdot \frac{1 + D \cdot i_g}{1 + (T - t_i) \cdot i_g} \cdot \delta_N + \frac{(1 - \lambda_T^u)}{(1 + i_f)^T} \cdot V_{\text{Nom}}. \quad (13)$$

3.2. *Coupon bonds*

We consider a priority coupon bond for which a coupon c has been agreed upon for each period as a percentage related to the nominal amount. The interest accrued before the declaration of bankruptcy is of equal ranking as the underlying claim (Ehricke, 2007, Section 39 RdNr. 13), so that the outstanding primary claim $C_{\text{CB},p}^{\text{Prim}}(t_i)$ of the coupon bond amounts to

$$C_{\text{CB},p}^{\text{Prim}}(t_i) = V_{\text{Nom}} \cdot (1 + CU) - \sum_{t=t_i}^{t_i+D-1} a_t \quad (14)$$

at the end of the bankruptcy proceedings. CU is the proportionate coupon for the period from the last coupon date until the institution's bankruptcy proceedings. In addition, there is an interest claim for the duration of the bankruptcy proceedings, which is based on the coupon's amount in the case of coupon bonds, so that the interest claim $C_{\text{CB},p}^{\text{Interest}}(t_i)$ of the coupon bond amounts to

$$C_{\text{CB},p}^{\text{Interest}}(t_i) = c \cdot \sum_{t=t_i}^{t_i+D-1} V_{\text{Nom}} - (D - t + t_i) \cdot a_t \quad (15)$$

at the end of the bankruptcy proceedings. Again, the principle of prohibition of compound interest applies. If priority and non-priority recovery rates are considered, this claim, which is derived from a priority coupon bond, results

in a payment of

$$\begin{aligned}
P_{CB,p}(t_i) = & V_{\text{Nom}} \cdot (1 + CU) \cdot \delta_V - \sum_{t=t_i}^{t_i+D-1} a_t \\
& + c \cdot \sum_{t=t_i}^{t_i+D-1} V_{\text{Nom}} - (D - t + t_i) \cdot a_t \cdot \delta_N \quad (16)
\end{aligned}$$

at the end of the bankruptcy proceedings. As a consequence, a coupon bond's value is

$$\begin{aligned}
V_{CB,p}^0 = & \sum_{t_i=1}^T \frac{(\lambda_{t_i}^u - \lambda_{t_i-1}^u)}{(1 + i_f)^{t_i}} \frac{P_{CB,p}(t_i)}{(1 + i_f)^D} + \sum_{c=t_i}^{t_i+D-1} \frac{a_c}{(1 + i_f)^{c-t_i}} \Bigg) \\
& + \sum_{t_i=1}^T \frac{(1 - \lambda_{t_i}^u) \cdot c \cdot V_{\text{Nom}}}{(1 + i_f)^{t_i}} + \frac{(1 - \lambda_T^u) \cdot V_{\text{Nom}}}{(1 + i_f)^T}, \quad (17)
\end{aligned}$$

with $P_{CB,p}(t_i)$ being determined according to Eq. (16).

When comparing this equation with that of the JT model (3), differences can be recognized once again: Although the payment of the recovery rate from the nominal amount in the JT model is made at the end of the proceedings and discounted over the entire term, the payment in the modified model is discounted by the duration of the bankruptcy proceedings plus the term to date (see also Fig. 3). Recovery rates on coupon payments in the JT model are discounted on the basis of the coupon dates, whereas the non-priority recovery rate on coupon payments in our model is discounted over the period until declaration of bankruptcy plus the duration of the bankruptcy proceedings. Compared with the JT model, the modified model can provide both higher and lower values. In the case of high coupon values, a non-priority recovery rate, and a long duration of the bankruptcy proceedings, a bond can — in the event of insolvency — result in a payment that is higher than without insolvency because creditors are granted compensation in the coupon's amount for the duration of the bankruptcy proceedings; this is in contrast to the JT model. The valuation by means of the presented model produces comparably lower results for bonds with, for example, short remaining terms, low coupons, and — at the same time — long durations of the bankruptcy proceedings.

Again, a comparison with the other two models seems reasonable. A comparison with the case that the recovery rate is a function that is dependent on the repayable amount at the time of declaring bankruptcy always results in values that are higher than in the presented model. First, this

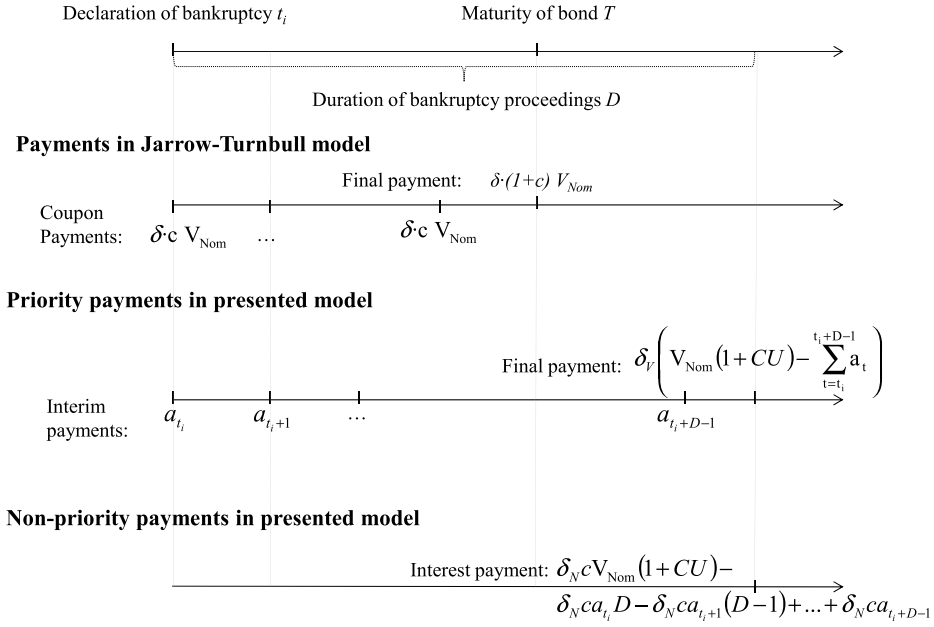


Fig. 3. Comparison of payments of coupon bonds after declaring bankruptcy.

Legend: Bond's maturity T , nominal amount V_{Nom} , recovery rate (JT) δ , recovery rate (priority) δ_V , recovery rate (non-priority) δ_N , a_t interims payment at time t , duration of bankruptcy proceedings D , coupon rate c , CU is the proportionate coupon for the period from the last coupon date until the institution's bankruptcy proceedings.

is because the coupon payment in the modified model is always of subordinate ranking. Second, the compound interest is not taken into consideration with regard to the coupon payment, so that compounding without compound interest and discounting with risk-free compound interest results in another effect that tends to decrease the recovery rate. If the presented model is compared with the assumption of Duffie (1998), there is a tendency to expect a higher value. Again, this results from the combination of subordination and the compound interest that is not considered.

A comparison with the assumption of Duffie and Singleton (1999), i.e., the recovery rate being a function of the value at the time bankruptcy is declared, produces results that are closer to those of the model presented in this paper. However, the rate of the statutory interest compared with that of the current market interest (risk-free interest) is crucial. Furthermore, the end of the bankruptcy proceedings can — in relation to the time of maturity — influence the result in that the value will be higher or lower depending on the coupon's amount compared with the risk-free interest rate.

The implications for non-priority coupon bonds are outlined subsequently for the sake of completeness. Because of a lack of interim payments and because there is only one uniform recovery rate δ_N , the claim $C_{CB,s}$ resulting from a non-priority coupon bond amounts to

$$C_{CB,s} = V_{\text{Nom}} \cdot (1 + CU) + V_{\text{Nom}} \cdot c \cdot D = V_{\text{Nom}} \cdot (1 + CU + c \cdot D) \quad (18)$$

at the time of the final payment, regardless of the time bankruptcy is declared; this leads to a repayment of

$$P_{CB,s} = V_{\text{Nom}} \cdot (1 + CU + c \cdot D) \cdot \delta_N \quad (19)$$

provided that the recovery rate is considered. A non-priority coupon bond's fair value $V_{CB,s}^0$ accordingly amounts to

$$\begin{aligned} V_{CB,s}^0 = & \sum_{t_i=1}^T \frac{(\lambda_{t_i}^u - \lambda_{t_i-1}^u)}{(1 + i_f)^{t_i+D}} \cdot P_{CB,s} + \sum_{t_i=1}^T \frac{(1 - \lambda_{t_i}^u) \cdot c \cdot V_{\text{Nom}}}{(1 + i_f)^{t_i}} \\ & + \frac{(1 - \lambda_T^u) \cdot V_{\text{Nom}}}{(1 + i_f)^T} \end{aligned} \quad (20)$$

with the two sums not being aggregated into one single sum for the sake of better traceability.

4. Empirical Analysis

4.1. Data set

Bonds of German DAX firms have been selected as the basis for the analysis because they generally guarantee sufficient liquidity and a more or less fair pricing as a result of their high issue volume.

The bond prices for simple, unsecured priority coupon bonds have been provided by ThomsonReuters Datastream[®]. This applies to 21 DAX firms, which issued a total of 103 priority bonds with a one-year interest rate frequency without optional features and for which bond prices are available for the period from January 1, 2006, to March 31, 2009 (with regards to the data distribution, see also Fig. A.2 in the Appendix).

The data set's descriptive statistics are also shown in Table 1.

All of the bonds surveyed have adequately long terms to have sufficient liquidity (see also Fig. A.3 in the Appendix). The coupon is sufficiently distributed (see Table 1) to make the potential postulated insolvency-related coupon effect visible.

Table 1 further includes data on the required credit spreads (*source*: Thomson Reuters; time series of the CMA), which in most cases are

Table 1. Descriptive statistics of the data set.

	Minimum	Mean	Median	Maximum	Standard Deviation	Bonds/CDS	Number of Observations
Coupon (%)	1.65	4.86	5.00	9.38	1.53	103	53,624
Term (years)	0.43	4.59	4.36	10.03	2.13	103	53,624
Number of bonds per issuer	1.00	4.90	4.00	19.00	4.56	103	53,624
CDS (1 year) (basis points)	1.00	54.62	13.80	2,629.16	142.93	21	17,067
CDS (2 years)	2.20	65.45	23.80	2,648.26	144.06	20	16,243
CDS (3 years)	3.50	70.37	29.20	2,331.44	140.49	21	17,067
CDS (4 years)	4.40	80.56	39.80	2,139.18	141.04	20	16,243
CDS (5 years)	5.50	84.20	44.30	2,070.70	138.72	21	17,067
CDS (6 years)	6.70	92.05	52.40	1,999.01	138.81	20	16,243
CDS (7 years)	7.70	91.45	54.20	1,947.10	134.31	21	17,067
CDS (8 years)	8.90	98.20	60.10	1,875.82	134.53	20	16,243
CDS (9 years)	9.90	99.91	62.80	1,906.64	134.26	20	16,243
CDS (10 years)	10.20	98.40	62.80	1,851.75	130.81	21	17,067

Explanation: The largest portion of the bonds (69%) has a coupon between 4 and 7%, with coupons between 5 and 6% (31 cases) and coupons between 6 and 7% (26 cases) occurring most often. A large portion of the bonds (66%) has a term between 3 and 9 years. Therefore, bonds with terms between 3 and 4 years (18.1%), 4 and 5 years (19.3%), and 5 and 6 years (13.0%) are predominant. Regarding terms 2, 4, 6, 8, and 9 years, no data is available for the German stock exchange (Deutsche Boerse). With regard to Merck, there are no CDS spreads available for any of those terms before December 4, 2006. Column 7: Line 2 to 4 statistics rely on the number of bonds observed, line 5 to 14 rely on the number of CDS available. Column 8: Line 2 to 4 statistics rely on the total quotations of bonds, line 5 to 14 statistics rely on the quotations of CDS available.

available for all issuers and terms for the entire period (see Table 2). A strong skewness to the right can be recognized in the credit spread distribution. This can be generally attributed to the high CDS spreads during the financial crisis.

4.2. Methodology

The equation generated in the preceding chapter seems better suited for valuating bonds as a result of the provisions of the insolvency law, but the question of which approach is chosen by the market participants has not been answered yet. To answer this question, coupon bonds that are traded on the market are assessed by means of both the model presented in Chapter 3 and the JT model. Because interim payments are made mostly on an ad hoc basis, they are not included in either of the models. Among the numerous input parameters, the default probability is a central one that has to be determined. Therefore, it is important to generate implicit default probabilities that are not dependent on a specific type of coupon. By means of using credit default swaps (CDSs), this is the only possibility because a recovery rate is not dependent on only one bond — according to the contractual details — but usually on more bonds (see Entrop *et al.*, 2010). Usually, more bonds or claims are selected at the time of issuance; in the case of insolvency, one of them is used to determine the recovery rate. The one who pays the premium of the CDS is entitled to make a delivery option. Because the payer of the premium is interested in keeping the recovery rate at a minimum level, and because there is a certain range of possible recovery rates, the expected recovery rate differs from the one assumed by the investors in their valuation of individual specific bonds. Therefore, it should be determined separately from the valuation of bonds.

In addition, the simple method is applied for specifying the recovery rate beforehand. In accordance with the suggestion of Altman and Kishore (1996), it is initially set at 30%. With the help of this assumption, the simple method can be used for determining an implicit default probability, which — in a next step — can be used for the valuation of the bonds regarding the various provisions. This is indeed a restrictive assumption, which is discussed further in Sec. 4.4.

For determining the implicit default probability, we apply the method of Rathgeber and Wang (2011), which determines the implicit default probability of a one-year CDS on the basis of its cash flow so that the swap's net present value is zero. The one-year default probability determined in this

way can — in a further step — be used to determine the one-year payments of a two-year CDS; hence, the two-year default probabilities can be determined. This method can also be applied if no CDSs exist for all terms; in this case, the subsequent probability is (simply) determined by omitting the preceding probability. By repeating this procedure until the 10-year default probability, we obtain 10 cumulative, implicit default probabilities that are then complemented by a default probability of zero for a period of zero years to determine — by means of a spline cubic interpolation — the implicit default probabilities also for terms of less than 1 year (see Rathgeber and Wang, 2011). In the rare cases in which terms of more than 10 years are available, the same spline is extrapolated beyond the 10-year period. The curves obtained in this way are all monotonically increasing; this is a necessary prerequisite for an arbitrage-free valuation.

Apart from the risk-free interest rate, which is determined by means of the Svensson method (see also, e.g., Steiner *et al.*, 2012, pp. 162–166) on the basis of German federal loans, the valuation of bonds also requires their cash flow and a recovery rate; this is in accordance with Eqs. (3) and (17). For determining the recovery rate there is the possibility of using historical recovery rates. If this alternative is chosen, a real, risk-averse measure is used for the recovery rate; however, the tested models require a risk-neutral measure for the recovery rate. Furthermore, there may be different recovery rate regimes for both models, so that a joint estimation of both models with only one recovery rate can result in systematic distortions. To be able to apply this method, the recovery rate is assumed to be the one expected factor that is independent of the default probability.

Similarly to Bakshi *et al.* (2006), we therefore revert to the determination of implicit recovery rates that are derived from bond prices. As to that method, the recovery rate¹⁴ is selected in a way that the difference between the bonds' model and market values is the variable that is to be minimized. In particular, the sum of the root-mean-square deviations RMSE of the relative price differences is minimized on a daily basis for all M bonds of an issuer by means of the Nelder–Mead Simplex method:

$$\text{RMSE} = \min_{\delta_V, \delta_N} \sqrt{\frac{1}{M} \sum_{i=1}^M \left(\frac{V_{\text{CB}}^0 - V_{\text{Market}}^0}{V_{\text{Market}}^0} \right)^2} \quad (21)$$

¹⁴In the GIC model, we use two recovery rates for bonds that are exclusively priority bonds and one recovery rate for priority and non-priority interest payments. The latter cannot occur ex-post, in a risk-neutral environment; however, it may occur *ex-ante*.

where V_{CB}^0 represents the calculated model value for the respective coupon bond and V_{Market}^0 represents the related market value. In accordance with Bakshi *et al.* (2006), the relative price differences are replaced by the yield rate differences in a modification of this method, with the main focus being placed on the analysis of the relative price differences. Because this may partly result in negative recovery rates (depending on the method), the recovery rates are restricted to the interval $[0,1]$ to calculate the root mean squared error (RMSE). A model's adaptability to market data can be detected by means of the sample's test static RMSE. To obtain also out-of-sample evidence, we apply a methodology that ignores one bond when estimating the recovery rate (i.e., $M - 1$ bonds are estimated); this approach is analogous to the yield curve estimation method (see Breedon *et al.*, 1996). Then, using the ignored bond as the basis, the RMSE is determined as a test static for the model's quality; each of the bonds is alternately excluded from the estimations once, so that M estimations are made per day.

Furthermore, the duration of the bankruptcy proceedings has to be determined to estimate the model presented in this paper. Similarly to Heyrath (2004, p. 136), the duration is assumed to be 5 years; however, this assumption is discussed further in a subsequent section.

4.3. Results

The implicit recovery rate for the basic case (in-sample test) is determined in a first step to determine the test static model quality. The implicit recovery rates are presented in Table 2. The recovery rates differ significantly depending on the period and company. The numerous implicit recovery rates that have been corrected to zero are particularly striking. In particular with regard to non-priority recovery rates, this number is even more extreme if the difference in the yield rate is used instead of the relative price difference for calculating the RMSE. Furthermore, it turns out that the recovery rates have to be slightly more adapted in the GIC model than they have to in the JT model; this can also be an indication of the model's quality. An analysis of the implicit recovery rates' time series shows that recovery rates had to be adapted in particular during the financial crisis (as to the robustness test, see Wilhelm and Brüning, 1992). Minimum values are not included in the table because they are always zero and therefore do not provide any insight.

Furthermore, the term structure curves of the (cumulated) implicit default probabilities are depicted in Fig. A.1 (see Appendix).

Table 2. Implicit recovery rates.

	JT Model (bond price)				GIC Model (bond price, priority)				GIC Model (bond price, non-priority)			
	Mean (%)	Maximum (%)	Standard Deviation (%)	Adjustment Rate (%)	Mean (%)	Maximum (%)	Standard Deviation (%)	Adjustment Rate (%)	Mean (%)	Maximum (%)	Standard Deviation (%)	Adjustment Rate (%)
Allianz	3.1	51.9	9.3	73.8	3.9	68.8	12.2	89.5	0.5	2.9	0.4	10.5
BASF	8.0	55.2	13.1	0.0	18.6	64.4	16.8	0.0	0.9	3.9	0.8	0.0
Bayer	0.9	32.4	4.0	94.8	1.4	42.3	5.5	69	0.6	4.0	0.6	6.9
Commerzbank	0.0	0.0	0.0	0.0	3.9	6.1	1.8	0.0	0.0	0.0	0.0	0.0
Daimler	10.6	51.0	14.3	30.5	2.3	39.0	4.7	75.7	2.1	18.3	2.9	0.0
Deutsche Bank	0.8	39.3	4.7	45.3	48.7	100.0	48.6	45.0	3.1	11.7	1.9	0.0
Deutsche Boerse	0.0	0.0	0.0	0.0	5.6	10.7	2.6	0.0	0.0	0.0	0.0	0.0
Deutsche Lufthansa	17.2	100.0	28.6	47.9	19.4	100.0	33.6	61.6	0.7	5.3	0.9	23.1
Deutsche Post	4.1	41.3	8.8	0.0	1.2	7.9	0.9	76.1	1.0	5.9	1.2	0.0
Deutsche Telekom	7.9	52.5	11.9	53.0	7.2	34.1	9.6	29.0	2.2	9.0	1.9	0.0
E.ON	1.3	31.3	4.6	0.0	2.5	46.6	3.9	61.8	2.2	7.2	1.7	0.0
Fresenius	30.9	56.9	13.5	0.0	41.1	77.3	18.7	0.0	0.3	4.2	0.9	28.0
Henkel	6.4	44.5	11.1	0.0	5.0	55.0	12.2	84.3	0.7	5.8	0.9	14.7
Linde	0.2	21.4	1.5	52.4	3.2	28.8	4.4	79.3	2.9	10.7	2.8	0.0
Merck	1.4	27.2	4.0	42.7	12.3	90.0	26.1	0.0	0.7	8.5	1.0	10.8
Metro	1.6	37.5	5.3	89.7	2.0	47.3	6.7	89.7	1.0	8.3	1.5	10.0
RWE	5.4	51.9	12.4	0.0	5.7	53.1	11.8	78.1	1.6	9.7	1.9	0.0
Siemens	3.3	62.2	9.7	28.6	9.6	83.8	19.2	0.0	1.0	6.9	1.2	9.6
Thyssen	5.9	33.1	8.8	47.6	30.3	94.0	22.7	0.0	1.3	14.4	2.1	0.0
TUI	36.9	57.4	12.6	0.0	50.9	100.0	34.8	14.6	7.7	41.8	6.8	0.0
Volkswagen	9.0	45.5	12.6	41.9	38.9	100.0	29.1	0.0	2.1	16.9	2.7	0.0

Explanation: JT (bond price) = descriptive statistics of the implicit recovery rates in the JT model (Eq. (3) δ); GIC (bond price, priority) = descriptive statistics of the implicit recovery rates of priority payments in the GIC model (Eq. (17) δ_V); GIC (bond price, non-priority) = descriptive statistics of the implicit recovery rates of non-priority payments in the GIC model (Eq. (17) δ_N); basic analysis with a recovery rate of 30% for CDS, duration of bankruptcy of 5 years and relative price variance in the JT model and, additionally, with a duration of the bankruptcy proceedings of 5 years and two recovery rates in the GIC model. Descriptive statistics are mean, maximum and standard deviation of the recovery rates. Furthermore, the adjustment rate reflects the portion of the recovery rates that have to be adjusted to the interval $[0,1]$.

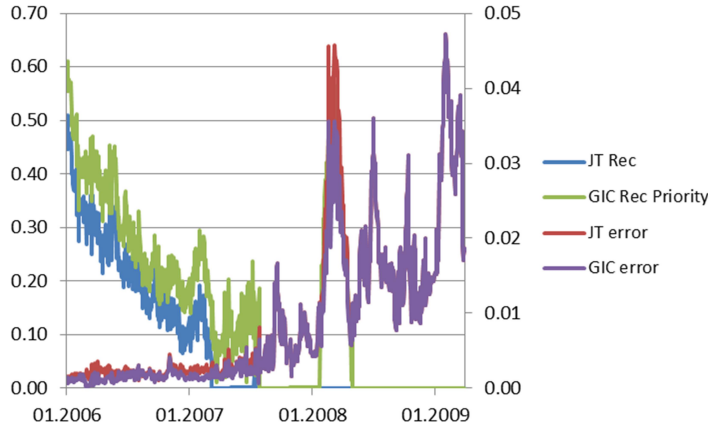


Fig. 4. Model errors (RMSE) obtained for Deutsche Telekom according to the JT model and the model presented in this paper (GIC).

Explanation: RMSE is calculated according to Eq. (21), errors and implied recovery rates (rec) are calculated according to Eq. (3) (JT) and Eq. (17) (GIC) (see also Tables 2 and 3).

In a next step, we analyze the adaption quality on a daily basis; therefore, we first use the RMSE of the relative price differences. For Deutsche Telekom, we obtain a curve as depicted in Fig. 4; this curve is very similar to those of most other issuers. On the one hand, Fig. 4 shows a dramatic increase in the errors (indicated in percent) for both models; this is a consequence of the financial crisis. These errors are at an extremely high level regarding the years 2008 and 2009. This increase can be explained by the liquidity crisis, which turned bonds into hard-selling instruments, which — depending on the stock exchange — lead to price distortions. It is remarkable that the increase can already be observed for the year 2007.

Furthermore, both models show a slight increase in the errors to a level of more than 5% already for the year 2007; apparently, the first heralds of the financial crisis had already been perceptible in mid-2007.

Finally, compared with the JT model, the presented model is — according to Eq. (17) — distinctively superior with regard to the subject of the analysis. This superiority is slightly less distinctive only on a few days during the financial crisis for which we could detect price distortions.

These indications are also confirmed by the systematic analysis of the two models as to all companies of the data set according to Table 3. With regard to some companies, a highly significant superiority of the presented model is revealed. This significance is detected by means of both the Wilcoxon rank-sum test analyzing the symmetry of the pair differences' distribution as to the median and a positive (arithmetic) mean of the pair differences

Table 3. Model errors (RMSE) of the two models (basic analysis).

	In-Sample Test				Out-of-Sample Test			
	Observations	GIC (error)	JT (error)	Significant Difference ("JT-GIC")	Observations	GIC (error)	JT (error)	Significant Difference ("JT-GIC")
Allianz	824	0.98%	0.98%	0%	1,477	2.99%	2.99%	0%
BASF	824	0.60%	0.70%	0.1%	3,484	1.30%	1.49%	0.19%***
Bayer	824	0.58%	0.60%	0.02%	—	—	—	—
Commerzbank	43	3.93%	3.93%	0%	—	—	—	—
Daimler	824	0.64%	1.02%	0.38%	5,672	2.77%	2.68%	−0.09%
Deutsche Bank	824	1.17%	1.24%	0.06%	15,011	1.45%	1.48%	0.03%***
Deutsche Boerse	250	5.58%	5.58%	0%	—	—	—	—
Deutsche Lufthansa	824	0.59%	0.70%	0.12%***	—	—	—	—
Deutsche Post	824	0.69%	0.74%	0.06%***	1,647	1.24%	1.13%	−0.11%
Deutsche Telekom	824	0.75%	0.96%	0.21%***	6,046	2.17%	2.30%	0.13%***
EON	386	1.54%	1.56%	0.02%	376	1.53%	1.69%	0.16%
Fresenius	815	0.20%	0.22%	0.02%***	—	—	—	—
Henkel	824	0.55%	0.53%	−0.02%***	25	3.82%	3.82%	0%
Linde	493	2.19%	2.15%	−0.03%	489	2.44%	2.52%	0.07%
Merck	587	0.46%	1.45%	0.99%***	344	0.41%	0.35%	−0.05%
Metro	824	0.81%	0.86%	0.05%	1,064	3.94%	3.93%	0%

Table 3. (Continued)

	In-Sample Test				Out-of-Sample Test			
	Observations	GIC (error)	JT (error)	Significant Difference ("JT-GIC")	Observations	GIC (error)	JT (error)	Significant Difference ("JT-GIC")
RWE	824	0.92%	0.86%	-0.06%	2,745	2.18%	2.21%	0.03%
Siemens	824	0.48%	0.92%	0.43%***	901	4.72%	5.10%	0.38%
Thyssen	824	1.97%	0.83%	-1.15%	1,710	1.39%	2.91%	1.52%***
TUI	824	1.08%	2.17%	1.09%***	1,647	3.33%	6.20%	2.86%***
Volkswagen	824	1.71%	1.31%	-0.39%	3,663	2.47%	2.76%	0.29%***
All	14,934	0.99%	1.08%	0.09%**	46,301	2.05%	2.27%	0.22%*

Explanation: GIC (error) = results of GIC model; JT (error) = results of JT model; * 10% significance level; ** 5% significance level; *** 1% significance level in accordance with Wilcoxon rank-sum test for the listed companies and Wilcoxon rank-sum test for clustered data (see line "All"). Error is calculated according to Eq. (21). GIC-model according to (17), JT according to Eq. (3). The difference is calculated as difference between the error (JT) and the error (GIC) and is afterwards averaged. The out-of-sample test is based on more observations than the in-sample test (sixth column compared with second column). This is due to the fact that for each day the number of estimations has to equal the number of bonds because in each of the estimations we ignore one bond, which is then used for determining the adaption quality. Due to the fact, less outstanding companies for five issuers an out-of-sample test was not possible (see also Table 2).

[difference: error (RMSE) according to the JT model minus error (RMSE) according to the GIC model]. With regard to some companies, the model presented in Sec. 3 even proves superior for all points in time of the analysis. The presented results can also be largely confirmed in the out-of-sample test (see Table 3). However, the out-of-sample test cannot be performed for a total number of five companies. In concrete terms, these five companies do not provide enough points in time at which two or more bonds are simultaneously quoted on the stock exchange.

Furthermore, the measuring errors of the presented model and of the JT model are mostly within a single-digit percentile range, with the measuring errors of the former usually being lower than those of the latter.

When considering which recovery rate produces a lower measuring error in the GIC model, both the in-sample and the out-of-sample tests show that the GIC model's error tends to be lower if the priority recovery rate is high compared with that of the JT model and vice versa. This relationship is robust and applies to almost all of the bond issuers (there are only two exceptions).

The overall result in the form of an aggregation of the individual results is not surprising. An analysis considering the clustering of the data proves that the clusters, however, are predetermined by the companies. The significance is verified by means of a rank-sum test for clustered data (Datta and Satten, 2005). All in all, these results also confirm the basic hypothesis that investors are rather guided by legal aspects.

4.4. *Robustness and discussion of the results*

At first, we have to answer the question to what extent the CDS market's implicit default probabilities apply to the bond market, or whether there is a systematic upward or downward distortion as to the pricing of CDSs resulting from the non-consideration of the so-called delivery option (Jankowitsch *et al.*, 2008). If this is true, it has to be answered whether this systematic distortion aims at a distortion in favor of one of the two models. To examine this, we exogenously assume the recovery rates for CDSs to be 20%, 40%, 50%, and 60% for the calculation of the CDS spreads' default probabilities. The results summarized in Table 4 reveal that this robustness test hardly influences our findings and that the presented model is superior to the JT model.

The duration of the bankruptcy proceedings is another pricing-relevant variable that is only estimated. Because of the discounting effect, it tends to

Table 4. Model errors of the two models when CDS recovery rates are varied (robustness test).

	Observations	CDS Recovery Rate 20% CDS Recovery Rate 40% CDS Recovery Rate 50% CDS Recovery Rate 60%		
		Significant Difference ("JT-GIC")	Significant Difference ("JT-GIC")	Significant Difference ("JT-GIC")
Allianz	824	0.35%	-0.01%*	-1.9%
BASF	824	0.41%	0.04%*	0.55%***
Bayer	824	0%	0.27%***	-0.12%***
Commerzbank	43	0%	0%	0.06%
Daimler	824	0.77%	0.28%	1.05%***
Deutsche Bank	824	2.73%***	0%***	0.01%***
Deutsche Boerse	250	0%**	0%	0.29%
Deutsche Lufthansa	824	0.02%	0.02%***	0.66%***
Deutsche Post	824	-0.33%	0.08%***	0.42%***
Deutsche Telekom	824	0.36%	0.14%***	0.21%***
EON	386	-1.3%	0.21%***	0.71%***
Fresenius	815	0.55%***	0%***	0%***
Henkel	824	0.16%***	-0.5%	0.52%***
Linde	493	-0.74%	-0.2%	0.82%***
Merck	587	-0.05%	1.35%***	-0.22%***
Metro	824	-0.04%	-0.34%	0.26%***
RWE	824	0.11%	0.18%***	0.73%***
Siemens	824	0.03%	0.03%	0.16%***
Thyssen	824	0.89%***	0.27%***	0.11%***
TUI	824	2.15%	0.37%***	0.29%***
Volkswagen	824	0.88%	-1.4%	0.25%***
All	14,934	0.44%***	0.02%***	0.22%***

Explanation: CDS recovery rates are varied for the calculation of the CDS spreads' default probabilities. GIC (error) = results of GIC model; JT (error) = results of JT model; * 10% significance level; ** 5% significance level; *** 1% significance level in accordance with Wilcoxon rank-sum test for the listed companies and Wilcoxon rank-sum test for clustered data (see line "All"). Error is calculated according to Eq. (21). GIC-model according to Eq. (17), JT according to Eq. (3). The difference is calculated as the difference between the error (JT) and the error (GIC) and was afterwards averaged (see also Table 2).

generate lower values for the presented model compared with the JT model, provided that the recovery rate is the same. Kranzusch and Icks (2010) find that the average duration of German bankruptcy proceedings is approximately 4 years; however, they point out that the average duration of the proceedings might be underestimated as a result of their analysis's methodology. Therefore, we assume an average duration of 4–6 years in the following analysis. Because the duration of the bankruptcy proceedings tends to be depicted by means of a low recovery rate for coupon bonds, it is not surprising that, because of the previous explanations, the modification of the duration of the bankruptcy proceedings does not affect the presented model's superiority (see Table A.1 in the Appendix).

A measuring error could also result from the findings being based on a pre-tax consideration. Investors focusing on an after-tax view might reach a different result. In the period surveyed, when the capped withholding tax had not been introduced yet, they might have preferred high capital gains and low-interest yields. Thus, such investors prefer bonds with low coupons to those with high coupons; as a consequence, these investors attribute higher values to bonds of the first type compared with those of the second type. However, this is identical to the view of the presented model in which investors tend to attribute a higher value to bonds with low coupons compared with those with high coupons as a result of the non-priority of not-due coupon payments. Therefore, the measured effect might be a purely tax-related coupon effect. The tax-related coupon effect is indeed controversial (see, e.g., Bühler and Rasch, 1994; Jaschke *et al.*, 2000; or Litzenberger and Rolfo, 1984); nevertheless, we apply a test that takes this effect into consideration. For that purpose, only 70% of the coupon is considered, which is in accordance with the former capital gains tax of 30%, and the interest rate used for discounting is reduced by the tax rate. In addition, the results summarized in Table 5 prove the presented model's superiority. Therefore, a purely tax-related coupon effect cannot be the reason for the revealed results. Furthermore, we also varied the norm according to Bakshi *et al.* (2006). Instead of minimizing the relative price variance, the effective yield's difference is minimized; this neither changes the results of the in-sample test, nor those of the out-of-sample test (also see Table 5).

Apart from the minimization of the root-mean-square distance (see, e.g., von Auer, 2007), which is quite common in economics, a comparison of the models is made in accordance with the Tschebyscheff norm, i.e., the quality is measured by means of the maximum price variance. This modification, too, does not change the findings (not indicated).

Table 5. Model error (RMSE) of the two models considering the German tax system, the effective yield, and a variation in the recovery rate for non-priority coupons (robustness test).

	Taxes	Effective Yield (in-sample test)	Effective Yield (out-of-sample test)	Recovery Rate of 0 for Non-Priority Payments
	Significant Difference ("JT-GIC")	Significant Difference ("JT-GIC")	Significant Difference ("JT-GIC")	Significant Difference ("JT-GIC")
Allianz	0.57%***	0%	0%	0%
BASF	0.45%***	0.01%	1.2%***	-0.05%
Bayer	-0.02%	0%	—	0.01%
Commerzbank	0%	0%	—	0%
Daimler	0.43%***	-0.01%	1.84%***	-0.18%
Deutsche Bank	0.14%	0.02%	0.98%***	0.01%
Deutsche Boerse	0%	0%	—	0%
Deutsche Lufthansa	0.19%***	0%	—	-0.03%
Deutsche Post	0.25%***	0%	0.87%***	-0.05%
Deutsche Telekom	0.24%***	-0.02%	1.82%***	-0.13%
EON	0.38%***	0.06%***	1.43%***	0.01%
Fresenius	0%***	0%***	—	-0.04%***
Henkel	-0.36%***	-0.03%	3%***	-0.16%
Linde	0.18%	0%	2.14%***	-0.03%
Merck	1.13%***	0.06%***	0.23%***	0.94%
Metro	0.16%	0.01%	2.93%***	0%
RWE	0.07%	0.02%**	1.83%***	-0.15%
Siemens	-0.1%	0.08%**	4.16%***	0.23%
Thyssen	-1.57%	0.21%***	2.28%***	-1.83%
TUI	0.77%***	-0.3%***	5.58%***	-3.21%
Volkswagen	0.03%	0.08%***	2.14%***	-0.49%
All	0.13%*	0.01%*	1.65%***	-0.3%

Explanation: The days correspond to the values indicated in Table 3. The “Taxes” column is a modification including an after-tax calculation, the “Effective yield” columns include a modification of the standard adaption, and the “Recovery rate” column is a modification in which the non-priority recovery rate is zero. GIC (error) = results of GIC model; JT (error) = results of JT model; * 10% significance level; ** 5% significance level; *** 1% significance level in accordance with Wilcoxon rank-sum test for the listed companies and Wilcoxon rank-sum test for clustered data (see line “All”). Error is calculated according to Eq. (21). GIC-model according to Eq. (17), JT according to Eq. (3). The difference is calculated as the difference between the error (JT) and the error (GIC) and was afterwards averaged. The out-of-sample test is based on more observations than the in-sample test (sixth column compared to second column). This is due to the fact that for each day the number of estimations has to equal the number of bonds because in each of the estimations we ignore one bond, which is then used for determining the adaption quality. Due to the fact, less outstanding companies for five issuers an out-of-sample test was not possible (see also Table 2).

Only if the non-priority recovery rates (i.e., those for outstanding coupons) are always set to zero, the presented GIC model's statistically significant superiority becomes weaker. This might indicate an over-adaption of the multi-parameter GIC model; however, this can be ruled out by means of the out-of-sample tests.

All in all, it is revealed that investors also take legal aspects into consideration when valuating default-risky bonds. The observations by Guha (2003), which are derived from U.S. bankruptcy proceedings and indicate that the insolvency quota only refers to the nominal value, can also be confirmed for the German law. In contrast, the findings (partly) contradict those of Khuong-Huu *et al.* (2008) and Bakshi *et al.* (2006), and, to a limited extent, those of Delianedis and Lagnado (2002) and Hull and White (2000).

5. Summary and Outlook on Future Research

The presented paper compares the JT model with the provisions of the German insolvency law. In accordance with the JT model, coupon bonds are valuated as a portfolio of zero-coupon bonds, which pay a recovery rate at the end of their term in the event of insolvency. The practical application of the German insolvency law, in contrast, differs in five valuation-relevant provisions:

- Whether the bond is a coupon bond or a zero-coupon bond;
- The ranking of the primary claim;
- The recovery rate for priority and non-priority claims;
- The duration of the bankruptcy proceedings; and
- The bond's remaining term at the time of the institution's bankruptcy proceedings.

Provided that these provisions are taken into account, modified values will be attributed to the bonds if the recovery rate is the same. A low value particularly results from a short remaining term, a long duration of the bankruptcy proceedings, and — with regard to zero-coupon bonds — a high statutory interest rate. Furthermore, there is an obvious coupon effect that is related to the insolvency law; as a result, zero-coupon bonds have the lowest value, bonds with high coupons have a lower value, and bonds with low coupons have a higher value.

These theoretical findings can be confirmed almost without exception by means of an empirical study of 103 corporate bonds over more than 800 trading days. Hence, the model presented in this paper proves to be superior

to the JT model, even if numerous input parameters are varied. In particular, these modifications refer to the duration of the bankruptcy proceedings, the pricing error (on the basis of the differences in the yield rates, see Sec. 4.1), and pricing differences that may occur as a result of the CDSs' delivery option. Furthermore, an after-tax calculation ruled out that the measured coupon effect is a tax-related coupon effect.

However, our findings leave room for further analyses. For example, a comparative static analysis could be conducted. A comparison of both models could derive hypotheses that provide insight into the question of which bonds the presented model offers huge advantages in the form of a lower measuring error. Some bond features, such as term and coupon price, might have an effect. Moreover, the delivery option mentioned previously could be included explicitly in the pricing of CDSs. In addition, other European countries also have insolvency provisions that differ from the JT model. If the results can be analogously confirmed for other countries, this will support the considerations that investors really include the insolvency law-dependent coupon effect in their decision-making process. As a result, the bond markets will largely prove efficient.

Appendix

Table A.1. Model error (RMSE) of the two models when the duration of the bankruptcy proceedings is varied.

	Observations	Duration of the Bankruptcy Proceedings (4 years)	Duration of the Bankruptcy Proceedings (6 years)
		Significant Difference ("JT-GIC")	Significant Difference ("JT-GIC")
Allianz	824	0.08%	0.08%
BASF	824	0.31%***	0.25%***
Bayer	824	0.56%***	0.33%***
Commerzbank	43	0%	0%
Daimler	824	0.58%***	0.57%***
Deutsche Bank	824	2.3%***	1.99%***
Deutsche Boerse	250	0%	0%
Deutsche Lufthansa	824	0.01%**	−0.03%
Deutsche Post	824	0.21%***	0.24%***
Deutsche Telekom	824	1.01%***	1.06%***
EON	386	0.57%*	0.57%*
Fresenius	815	0.06%***	0.06%***

RECOVERY RATE IN THE EVENT OF AN ISSUER'S INSOLVENCY

Table A.1. (Continued)

	Observations	Duration of the Bankruptcy Proceedings (4 years)	Duration of the Bankruptcy Proceedings (6 years)
		Significant Difference ("JT-GIC")	Significant Difference ("JT-GIC")
Henkel	824	−0.01%	0.14%***
Linde	493	0.78%*	0.78%*
Merck	587	1.42%***	0.48%***
Metro	824	0.17%	0.23%
RWE	824	0.69%***	0.74%***
Siemens	824	0.83%***	0.4%
Thyssen	824	2.61%***	2.86%***
TUI	824	1.54%***	−0.37%***
Volkswagen	824	2.2%***	2.23%***
All	14,934	0.84%***	0.67%***

Explanation: Duration of bankruptcy varies from 4 to 6 years. GIC (error) = results of GIC model; JT (error) = results of JT model; * 10% significance level; ** 5% significance level; *** 1% significance level in accordance with Wilcoxon rank-sum test for the listed companies and Wilcoxon rank-sum test for clustered data (see line "All"). Error is calculated according to Eq. (21). GIC-model according to Eq. (17), JT according to Eq. (3). The difference is calculated as the difference between the error (JT) and the error (GIC) and was afterwards averaged (see also Table 2).

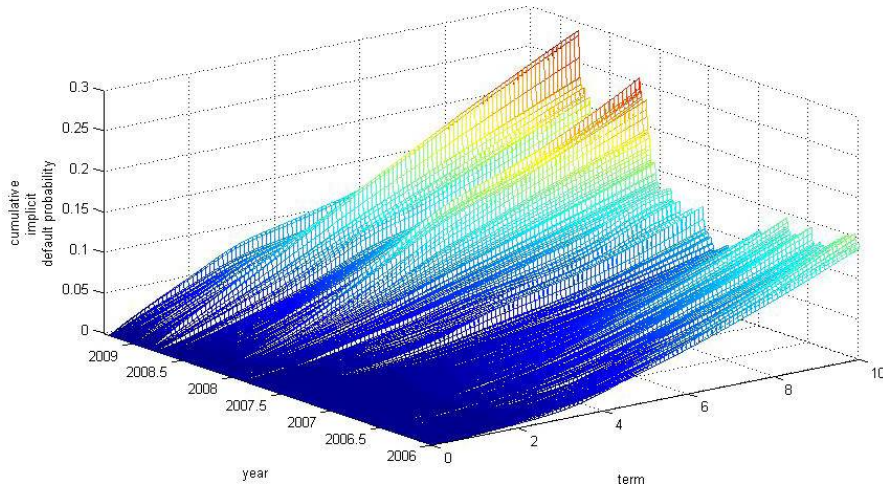


Fig. A.1. Term structure curves of the (cumulated) implicit default probabilities.

Explanation: Term structure curve of the CDS spreads' implicit default probabilities for Deutsche Telekom from 2006 to 2009. A spline cubic interpolation is applied; the recovery rate for CDS is assumed to be 30%.

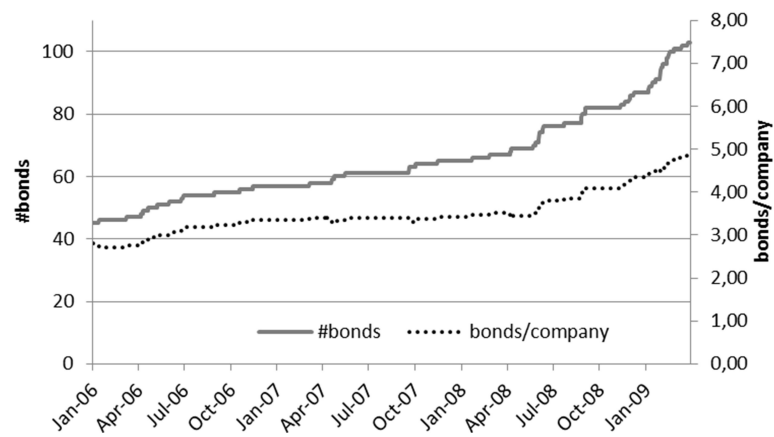


Fig. A.2. Number of the bonds surveyed and bonds per company on a daily basis over the entire period surveyed.

Explanation: Number of bonds of the entire data set of the period surveyed; in addition, number of bonds in relation to the number of companies surveyed over that period (full sample: 21 companies and 103 outstanding bonds).

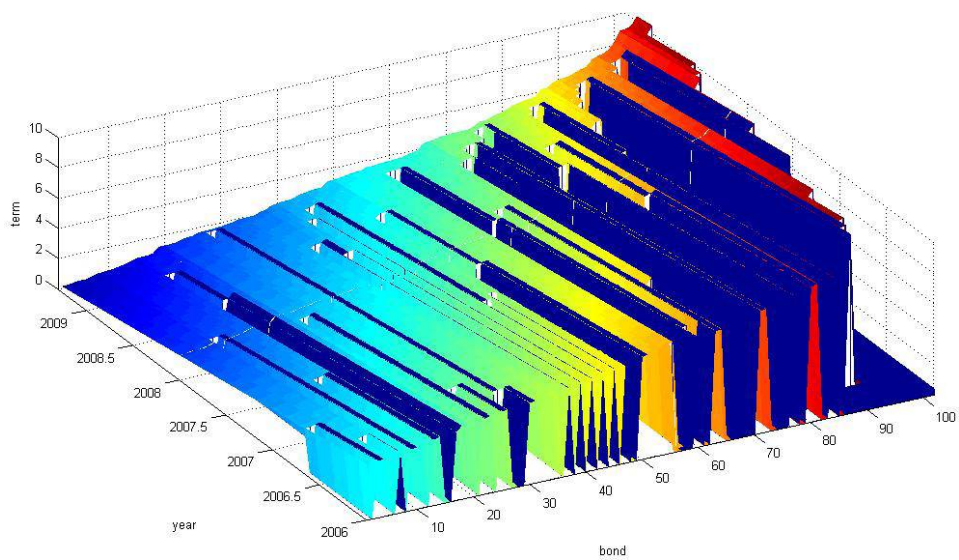


Fig. A.3. Terms of the bonds surveyed on a daily basis over the entire period surveyed.

Explanation: Terms of the bonds of the entire data set of the period surveyed (full sample: 103 outstanding bonds).

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