Why Germany Was Supposed To Be Drawn in the Group of Death and Why It Escaped

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An explanation of the World Cup draw and its potential effects on who advances to the next round.

Millions of football fans all over the world cast their eyes toward Leipzig, Germany, to follow the 2006 FIFA World Cup Draw December 9, 2005. Prior to the draw, 32 teams qualified for the World Cup. The draw divides those teams into eight groups, labeled A through H, of four teams each. Each group of four plays a round-robin—every team plays every other team, for a total of six games within the group—and the top two teams in each group advance to the next stage. The question we want to raise here is about the mathematical fairness of the draw. Did all the top teams have the same chance of being placed in a group with a difficult opponent? To answer this question, we will take a look at the FIFA procedure for the draw.

Pot 1
Germany (seeded A1)
Brazil (seeded F1)
England
France
Italy
Spain
Argentina
Mexico

Pot 2
Togo
Angola
Côte d'Ivoire
Tunisia
Ghana
Australia
Ecuador
Paraguay

Pot 3
Croatia
Czech Republic
Netherlands
Poland
Portugal
Sweden
Switzerland
Ukraine

Pot 4
Iran
Japan
Korea Republic
Saudi Arabia
Costa Rica
USA
Trinidad and Tobago

Extra Pot: Serbia and Montenegro

Figure 1. Allocation of the pots

FIFA Rules

There are two main goals in assigning teams to groups: avoid early matches between title contenders and avoid putting teams from the same continental zone into the same group. To accomplish these goals, the 32 teams were divided into four "pots," labeled 1 through 4, of eight teams each. The first pot contained the eight seeded teams. The 24 remaining teams were assigned to pots 2, 3, and 4 to achieve the best possible geographical distribution between the groups. There also existed a special pot containing Serbia Montenegro as the lowest-ranked team from Europe. Then, one team from each pot was put into each group from A through H. Teams from Pot 1 would occupy Position 1 in the groups; teams from Pot 2 would occupy Position 2, etc.

Group A
Germany
Ecuador
Poland
Costa Rica

Group B
England
Paraguay
Sweden
Trinidad and Tobago

Group E
Mexico
Angola
Portugal
Iran

Group F
Brazil
Australia
Croatia
Japan

Group C
Argentina
Côte d'Ivoire
Netherlands
Serbia and Montenegro

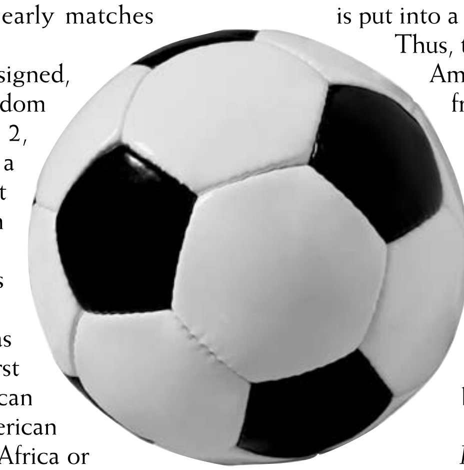
Group D
Italy
Ghana
Czech Republic
USA

Group G
France
Togo
Switzerland
Korea Republic

Group H
Spain
Tunisia
Ukraine
Saudi Arabia

Figure 2. Result of the FIFA 2006 draw

Moving teams from pots to groups was accomplished by a random draw. The result of that draw, and therefore the composition of the groups, is shown in Figure 2. The first teams to be drawn were the eight teams from Pot 1; they were assigned to Position 1 in groups A through H. Germany, as the host, was pre-assigned to Group A. Brazil, as the reigning world champion, was pre-assigned to Group F. The other six teams in Pot 1 were assigned to the top positions of groups B, C, D, E, G, and H, in the order in which they were randomly drawn. Assigning the eight teams in Pot 1 to the eight groups accomplished the goal of avoiding early matches between likely contenders.



After the first eight teams were assigned, the remaining 24 were drawn in random order. As soon as a team from pots 2, 3, and 4 was drawn, it was placed in a group. The first team drawn from a pot was placed in Group A, the second in Group B, etc. However, two modifications had to be made for drawing teams from Pot 2.

1. If an African team or Australia was drawn, it had to be placed in the first available group with a South American team until the groups with South American teams were covered with teams from Africa or Australia. Suppose Togo was drawn first. We can see from Figure 2 that Group C, headed by Argentina, is the first group with a South-American team. So Togo would be placed in Group C.

2. If a South-American team was drawn into a group with another South-American team, it was automatically moved to the next available group.

Fairness of the Draw

Those are the rules of the draw. Now, we would like to assess the fairness, in a mathematical sense, of those rules. In particular, we will compare the probability of Germany drawing a difficult team into its group, Group A, to the probability of Italy drawing a highly troublesome team into its group, Group D.

Of course, the definition of a difficult team is subjective. One might base it on the FIFA Coca Cola World Ranking. By this criterion, Tunisia is the most difficult team in Pot 2. Or, one could define difficult teams by their performance in previous FIFA World Cup Competitions. By this criterion, Paraguay is the most difficult team in Pot 2. For the purpose of this paper, we take South-American teams to be the most difficult. We think this is a reasonable decision because Ecuador and Paraguay, the two South-American teams in Pot 2, challenged the reigning champion, Brazil, in the qualifications and were successful in all relevant home matches in the hard South-American qualification group. We focus on only Pot 2 because the draw of Pot 3 is, by definition, fair and that of Pot 4 has no decisive exception rule. Thus, we will compare Germany and Italy's chances of drawing a South-American team from Pot 2 into their groups. Italy (I) stands as a proxy for all non-South-American teams.

Let A denote an African/Australian team, S denote a South-American team, and G denote Germany. To begin, we calculate $P_G \equiv \Pr(GvS)$, the probability that a South-American team from Pot 2 is drawn into Group A. And to calculate P_G, we must look more closely at Pot 2. Notice Pot 2 has six African/Australian and two South-American teams. If a South-American team is drawn first, it is put into Group A. But, if an African/Australian team is drawn first, then modification #1 comes into play and that team is put into the first group headed by a South American-team—Group C as it turned out. Similarly, if a South-American team is drawn second, it is put into Group A. But, if an African/Australian team is drawn second, then modification #1 comes into play and that team is put into a group headed by a South-American team. Thus, the only way Germany can avoid a South-American team is if the first three draws are all from Africa. Therefore,

$$P_G = 1 - 6/8 \cdot 5/7 \cdot 4/6 \approx 64.29\%.$$

Now, we calculate $P_I \equiv \Pr(IvS)$, the probability that a South-American team from Pot 2 is drawn into the group headed by Italy. Italy was not predetermined to be in Group D; it could have been in any of B, C, D, E, G, or H. So, we break up the calculation as:

$$P_I = \Pr(IvS \cap \text{Italy in Group B}) + \Pr(IvS \cap \text{Italy in Group C}) + \ldots + \Pr(IvS \cap \text{Italy in Group H}).$$

Each of these terms is complicated. We calculate them in three sets.

Term 1.

Suppose Italy has been assigned to Group B. The next team drawn into Group B will be decided by the first four draws. The possibilities for the first four draws are:

AAAA AAAS AASA ASAA SAAA
AASS ASAS SAAS ASSA SASA SSAA

The ones that give a South-American team to Group B are:

AAAS AASS ASAS
SAAS ASSA SASA SSAA

and their probabilities are:

$$\frac{6}{8}\frac{5}{7}\frac{4}{6}\frac{2}{5}+6\cdot\frac{6}{8}\frac{5}{7}\frac{2}{6}\frac{1}{5}\approx 35.71\,\%t \qquad .$$

Thus, the probability of Term 1 is:

$\Pr(IvS \cap \text{Italy in Group B})=$

$\Pr(IvS \mid \text{Italy in Group B}) \cdot \Pr(\text{Italy in Group B}) \approx \frac{35.71}{6}\%$

$\approx 5.95\%$.

Suppose Italy is drawn into Group C. Then, there exist two cases: Either a South-American team (Argentina per force) or a non-South-American team is placed into Group B. If Argentina is placed into Group B, then, because of modification #2, the scenario is equivalent to Italy being in Group B. Consequently, $\Pr(IvS \mid \text{Argentina in B and Italy in C}) \approx 35.71\%$.

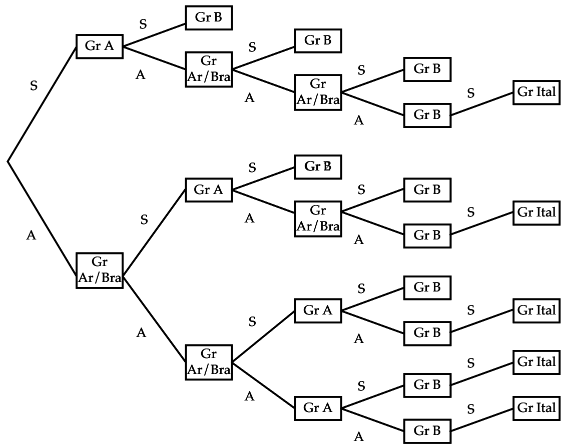


Figure 3. Scenarios for a South-American (S) team going into Group C if Italy (Ital) is in Group C and Argentina (Ar) is not in Group B

The first four scenarios in the tree show that IvS occurs whenever the first four draws include exactly one S team and the fifth draw is the other S team. The probability of such a scenario is $\left(\binom{2}{1}\binom{6}{3}\Big/\binom{8}{4}\right)\times\frac{1}{4}=\frac{1}{7}$.

The fifth scenario in the tree occurs when the first four teams are African/Australian and the fifth is South-American. Its probability is $\frac{6}{8}\times\frac{5}{7}\times\frac{4}{6}\times\frac{3}{5}\times\frac{2}{4}=\frac{3}{28}$.

Therefore,

Pr(IvS|Argentina not in B and Italy in C) = $\frac{1}{7}+\frac{3}{28}=\frac{1}{4}$.

Therefore, **Term 2** is:

Pr(IvS $\cap$ Italy in C) =

Pr(IvS $\cap$ Italy in C $\cap$ Argentina in B)

+ Pr(IvS $\cap$ Italy in C $\cap$ Argentina not in B)

$\approx 35.71\%\times\frac{1}{6}\times\frac{1}{5}+25\%\times\frac{4}{6}\times\frac{1}{5}\approx 4.52\%$.

Terms 3–6.

The reasoning for all these terms is similar. We illustrate with Term 3 and calculate Pr(IvS $\cap$ Italy in Group D). Again, there are two cases: Argentina is in B or C (ahead of Italy) or Argentina is in E, G, or H (behind Italy). The first case is determined by the first five teams drawn from Pot 2. More than two of those teams must be African/Australian, and the first two African/Australian teams will be assigned to the groups headed by Argentina and Brazil. Therefore, the fifth team drawn will be assigned to the group headed by Italy, so

Pr(IvS $\cap$ Italy in Group D|Argentina ahead of Italy)

= Pr(fifth team drawn is South-American) = $\frac{1}{4}$.

Similarly, if Argentina is in G or H, then the situation is determined by the first six teams drawn. More than two of those teams will be African/Australian; the first two African/Australian teams will be assigned to Argentina and Brazil, so Italy will get the sixth team drawn. Therefore,

Pr(IvS $\cap$ Italy in Group D|Argentina behind Italy)

= Pr(sixth team drawn is South-American) = $\frac{1}{4}$.

By similar reasoning, terms 3–6 are identical and their sum is $\frac{1}{4}\times\frac{4}{6}\approx 16.67\%$.

Putting everything together, we have P_I = Term 1 + … + Term 6 ≈ 5.95% + 4.52% + 16.67% ≈ 27.14%.

As our results reveal, the probability that Italy faces a South-American team, and thereby a difficult team in the sense of this paper, is much lower than that of Germany.

As mentioned before, we would like to elaborate on our choice and consequences of the definition of a difficult team. Indeed, there are other difficult teams in Pot 2. Ghana and Ivory Coast, especially, attracted a lot of betting dollars and could be considered difficult. Including them, we have four troublesome teams (tT) in Pot 2, which leads to an overall increase in the probabilities computed above. To see whether such considerations matter for our comparison of Germany and Italy, we summarize the relevant calculations.

Pr(Italy vs. tT)= $\frac{1}{6}$ Pr(Italy vs. tT|Italy in B)

+ $\frac{1}{6\cdot 5}$ Pr (Italy vs. tT|Argentina in B and Italy in C)

+ $\frac{4}{6\cdot 5}$ Pr (Italy vs. tT $\cap$ Argentina not in B and Italy in C)

+ $\frac{4}{6}$ Pr (Italy vs. tT|Italy in D or higher)

=9.52%+1.90%+6.67%+33.33%=51.43%

and

Pr= (German vs. tT) = 1 − Pr (German vs. $\overline{tT}$) = 1 − 23.81% = 76.19%.

So, the conclusion is stable: There remains a huge difference between the probability of Italy, 51.43%, and that of Germany, 76.19%, facing a difficult team.

Conclusion and Outlook

The probability that Germany draws a difficult team is significantly higher (64.29%>27.14% or 76.19%>51.43%) than for Italy. More generally, judging from the FIFA 2006 draw procedure, Germany was likely to draw a difficult group, while other teams, such as Italy, were not.

Following the actual draw, Germany's team manager wore a satisfied smile, even though his group included Ecuador—a South-American team from Pot 2. Meanwhile, the Italian coach raised an eyebrow. Were these reactions justified?

At the end of the round robin and out of Pot 2, only Ecuador, Ghana, and, surprisingly, Australia qualified for the next round. The eight seeded teams—five teams from Pot 3 and no team from Pot 4—survived the first round. Two teams from Pot 3—Portugal and Ukraine—and none from Pot 2 reached the quarter finals; Portugal even made it to the semifinals. All in all, the key to a group of death lay mainly in Pot 3, which was not subject to the FIFA exception rule and could thus be considered a fair draw. Therefore, the unfairness of Pot 2 seems to have had a minor influence. One can consider this as the explanation for the reaction of the two coaches.