What makes individual investors exercise early? Empirical evidence from non-tradable fixed-income products[☆]

Mathias Eickholt[a], Oliver Entrop[a,*], Marco Wilkens[b]

[a] University of Passau, Innstraße 27, Passau 94032, Germany
[b] University of Augsburg, Universitätsstraße 16, Augsburg 86159, Germany

JEL classification:
G10
G11
G13

Keywords:
Early exercise
Failure to exercise
Liquidity demand
Putable bond

A B S T R A C T

This paper studies the early exercise behavior of individual investors in non-tradable German governmental putable bonds. Analyzing holding and exercise decisions of more than 220,000 individual investors at a single-account level over 13 years, our major findings are: (i) Individual investors use their early exercise right predominantly at times that are not optimal according to standard option pricing theory. (ii) Only very few attractive exercise opportunities are exploited over time. (iii) Both exercises and failure to exercise differ significantly among investor groups, are related to their personal characteristics as well as product characteristics and environmental circumstances, and are subject to cognitive biases. They tend to be persistent over time on the investor level. (iv) The demand by investors for liquidity and financial flexibility is a more important motive for investment and exercise than performance seeking.

1. Introduction

The right to exercise early before maturity is a common feature of many non-tradable investment products such as savings bonds, redeemable long-term deposits, (non-tradable) putable bonds or other financial innovations offered by banks to individual investors. While exercise behavior has been analyzed comprehensively with regard to American equity options (e.g., Overdahl and Martin, 1994; Finucane, 1997; Poteshman and Serbin, 2003; Pool et al., 2008; Barraclough and Whaley, 2012) and mortgages (e.g., Green and Shoven, 1986; Schwartz and Toruos, 1989; Stanton, 1995; Deng et al., 2005), little is known about individual investors' use of early exercise rights in these kinds of fixed-income investment products (which may be due to a lack of data at the investor-specific level). This is particularly remarkable in view of the products' relevance and because investors' exercise behavior has direct implications for the pricing of the products by the issuing institutions and for their liquidity management.[1]

While we can — based on the above-mentioned literature — certainly expect that individual investors' exercise behavior in non-tradable fixed-income products also differs from the behavior predicted by standard option pricing theory (OPT) in frictionless markets, we cannot necessarily expect that the results for equity options and mortgages directly transfer to non-tradable fixed-income investment products. Mortgages are loans and the decision to redeem them early is obviously different from exercising an investment early. Tradable American equity options can also be sold on the secondary market while the only way to monetize the non-tradable investment products considered above is early exercising.[2] Thus, the decision to exercise a non-tradable product early is equivalent to selling the product.

The purpose of this paper is to further our understanding of individual investors' exercise behavior by providing a comprehensive empirical analysis of their use of early exercise rights in German governmental non-tradable putable bonds (German Federal Savings Notes, GFSN). GFSN were introduced in the 1970s for savers as an alternative to banks' long-term deposits and consequently attracted a similar clientele, which is why we can expect that our results are transferable to similar types of products. Benefitting from a unique, not-publicly available data set comprising the decisions

☆ An earlier version of this paper circulated under the title "What makes individual investors exercise early? Empirical evidence from the fixed-income market".

* Corresponding author.
E-mail address: oliver.entrop@uni-passau.de (O. Entrop).

1 We will discuss the implications of our findings for the pricing in the conclusion.

2 As an additional difference, equity options and non-tradable fixed-income investment products are certainly bought for a variety of motives and do not necessarily attract identical clienteles.

of more than 220,000 individual investors to hold or exercise GFSN over a time horizon of 13 years, we conduct a detailed analysis at a single-account level.

To the best of our knowledge we are the first to consider the early exercise behavior in non-tradable fixed-income investment products. We basically cover three research questions. First, we analyze determinants of early exercises. In this context we focus on the following dimensions: value orientation in exercising according to OPT, investment history, environmental circumstances, product characteristics and investors' personal characteristics.

Second, we examine the exercise quality and its determinants. More specifically, we (i) analyze how often the empirically observed exercise decisions are at points in time that are optimal according to OPT, and (ii) how often investors fail to exploit attractive opportunities, i.e. investors do not exercise at points in time where it would be optimal according to OPT. We stress that these two subquestions are *not* two sides of the same coin. As OPT assumes frictionless markets and does not consider, e.g., investors' portfolios, tax incentives and their potential liquidity constraints, an exercise that is not optimal according to OPT is not necessarily irrational or suboptimal for the investor. In fact, if, for example, an investor requires liquidity it can be optimal to exercise the GFSN at a point in time that is suboptimal according to OPT rather than taking an expensive loan. On the other hand, it is hard to imagine good economic reasons for not exercising at points in time that are optimal according to OPT as investors could simply shift their investment into more favorable fixed-income products.

Third, we seek to discover individual investors' general motives for investment and exercise with regard to such products.

As indicated, we start our analysis by examining what determines individual investors' early exercises of GFSN. Using pooled and random-effects logit regressions, our main findings are as follows: first, value orientation is an important determinant. The probability of investors exercising the option increases significantly with the diminishing ratio of present value to exercise value. This implies that investors are sensitive to GFSNs' value and that their exercise behavior is not fully detached from standard theory. However, there are many systematics that do not reflect behavior predicted under standard OPT assumptions. In fact, second, the exercise probability is linked to the investment and decision history. For example, we observe delays of up to six months in investors' reactions to market changes. Third, strong movements in the equity market are accompanied by enhanced exercise activities in GFSN, which indicates that individual investors use the early exercise right to liquidate investments so as to participate in attractive growth phases of other markets. Similarly, our results suggest — in an analogy to the equity market (e.g., Badrinath and Lewellen, 1991; Grinblatt and Keloharju, 2001; Ivkovic et al., 2005; Liedtka and Nayar, 2012) — that the exercise option is broadly used to optimize tax payments. Fourth, we find differences in the exercise probability related to product characteristics. Besides a product's maturity, the steepness of the coupon structure and the time until the next coupon payment also seem to have an influence on individual investors' decision-making. Our analysis shows that — valuation, maturity and all else being equal — investors prefer to keep products with a high final coupon payment rather than products with a flatter coupon structure, which we attribute to psychological reasons. Moreover, it becomes obvious that investors hesitate to exercise a GFSN shortly before a coupon payment date, which — referring to the mental accounting concept (e.g., Shefrin and Statman, 1984; Thaler, 1999) — implies that investors differentiate between accrued interests and cash payouts. Thus, exercise decisions seem also to be subject to cognitive biases. Fifth, the exercise frequency varies depending on an investor's portfolio and personal characteristics. For instance, we note that experience in early exercising a GFSN investment leads to an increased probability of another exercise, which might be due to lower information costs or learning effects (see Nicolosi et al., 2009). As another example, we find that investors between 20 and 40 years of age are significantly more likely to exercise than other investors groups. We ascribe this to a potentially greater demand for liquidity at that stage of life due to, e.g., starting a family.

Next, we study the exercise quality and its determinants. Evaluating all decisions to hold or exercise early for each month and each product, our first result is that the broad majority of early exercises (between 76% and 86%) occur at times when standard option pricing theory would dictate that it is not advantageous, i.e. investors exercise at times when the exercise value lies (significantly) below the continuation value. This behavior tends to be persistent over time on the investor level and is significantly related to personal characteristics.

We also find that individual investors regularly fail to exploit (more) attractive exercise opportunities. Throughout our sample period — given monthly granularity — more than 98% of exercise rights in GFSN that should have been used according to OPT were neglected. This share of failure-to-exercise observations is considerably larger than in comparable studies for the option market (e.g., Poteshman and Serbin, 2003; Pool et al., 2008; Barraclough and Whaley, 2012). Most interestingly, failure to exercise is also related to product and personal characteristics. Investors refrain from using an attractive exercise opportunity more often when the next coupon payment is shortly due, the coupon structure is steep and duration is low, which we interpret as resulting from cognitive biases like above. This behavior also depends on the age cohort. Also failure to exercise turns out to be persistent over time.

Not surprisingly, we calculate an average negative excess return from exercising of circa -0.117% to -0.300% per year for our sample and conclude that only very few individual investors achieve a better performance through early exercising compared to a buy-and-hold strategy. We find this result particularly remarkable. Even though it is clear that investors do not use the early exercise right optimally in the sense of OPT the deviation from the optimal strategy is so large that exercising does not add financial value compared to a buy-and-hold strategy but reduces it. While this holds true for almost all investor subgroups, we again find differences depending on personal characteristics.

While the literature typically relates failure to exercise at optimal points in time to irrationality (e.g., Poteshman and Serbin, 2003; Pool et al., 2008; Barraclough and Whaley, 2012), this cannot necessarily be claimed for the suboptimal exercises according to OPT as discussed above.[3] But, as noted in our first analysis, the early exercise right is for example frequently used to optimize tax payments. Further, it is very likely that investors use the exercise right for liquidity reasons. Our third research interest is to further explore the general motives of individual investors' in exercising GFSN. For this, we run an exploratory factor analysis based on consolidated information about investors' GFSN investment and exercise strategies. Passing several robustness checks, the factor analysis isolates four to five latent factors that we interpret as follows: the first and most important factor comprises investors' wish for liquidity and financial flexibility. The second factor represents the importance of the mid- and long-term value of an investment. The third factor isolates the desire for performance and is associated with positive excess returns. In contrast, a fourth factor describes an active "trading" strategy that does not however result in higher investment yields, for which reason we label this factor "activism". Finally, the fifth factor represents an investor's

[3] It should be noted that even missing attractive exercise opportunities can be rationalized if, for example, continuous monitoring of the investment is costly (e.g., Stanton, 1995; Barraclough and Whaley, 2012; Liao et al., 2014) or transaction costs exist (e.g., Stanton, 1995; Finucane, 1997; Koziol, 2006).

responses to changes in taxation. Not surprisingly, the relevance of these identified latent factors differs depending on investor groups. Still, the overall results of the factor analysis suggest that individual investors in GFSN interpret their right to exercise early as more of a financial flexibility feature than as an option to improve the investment performance.

The paper proceeds as follows. In Section 2 we introduce the data set, present the valuation algorithm, analyze the values at issuance of GFSN and describe the variables for our empirical analysis. Section 3 analyzes the determinants of early exercising (research question one, Section 3.1), evaluates the exercise quality of individual investors' exercise behavior including the resulting performance (research question two, Section 3.2) and discusses implications (Section 3.3). Section 4 runs a factor analysis to gain further insights into individual investors' motives and decision-making (research question three). Section 5 concludes.

2. Data and valuation

2.1. Product description

For our analyses we are able to utilize a large and unique data set from the German Finance Agency ("Bundesrepublik Deutschland Finanzagentur GmbH")[4], which covers investment and early exercise decisions of individual investors in German Federal Saving Notes ("Bundesschatzbriefe", in the following GFSN). GFSN are basically standard non-tradable fixed-rate putable bonds issued by the Federal Republic of Germany for financing the government,[5] which — most interesting for our study — incorporate an early exercise option that gives the investor the right to reclaim his investment at any time during maturity after an initial one-year blocking period.[6] No fees or penalty payments are charged in the case of an early exercise. At each issuance date two types of GFSN are offered: Type A, a step-up bond with a maturity of six years and yearly coupon payments and Type B, a step-up bond with a maturity of seven years where all coupons are accrued and paid at maturity.

To formalize this description let us assume that the notional value is 1, $i=0$ is the issuance date, c_i is the coupon in $i=1,\dots,T$, with $c_{i+1}\geq c_i$ and where $T=6$ or $=7$ is the maturity date for Type A or Type B GFSN, respectively. For Type A the coupons c_i are paid yearly in i and the final payment in T is $1\cdot(1+c_6)$. For Type B, the coupons are accrued and paid at maturity, i.e. the final payment equals $1\cdot\prod_{j=1}^{7}(1+c_j)$.[7]

If a GFSN is exercised early in $t\in[1,T]$ the payment in t, i.e. the exercise value, consists of notional value plus accrued interest since the last coupon payment for Type A, and notional value plus accrued interest since issuance for Type B. That means the exercise value EV_t is given by:

$$EV_t=\begin{cases}1\cdot\left(1+c_{\lfloor t\rfloor^*+1}(t-\lfloor t\rfloor^*)\right) & \text{for Type A}\\ 1\cdot\prod_{j=1}^{\lfloor t\rfloor^*}(1+c_j)\left(1+c_{\lfloor t\rfloor^*+1}(t-\lfloor t\rfloor^*)\right) & \text{for Type B ,}\end{cases} \quad (1)$$

where $\lfloor x\rfloor^*=\max\{k\in\mathbb{N}_0:k<x\}$ is the largest non-negative integer smaller than x.

At each point in time there is one Type A and one Type B GFSN that can be bought by investors. These two GFSN coincide in their first six coupon payments. After a certain period of time — that is ex ante not known but is usually after market interest rates have changed significantly — the current issuance is closed and new GFSN of both types with a new coupon structure can be bought. The average length of time an issuance is offered is 45 days in our sample period. Investors can purchase the current issuance of a GFSN at nominal value plus accrued interests. That means if the issuance date is $i=0$ and the GFSN is bought in $0\leq s<1$ the price is $1\cdot(1+c_1\,s)$.

GFSN can be acquired in two ways. First, it is possible to invest directly through the German Finance Agency at no cost via telephone, postal order or online order. The GFSN are then booked in a cost-free account at the German Finance Agency. Second, GFSN can be purchased at banks, which however typically claim custody fees for administering GFSN positions. Investors can shift their investments from an account at a bank to the cost-free account at the German Finance Agency at any time. GFSN are sold exclusively to individual investors,[8] hence we do not have to distinguish in our analysis between different investor types. As GFSN cannot be traded on a secondary market the only possibility of monetizing an investment is to use the exercise right and return the product to the issuer.

2.2. Valuation and value at issuance

Although the focus of this study is on the early exercise behavior of investors once they have bought a GFSN, it is worth first taking a closer look at GFSNs' values at issuance, i.e. at the question whether the coupons are set in a way that can be assumed to be market-consistent, given the early exercise right, and, thus whether they are attractive for investors.

Due to the early exercise right a GFSN is an American-type security. According to standard option pricing theory, calculating its value is, given the standard regularity conditions, basically an optimal stopping problem and is determined by an optimal exercise strategy that maximizes the expected discounted payoffs under the martingale measure (e.g., Duffie, 2001, pp. 183–185). In the following we describe Type B GFSN as this makes formulas shorter, although interim coupon payments (of Type A) can easily be integrated. We present the valuation problem generally enough to calculate values at issuance as well as at any point until maturity. The latter is necessary for the subsequent sections where we analyze investors' exercise behavior.

Given the exercise value EV_t in Eq. (1), the present value of a GFSN Type B in $t\in[0,T]$ is given by:[9]

$$PV_t=\sup_{\tau\in\mathcal{T}_t}E_t^Q\left(\exp\left(-\int_t^{\tau}r(s)ds\right)EV_\tau\right) \quad (2)$$

$$=E_t^Q\left(\exp\left(-\int_t^{\tau_t^{opt}}r(s)ds\right)EV_{\tau^{opt}}\right), \quad (3)$$

where $r(s)$ denotes the short rate process that generates the filtration $(\mathcal{F}_s)_{0\leq s\leq T}$. Q is the spot martingale measure and $E_t^Q(\cdot)$ represents the respective expectation operator conditional on the information set $\mathcal{F}_t$ available in t. $\mathcal{T}_t$ is the set of all $(\mathcal{F}_s)$-stopping times

[4] The German Finance Agency is a state-owned central service agency for Germany's governmental borrowing.

[5] While GFSN accounted for a significant share of Germany's borrowing until the 1990s, the relevance has decreased since then. Finally, due to high administrative costs, the German government decided in 2012 to stop offering products exclusively for individual investors and stopped issuing new GFSN. Nevertheless, similar products are still sold by, e.g., banks.

[6] An additional restriction applies: the maximal exercise value per investor is capped at € 5000 within 30 interest days, which we ignore in the following.

[7] It should be noted that the Type B GFSN has a zero-bond structure because coupons are accrued until maturity but it is not a zero-bond in the classical sense. The latter has only a face value that is paid at maturity but carries no coupons.

[8] Additionally, GFSN can also be acquired by resident institutions serving public benefit, charitable or religious purposes, which we neglect in the following due to the very small share of overall investments.

[9] It should be noted that the exercise value at maturity T equals the final payment of the GFSN.

Table 1
GFSN coupon structure and spot rates.

		Type A and Type B GFSN						Type B GFSN
		Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Yearly coupon in %	Mean	2.745	3.199	3.627	4.034	4.409	4.821	4.944
	Median	2.750	3.250	3.625	4.000	4.250	4.500	4.750
	Min.	1.000	1.500	2.000	2.500	3.250	3.500	3.500
	Max.	4.500	4.750	5.500	6.250	6.750	7.500	7.500
	Std. dev.	0.810	0.811	0.805	0.819	0.856	0.927	0.924
Spot rate at issuance in %	Mean	3.387	3.540	3.700	3.853	3.994	4.122	4.235
	Median	3.429	3.599	3.779	3.898	3.936	4.021	4.117
	Min.	1.909	2.061	2.207	2.383	2.562	2.730	2.882
	Max.	5.119	5.216	5.267	5.293	5.608	5.940	6.202
	Std. dev.	0.821	0.766	0.727	0.709	0.707	0.713	0.724

The table exhibits statistics on the coupon offerings for Type A and B GFSN for all 204 issuances (102 Type A, 102 Type B) in our sample period from July 1996 to February 2009 and on the corresponding German spot rates. For year 1 to 6 identical coupons are offered for Type A and B at each issuance date, whereas the coupons in year 7 are only applicable for Type B. The spot rates represent the term structure of interest rates on listed Federal securities at the respective issuance date according to the Deutsche Bundesbank.

with values in the time interval $[\max\{1, t\}, T]$ where the left interval boundary takes the initial one-year blocking period into account. $\tau_t^{opt} \in \mathcal{T}_t$ is an optimal stopping time where the supremum in Eq. (2) is reached. It can be characterized by the first time the value process touches the exercise value:

$$\tau_t^{opt} = \inf\{s \geq \max\{1, t\} : PV_s = EV_s\}. \qquad (4)$$

For numerically proxying Eq. (2) and (4) we make use of a discrete-time version of the optimal stopping problem (2) that is provided in the Appendix. The first step in the valuation, however, is to model the underlying interest rate dynamics, i.e. specify the short rate process. To avoid the need to recalibrate the term structure model at every point in time, which would be the case in common HJM models or LIBOR market models (see, e.g., Hull, 2006), and to achieve consistent dynamics over time, we choose to apply a more sophisticated model of the class of (essentially) affine term structure models according to Dai and Singleton (2000), which determines the interest term structure throughout the whole sample period based on a time-invariant function of only a small set of common state variables. Egorov et al. (2006) and Tang and Xia (2007) show that a three-factor essentially affine model $EA_1(3)$, with one factor affecting the conditional variance matrix, provides empirically the best performance for different countries, including Germany, while it also allows for very flexible specifications. We calibrate this model to monthly German Federal term structures over the period of July 1996 to February 2009 via a Kalman filtering algorithm together with quasi-maximum likelihood, under the assumption that the rates of all yearly maturities of 1 to 10 years are not perfectly observed. This is basically the same procedure as used by, e.g., Hördahl and Vestin (2005).

Next, we follow the standard approach and solve the discrete-time optimal stopping problem via stochastic dynamic programming (e.g., Glasserman, 2004). Here we apply the least-squares Monte Carlo simulation approach as suggested by Longstaff and Schwartz (2001), carrying out simulations with 10,000 paths, whereby we use Euler discretization of the short rate process on a monthly basis and take the first four monomials as basis function. The Appendix provides a detailed description of our simulation procedure.

Before we apply the valuation algorithm presented above, we examine the coupon offering of GFSN at issuance. Table 1 provides an overview of GFSN coupon structures in comparison to prevailing German federal spot rates at issuance in our sample period of July 1996 to February 2009. Typically, the offered coupons lie below the corresponding spot rates for the first years but rise above market rates towards maturity. Yet in accordance with changes in market conditions, the coupon structures of different issuances vary, implying a significant variance in coupon offerings over time for both Type A and B.

For analyzing the present values at issuance, we calculate the present value of the currently offered GFSN (assuming a notional value of 1) at the first trading day of each month. Fig. 1 visualizes the results. The black dots show the respective present values separately for Type A and B. Obviously, there is a significant variation in these values at issuance over time which means that the respective GFSN are equipped with coupon structures that are differently attractive for investors, given the respective prevailing market interest rates.

This is particularly reflected in the present value ex option (grey dots) which is simply the discounted fixed-payment stream, ignoring the early exercise right. There are some phases where the value ex option of the offered GFSN lies significantly below par, e.g. even lower than 0.98 or 0.96. In these cases it is likely that it will be optimal to rationally use the early exercise right directly after the initial one-year blocking period, which implies that the value of the option as the difference between the present value and the present value ex option is quite high.[10] If, however the value ex option is above par, the exercise right is not in the money (and thus less likely to get into the money within short time) and the option value is accordingly small(er). The present values of Type B GFSN are always above the respective Type A values because of the longer maturity which increases the option value.

All in all, the GFSN (especially Type B) in our study are very often an attractive investment in terms of their present value. However, this requires the investors to use the early exercise right fully rationally in accordance with standard option theory. If an investor sees GFSNs as a buy-and-hold investment, they are often unattractive as he forgoes the option value. This is reflected in the present value ex option being most often below par.

For completeness, the lower part of Fig. 1 depicts the sold volume of each GFSN issuance in mio. EUR. There is a falling trend over time. In a separate analysis (not reported here) we do not find evidence for a link between values and volumes. Further, the sold volume of GFSN of Type B is much smaller than those of type A. This may be linked to the different tax treatment and/or mental accounting:[11] while coupons of the Type B GFSN are taxed together at the same time, Type A coupons are taxed yearly, mean-

[10] It should be noted that the present value at issuance has a lower boundary at $(1 + \text{first coupon})/(1 + 1\text{-year spot rate})$ as the GFSN can be exercised after 1 year for the first time.

[11] See Shefrin and Statman (1984) and Thaler (1999) for analogous considerations in case of stocks and dividends.

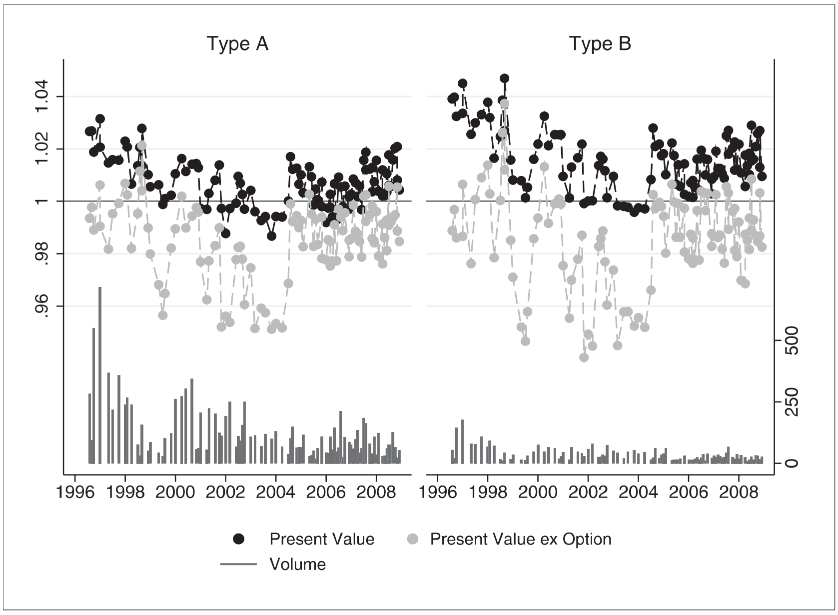

Fig. 1. Value at issuance of GFSN and volumes. This figure shows the present value (black dots) and the respective value without the early exercise right ("ex Option", grey dots) of the currently offered GFSNs on a monthly basis, separately for Type A and B, assuming a notional value of 1. The present value is calculated via the algorithm described in Section 2.2, the present value ex option equals the discounted fixed-payment stream. The valuation is carried out on the first trading day each month and is shown on the left y-axis. Volume (grey bars) is the EUR-volume of GFSN that are sold to investors each month. The right y-axis shows the volume in mio. EUR.

ing that personal tax allowances can be utilized each year. Further, investors may prefer regular payments from their securities to accumulating coupons.[12]

2.3. Summary statistics

Our data set comprehends — on a daily and single-account basis — all GFSN transactions and early exercise decisions of 223,017 individual investors booked between July 1996 and February 2009 in an account at German Finance Agency.[13] Overall, these transactions are spread over 204 issuances of Type A and B GFSN in the observation period on 102 issuances dates that have already been analyzed in the previous section. In addition, we have some information on each investor's personal characteristics. This detailed data structure on an individual account and daily level allows us to link individual transactions to, e.g., an investor's age, gender or investment history. Table 2 presents selected summary statistics for the investor sample.

In general, we have a broad and relatively balanced investor base that is very similar for both Type A and Type B GFSN structures. For instance, there is an almost equal representation of male (33.469% for Type A, 38.406% for Type B) and female (38.362%, 37.995%) investors (no information for 28.169%, 23.598%). Similarly, we have a wide range of investor ages (on average 43.551 years, 32.711 years) with a slightly positively skewed distribution, i.e. there are somewhat more older than younger investors in our data set. There is also a significant share of very young investors (in particular for the zero-bond structure of Type B GFSN), which we mainly interpret as savings accounts in a child's name for, e.g., education costs. Concerning investors' educational background, the data provides us with information on doctoral degrees and professorships, which taken together form 4.150% (3.499%) of the investors. Geographically, the investor base is widespread over Germany. We classify all investors into four clusters of residence areas[14] based on the first two digits of an investor's zip code, which we have in the data, and conclude that the majority of investors in GFSN live in less populated areas (51.371%, 51.417%), while a smaller number (15.380%, 15.089%) lives in highly populated areas or cities. Finally, we note that more than half of the investors prefer to acquire GFSN indirectly (59.181%, 54.011%) via banks and transfer their accounts later to the German Finance Agency, even though typically custody and administration are only cost-free at the German Finance Agency.[15] Next, Table 3 summarizes the activities of our investor sample.

The statistics show that most investors invested rarely or only once in GFSN during the sample period. The average number of investments is comparatively small with only 3.147 investments per investor for Type A and 2.790 for the less popular structure of Type B GFSN. Furthermore, we find small median overall investment volumes of only € 10,226 and € 4090 respectively, whereby the high variances for both product types indicate that there are also investors with significantly higher investment volumes and trading frequencies. In general, we attribute the low activity

[12] While we focus in this study on early exercise decisions, a detailed analysis of the buying behavior would be a separate study on its own. As pointed out by Barber and Odean (2008) for the equity market, buying and selling decisions are different choice problems. When investing money investors can choose from a large range of securities while they can only choose from the securities they hold when they want to monetize an investment.

[13] These accounts stand for approximately 64% of the overall outstanding volume on average. Our data set is a randomly drawn sample of the original data that represents circa 25% of all accounts at the German Finance Agency. We exclude accounts with an average transaction volume below € 300 since for such small volumes the early exercise right is of comparatively low importance.

[14] As a proxy we use the average population density per sq. km according to German Federal Statistics Office (2009) and define the following clusters: <250 = sparsely, 250–750 = moderately, >750 = highly populated area. In a fourth cluster we group investors living in large cities, which we identify by their short double-digit zip codes that are allocated only to major cities in Germany.

[15] Indirect distribution means in this paper that an investor purchases a GFSN at a bank and transfers his investment later to the German Finance Agency. In contrast, we define the preferred distribution channel as direct if an investor executes at least one direct transaction in the overall observation period. We are aware that this classification might produce a bias towards direct distribution since some investors use both channels. On the other hand we do not have direct transactions in our data set from 1996 to 1999 as these were not possible during these times.

Table 2
Statistics on investor base.

	Type A GFSN		Type B GFSN	
	Percent	Absolute	Percent	Absolute
Personal characteristics				
Gender				
Male	33.469	52,717	38.406	35,595
Female	38.362	60,425	37.995	35,214
n/a	28.169	40,370	23.598	21,871
Age				
0 to 20 years	18.978	29,892	38.515	35,696
21 to 40 years	23.665	37,276	20.986	19,450
41 to 60 years	28.127	44,304	24.455	22,665
61 to 100 years	25.960	40,890	13.166	12,202
n/a	3.270	5150	2.878	2667
Doctoral degree				
Doctorate or professorship	4.150	6536	3.499	3243
No doctoral degree	95.850	150,976	96.501	89,437
Geographical location				
City	4.165	6560	3.835	3554
Highly populated	11.215	17,665	11.254	10,430
Moderately populated	28.547	44,965	29.252	27,111
Sparsely populated	51.371	80,916	51.417	47,653
n/a	4.702	7406	4.243	3932
Preferred distribution(1)				
Indirect (at banks)	59.181	93,217	54.011	50,057
Direct (via the German Finance Agency)	40.819	64,295	45.989	42,623
Overall	100.000	157,512	100.000	92,680

The table exhibits information on personal characteristics of all 223,017 individual investors in our data sample, whereof 27,175 investors hold both Type A and Type B GFSN. Investors are clustered in residence areas according to the first two digits of their zip code using an urbanization index and population density figures from German Federal Statistics Office (2009). We classify the population density per sq km as follows: <250 = sparsely, 250–750 = moderately, >750 = highly populated. (1) We define the preferred distribution channel as direct if an investor executes at least one direct transaction. Indirect means that an investor purchases a GFSN at a bank and later transfers his investment to an account at the German Finance Agency.

Table 3
Statistics on investors' financial activities.

	Type A GFSN					Type B GFSN				
	Mean	Med.	p5	p95	St. dev.	Mean	Med.	p5	p95	St.dev.
Investments per investor										
Number of investments	3.147	1.000	1.000	11.000	5.200	2.790	1.000	1.000	9.000	5.139
Overall investment volume in €	23,053	10,226	1019	85,569	45,614	11,526	4090	511	44,945	30,593
Volume per investment in €	8818	5266	639	25,565	14,789	5085	2505	460	17,663	6,693
Early exercises per investor										
Number of exercises	0.526	0	0	2.000	1.491	0.517	0	0	2.000	1.483
Overall exercise volume in €	1812	0	0	10,178	4761	1284	0	0	6647	3547
Volume per exercise in €	4094	4090	511	10,000	2742	3017	2540	511	7669	2610
Early exercises over time										
Exercise rate per month and GFSN in %	0.626	0.309	0.081	2.418	1.058	0.518	0.327	0.081	1.693	0.671
Exercise volume per month and GFSN in €	160,356	80,323	2711	570,448	296,062	55,242	26,212	2228	209,374	296,062

The table exhibits statistics on the investments and early exercises of all 223,017 individual investors in 204 issued GFSN (102 Type A, 102 Type B) during our sample period from July 1996 to February 2009. The early exercise rate is defined as the monthly ratio of number of exercises to the number of current investments in this GFSN.

level in our sample to the strict restriction of GFSN to individual investors. With institutional investors we would expect considerably more transactions and higher investment sums.

The right to redeem early a GFSN investment is frequently used (fourth to sixth row). On average each investor in Type A GFSN exercised early 0.526 times (0.517 for Type B) throughout the sample period resulting in a mean overall exercise volume of € 1812 (€ 1284). Again the standard deviation is high with 1.491 (1.483) exercises per investor, which is also emphasized by the median of 0 (0) and the 95 percent quantile of 2.000 (2.000) exercises per investor account. The high variance here is a first indicator that individual investors use early exercise rights very heterogeneously.

Finally, the last two rows of Table 3 outline investors' use of the exercise option over time. As proxy we calculate a monthly early exercise rate per GFSN, defined as the ratio of number of exercises to the number of overall investments ($\frac{\text{Number of exercises}}{\text{Number of investments}}$). On average this rate amounts to circa 0.626% (0.518%) per month and GFSN, whereby the median value is smaller due to several exercise peaks throughout our observation period. Interestingly, a quite constant base exercise rate seems to exist over all GFSN. Even the 5 percent quantile of the exercise rates still lies at around 0.081% (0.081%), which implies that exercises occur independently of the market environment.

Table 4
Variables considered for analysis of determinants.

Abbr.	Variable description	Type A GFSN Mean	Type A GFSN St.dev.	Type B GFSN Mean	Type B GFSN St.dev.
Value orientation					
PVEV	Ratio of present value to exercise value	1.034	0.027	1.0511	0.048
Investment history					
MISSEDALL	Number of non-exercise months with PVEV=1 since issuance	2.869	6.044	1.726	5.022
MISSED6	Number of non-exercise months with PVEV=1 within the last 6 months	0.769	1.649	0.427	1.291
VOLUME	Investment volume in €	7513	11,165	3972	8025
Environmental circumstances					
CDAX10+	Dummy, changes in CDAX > +10% within a month	0.033	0.177	0.031	0.174
CDAX10-	Dummy, changes in CDAX < -10% within a month	0.080	0.271	0.081	0.273
CDAX25+	Dummy, changes in CDAX > +25% within 6 months	0.094	0.292	0.095	0.293
CDAX25-	Dummy, changes in CDAX < -25% within 6 months	0.085	0.278	0.087	0.281
VOLDAX	45-day option-implied volatility of DAX measured by VDAX	24.045	9.846	23.888	9.877
VOLINT	Volatility of German 5-year spot rate estimated from weekly returns over a 6-month window	0.012	0.002	0.012	0.002
NEWMARKET	Dummy, introduction of new stock market “New Market” in Germany (January 1998 to December 2000)	0.197	0.398	0.176	0.381
TAX99	Dummy, change in tax legislation 2000 (November-December 1999)			0.012	0.109
TAX06	Dummy, change in tax legislation 2007 (November-December 2006)			0.017	0.129
YEAREND	Dummy, end of the year (each December)			0.086	0.280
Product characteristics					
UPSTEEP	Average yearly growth of coupon payments until maturity (last coupon minus current coupon divided by years to maturity)	0.005	0.002	0.004	0.002
DURATION	Fisher-Weil duration of GFSN without option in years	2.585	1.315	3.300	1.711
COUPON	Dummy, coupon payment upcoming within the next 60 days	0.090	0.285		
BLOCK	Dummy, first month after blocking period	0.023	0.151	0.021	0.142
LIFETIME	Lifetime since issuance in years	2.315	1.450	2.672	1.711
Portfolio characteristics					
INVESTS	Number of former investments in GFSN	7.397	10.127	8.231	11.668
INVESTSUM	Sum of overall investments in € so far	40,087	73,561	27,156	64,055
EXERCISED	Dummy, investor has exercised early a GFSN once before	0.104	0.306	0.108	0.311
PERFORMED	Dummy, investor has exercised early a GFSN once before with PVEV=1	0.037	0.189	0.032	0.176
N		19.089m		11.467m	

The table shows the variables and ratios we consider in our analysis of determinants of individual investors' exercise behavior. Overall 19.089m monthly observations are considered for Type A GFSN (11.476m for Type B). The one-year blocking period at beginning of a GFSN is excluded. Data sources are Deutsche Bundesbank for interest term structures, Thomson One Banker for environmental and equity market variables and the German Finance Agency for GFSN data. The descriptive statistics are calculated based on all observations of the data set, wherefore the figures differ from, e.g., the analysis on an individual investor level (see Table 3).

2.4. Variables

To investigate individual investors' decisions in the course of time and to identify determinants of early exercises we convert the described transaction data to a longitudinal structure on a monthly basis. The resulting data panel comprises circa 31 million decisions by the investor base to hold or exercise a GFSN. We comprehend the data set with several variables and ratios (Table 4 contains a detailed overview) in the following categories to account for potential influences on investors' decisions: value orientation, investment history, environmental circumstances, product characteristics, portfolio and personal characteristics.

Value orientation

We use the ratio (PVEV) of present value of a GFSN (*PV*, see Eq. (2)) to its exercise value (*EV*, see Eq. (1)) to measure the financial advantage (or disadvantage) of a potential early exercise according to OPT:[16]

$$PVEV = \frac{Present\ value\ of\ GFSN}{Exercise\ value\ of\ GFSN} = \frac{PV}{EV}. \quad (5)$$

As discussed in Section 2.2 it is optimal to use the early exercise right as soon as the exercise value equals the present value which is equivalent to the exercise value being above or equal to the continuation value. This means only an early exercise at a PVEV-ratio of 1 can be classified as optimal according to OPT,[17] whereas an exercise is the more “disadvantageous” the higher the PVEV-ratio rises above 1.[18]

Of course, we do not assume that individual investors apply such a sophisticated model. It may be even questionable whether they apply a simple, but proper, option pricing model in their decision making at all. Nevertheless, using an advanced model allows us to compare their exercising behavior with the theoretical optimal behavior that professional investors can be expected to show and to analyze respective determinants of (in this respect) suboptimal exercises.

Investment history

Considering the investment history allows us to set individual investors' behavior in relation to, e.g., former valuation movements and former decisions. The variable MISSEDALL measures for each GFSN and month how many attractive exercise opportunities an investor has ignored so far, quantified by the number of months when the PVEV-ratio equaled 1. MISSED6, defined as the number of attractive exercise months omitted within the last half year, con-

[16] As an alternative measure we also used the difference between option value and intrinsic option value, i.e. *option value* − (*exercise value* − *present value ex option*). This is basically the market-based rule as described by Pool et al. (2008), but the market price is replaced by the model present value. This variable equals zero if an exercise is optimal according to OPT, while it is positive otherwise. Our results are robust to replacing PVEV by this variable, but the variable COUPON loses significance in the logit regression reported in Table 5 for Type A GFSN.

[17] For simplicity, we define an exercise decision as optimal if the PVEV-ratio equals 1 at the exercise date regardless of the investment history, even though the exercise decision is theoretically only optimal if the exercise happens at the first feasible opportunity. This means, the estimated numbers in this paper represent a lower bound for the share of exercise decisions that are not in line with OPT.

[18] Recall that an early exercise does not induce any costs or penalty payments.

trols for short-term effects and examines whether investors react sluggishly to former exercise opportunities, which is a well known pattern for small traders in equity products (e.g., Hvidkjaer, 2011). Finally, we also incorporate the investment volume (VOLUME).

Environmental circumstances

We control for four environmental factors. First, we consider the development of the German stock index CDAX as proxy for potential investment yields in the equity class. Based on the assumption that mainly large changes of the CDAX may have an influence on early exercises in the fixed-income market, the dummy variables CDAX10+ and CDAX10- signal increases or decreases of the CDAX of more than 10% within one month. Focusing on longer trends, the dummy variables CDAX25+ and CDAX25- cover index changes of more than 25% within 6 months. Second, VOLDAX and VOLINT measure the option-implied 45-day volatility of the German stock market (using the VDAX index) and the volatility of the German 5-year spot rate estimated from weekly returns over a 6-month window to encompass investors' reaction to uncertainties in the markets. Third, the dummy variable NEWMARKET controls for abnormal capital outflows to the "New Market" between 1998 and 2000, a new stock segment in Germany which attracted considerable attention among individual investors during this period. Fourth, we account for tax effects, whereby we focus on Type B GFSN since changes in the tax legislation have a much higher relevance for products with a zero-bond structure — as all coupon payments are taxed at the same time — than for coupon-paying products for which personal tax allowances can be utilized each year. During our observation period two major changes occurred in the German tax legislation. In both years 2000 and 2006 individual investors' tax allowances were severely reduced, which might have made it attractive for investors to exercise GFSN positions early shortly before the new regulations became effective to optimize personal tax debts. Accordingly, our variables TAX99 and TAX06 are designed to capture potential extraordinary exercise activities in November and December 1999 and 2005, respectively. In addition, the dummy YEAREND controls for tax-motivated early exercises at the end of the year when individual investors typically review their tax burdens and allowances.

Product characteristics

While the general structure of GFSN remains constant over all issuances in our sample period, the offered coupons are typically adapted to current market circumstances with every issuance. Accordingly, we have a broad range of coupon structures among the 204 GFSN offerings in our sample. To determine if visual or psychological factors, such as the shape of the coupon structure or shortly awaited coupon payments affect an investor's behavior, we consider three variables. First, we calculate the average yearly growth of coupon payments until maturity as an indicator for the steepness of the coupon structure (UPSTEEP). Second, we compute for each GFSN and month the duration in years (DURATION), which we interpret as the weighted average time until an investor receives all coupon payments and the initial investment. For the calculation we refer to the definition of Fisher and Weil (1971). Third, the dummy variable COUPON, which is only relevant for Type A GFSN, signals if a coupon payment occurs within the next 60 days.

Lastly, we incorporate the dummy variables BLOCK and LIFETIME (for Type A GFSN). BLOCK controls for potential increased exercise activities in the first month after the initial one-year blocking period. LIFETIME is the elapsed time since issuance.[19]

Portfolio characteristics

Besides personal attributes, such as age or gender, we also account for an investor's experience in GFSN. We define four proxies: the number of former personal investments in GFSN (INVESTS), the sum of overall investment volumes so far (INVESTSUM), a dummy variable indicating whether the investor has made use of the early exercise right once before for a GFSN investment (EXERCISED) and a dummy variable signaling if he has exercised early at an economically opportune time according to OPT once before (PERFORMED) as indicated by a PVEV-ratio at exercise of 1.

3. Analysis of early exercise decisions

3.1. Determinants

In this section we use the described panel data set to examine determinants of individual investors' early exercise decisions. We apply pooled and random-effects logit regressions to estimate the influence of our above-mentioned variables (see Table 4) on the probability of an exercise. For the panel regression we consider that each GFSN per investor represents an own group of observations, whereby the number of observations ranges from 1 for GFSN that are exercised early in the first month after the blocking period, to a maximum of 60 (72 for Type B) for investments held until maturity.

Our analyses are positioned at the very low end of the logit distribution due to the small number of exercise events compared to non-exercise events (while GFSN in our sample are in general frequently exercised, the exercise rate on a monthly level still lies below 1% per month, as shown in Table 3), which could lead to biased coefficient estimates and standard errors (see King and Zeng, 2001). Yet, rare-event regressions as introduced by King and Zeng (2001) result in only minor deviations in the estimates, wherefore we choose to continue with standard estimators due to the better handling and improved comparability. We also run several robustness checks (not reported here) based on time and investor subsamples, which lead to consistent findings. Table 5 contains the regression results on our overall data set and the respective average marginal effects (dy/dx) for the pooled regression. Average marginal effects are displayed at 1e+2 as some are quite small. This might raise concerns in the reader over the economic relevance of some effects found. However, many marginal effects turn out to be significantly larger when they are calculated at high or low quantiles of the (continuous) covariates.[20] We use robust standard errors in this and all following regressions (see Huber, 1967; White, 1980; 1982).

We emphasize six regression results: first, the probability of the option being exercised is negatively related to the PVEV-ratio. A decreasing PVEV-ratio goes hand-in-hand with an increasing exercise probability, which implies that the average investor is sensitive to changes in the value of his investment. Second, the investment history has a statistically significant influence. The positive coefficients for MISSED6 and MISSEDALL indicate that the probability of an exercise increases with the number of missed attractive exercise opportunities. While the influence of MISSED6 might be attributed to delayed or sluggish responses to earlier market or value changes, the positive loading for MISSEDALL is a more surprising result, as investors who aim to maximize their investment performance should utilize one of the first attractive exercise opportunities to shift investments or re-arrange their portfolio. We further investigate this issue and how frequently investors exploit arising attractive exercise chances according to OPT in Section 3.2.

[19] Due to the zero-bond structure of Type B GFSN the variables LIFETIME and DURATION are for this type perfectly correlated, wherefore we neglect the LIFETIME variable for Type B in the following regressions. For Type A GFSN the lifetime is also correlated to the duration, which might potentially result in inflated standard errors. However, running the following regressions with orthogonalized regressors leads to similar findings, so that we decided to continue with the described variables.

[20] Respective results are available upon request from the authors.

Table 5
Determinants of early exercise.

	Type A GFSN			Type B GFSN		
	Observations per investment			Observations per investment		
	Mean	Min.	Max.	Mean	Min.	Max.
	42.370	1.000	60.000	48.770	1.000	72.000
	Logit Coefficient	*RE logit* Coefficient	Margin(1)	*Logit* Coefficient	*RE logit* Coefficient	Margin(1)
Value orientation						
PVEV	-13.446***	-12.851***	-4.820	-7.650***	-3.296***	-2.804
Investment history						
MISSEDALL	0.022***	0.049***	0.008	0.001	0.039***	0.000
MISSED6	0.059***	0.171***	0.021	0.144***	0.224***	0.053
VOLUME	0.011***	0.023***	0.004	0.014***	0.031***	0.005
Environmental circumstances						
CDAX10+	0.261***	0.323***	0.105	0.289***	0.358***	0.121
CDAX10-	0.157***	0.124***	0.060	0.242***	0.194***	0.099
CDAX25+	0.131***	0.122***	0.050	0.318***	0.408***	0.132
CDAX25-	-0.377***	-0.510***	-0.116	-0.154***	-0.268***	-0.053
VOLDAX	0.005***	0.007***	0.002	-0.003**	-0.003*	-0.001
VOLINT	-86.190***	-84.767***	-30.897	-49.856***	-70.138***	-18.272
NEWMARKET	1.010***	1.299***	0.489	0.991***	1.357***	0.500
TAX99				0.482***	0.584***	0.222
TAX06				0.999***	1.118***	0.588
YEAREND				0.349***	0.405***	0.146
Product characteristics						
UPSTEEP	-4.943*	59.881***	-1.772	-78.250***	-68.645***	-28.678
DURATION	4.346***	12.028***	1.558	0.204***	-0.177***	0.075
COUPON	-0.140***	-0.089***	-0.048			
BLOCK	0.368***	-0.259***	0.154	0.407***	-0.617***	0.179
LIFETIME	3.537***	11.025***	1.268			
Portfolio characteristics						
INVESTS	-0.046***	-0.113***	-0.017	-0.046***	-0.107***	-0.017
INVESTSUM	-0.005***	-0.009***	-0.002	-0.006***	-0.010***	-0.002
EXERCISED	2.979***	6.445***	3.208	2.977***	6.497***	3.287
PERFORMED	0.428***	1.047***	0.179	0.235***	0.618***	0.094
Personal characteristics						
Gender						
Female	-0.059***	-0.144***	-0.021	-0.105***	-0.169***	-0.038
n/a	0.046***	0.048*	0.017	0.041**	0.068**	0.016
Age						
21 to 40 years	0.484***	0.870***	0.185	0.641***	1.014***	0.241
41 to 60 years	0.240***	0.362***	0.081	0.427***	0.652***	0.144
61 to 100 years	-0.008	-0.126***	-0.002	0.294***	0.348***	0.093
n/a	-4.228*	-6.579***	-0.304	-4.004***	-6.726***	-0.272
Doctoral degree						
Doctorate or professorship	-0.125***	-0.244***	-0.043	-0.122***	-0.327***	-0.043
Geographical location						
City	0.002	0.118**	0.000	-0.022	0.010	-0.008
Highly populated	-0.045***	-0.016	-0.016	0.025	0.025	0.010
Moderately populated	-0.018*	-0.014	-0.007	-0.054***	-0.053**	-0.020
n/a	0.016	0.003	0.006	-0.028	-0.109*	-0.010
Preferred distribution						
Direct	-0.221***	-0.526***	-0.080	-0.114***	-0.290***	-0.042
Constant	-11.204***	-51.235***		1.599***	-3.465***	
Groups (Investments)		450,526			235,307	
N	19.089m	19.089m		11.467m	11.467m	
Pseudo-R^2 in %	17.00	19.78		15.17	19.27	

The table exhibits the results of a pooled logit and a random-effects logit regression on individual investors' exercise decisions in Type A and Type B GFSN. Only investments and decisions after the one-year blocking period are considered. For the pooled regressions robust standard errors are used. For the panel estimation we apply the Gauss-Hermite algorithm with 4 integration points. The lifetime variable is not considered for Type B GFSN as it is perfectly correlated to the respective product's duration. The marginal effects (dy/dx) are calculated as average marginal effects based on the pooled logit regression. Pseudo-R^2 is the percentage improvement in the log-likelihood achieved by our model compared to a constant-only model. *** signals statistical significance at the 0.1%, ** at the 1% and * at the 5% level. (1) Average marginal effects, displayed at 1e+2.

Third, environmental influences have a significant impact. We find that large movements in the equity segment (CDAX) and the launch of the "New Market" in Germany are accompanied by increasing exercise activity, which suggests that investors use the early exercise right to liquidate investments so as to participate in attractive growth phases of other markets. On the other hand, the exercise probability diminishes in times of severe equity market drops (CDAX25-) and high volatility in the interest term structure (VOLINT). We trace this to a higher attractivenesses of fixed-income investments in times of bear equity markets and in times of higher uncertainty. Finally, the significantly positive coefficient for the tax variables indicates that — in an analogy to the equity market (e.g., Badrinath and Lewellen, 1991; Grinblatt and Keloharju, 2001; Ivkovic et al., 2005; Liedtka and Nayar, 2012) — individual investors use the exercise right broadly to react to tax changes and to optimize their tax debt.

Fourth, the probability of an early exercise depends on product characteristics. We find a in general positive influence of the duration (DURATION) and a (in most cases) negative loading regarding the steepness of the upcoming coupon structure (UPSTEEP). Additionally, we note that investors hesitate to exercise early shortly before a coupon payment date (COUPON). Such behavior cannot be justified with standard economic arguments as, according to theory, an exercise decision should only be based on the current valuation regardless of the upcoming coupon structure. Similarly, there is no structural advantage to waiting until a coupon payment, as accrued interests are considered at an early exercise. Hence, we interpret the empirical patterns related to the product structure as psychological effects. Apparently, individual investors value GFSN with increasing coupon payments more highly than almost identical products that offer a flatter coupon structure. Moreover, the reduced exercise probability before coupon payment dates implies that investors differentiate between accrued interests and cash payouts and thus use mental accounting (e.g., Shefrin and Statman, 1984; Thaler, 1999). Lastly, we note that there is a peak in exercise rates in the first month after the blocking period.

Fifth, the early exercise frequency differs among investor groups. The regressions reveal that experience in exercising is associated with a higher early exercise probability (EXERCISED). This effect is even stronger if the investor has exercised at an attractive point in time according to OPT (PERFORMED). We attribute both effects to experienced investors having greater financial literacy, lower information costs and less inhibition in using the right to exercise. On the other hand, our two more general portfolio variables — the number of investments and the overall investment volume so far — show negative coefficients, which might be due to less exercise activity of the part of wealthy investors (see also Dhar and Zhu, 2006).

Sixth, male investors and investors between 20 and 40 years of age use the exercise right more often, while investors holding a doctoral degree and investors who acquire directly through the German Finance Agency are slightly less likely to exercise. An investor's geographical location has no statistical influence.

Our main conclusion in this first analysis is that a broad range of factors cause individual investors to exercise early. Value orientation is only one determinant among several other influence factors. Hence, the next section focuses on examining the exercise quality of investors in more detail.

3.2. *Exercise quality*

In this section we investigate the quality of individual investors' exercise behavior according to OPT and analyze respective determinants. According to OPT a putable bond should be exercised as soon as its present value equals the exercise value, which implies a PVEV-ratio of 1. In contrast, no early exercises should occur if the PVEV-ratio is greater than 1. We use this definition to determine how many early exercise decisions in our data set are in line with OPT (Section 3.2.1) and how many optimal exercise opportunities investors forgo over time (Section 3.2.2). Finally, we analyze the performance investors achieve with their exercise behavior (Section 3.2.3).

3.2.1. *Exercises according to OPT*

The left part of Panel A in Table 6 shows that the majority of early exercises take place when PVEV is greater than 1; we find 75.674% for Type A and even 86.061% for Type B. In other words, only 24.326% (13.939%) of the exercise decisions in our data set comply with standard OPT, namely that exercises should occur only at a PVEV-ratio of 1. Instead, we observe exercises at a broad range of valuations. In fact, a more detailed analysis (not reported here) reveals that the largest share of exercises (45.778% and 50.743%) coincides with a PVEV-ratio of greater than 1.03.

In the next step we form a dummy variable indicating whether or not an early exercise is in line with OPT. To analyze what, if any, differences in the respective exercise behavior exist between selected investor groups, we run logit regressions on portfolio and personal characteristics of investors, whereby we control also for the investment volume, for tax influences and for a potentially increased demand in the first month after the blocking period. The results are reported in Panel B of Table 6, left part. We stress some distinctive features here. The negative coefficient for BLOCK in the first regression implies that in the first month after the blocking period a significantly higher number of exercises than usual are not in line with OPT, which we attribute to accumulated requests to exercise, due perhaps to liquidity constraints, that pile up until the first exercise possibility.

The positive influence of PERFORMED shows that investors who have already exercised in line with OPT once before are much more likely to make further analogous exercise decisions. On the other hand the negative loading for EXERCISED in the first regression implies that a non-OPT-consistent decision to exercise (i.e. the PERFORMED dummy is 0) is regularly followed by a decreasing probability of an OPT-consistent exercise decision. These two findings together indicate that the exercise quality of investors in relation to OPT is in principle persistent over time.

Looking at personal characteristics, we find that female investors tend to act more in line with OPT. Differences also exist related to the age of an investor. Most strikingly, we observe that investors between 20 and 40 years of age employ their exercise rights considerably more often at points in time that are not consistent with OPT. A possible reason might be a higher demand for liquidity in that stage of life due to, e.g., starting a family.

We do not find that investors' geographical location has any statistical influence. However, differences appear regarding the preferred distribution channel. Investors who prefer to acquire directly through the German Finance Agency make exercise decisions that are better according to OPT. This is consistent with the usually higher financial literacy of investors who skip intermediaries. As our last finding, we note that investors holding a doctoral degree behave similarly to the just-described direct investors, which may be due to a higher financial literacy or to their lower liquidity constraints, as doctoral degrees are typically associated with an above-average income in Germany.

3.2.2. *Failure to exercise*

We use the calculated PVEV-ratio per month and GFSN to determine how frequently individual investors take advantage of attractive exercise opportunities. This means we extend our analysis beyond the investor's final early exercise and also consider all previous decisions to continue holding the investment. The results are presented in Table 6, right part of Panel A. In line with our former findings the results reveal that individual investors act much less in line with OPT than we would expect from, for example, institutional investors. In fact, overall only 1.856% (1.829%) for Type A (Type B) of the attractive exercise opportunities that arise are utilized by individual investors. In other words, this implies that investors in both GFSN types waive more than 98% of their chances to increase the investment yield through re-arranging their portfolio, resulting in significant opportunity costs. This finding is also supported by the above-defined variable MISSEDALL (see Table 4), which counts for each investment at maturity or at an early exercise how many attractive exercise months the investor has let pass and instead decided to continue holding the investment. In

Table 6
Exercise quality.

Panel A	Distribution of early exercises in %				Attractive exercise opportunities in %			
	Type A GFSN		Type B GFSN		Type A GFSN		Type B GFSN	
	PVEV=1	PVEV>1	PVEV=1	PVEV>1	Exploited	Missed	Exploited	Missed
	24.326	75.674	13.939	86.061	1.856	98.144	1.829	98.171

Panel B	*Logit*: Early exercise optimal according to OPT (PVEV=1, yes/no)				*Logit*: Failed to exercise when PVEV=1 (yes/no)			
	Type A GFSN		**Type B GFSN**		**Type A GFSN**		**Type B GFSN**	
	Coefficient	Margin(1)	Coefficient	Margin(1)	Coefficient	Margin(1)	Coefficient	Margin(1)
Investment history								
MISSEDALL					0.002	0.002	0.030***	0.030
MISSED6					-0.068***	-0.062	-0.033***	-0.034
VOLUME	-0.011***	-0.192	-0.011***	-0.176	-0.006***	-0.006	-0.011***	-0.011
Environmental circumstances								
TAX99			-0.784***	-10.715			0.387***	0.338
TAX06			0.425***	7.369			-0.508***	-0.641
YEAREND			0.378***	6.432			-1.050***	-1.599
Product characteristics								
UPSTEEP					26.743***	24.315	150.609***	154.622
DURATION					-2.398***	-2.181	-0.395***	-0.406
COUPON					0.102***	0.090		
BLOCK	-1.702***	-25.760	-1.041***	-14.138				
LIFETIME					-1.546***	-1.406		
Portfolio characteristics								
INVESTS	0.012***	0.204	0.012***	0.198	0.043***	0.039	0.043***	0.045
INVESTSUM	0.003***	0.051	0.003***	0.044	0.002***	0.002	0.002***	0.002
EXERCISED	-0.624***	-10.499	-0.562***	-8.854	-2.403***	-4.059	-2.455***	-4.912
PERFORMED	1.873***	37.995	1.778***	36.875	-1.240***	-1.646	-1.257***	-1.979
Personal characteristics								
Gender								
Female	0.050**	0.882	0.050	0.804	0.045***	0.040	0.118***	0.117
n/a	0.008	0.136	0.030	0.481	-0.121***	-0.115	-0.119***	-0.131
Age								
21 to 40 years	-0.641***	-11.223	-0.650***	-10.494	-0.149***	-0.145	-0.241***	-0.259
41 to 60 years	-0.310***	-5.648	-0.389***	-6.621	-0.048*	-0.045	-0.098***	-0.099
61 to 100 years	-0.006	-0.116	-0.182***	-3.215	0.109***	0.095	-0.027	-0.026
n/a	0.196	3.727	0.231	4.380	3.960***	0.948	3.620***	0.999
Doctoral degree								
Doctorate or professorship	0.069	1.221	0.157**	2.601	0.077**	0.068	0.062	0.063
Geographical location								
City	-0.012	-0.203	0.019	0.314	0.070*	0.063	0.094	0.094
Highly populated	0.020	0.346	-0.009	-0.142	0.032	0.030	0.035	0.036
Moderately populated	0.013	0.222	0.037	0.599	0.010	0.010	0.040	0.041
n/a	0.083	1.461	-0.093	-1.468	-0.026	-0.025	0.090	0.091
Preferred distribution								
Direct	0.999***	18.462	0.698***	11.506	0.414***	0.409	0.335***	0.364
Constant	-0.767***		-1.228***		15.105***		5.803***	
N	82,787	82,787	47,900	47,900	3.028m	3.028m	1.114m	1.114m
Pseudo-R^2 in %	19.31		12.90		20.49		18.79	

The table shows four economic analyses of individual investors' exercise behavior in GFSN. In the left part of Panel A we compute the share of early exercises in our data set that are in line with OPT, indicated by PVEV=1 where PVEV>1 capture the remaining exercises. Second, in the right part of Panel A we calculate the percentage of exploited and missed exercise opportunities that are in line with OPT (PVEV=1). Third, the left part of Panel B presents the results of a pooled logit regression on a dummy variable that indicates if an exercise is in line with OPT. The second column displays the corresponding average marginal effects (dy/dx). Fourth, the right part of Panel B exhibits the results of a pooled logit regression on a dummy variable that indicates if an investor fails to use an attractive exercise opportunity, i.e. when PVEV=1. Again, the next column shows the average marginal effects (dy/dx). Robust standard errors are used. Pseudo-R^2 is the percentage improvement in the log-likelihood achieved by our model compared to a constant-only model. *** signals statistical significance at the 0.1%, ** at the 1% and * at the 5% level. (1) Average marginal effects, displayed at 1e+2.

a separate analysis not shown here, we find that at an early exercise of a Type A GFSN an investor has already missed on average 3.920 economically reasonable exercise opportunities (2.425 for Type B), of which 1.609 (1.000) are within the last half year. While such a general failure to exercise is also well documented for retail investors in equity derivatives (Pool et al., 2008; Barraclough and Whaley, 2012), the incidence of missed exercises in our study is significantly higher. A possible reason might be the more conservative character of fixed-income products compared to other investment classes, as these are presumably preferred by a different type of investor with other investment horizons and motives.

Next, we use a dummy variable that denotes if an investor fails to exercise at an attractive opportunity or not and run logit regressions again. In this case we also consider the investment history as independent variable (MISSEDALL, MISSED6) to determine whether earlier decisions to forgo similar opportunities have an influence. Moreover, we control for the effect of a shortly upcoming coupon payment date (COUPON), account for different product characteristics (UPSTEEP, DURATION) and incorporate the lifetime of a GFSN.

Beginning with the investment history, the regression output in Table 6, right part of Panel B, supports our former finding that investors tend to respond sluggishly to market changes. The likelihood of an investor missing a good opportunity to exercise declines if that investor has recently left out some attractive exercise chances (MISSED6). Regarding the product variables, we observe that the probability that an attractive exercise opportunity will be ignored is higher if the product characteristics of the respective GFSN match investors' preferred payment profile (see Section 3.1), i.e. if the GFSN has a steeply rising coupon structure and a low capital duration. Similarly, the positive coefficient for COUPON corresponds to the already discussed tendency among individual investors to ignore (attractive) early exercise opportunities shortly before coupon payment dates. The negative loading for the LIFETIME variable for Type A GFSN indicates that the probability of a failure to exercise increases over time, which might perhaps be due to a lack of attention to products that mature in the near future. Further, we observe that the probability of an investor missing an exercise opportunity drops significantly if he has exercised a GFSN before and decreases even more if his earlier exercise decision was in line with OPT (i.e. both the EXERCISED and the PERFORMED dummy equal 1). This implies that there is also a certain persistence in the failure to exercise.

Looking at personal characteristics, female investors tend to miss more attractive exercise opportunities. Recall the finding from the previous section that they act more in line with OPT if they exercise. Together this suggests that they tend to act more cautiously and thus miss more potential chances than men, but achieve economically better results if they do decide to exercise. This interpretation is in line with several studies on individual investors' trading behavior in stocks (e.g., Barber and Odean, 2001; Grinblatt and Keloharju, 2001). We find a similar pattern for investors holding a doctoral degree and for direct investors.

Finally, differences are also related to the age of an investor. Investors between 20 and 40 years of age tend to miss attractive exercise opportunities less often than other age groups. Like in the previous section we do not find any influence attached to investors' geographical location.

3.2.3. *Excess returns of exercising*

To quantify the impact of investors' exercise strategies on their realized return, we determine in this section the performance of each GFSN investment and each respective exercise in our data set. As performance measure we use the difference ("excess return") between the yearly internal rate of return of the empirically observed exercise behavior and the yearly internal rate of return of a buy-and-hold strategy. In the calculation of the former we assume that at an early exercise the whole investment is directly reinvested in a fixed-income product with the same (remaining) maturity that pays the fair market par-yield.[21] Moreover, we ignore any potential transaction costs related with the re-investment. Table 7 provides the results.

As already indicated by the low number of exercises that are optimal in the sense of OPT and the high number of missed attractive exercise opportunities (see Panel A of Table 6), the results show that only few investors achieve a positive yearly excess return through exercising GFSN. On average, exercises in our data sample lead to negative excess returns against a buy-and-hold strategy of circa -0.117% per year for Type A and circa -0.300% for Type B GFSN.[22]

Not surprisingly, a separate analysis (not reported here) again finds differences in investors' exercise performance related to personal and investment characteristics, whereby the general patterns are in line with our former discussions. For example, investors between 20 and 40 years of age show an even poorer exercise performance (mean excess returns of -0.302% and -0.438%) than, e.g., investors between the ages of 0 and 20 (-0.029% and -0.243%). Investors who prefer direct distribution (0.119% and -0.096%) markedly outperform those who prefer to invest at banks (-0.372% and -0.498%). Finally, we note that the best exercise performance for both GFSN types tends to be achieved by investors who exercise early only with regard to selected investments, whereas investors who have a high ratio of early exercises with all their investments regularly realize poor excess returns.

Table 7
Excess returns of early exercising.

	Individual investors' excess return of early exercising in % p.a.	
	Type A GFSN	Type B GFSN
Mean	-0.117	-0.300
Median	-0.098	-0.289
p5	-1.306	-1.429
p10	-1.017	-1.124
p90	0.709	0.572
p95	1.023	0.868
St. dev	0.704	0.711
N	82,787	47,900

The table shows statistics on the average yearly excess return of early exercising based on the exercise decisions of all 223,017 individual investors in 204 issued GFSN (102 Type A, 102 Type B) during our sample period from July 1996 to February 2009. For the calculation we assume that an investment exercised early is directly reinvested in a product with an identical remaining maturity paying the market yield. The excess return is then calculated as the difference between the yearly internal rate of return of the exercise strategy and a buy-and-hold strategy.

3.3. *Discussion*

Summing up key empirical results of our previous analyses, we first note that early exercises are linked to a GFSN's value, which implies that investors' behavior is not fully uncoupled from standard theory and performance orientation. Nevertheless, a broad range of factors from investment history and environmental circumstances to product, portfolio and personal characteristics are strongly linked to the exercise decision, resulting in average returns that lie below the returns of a simple buy-and-hold strategy.

We find that the number of failure-to-exercise events is considerably larger than in comparable studies for the option market such as Poteshman and Serbin (2003), Pool et al. (2008) and Barraclough and Whaley (2012). The literature usually refers to this behavior as irrationality (e.g., Poteshman and Serbin, 2003; Pool et al., 2008; Barraclough and Whaley, 2012), but acknowledges that it may still be related to rational decision-making in the case of significant monitoring (e.g., Stanton, 1995; Barraclough and Whaley, 2012; Liao et al., 2014), transaction (e.g., Stanton, 1995; Finucane, 1997; Koziol, 2006) or learning costs (e.g., Barraclough and Whaley, 2012). The latter may be of specific relevance here, as valuing putable bonds requires some non-trivial financial sophistication while direct transaction costs do not exist for the GFSNs we consider. On the other hand, it is questionable why a fully rational, performance-seeking investor should buy a product where he anticipates that high monitoring or learning costs will prevent him from using the early exercise right optimally. Still, parts of our results support the irrationality argument, as they imply that investors suffer from cognitive biases in relation to product characteristics such as the steepness of the coupon structure.

Further, our analyses show that investors deviate from the optimal exercise strategy according to OPT to a large extent when they exercise. This also manifests in the average negative excess return. As stressed before, this observed behavior is not necessarily irrational if, for example, investors aim at optimizing tax payments or suffer from liquidity constraints. In fact, our findings suggest

[21] This implies that the fictive re-investment product has a market value of 1.

[22] It should be noted that excess returns would be even more negative if they were calculated against an optimal exercise strategy instead of a buy-and-hold strategy.

that exercising is related to tax rule changes and the use of tax allowances. While it is very likely and economically plausible that liquidity needs are a (another) key driver of early exercising for individual investors, much uncertainty is obviously attached to judging investors' liquidity needs without having access to their full financial and non-financial portfolio, their consumption objectives and potential liquidity constraints. However, some patterns in our results seem to support the liquidity argument. First, the significantly higher probability of early exercise of a GFSN by investors between the ages of 20 to 40 might be due to their higher liquidity requirements at that stage of life. Second, the higher early exercise activity observed in the first month after the initial blocking period can be attributed to investors' demand for liquidity that has accrued over the first twelve months of maturity. Third, the increased exercise rates during phases of strong growth in the equity markets can be interpreted as a direct reaction of investors in GFSN who use the early exercise right to liquidate their investments so as to benefit from bullish trends in other markets. Fourth, we observe a base exercise rate as reported in Section 2.3 that is independent of value and other factors such as environmental circumstances. This, of course may occur due to irrationality, but will also be observed if investors exercise independently for liquidity reasons.

4. Investment and exercise motives

Based on our previous analyses and the discussion above, we aim to cast further light on the motives of individual investors for holding and exercising GFSN in this section. We perform our analysis in two steps. First, we compile a new data set that describes the average investment and exercise strategy of each individual investor in our data set. We then conduct an exploratory factor analysis to extract the main dimensions of variation in the data and to isolate latent factors that drive investors' investment and exercise behavior. Second, we examine the relevance of the identified latent factors for each investor and determine whether there are any statistically significant relationships to personal and financial attributes.

4.1. Exploratory factor analysis

While we have concentrated in this paper so far on decisions and transactions at an individual account and product level, we now focus on the general behavior of individual investors. Hence, our first step is to create a new data set that consolidates the individual transaction data for each investor into a few explicit characteristics of behavior on the investor level. Table 8 provides an overview of the variables we calculate for each account, whereby the selection adheres closely to the identified determinants of early exercising (see Table 5).

In short, we consider three kinds of variable. First, we compute economic indicators and ratios that describe an investor's average investment and exercise strategy. For instance, we determine what percentage of his exercises occurs at times when the PVEV-ratio equals 1 (PVEVLOW), which implies — as discussed — an optimal exercise according to OPT. Similarly, we calculate the variables PVEVMED and PVEVHIGH, which represent the share of early exercises at medium ($1<\text{PVEV}\leq 1.03$) or high PVEV-ratios ($\text{PVEV}>1.03$). Second, we account for the average reaction to environmental influences, such as the percentage of exercise opportunities used when the CDAX has moved by more than 25% during the previous 6 months or the percentage of investments an investor liquidates in the months before changes in the tax environment become effective. Third, we subsume information on the products an investor chooses to purchase. Here, we consider, for instance, the average steepness of the coupon structure over all chosen investments or the average present value of the GFSN without the option right. Finally, we also incorporate an investor's number of investments and early exercises.

In this analysis we have to cope with missing data, because some of the defined variables in Table 8 depend on economic or environmental circumstances that may not have arisen during an individual's investment period. For example, we cannot calculate the percentage of exercised investments in the months before changes in the tax legislation become effective if an investor did not hold a GFSN at such a time. According to the classification of Rubin (1976), such data is missing at random (MAR), since its absence is not related to the value of the respective variable but only to other variables, such as the market or tax environment. The literature suggests several methods of handling data missing at random (see, e.g., Allison, 2002; Enders, 2010), yet there seems to be no best commonly accepted approach. We apply multiple imputation algorithms and the full-information maximum likelihood method here, but find only small differences in the results. We proceed with the likelihood estimator for efficiency reasons, whereby we implicitly assume that individual investors, for whom we may lack some data, do not systematically make decisions that are widely different from the other investors in our data set, which seems reasonable considering the broad and large investor base.

An exploratory factor analysis, using the maximum-likelihood extraction method and applying robust standard errors to correct for non-multivariate normally distributed data (see Yuan and Bentler, 2000), isolates four factors that primarily drive the investment and exercise behavior of Type A GFSN investors, while five factors seem most suitable for Type B GFSN investors (see Table 9).[23] For both analyses the model fit is satisfying. The standardized root mean square residual (SRMR) lies at 5.2% for Type A GFSN (3.7% for Type B), the root mean square error of approximation (RMSEA) lies at circa 6.5% (5.0%) and the comparative fit index (CFI) approaches 88% (92%). Overall, the identified factors account cumulatively for circa 55.227% (59.000%) of the overall variance. Table 9 contains details on the factor selection and on the rotated standardized factor loadings, which we estimate via the direct geomin oblique rotation algorithm.

The factor analysis returns distinct and strong loading patterns for all identified factors. It is remarkable that the estimated factors and factor structures are very similar for the independently conducted analyses of Type A and Type B GFSN investors, which we take to indicate the economic robustness of our results. The first and most important factor, which in the case of Type A GFSN accounts for about 19.153% of the variance in all variables (17.819% for Type B), shows positive loadings for all three PVEV variables. This indicates that the factor is related to both according to OPT optimal and suboptimal exercise decisions, which is plausible if exercises are triggered by exogenous factors such as the introduction of new market segments or strong movements in the equity market. In fact, for this factor we observe statistical positive coefficients for all market variables (CDAX10, CDAX25 and NEWMARKET) and for the BLOCK variable, which indicates an increased exercise rate in the first month after the blocking period. Based on these observations and on our discussion above, we interpret the first factor as being due to an investor's need for liquidity and financial flexibility.

The second factor represents the importance of the mid- and long-term value of an investment and thus stands for a conservative strategy. It is associated with an investment behavior focusing on steep coupon structures and a high valuation at issuance.

[23] To determine the number of latent factors, we consider the Kaiser-criterion (Kaiser, 1960), a parallel analysis and a scree plot. For a clear presentation of the results we decided to keep factors with an eigenvalue larger than 1.0, although the results do not differ significantly in the case of other selection criteria.

Table 8
Variables calculated to summarize an investor's investment and exercise behavior.

Abbr.	Variable description	Type A GFSN Mean	Type A GFSN St.dev	Type B GFSN Mean	Type B GFSN St.dev
Value and performance orientation					
PVEVHIGH	Percentage of early exercises employed where PVEV > 1.03	0.147	0.334	0.163	0.348
PVEVMED	Percentage of early exercises employed where 1<PVEV ≤ 1.03	0.056	0.205	0.066	0.221
PVEVLOW	Percentage of early exercises employed where PVEV = 1	0.087	0.260	0.082	0.250
Ø EXCESSRETURN	Average excess return of early exercising p.a.	-0.001	0.004	-0.001	0.004
Ø MISSEDALL	Average number of missed attractive exercise opportunities before early exercise / maturity	5.790	5.823	4.383	5.086
Environmental circumstances					
CDAX10	Percentage of exercise opportunities used when CDAX moved by more than +/-10% in last month	0.008	0.047	0.007	0.042
CDAX25	Percentage of exercise opportunities used when CDAX moved by more than +/-25% over last 6 months	0.008	0.042	0.007	0.038
NEWMARKET	Percentage of exercise opportunities used in phase of "New Market" in Germany (January 1998 to December 2000)	0.009	0.060	0.008	0.052
TAX9906	Percentage of Type B GFSN exercise opportunities used in November / December 1999 and 2006			0.012	0.089
YEAREND	Percentage of Type B GFSN exercise opportunities used in December			0.007	0.042
Product characteristics					
Ø VALUE	Average present value of GFSN investments at issuance	1.012	0.008	1.019	0.011
Ø BOND	Average present value of GFSN without option at issuance	0.987	0.010	0.986	0.014
Ø STEEPNESS	Average steepness of coupon structure of GFSN investments at issuance	0.027	0.010	0.027	0.010
Ø DURATION	Average Fisher-Weil duration of GFSN investments without option at issuance in years	5.544	0.188		
BLOCK	Percentage of exercise opportunities used in first month after the blocking period	0.022	0.116	0.018	0.099
Portfolio characteristics					
INVESTS	Number of investments in GFSN	3.755	6.210	4.039	6.961
EXERCISES	Number of early exercises	0.643	1.773	0.735	1.982
N		157,512		92,680	

The table gives an overview of the calculated variables and ratios we use to describe an individual investor's average investment and exercise strategy. The excess return is calculated according to the definition in Table 7. The descriptive statistics summarize an investor's average investment and exercise strategy over his whole portfolio, wherefore the figures differ from, e.g., the analysis on an individual investor level (see Table 3). The duration is not considered for Type B GFSN (zero-bond structure) as it always equals maturity.

Table 9
Exploratory factor analysis (EFA) on consolidated investment and exercise behavior.

	Factor 1		Factor 2		Factor 3		Factor 4		Factor 5
	Liquidity / financial flexibility		*Value*		*Performance*		*Activism*		*Tax*
	Type A	Type B	Type A	Type B	Type A	Type B	Type A	Type B	Type B
Value and performance orientation									
PVEVHIGH	0.563*	0.519*	0.036*	0.028*	-0.415*	-0.491*	0.105*	0.100*	0.160*
PVEVMED	0.022*	0.017*	-0.028*	-0.056*	-0.104*	-0.081*	0.182*	0.217*	0.131*
PVEVLOW	0.111*	0.099*	-0.063*	-0.082*	0.285*	0.285*	0.347*	0.318*	0.130*
Ø EXCESSRETURN	-0.068*	-0.051*	-0.050*	-0.056*	1.402*	1.353*	0.042*	0.024*	-0.024*
Ø MISSEDALL	-0.313*	-0.231*	-0.122*	-0.159*	0.072*	0.084*	0.032*	0.012	-0.113*
Environmental circumstances									
CDAX10	0.389*	0.398*	0.022*	0.016*	-0.124*	-0.104*	0.043*	0.073*	-0.027
CDAX25	0.449*	0.478*	0.020*	0.026*	-0.024*	-0.051*	0.024*	0.043*	-0.023*
NEWMARKET	0.556*	0.508*	0.070*	0.072*	-0.138*	-0.135*	-0.041*	-0.041*	0.038*
TAX9906		0.051*		0.019*		-0.122*		0.031*	0.695*
YEAREND		0.073*		0.026*		-0.100*		0.050*	0.588*
Product characteristics									
Ø VALUE	0.020*	0.021*	1.666*	1.336*	-0.051*	-0.070*	-0.032*	-0.092*	0.000
Ø BOND	0.065*	0.007	0.462*	0.564*	-0.029*	-0.021*	-0.102*	0.023*	0.020*
Ø STEEPNESS	0.089*	0.076*	0.311*	0.424*	-0.104*	-0.113*	-0.227*	-0.164*	0.027*
Ø DURATION	-0.080*		0.062*		0.031*		0.382*		
BLOCK	0.678*	0.673*	0.023*	0.002	-0.007	0.029*	-0.013*	-0.013*	-0.009
Portfolio characteristics									
INVESTS	0.004	-0.003	-0.093*	-0.129*	0.039*	0.053*	0.768*	0.652*	-0.040*
EXERCISES	0.248*	0.232*	-0.060*	-0.045*	-0.044*	-0.041*	0.734*	0.808*	0.073*
Eigenvalue	2.873	2.851	2.424	2.373	1.627	1.546	1.360	1.206	1.464
Proportion in %	19.153	17.819	16.160	14.831	10.847	9.663	9.067	7.583	9.150
Cumulative in %	19.153	17.819	35.313	32.650	46.160	42.313	55.227	49.850	59.000
Indicators for model fit									
SRMR	0.052	0.037							
RMSEA	0.065	0.050							
CFI	0.879	0.923							

The table shows the results of an exploratory factor analysis of individual investors' investment and exercise strategies in Type A and Type B GFSN. The results are based on the maximum-likelihood factor extraction method, whereby we keep factors with an eigenvalue larger than 1.0 (Kaiser-criterion). The geomin oblique algorithm is applied to rotate factors. RMSEA stands for root mean square error of approximation, SRMR for standardized root mean square residual and CFI for comparative fit index. * signals statistical significance at the 0.1% level. To make the layout clearer, we do not report other significance levels. Nearly all coefficients are significant at the 0.1% level in any case.

Table 10
Relation of factor scores to investors' personal characteristics.

	Factor 1		Factor 2		Factor 3		Factor 4		Factor 5
	Liquidity / financial flexibility		*Value*		*Performance*		*Activism*		*Tax*
	Type A	Type B	Type A	Type B	Type A	Type B	Type A	Type B	Type B
Personal characteristics									
Gender									
Female	-0.046***	-0.055***	-0.120***	-0.099***	0.008	0.024	-0.030***	-0.033***	-0.009
n/a	0.018**	0.013	0.042*	0.030	0.016	-0.038*	0.044***	0.039***	0.028***
Age									
21 to 40 years	0.180***	0.206***	0.011	-0.013	-0.347***	-0.309***	0.036***	0.101***	0.100***
41 to 60 years	0.085***	0.111***	-0.211***	-0.216***	-0.172***	-0.118***	0.091***	0.157***	0.056***
61 to 80 years	0.023**	0.078***	-0.457***	-0.476***	0.012	0.029	0.031***	0.129***	0.067***
n/a	-0.094***	-0.148***	-0.324***	0.078*	0.007	0.198***	-0.075***	-0.014	-0.096***
Doctoral degree									
Doctorate of professorship	-0.059***	-0.073***	-0.024	-0.191***	0.050*	0.080**	0.043***	0.024*	-0.072***
Geographical location									
City	-0.034**	-0.019	-0.046	-0.222***	-0.025	0.047	-0.005	-0.039***	-0.029*
Highly populated	-0.001	0.018	0.062**	-0.046*	0.032**	0.028	0.030***	0.004	-0.001
Moderately populated	0.012*	0.016*	-0.050***	-0.001	0.018	0.025	0.030***	0.024***	-0.013*
n/a	-0.037**	-0.026	0.574*	0.459***	0.147***	0.111***	0.005	0.001	-0.036**
Preferred distribution									
Direct	-0.029***	-0.042***	-1.026***	-1.561***	0.142***	0.101***	0.477***	0.228***	-0.006
Average volume per investment									
€ 1,000-3,000	0.048***	0.041***	0.195***	0.059***	-0.008	-0.073***	-0.121***	-0.054***	0.053***
€ 3,000-10,000	0.093***	0.101***	0.253***	0.061***	0.055***	-0.150***	-0.176***	-0.084***	0.143***
> € 10,000	0.085***	0.071***	0.224***	-0.065**	-0.004	-0.151***	-0.276***	-0.172***	0.128***
Constant	-0.112***	-0.084***	0.274***	0.702***	0.050**	0.112***	-0.079***	-0.184***	-0.084***
N	157,512	92,680	157,512	92,680	157,512	92,680	157,512	92,680	92,680
R^2 *in %*	1.07	1.65	3.65	14.86	0.96	0.96	12.32	5.98	1.21

The table exhibits regression results for the estimated four (five) latent factor scores on the personal characteristics (as detailed in Table 2) of all 157,512 individual investors in Type A GFSN and of all 92,680 individual investors in Type B GFSN. The average investment volume is defined in four clusters. Robust standard errors are used. *** signals statistical significance at the 0.1%, ** at the 1% and * at the 5% level.

The third latent factor consolidates the desire for high investment performance. It loads strongly on the ratio of early exercises at a PVEV-ratio of 1 (PVEVLOW) and accordingly on positive excess returns of exercising (EXCESSRETURN). In contrast, the fourth factor summarizes a highly active investment behavior with a high number of early exercises that, however, do not result in positive excess returns — hence we label this factor "activism". Finally, the fifth factor, which appears only for Type B GFSN, captures the sensitivity to changes in the tax regulation. It loads mainly on the tax (TAX9906) and the year-end variable (YEAREND).

Overall, we note that the results of the factor analysis coincide with our previous finding that the desire for a better yield performance is only one of several factors in the decision-making of individual investors. In fact, the factor analysis suggests again that other motives, such as the need for financial flexibility, play a more important role.

4.2. Importance of latent factors

Following the general analysis, we also estimate personal factor scores for each individual investor in our data set. To determine whether any differences exist in the relevance of latent factors related to personal characteristics, we regress the computed scores on selected personal and financial characteristics (average volume per investment). Table 10 shows the results.

We emphasize five regression results relevant to points discussed in this paper. First, the relevance of the financial flexibility factor is most strongly pronounced for investors between 20 and 40 years of age. Additionally, this factor appears to be more important for male investors and for higher average investment volumes. Second, the desire for value mainly drives the decisions of investors with larger investments, who prefer to acquire GFSN directly at banks. We also observe that this factor is of higher relevance for investors younger than 20, compared to other age groups, which we attribute to GFSN accounts that are established as savings accounts in a child's name. Third, the performance factor is strongly marked for direct investors who omit any intermediary and presumably have a higher average financial literacy. In contrast, performance seems to be a less important motive for many investors between the ages of 20 and 40, which corresponds to the high relevance of the financial flexibility factor for this investor group. Fourth, activism is stronger associated with male direct investors and is closely related to low average investment volumes. As for all other factors, we find no clear relevance attached to an investor's residence area. Fifth, the tax factor is more relevant for investors with higher investment volumes, which seems very reasonable.

5. Conclusion

In this paper we analyzed individual investors' empirical use of early exercise rights in German Federal Savings Notes (GFSN). In short, we find that a broad range of economic and environmental factors as well as product characteristics determine investors' exercise decisions, whereby distinct differences exist among the investor base that depend on portfolio and personal characteristics. Still, most investors have in common their use of the exercise right at times that are not optimal according to standard option pricing theory and a frequent failure to exploit more favorable, i.e. optimal, exercise opportunities.

This failure to exercise can hardly be justified by standard reasons as investors could simply shift their money to more favorable fixed-income products. However, we note that monitoring or learning costs may prevent individual investors from following optimal strategies. We find systematics in this behavior that are related to product and personal characteristics. In fact, investors miss attractive exercise opportunities more often when the next coupon payment is shortly due, the coupon structure is steep and duration is low, which suggests that they suffer from cognitive biases. This behavior also depends on the age cohort. Finally, failure to exercise tends to be persistent on the investor level.

Exercising at a point in time that is not optimal according to standard option pricing theory may be reasonable from the investor's point of view, for example because of tax reasons or liquidity constraints. The observed behavior and several empirical patterns suggest that for individual investors performance is in general a less important motivator of exercise decisions than it presumably would be for, e.g., professional or institutional investors in similar product. In fact, our results are consistent with the hypothesis that the need for financial flexibility and liquidity is a major motive for early exercising. The results of an exploratory factor analysis support this hypothesis. Indeed, while we identify five latent factors that mainly drive individual investors' investment and exercise strategy, the most important factor can be interpreted as the desire for flexibility and liquidity.

As already noted at the beginning, the GFSN analyzed in this paper are similar to bank products that include an early exercise right and were designed to attract a similar clientele. Hence, we can expect that our results and implications apply to a more general set of products and are not limited to GFSN. Given this, we emphasize that sophisticated liquidity management is highly important for the issuing institutions that offer such products to individual investors. Issuers must anticipate that investors will use their exercise rights not necessarily to optimize their investment yields, but in response to other influences such as environmental changes or liquidity constraints. This means early exercises frequently occur at times not predicted by standard theory. Additionally, information on the personal characteristics of the investor base can be used to refine predictions of early exercise activity. For instance, the exercise behavior in our sample differs particularly with regard to gender, age and the preferred distribution channel of an investor.

Furthermore, as individual investors fail to exercise the broad majority of attractive early exercise opportunities and frequently use the option right — as noted — at suboptimal times according to option pricing theory, the empirical value of the early exercise right for the issuer will be in general lower than its financial fair value when standard theory is applied. Hence, issuers gain a (significant) financial margin in pricing such products for individual investors with standard financial models. Moreover, our analyses imply that issuers can increase this margin with what is commonly referred to as behavioral financial engineering, i.e. by offering specific product designs and exploiting the cognitive biases of individual investors. For instance, there are indications that individual investors prefer to keep products with a high final coupon payment rather than financially fully equivalent products with a flatter coupon structure. Accordingly, our results suggest that investors more often fail to use attractive exercise opportunities that arise for bonds with specifically designed coupon structures, which is obviously advantageous for the issuer.

Acknowledgements

We thank the participants of the Financial Management Association (FMA) Annual Meeting 2014 in Nashville, of the International Finance and Banking (IFABS) Society Conference 2014 in Lisbon and of the Joint Conference of the Annual Meeting of the German Finance Association (DGF) and the Symposium on Finance, Banking, and Insurance 2014, Karlsruhe, of seminars at the University of Marburg and the University of Passau, and two anonymous referees for very helpful comments and suggestions on an earlier draft of this paper. All remaining errors are our own. We thank the German Finance Agency ("Bundesrepublik Deutschland Finanzagentur GmbH") for providing data and supporting this study.

Appendix A. Least squares Monte Carlo simulation

We discretize the time interval [0, T] into N equidistant time steps. $\mathbb{T} = \{t_0 = 0, t_1, \ldots, t_N = T = 7\}$ with $t_i - t_{i-1} = \Delta t$ is the set of points in time we consider. We assume that $t_b = 1 \in \mathbb{T}$ so that the end of the initial blocking period is part of the points in time considered which implies that $T/\Delta t$ is an integer. The notation is analogous to Section 2.2, but $\mathcal{T}_{t_n}$, of course, only covers respective stopping times with values in $\mathbb{T}$. Setting the interest rate over the period $[t_i, t_{i+1})$ to $r(t_i)$ the discrete-time approximation of Eq. (2) for $t_n \in \mathbb{T}$ is given by:

$$PV_{t_n} = E^Q_{t_n}\left(\exp\left(-\sum_{s\in\{t_n,\ldots,\tau^{opt}_{t_n}-\Delta t\}} r(s)\Delta t\right) EV_{\tau^{opt}_{t_n}}\right) \tag{A.1}$$

$$= \sup_{\tau\in\mathcal{T}_{t_n}} E^Q_{t_n}\left(\exp\left(-\sum_{s\in\{t_n,\ldots,\tau-\Delta t\}} r(s)\Delta t\right) EV_\tau\right) \tag{A.2}$$

$$\text{with}\quad \tau^{opt}_{t_n} = \inf\{\mathbb{T} \ni s \geq \max\{t_b, t_n\} : PV_s = EV_s\}. \tag{A.3}$$

Following the standard dynamic programming approach (see, e.g., Glasserman, 2004, pp. 424–425, for details) the present value PV_{t_n} can be recursively determined via backward induction by:

$$PV_{t_N} = EV_{t_N} \tag{A.4}$$

$$PV_{t_n} = \max\{EV_{t_n}, CV_{t_n}\} \qquad \text{for } t_b \leq t_n < t_N \tag{A.5}$$

$$PV_{t_n} = E^Q_{t_n}\left(\exp\left(-\sum_{s\in\{t_n,\ldots,t_b-\Delta t\}} r(s)\Delta t\right) PV_{t_b}\right) \qquad \text{for } 0 \leq t_n < t_b \tag{A.6}$$

with

$$CV_{t_n} = \exp(-r(t_n)\Delta t) E^Q_{t_n}(PV_{t_{n+1}}). \tag{A.7}$$

At maturity the present value is the exercise value (see Eq. (A.4)). At any t_n before maturity but after the initial blocking period (see Eq. (A.5)), it equals the maximum of the exercise value and the continuation value CV_{t_n} that is defined as the discounted expected present value in t_{n+1} conditional on t_n according to Eq. (A.7).[24] If t_n is inside the blocking period (see Eq. (A.6)) the present value is simply the conditional expectation of the discounted present value in t_b. An optimal stopping time is then given by:

$$\tau^{opt}_{t_n} = \inf\{\mathbb{T} \ni s \geq \max\{t_b, t_n\} : EV_s \geq CV_s\}, \tag{A.8}$$

that is the first point in time at which an early exercise is possible where the exercise value is equal to or exceeds the continuation value.

For solving the dynamic programming problem we apply least squares Monte Carlo simulation according to Longstaff and Schwartz (2001) but use Glasserman (2004)'s interleaving estimator to mitigate the effects of a potential high bias due to the backward induction approach and of a potential low bias due to a suboptimal stopping rule.[25] The procedure is as follows:[26]

[24] It should be noted that the discount factor $\exp(-r(t_n)\Delta t)$ in Eq. (A.7) is outside the expectation operator here as $r(t_n)$ is $\mathcal{F}_{t_n}$-measurable.

[25] Glasserman (2004) provides a detailed analysis of these potential biases.

[26] For ease of notation we present the procedure for calculating the present value in $t_0 = 0$. The value at any other point in time t_n can easily be obtained by starting the simulation in t_n rather than t_0.

1) Simulate K paths of the short rate process $r(s)$ with step-size Δt. Let $r_k(s)$ denote the simulated short rate in s of path k. Each path k starts with $r(0)$, i.e. $r_k(0) = r(0)$.
2) For each path k, set the estimate of the present value $\widehat{PV}_{kt_N}$ in t_N to the exercise value, i.e. $\widehat{PV}_{kt_N} = EV_{t_N}$.
3) Assume for a given $t_n \geq t_b$ that the present value $\widehat{PV}_{kt_j}$ in t_j has been estimated for all $t_j > t_n$ and each path k.
   3.1) Run the following cross-sectional OLS-regression over all paths k with some $I > 0$:

$$\exp(-r_k(t_n)\Delta t)\widehat{PV}_{kt_{n+1}} = \sum_{i=0}^{I-1} \beta_{it_n} r_k(t_n)^i + \epsilon_k. \quad (A.9)$$

   Define the continuation value in t_n as a function of the short rate in t_n via:

$$\widehat{CV}_{t_n}(r(t_n)) = \sum_{i=0}^{I-1} \beta_{it_n} r(t_n)^i. \quad (A.10)$$

   3.2) For each k, simulate *one* new path $r_k^*(t_j), n \leq j < N$, starting with $r_k(t_n)$, i.e. $r_k^*(t_n) = r_k(t_n)$. Define

$$\tau_{kt_n}^{*opt} = \inf\left\{t_j > t_n : EV_{t_j} \geq \widehat{CV}_{t_j}\left(r_k^*(t_j)\right)\right\}. \quad (A.11)$$

   Set

$$\widehat{PV}_{kt_n}^* = \exp\left(-\sum_{s\in\left\{t_n,\dots,\tau_{kt_n}^{*opt}-\Delta t\right\}} r_k^*(s)\Delta t\right) EV_{\tau_{kt_n}^{*opt}}. \quad (A.12)$$

   3.3) Set the estimate of the present value in t_n for path k via:

$$\widehat{PV}_{kt_n} = \begin{cases} EV_{t_n} & \text{for } EV_{t_n} \geq \widehat{CV}_{t_n}(r_k(t_n)) \\ \widehat{PV}_{kt_n}^* & \text{for } EV_{t_n} < \widehat{CV}_{t_n}(r_k(t_n)) \end{cases}. \quad (A.13)$$

4) The estimate of the present value in $t_0 = 0$ is then given by:

$$\widehat{PV}_0 = \frac{1}{K}\sum_{k=1}^{K} \exp\left(-\sum_{s\in\{t_0,\dots,t_b-\Delta t\}} r_k(s)\Delta t\right)\widehat{PV}_{kt_b}. \quad (A.14)$$

The interleaving estimator manifests in step 3.3), where the present value is not set to the continuation value if the exercise value is below the continuation value. Instead it is obtained by applying the optimal stopping time to an independent new path of short rates as shown in step 3.2).

There are some degrees of freedom when implementing the above approach. For reasons of numerical time efficiency we set $\Delta t = 1/12$ and use $K = 10,000$ runs. In the regression (A.9) we set $I = 4$, i.e. we use the first four monomials as basis functions, as numerical tests have shown that more monomials do not change the results much. The results are also stable using alternative basis functions such as Laguerre and Hermite polynomials and to a higher number of runs.

References

Allison, P.D., 2002. Missing data. Quant. Appl. Soc. Sci. 136.
Badrinath, S.G., Lewellen, W.G., 1991. Evidence on tax-motivated securities trading behavior. J. Financ. 46 (1), 369–382.
Barber, B.M., Odean, T., 2001. Boys will be boys: gender, overconfidence, and common stock investment. Quart. J. Econ. 116 (1), 261–292.
Barber, B.M., Odean, T., 2008. All that glitters: the effect of attention and news on the buying behavior of individual and institutional investors. Rev. Financ. Stud. 21 (2), 785–818.
Barraclough, K., Whaley, R.E., 2012. Early exercise of put options on stocks. J. Financ. 67 (4), 1423–1456.
Dai, Q., Singleton, K.J., 2000. Specification analysis of affine term structure models. J. Financ. 55 (5), 1943–1978.
Deng, Y., Zheng, D., Ling, C., 2005. An early assessment of residential mortgage performance in china. J. Real Estate Financ. Econ. 31 (2), 117–136.
Dhar, R., Zhu, N., 2006. Up close and personal: investor sophistication and the disposition effect. Manage. Sci. 52 (5), 726–740.
Duffie, D., 2001. Dynamic asset pricing, 3rd edition Princeton University Press.
Egorov, A.V., Hong, Y., Haitao, L., 2006. Validating forecasts of the joint probability density of bond yields: can affine models beat random walk? J. Econom. 135 (1–2), 255–284.
Enders, C.K., 2010. Applied missing data analysis, 1st edition The Guilford Press.
Finucane, T.J., 1997. An empirical analysis of common stock call exercise: a note. J. Bank. Financ. 21 (4), 563–571.
Fisher, L., Weil, R.L., 1971. Coping with the risk of interest-rate fluctuations: returns to bondholders from naive and optimal strategies. J. Busin. 44 (4), 408–431.
German Federal Statistics Office, 2009. Regionaldatenbank Deutschland. Online publication.
Glasserman, P., 2004. Monte carlo methods in financial engineering, 1st edition Springer.
Green, J., Shoven, J.B., 1986. The effects of interest rates on mortgage prepayments. J. Money Credit Bank. 18 (1), 41–59.
Grinblatt, M., Keloharju, M., 2001. What makes investors trade? J. Financ. 56 (2), 589–616.
Hördahl, P., Vestin, D., 2005. Interpreting implied risk-neutral densities: the role of risk premia. Rev. Financ. 9 (1), 97–137.
Huber, P.J., 1967. The Behavior of Maximum Likelihood Estimates under Nonstandard Conditions. In: Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability. University of California Press, pp. 221–233.
Hull, J.C., 2006. Options, futures and other derivatives, 6th edition Prentice Hall.
Hvidkjaer, S., 2011. A trade-based analysis of momentum. Rev. Financ. Stud. 19 (2), 457–491.
Ivkovic, Z., Poterba, J., Weisbenner, S., 2005. Tax-motivated trading by individual investors. Am. Econ. Rev. 95 (5), 1605–1630.
Kaiser, H.F., 1960. The application of electronic computers to factor analysis. Educ. Psychol. Meas. 20 (1), 141–151.
King, G., Zeng, L., 2001. Logistic regression in rare events data. Politic. Anal. 9 (2), 137–163.
Koziol, C., 2006. Empirical exercise behavior of warrant holders and its consequences for warrant values. Intern. J. Theoret. Appl. Financ. 9 (2), 245–268.
Liao, L., Li, Z., Zhang, W., Zhu, N., 2014. Exercise to lose money? Irrational exercise behavior from the Chinese warrants market. J. Futures Markets 35 (5), 399–419.
Liedtka, S.L., Nayar, N., 2012. Early Exercise of Call Options to Accelerate Application of Lower U.S. Tax Rates on Capital Gains. In: Stock, T. (Ed.), Advances in Taxation. Emerald Group Publishing Limited, pp. 107–134.
Longstaff, F.A., Schwartz, E.S., 2001. Valuing American options by simulation: a simple least-squares approach. Rev. Financ. Stud. 14 (1), 113–147.
Nicolosi, G., Peng, L., Zhu, N., 2009. Do individual investors learn from their trading experience? J. Financ. Markets 12 (2), 317–336.
Overdahl, J.A., Martin, P.G., 1994. The exercise of equity options: theory and empirical tests. J. Derivat. 2 (1), 38–51.
Pool, V.K., Stoll, H.R., Whaley, R.E., 2008. Failure to exercise call options: an anomaly and a trading game. J. Financ. Markets 11 (1), 1–35.
Poteshman, A.M., Serbin, V., 2003. Clearly irrational financial market behavior: evidence from the early exercise of exchange traded stock options. J. Financ. 58 (1), 37–70.
Rubin, D.B., 1976. Inference and missing data. Biometrika 63 (3), 581–592.
Schwartz, E.S., Toruos, W.N., 1989. Prepayment and the valuation of mortgage backed securities. J. Financ. 44 (2), 375–392.
Shefrin, H.M., Statman, M., 1984. Explaining investor preference for cash dividends. J. Financ. Econ. 13 (2), 253–282.
Stanton, R., 1995. Rational prepayment and the valuation of mortgage-backed securities. Rev. Financ. Stud. 8 (3), 677–708.
Tang, H., Xia, Y., 2007. An international examination of affine term structure models and the expectations hypothesis. J. Financ. Quant. Anal. 42 (1), 41–80.
Thaler, R.H., 1999. Mental accounting matters. J. Behav. Decis. Mak. 12 (3), 183–206.
White, H., 1980. A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. Econometrica 48 (4), 817–838.
White, H., 1982. Maximum likelihood estimation of misspecified models. Econometrica 50 (1), 1–25.
Yuan, K.H., Bentler, P.M., 2000. Three likelihood-based methods for mean and covariance structure analysis with nonnormal missing data. Sociol. Methodol. 30 (1), 165–200.