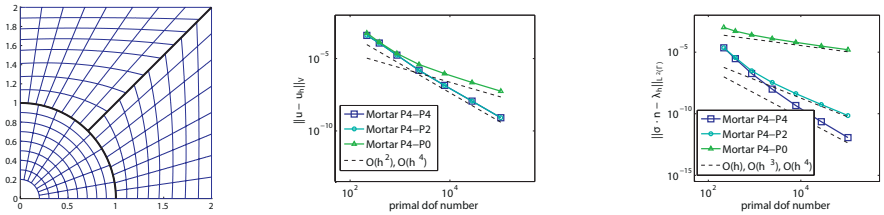

FIGURE 1. Error decay for different dual degrees, originally presented in [1]. Left: Mesh and domain partitioning. Middle: Error of the primal variable in the broken H^1 norm. Right: Error of the dual variable in the L^2 norm.

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Multiscale Petrov-Galerkin Finite Element Method for High-Frequency Acoustic Scattering

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(joint work with Daniel Peterseim)

The Helmholtz equation in an open bounded Lipschitz polygon $\Omega \subseteq \mathbb{R}^d$ ($d \in \{1,2,3\}$) with outer unit normal ν reads

$$\begin{aligned} -\Delta u - \kappa^2 u &= f \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \Gamma_D, \\ \nabla u \cdot \nu - i\kappa u &= g \quad \text{on } \Gamma_R. \end{aligned} \tag{1}$$

Here, the boundary $\partial\Omega$ is decomposed into disjoint parts $\partial\Omega = \Gamma_D \cup \Gamma_R$. Typically, the Dirichlet boundary Γ_D refers to a sound-soft obstacle whereas the Robin boundary Γ_R results from truncation of the full space problem to the bounded domain Ω. It is well known that standard finite element approximations to (1) exhibit the so-called *pollution effect* [1], which means that the ratio of the error of the finite element method and the best possible approximation in the finite element space becomes arbitrarily large as the real parameter $\kappa > 0$ (the wavenumber) increases. The mesh-size H for an accurate representation of the wave usually requires a fixed number of grid points per wavelength, written $\kappa H \approx 1$. The stability of the finite element method, however, requires a much finer mesh-size h with $h\kappa^\alpha \approx 1$ for some $\alpha > 1$. This makes high-frequency scattering simulations with standard methods problems computationally costly.

The talk presents a pollution-free Petrov-Galerkin multiscale finite element method for the Helmholtz problem with large wave number κ. The proposed method employs standard continuous Q_1 finite elements at a coarse discretization

scale H as trial functions, whereas the test functions are computed as the solutions of local problems at a finer scale h. The diameter of the support of the test functions behaves like mH for some oversampling parameter m. Provided m is of the order of $\log(\kappa)$ and h is sufficiently small, the resulting method is stable and quasi-optimal in the regime where H is proportional to κ^{-1}. In homogeneous (or more general periodic) media, the fine scale test functions depend only on local mesh-configurations. Therefore, the seemingly high cost for the computation of the test functions can be drastically reduced on structured meshes. Numerical experiments in two and three space dimensions give empirical insight in the dependence of the parameters H, h, and m.

The talk is based on the recent works [2, 3].

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A Plane Wave Virtual Element Method for the Helmholtz Problem

Ilaria Perugia

(joint work with Paola Pietra, Alessandro Russo)

The virtual element method (VEM) is a generalisation of the finite element method recently introduced in [2, 3], which takes inspiration from mimetic finite difference schemes, and allows to use very general polygonal/polyhedral meshes.

My talk was concerned with a new method introduced in [15], based on inserting plane wave basis functions within the VEM framework in order to construct an H^1-conforming, high-order method for the discretisation of the Helmholtz problem, in the spirit of the partition of unity method (PUM, see e.g., [12, 13]).

Plane wave functions are a particular case of Trefftz functions for the Helmholtz problem, i.e., functions belonging to the kernel of the Helmholtz operator. Inserting Trefftz basis functions within the approximating spaces in finite element discretisations of the Helmholtz problem allows to obtain, compared to standard polynomial spaces, similar accuracy with less degrees of freedom, mitigating the the strong requirements in terms of number of degrees of freedom per wavelength due to the pollution effect [1]. There are in the literature several finite element methods for the Helmholtz problem which make use of Trefftz functions (for details, see the recent survey [8]). Besides the above mentioned PUM, which is H^1-conforming, other approaches use discontinuous Trefftz basis functions and impose interelement continuity with different strategies: by least square formulations [17, 14]); within a discontinuous Galerkin (DG) framework, like the ultra weak variational formulation [5, 4] or its Trefftz-DG generalisation [9]; by the use