

# Stochastic Processes of Demarkovization and Markovization in Chaotic Signals of Human Brain Electric Activity from EEGs during Epilepsy<sup>¶</sup>

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## 1. INTRODUCTION

We develop a new approach that could provide us with a powerful means of discrete time series analysis and processing. The subject of our study is human electroencephalogram (EEG) records, because we address our work to those who are interested in signal processing in live complex systems. In studying natural complex systems, very little is usually known about their internal structure and the relationship between their components. The time series describing the dynamics of one or several parameters are typically used for obtaining diagnostic information. The received information is inadequate for describing all the degrees of freedom of this system. Quantitative and qualitative methods proposed recently allow constructing the framework for the description of natural complex systems. It allows diagnosing diseases without going into detail of the internal structure underlying natural complex systems. A similar approach can be used to describe and investigate diversified complex systems as they are related only to the concepts of this framework. Here, we present the results of applying a new framework involving ideas of discrete non-Markovian stochastic processes to the analysis of electric potentials of brain. It turns out that discussing the results in terms of demarkovization and markovization is the best way to uncover the features of seizure dynamics.

Brain cells communicate by producing tiny electrical impulses. In an EEG, electrodes are placed on the

scalp over multiple areas of the brain to detect and record the electrical pulses within the brain. The EEG is used to help diagnose the presence and type of seizure disorders, confusion, head injuries, brain tumors, infections, degenerative diseases, and metabolic disturbances that affect the brain.

It is well known that epilepsy is one of the most serious diseases of the human brain [1, 2]. The dynamics of the electric signals accompanying it belongs to a class of nonlinear, nonstationary, and nonergodic processes of complex systems of a live nature [3, 4]. The discrete and non-Markovian properties of time variation of the signals and the sudden alternation of the behavior regimes must be taken into account in analyzing the electrical activity of brain potentials. Together with the fast change of chaotic and regular modes in the behavior of the system, this creates serious problems for the diagnosis and treatment of patients with epilepsy seizure. This is why traditional methods of nonlinear dynamics, such as the Lyapunov exponent, Kolmogorov–Sinai entropy, and correlation and fractal dimensions, are not sufficiently sensitive for the purpose of distinction between different chaotic regimes in epilepsy.

## 2. BASIC THEORY

Our approach is based on the recent theory for stationary [5] and nonstationary cases [6] of discrete stochastic processes in complex systems. We analyze the stochastic process on the basis of the chain of the cou-

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pled non-Markovian discrete equations for the initial discrete time correlation function (TCF)  $a(t)$  ( $t = m\tau$ ),

$$\begin{aligned} \frac{\Delta a(t)}{\Delta t} &= \lambda_1 a(t) - \tau \sum_{j=0}^{m-1} M_1(j\tau) a(t-j\tau), \\ \frac{\Delta M_1(t)}{\Delta t} &= \lambda_1 M_1(t) - \tau \sum_{j=0}^{m-1} M_2(j\tau) M_1(t-j\tau), \quad (1) \\ \frac{\Delta M_2(t)}{\Delta t} &= \lambda_3 M_2(t) - \tau \sum_{j=0}^{m-1} M_3(j\tau) M_2(t-j\tau), \end{aligned}$$

where  $\lambda_n$  is the eigenvalue spectrum of the Liouville operator  $i\hat{L}$  and  $\lambda_n$  are the general relaxation parameters,

$$\lambda_n = i \frac{\langle \mathbf{W}_n^* L \mathbf{W}_n \rangle}{\langle |\mathbf{W}_n|^2 \rangle}, \quad \lambda_n = - \frac{\langle \mathbf{W}_{n-1} (i\hat{L} - \lambda_{n+1}) \mathbf{W}_n \rangle}{\langle |\mathbf{W}_{n-1}|^2 \rangle}.$$

The kinetic nonlinear finite-difference Eqs. (1) are analogous to the well-known chain of kinetic equations of the Zwanzig–Mori (ZM) type. These ZM equations play a fundamental role in the modern statistical mechanics of nonequilibrium phenomena with continuous time. Kinetic Eqs. (1) can be considered a discrete-difference analogy of hydrodynamic equations for physical phenomena with discrete time. By analogy with [5–7], we define the generalized nonlinear non-Markovity parameter in the frequency-dependent case as

$$\epsilon_i(\omega) = \left\{ \frac{\mu_{i-1}(\omega)}{\mu_i(\omega)} \right\}^{1/2}, \quad (2)$$

where  $i = 1, 2, \dots$  and  $\mu_i(\omega)$  is the power spectrum of the  $i$ th memory function. It is convenient to use this parameter for quantitative description of long-range memory effects in the system considered together with memory functions defined above. The values of  $\epsilon_i(\omega)$  allow us to obtain a quantitative estimate of non-Markovity effects and the statistical collective memory in the chaotic changes of the experimentally measured EEG data. The parameters  $\epsilon_i(\omega)$  allow classification of all the observed processes into three important types [5]. A Markov process corresponds to the situation where the non-Markovity parameter takes an indefinitely large value  $\epsilon_i(\omega) \rightarrow \infty$ , and the quasi-Markov processes correspond to the case where  $\epsilon_i(\omega) > 1$ . The limit case  $\epsilon_i(\omega) \approx 1$  describes non-Markovian processes. In this case, the time scale of memory processes and the correlation dynamics (or the nearest junior and senior memory function) coincide with each other.

### 3. EXPERIMENTAL DATA

We quantitatively demonstrate the stochastic description of the time–frequency peculiarities of epilepsy. We use experimental data [8] on human EEGs. These files show tonic–clonic seizures of two subjects recorded with a scalp–right–central (C4) electrode (linked earlobes reference). It contains a total of 3 min with about 1 min pre-seizure, the seizure, and some post-seizure activity. The sampling rate is 102.4 Hz (see the papers cited in [8] for more details).

### 4. NUMERICAL CALCULATIONS

We consider a discrete time series of the electric activity as a one-point stochastic process

$$X = \{ x(T), x(T+\tau), x(T+2\tau), \dots, x(T+k\tau), \dots, x(T+\tau N-\tau) \}. \quad (3)$$

It is convenient to introduce the normalized time correlation function for the quantitative description of time series,

$$\begin{aligned} a(t) &= \frac{1}{(N-m)\sigma^2} \\ &\times \sum_{j=0}^{N-1-m} x(T+j\tau) x(T+(j+m)\tau), \end{aligned} \quad (4)$$

where  $\sigma^2$  is the variance,  $N$  is the number of measurements, and  $\tau$  is a finite discretization time. The key element of the theory consists in transition from continuous values, variables, and equations to discrete ones. We then obtain a Liouville-like equation of motion for multidimensional state vectors. We can use the method of projection operators in a finite-dimensional vector space. This allows splitting the Liouville-like discrete equation of motion into two mutually orthogonal subspaces, one of which is relevant and the other is irrelevant to discrete time correlations. We have also developed the method for obtaining the set of dynamic orthogonal variables by the Gram–Schmidt orthogonalization procedure.

Dynamical orthogonal variables were calculated from initial time series (3) by the formulas (see [5, 6])

$$\begin{aligned} \hat{\mathbf{W}}_0 &= \mathbf{A}_k^0, \quad \hat{\mathbf{W}}_1 = \left( \frac{\Delta}{\Delta t} - \lambda_1 \right) \mathbf{A}_k^0, \\ \hat{\mathbf{W}}_2 &= \left( \frac{\Delta}{\Delta t} - \lambda_2 \right) \mathbf{W}_1 + \lambda_1 \mathbf{A}_k^0 \\ &= \left[ \left( \frac{\Delta}{\Delta t} \right)^2 - \frac{\Delta}{\Delta t} (\lambda_1 + \lambda_2) + \lambda_1 \lambda_2 + \lambda_1 \right] \mathbf{A}_k^0, \\ \hat{\mathbf{W}}_3 &= \left( \frac{\Delta}{\Delta t} - \lambda_3 \right) \mathbf{W}_2 + \lambda_2 \left( \frac{\Delta}{\Delta t} - \lambda_1 \right) \mathbf{A}_k^0, \end{aligned} \quad (5)$$

where the parameters  $\lambda_i$  and  $\Lambda_i$  were calculated using (2). Simple, but cumbersome calculations show that the first short-memory function  $m_n(t)$  represents a normalized TCF of the first dynamic variable  $W_n$ ,

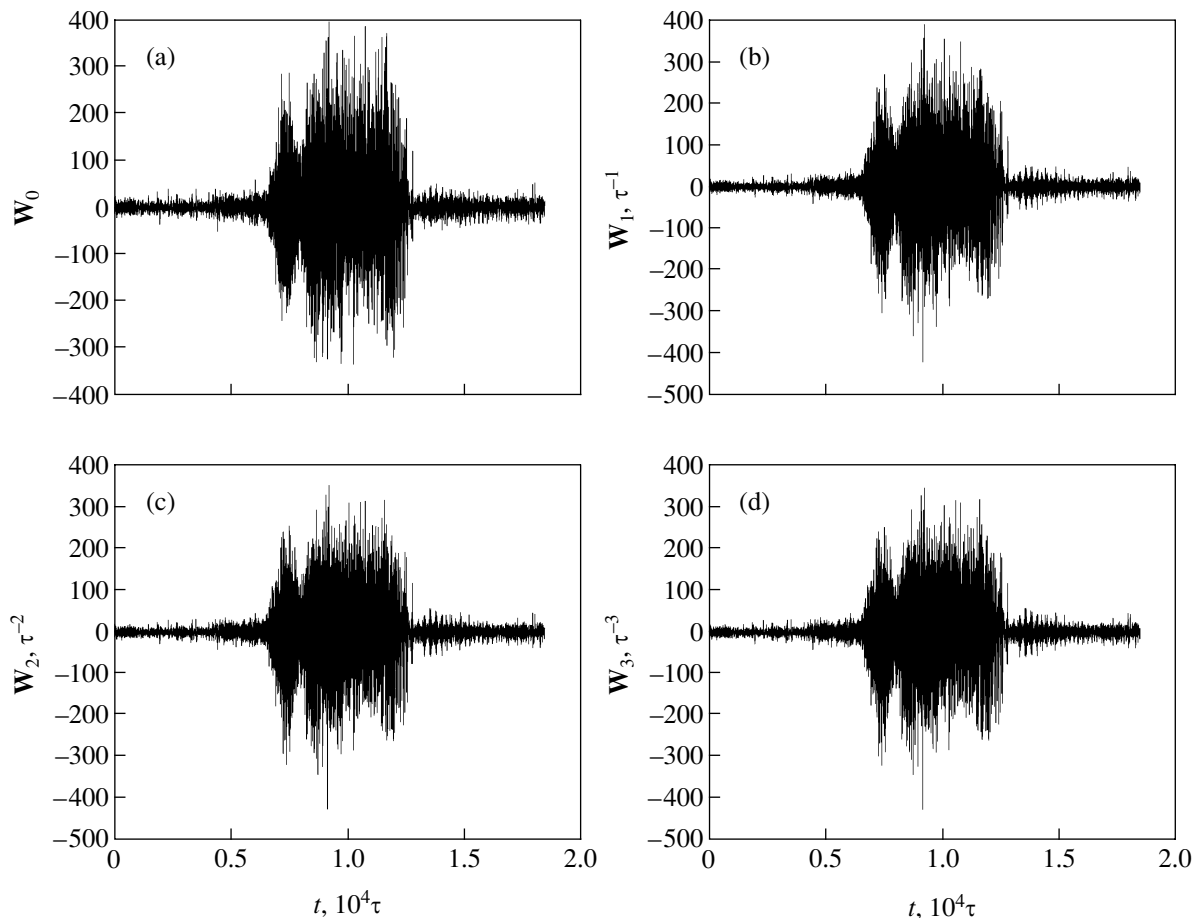
$$m_n(t) = \frac{\langle \mathbf{W}_n(0)\mathbf{W}_n(t) \rangle}{\langle \mathbf{W}_n(0) \rangle^2}. \quad (6)$$

We then obtain a chain of finite-difference discrete non-Markovian kinetic equations for the initial time correlation function and memory functions of various orders. We note that all the involved kinetic and relaxation parameters, the time correlation function, and the memory functions can easily be found and calculated directly from the experimental time series. The spectra of memory functions were calculated using the fast Fourier transform.

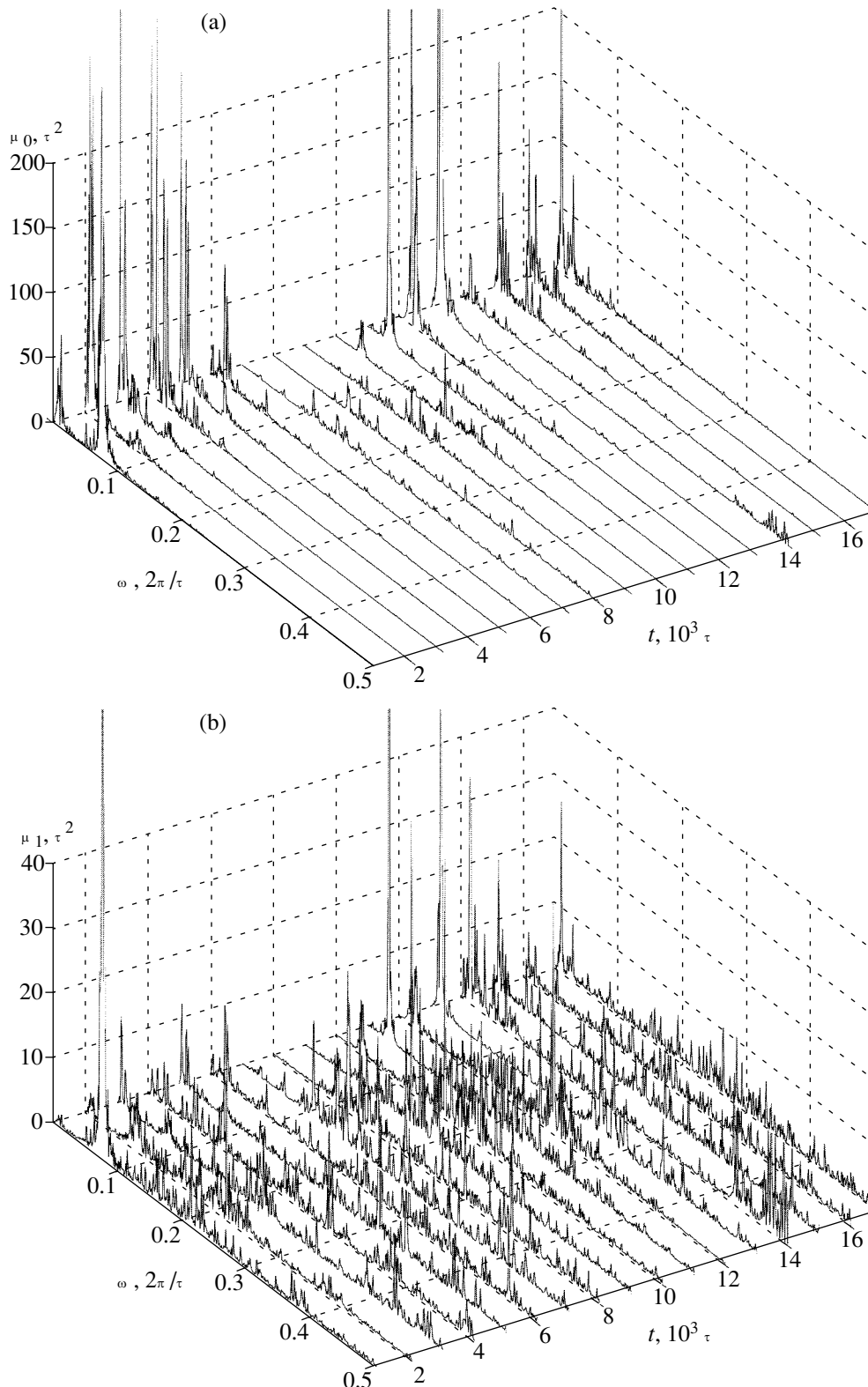
## 5. NON-MARKOVIAN PROPERTIES OF EEGs

We have analyzed the time and frequency evolution of the signals during tonic-clonic seizure by means of the time-window technique. We find that the memory function spectra and the statistical spectrum of the non-Markovity parameter are valuable for quantitative and qualitative analysis of epileptic seizures. Numerical parameters based on the theory of discrete non-Markovian processes provide quantitative information about the state of the brain before, during, and after the seizure.

Non-Markovian properties are known to play an essential role in the time dynamics of complex systems. On the basis of our theory [5, 6], we can calculate memory functions  $M_i(t)$ ,  $i = 0, 1, 2, 3$  directly from experimental data by Eqs. (2.41)–(2.46) in [6]. We analyze the properties of memory functions by calculating their power frequency spectra. For a quantitative estimation



**Fig. 1.** Time record of the first four orthogonal variables  $W_0$  (a),  $W_1$  (b),  $W_2$  (c), and  $W_3$  (d) of the sampling of electric activity during the tonic-clonic seizure under study. The difference in the dimensions of the four variables must be taken into account in analyzing the scales. The general form of all signals has definite similarity. Simultaneously, some differences in time behavior,  $W_i(t)$ , are made evident, especially for the states before and after the seizure. We emphasize that the whimsical entanglement of regular and chaotic components is omnipresent in the time-recording window of all the signals. We also note that the difference between the raw EEG data before, during, and after the seizure is sufficiently dramatic. However, simple registration of this fact does not allow us to reveal such subtle features of EEG spectra as the presence or absence of the chaotic or regular components in the signal.



**Fig. 2.** The window-time behavior of the power spectra  $\mu_i(\omega)$ ,  $i = 0$  (a), 1 (b), 2 (c), and 3 (d), for the considered sampling with the tonic-clonic seizure from the short-time window dynamics of the human brain electric activity. The sharp reduction (by almost one order) of intensity of the low-frequency components of the spectra (in the region of  $\delta$  and  $\theta$  rhythms) attracted our attention at the transition from  $\mu_0$  to  $\mu_1$ ,  $\mu_2$ , and  $\mu_3$ . The spectra  $\mu_i(\omega)$ ,  $i = 1, 2, 3$ , contain rather strong noises distributed at regular intervals in the entire frequency region. The intensity in the region of  $\delta$  and  $\theta$  rhythms sharply decreases in the first half of the seizure (7th, 8th, 9th and, in part, 10th windows) in all  $\mu_i$ ,  $i = 0, 1, 2, 3$ . The sharp increase in the intensity in the low-frequency region of the spectrum by almost 100 times (in the regions of  $\delta$  and  $\theta$  rhythms) is observed in the second half of the seizure (11th, 12th, and 13th windows).

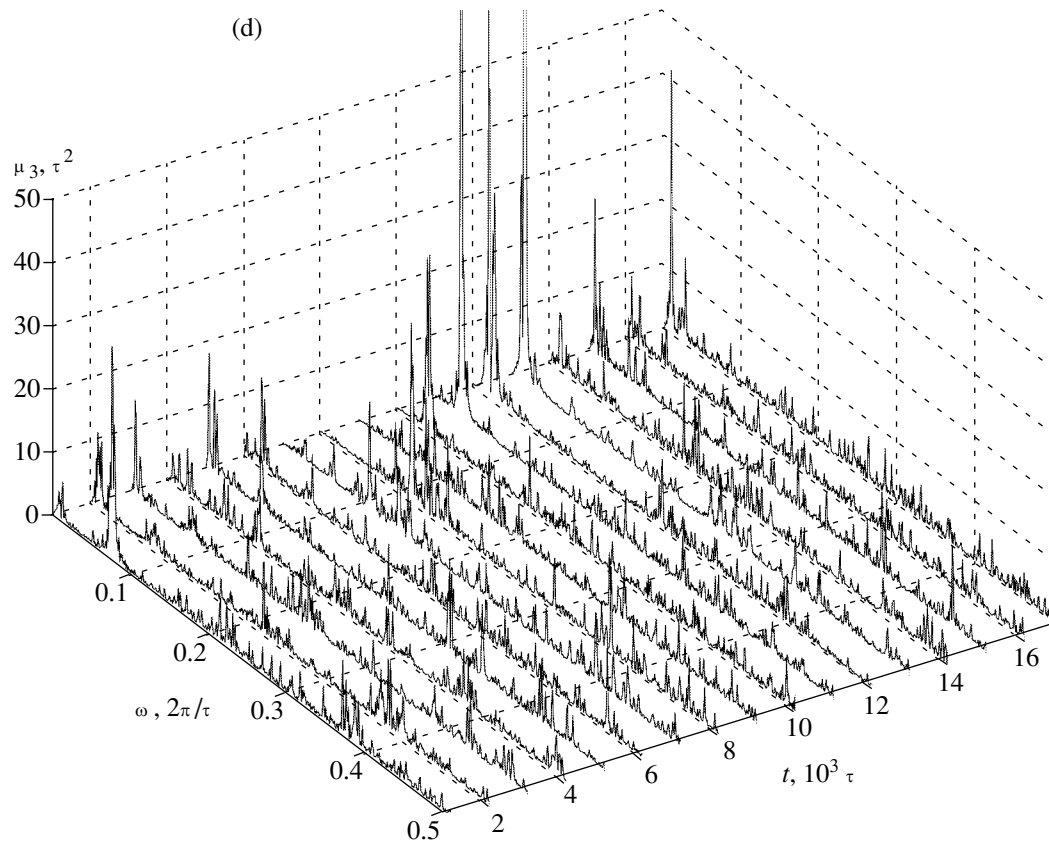
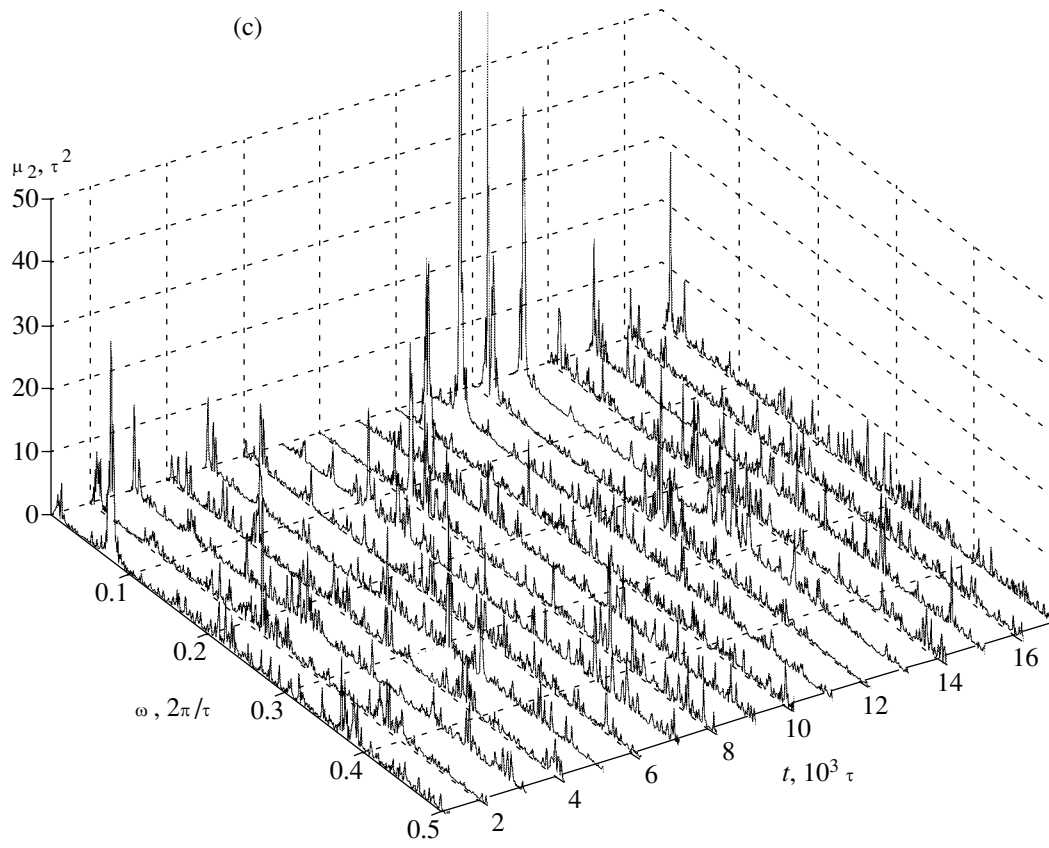


Fig. 2. (Contd.)

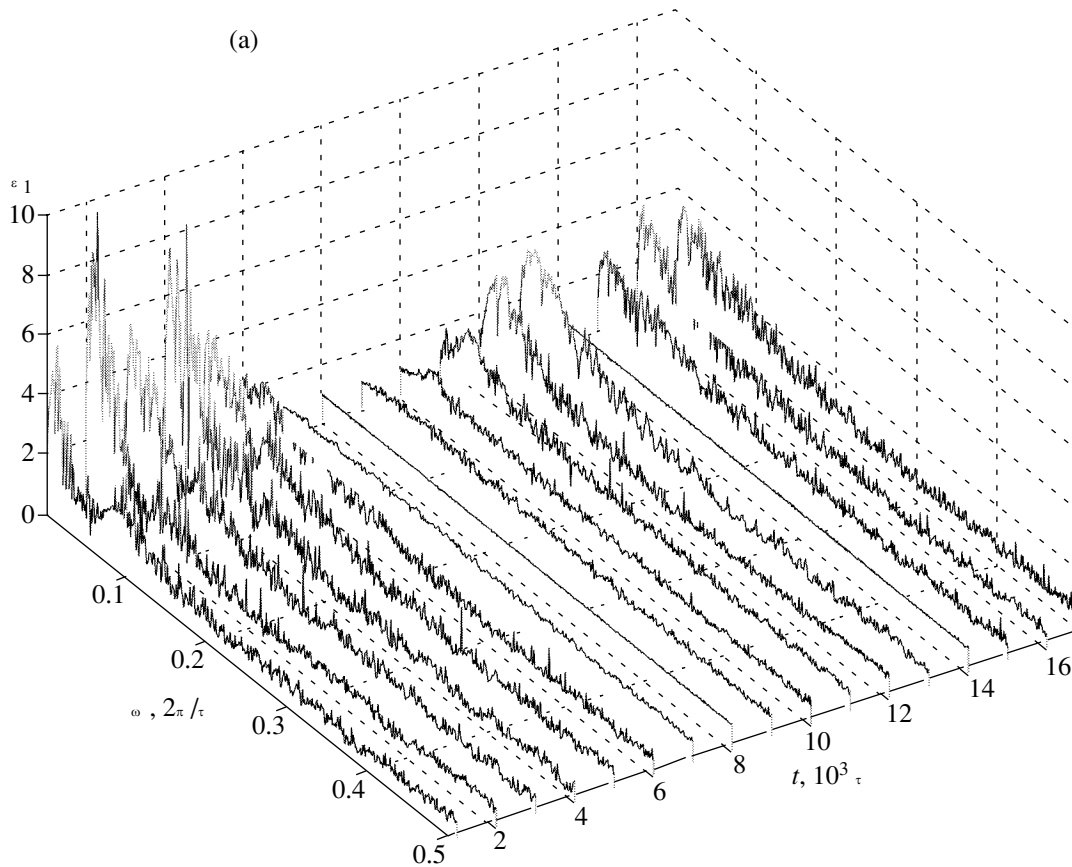
of the non-degree of Markovity, we use the frequency-dependent generalized non-Markovity parameter  $\varepsilon_i(\omega)$  introduced by us previously [5]. From the theory in [5, 6], we can also calculate the quantitative values of the kinetic and relaxation parameters  $\lambda_1, \lambda_2, \lambda_3, \Lambda_1$ , and  $\Lambda_2$  that give additional information on the properties of the complex system under study.

For the observed EEG spectra, we divide the entire time evolution data into nonoverlapping epochs of 1024 data points each. The dynamics of the first four dynamical orthogonal variables  $W_0, W_1, W_2$ , and  $W_3$  of the entire data set is presented in Fig. 1. For each epoch, we have calculated the power spectra of the first four memory functions  $\mu_0(\omega), \mu_1(\omega), \mu_2(\omega), \mu_3(\omega)$  and the

first three points of statistical spectra of the non-Markovity parameter  $\varepsilon_1(\omega), \varepsilon_2(\omega)$ , and  $\varepsilon_3(\omega)$  [5]. The time evolution of the spectra is shown in three-dimensional diagrams (Figs. 2 and 3). The time evolution of the numerical parameters  $\lambda_1, \lambda_2, \lambda_3, \Lambda_1, \Lambda_2$  is presented in Fig. 4.

We emphasize that strong demarkovization of the stochastic changes of brain electrical potentials with decreasing numerical values  $\varepsilon_i$  to the point of unity is exhibited during a tonic-clonic seizure. The chaotic regime of the system is then replaced by the steady non-Markovian-state regime.

It can be seen from Figs. 1a–1d that the time evolution of the dynamic orthogonal variables  $W_i, i = 0, 1, 2$ ,



**Fig. 3.** The window-time behavior of the first three points of the non-Markovity parameter  $\varepsilon_i(\omega), i = 1$  (a), 2 (b), and 3 (c), for the long sampling including the tonic-clonic seizure during epilepsy. For the state before the seizure, the quasi-Markovian behavior ( $\varepsilon_1 \sim 10$ ) of the first point  $\varepsilon_1(\omega)$  in the low-frequency region (with  $\delta$  and  $\alpha$  rhythms) is obvious. The beginning of the seizure (7th, 8th, 9th, and 10th windows) exhibits a strong non-Markovity ( $\varepsilon_1 \sim 1$ ) on all frequencies of the full spectrum. A weak non-Markovity in the region of  $\delta$  and  $\alpha$  rhythms ( $\varepsilon_1 \rightarrow 4$ ) is found during the seizure. A strong non-Markovity on all frequencies is established immediately after termination of the seizure (14th window). The frequency behavior of  $\varepsilon_3(\omega)$  is characterized by steady non-Markovity ( $\varepsilon_3 \rightarrow 1$ ) in all the windows and in the entire frequency region. A weak quasi-Markovian noise (in the region of  $\alpha$  and  $\beta$  rhythms) appears before the seizure (2nd and 5th windows) and at the end of the seizure (12th, 13th, and 14th windows). The behavior of parameter  $\varepsilon_2(\omega)$  is rather peculiar. A strong non-Markovity ( $\varepsilon_2 \sim 1$ ) appears long before the seizure (3rd, 4th, 5th, and 6th windows). Further development of the seizure is accompanied by a slight noise in  $\varepsilon_2(\omega)$  in the region of  $\alpha$  and  $\beta$  rhythms. The termination of the seizure results in a strong non-Markovity ( $\varepsilon_2 \sim 1$ ) in the 13th window. Noisiness in the entire frequency range of the 14th window then occurs. The steady non-Markovity ( $\varepsilon_2 \sim 1$ ) is appreciable in the 15th, 16th, and 17th windows, appearing after termination of the seizure. The low-frequency (in the region of  $\delta$  rhythms with  $\varepsilon_2 \sim 1$ ) and high-frequency (in the top border of the  $\gamma$ -spectrum with  $\varepsilon_2 \sim 3$ ) sites of the spectrum are intensely noisy.

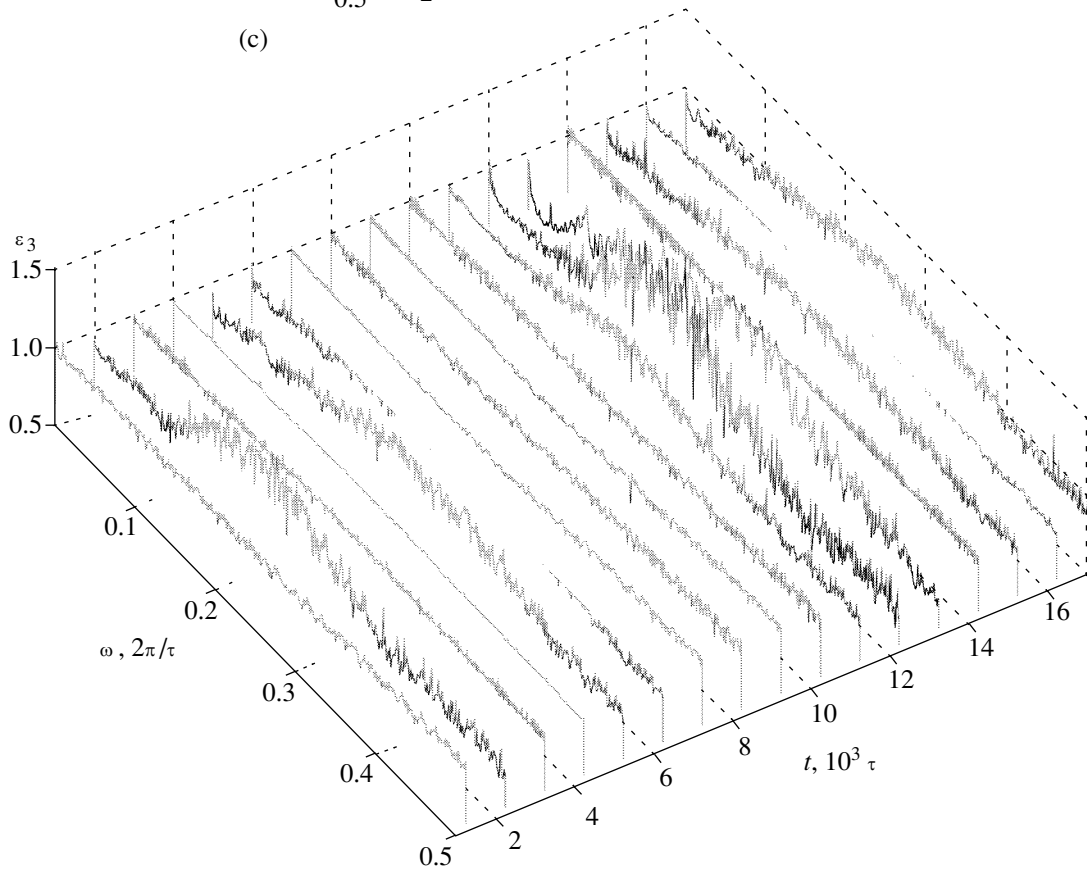
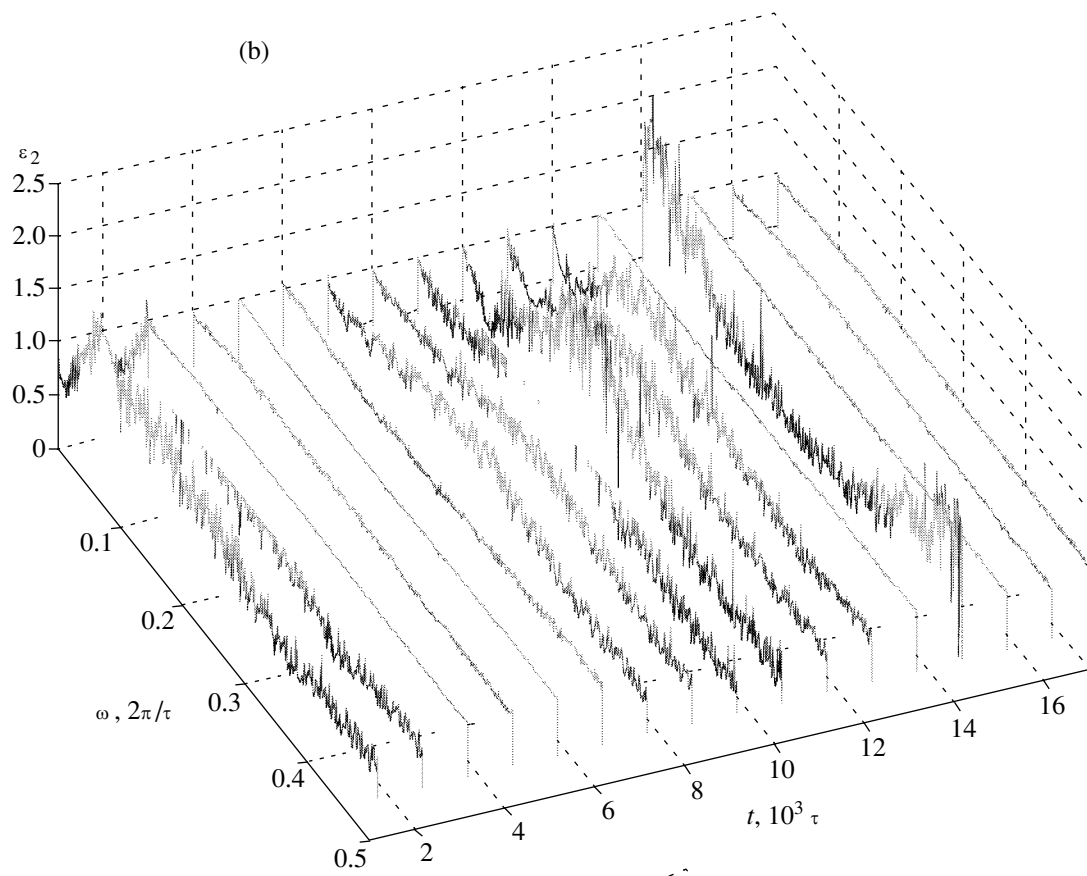
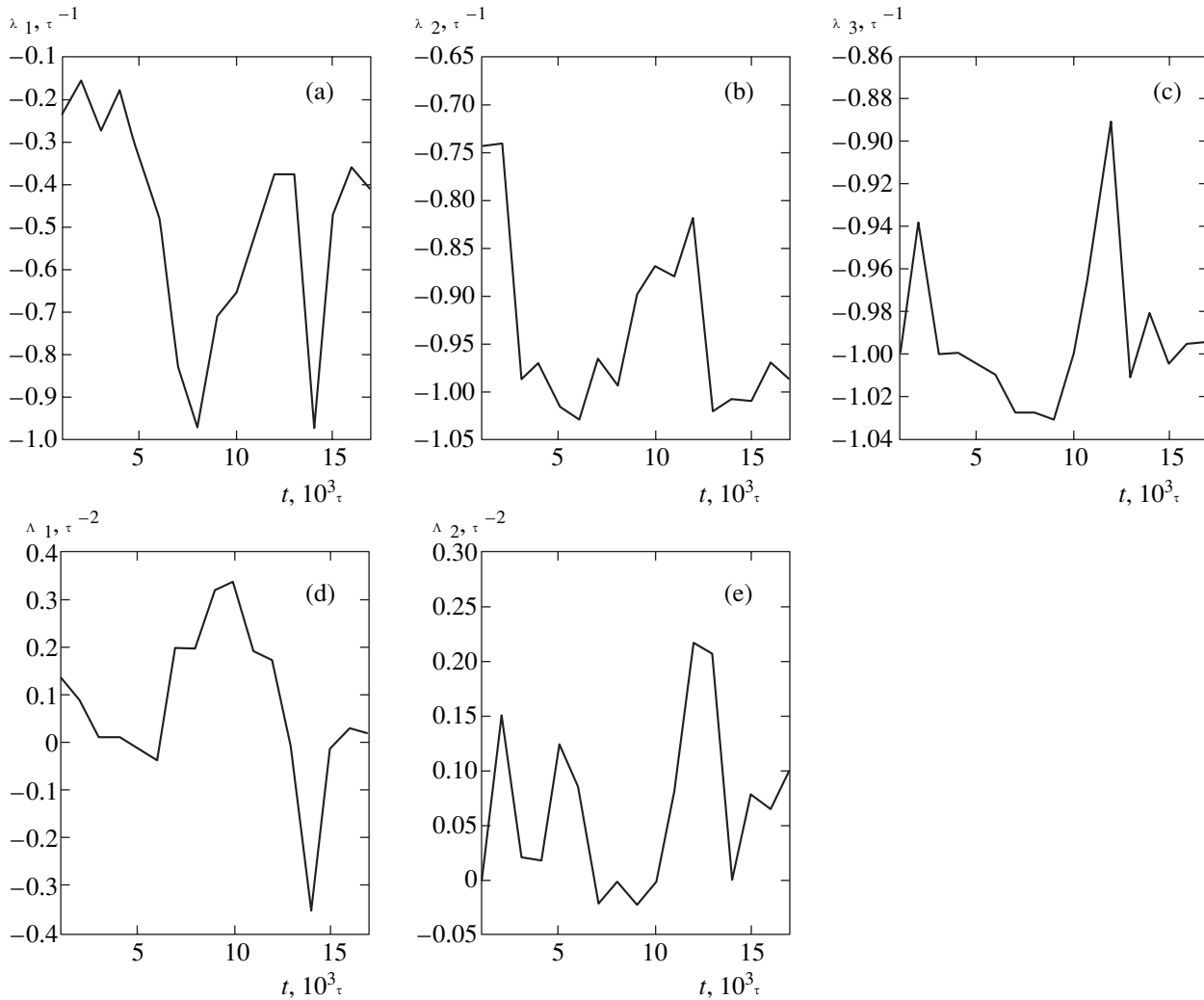


Fig. 3. (Contd.)



**Fig. 4.** The window-time behavior of the kinetic ( $\lambda_1$  (a),  $\lambda_2$  (b), and  $\lambda_3$  (c)) and relaxation ( $\lambda_1$  (d) and  $\lambda_2$  (e)) parameters for the time sampling at epilepsy with the tonic–clonic seizure. The kinetic parameters  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  are always negative and increase with seizure. The relaxation parameters  $\lambda_1$  and  $\lambda_2$  change sharply with the sign change from the beginning of the seizure. The most dramatic changes in the behavior of  $\lambda_1$  and  $\lambda_2$  occur during the seizure in the opposite directions.

3, can be smoothed. Therefore, the scales of these variables before and during the seizure are practically identical. The beginning of the seizure (see Figs. 2a–2d) is characterized by a sharp recession of low-frequency peaks in the spectrum  $\mu_0(\omega)$  (7th–10th windows); these peaks in  $\mu_0(\omega)$  rise sharply at the end of the seizure and immediately after the seizure. The spectra of  $\mu_j(\omega)$ ,  $j = 1, 2, 3$ , differ by white noise and low-frequency bursts on the tail of the seizure. These bursts are most appreciable in the behavior of the spectra  $\mu_2(\omega)$  and  $\mu_3(\omega)$ .

The behavior of the first three points in the statistical spectrum of the non-Markovity parameter  $\varepsilon_i(\omega)$ ,  $i = 1, 2, 3$  (see Fig. 3), turn out to be most indicative and demonstrative. The state before the seizure can be considered quasi-Markovian in the 1st–6th windows for the first level in the low-frequency region (here,  $\varepsilon_1(\omega)$  reaches a value of 10) and in the 1st and 2nd windows

for the second level ( $\varepsilon_2(\omega) \sim 1.5$ ). The beginning of the seizure (the 7th and 8th windows) is accompanied by the strong non-Markovity of the first level ( $\varepsilon_1(\omega) \approx 1$ ). The increase in low-frequency non-Markovity on the first ( $\varepsilon_1 \sim 3.8$ ), second ( $\varepsilon_2 \sim 1.5$ ), and the third ( $\varepsilon_3 \sim 1.5$ ) relaxation levels is visible at the end of the seizure (10th–13th windows). The behavior on the third level with a value of  $\varepsilon_3 \approx 1$  can be considered non-Markovian.

Non-Markovian relaxation behavior on the second level is noteworthy (see Fig. 3b). The strong non-Markovity ( $\varepsilon_2 \approx 1$ ) in the entire frequency region appears long before the seizure in the range from the third to the sixth windows. The weak noise at the mean frequencies is appreciable during the seizure (10th–12th windows). The ending of the seizure coincides with the non-Markovian 13th and quasi-Markovian 14th windows. The



appearance of a strongly pronounced non-Markovian state on the second level with a value of  $\varepsilon_2 \approx 1$  is therefore a clear precursor to the seizure. It is significant that a similar precursor is absent in other non-Markovian markers.

The relaxation ( $\lambda_1, \lambda_2$ , and  $\lambda_3$ ) and kinetic ( $\Lambda_1$  and  $\Lambda_2$ ) parameters calculated with the formulas of the theory (see Fig. 4) are very sensitive to approaching the seizure. All the parameters  $\lambda_i, i = 1, 2, 3$ , always remain negative and change within wide limits:  $(-0.97 \leq \lambda_1 \leq -0.15, -1.03 \leq \lambda_2 \leq -0.74, \text{ and } -1.03 \leq \lambda_3 \leq -0.89)$  in units of  $\tau^{-1}$ . Parameters  $\Lambda_1$  and  $\Lambda_2$  change sign at the time of the seizure. This corresponds to alternation in the type of solution to the discrete nonlinear kinetic stochastic equation (see Eqs. (2.56)–(2.58) in [6]). All of the above parameters are sensitive to approaching the seizure. A sharp decrease in the values and the sign alternation of  $\lambda_i$  and  $\Lambda_i$  can also be considered a quantitative precursor to a seizure.

Therefore, the increase in parameters  $\varepsilon_i(\omega)$  can be considered a markovization of the stochastic process. It may signify an increase in the chaotic components of EEG signals. Simultaneously, a decrease in  $\varepsilon_i(\omega)$  to unity is related to demarkovization of the process under study and to an increase in the regular components of the signals. It is obvious from Figs. 1–4 that the specific alternations, fast and sudden changes of chaotic and regular regimes, are inherent features of the stochastic variation of electric potentials during epileptic seizure.

## 6. CONCLUSIONS

We have clearly demonstrated that the set of kinetic, relaxation, dynamic, and spectral parameters and characteristics of a discrete stochastic process are valuable for quantification of stochastic processes of markoviza-

tion and demarkovization in EEG data and for prediction of and precursor to epileptic seizure. Because a similar situation is typical of the majority of the phenomena in live systems, our findings are most relevant for life sciences.

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