

# The price-setting behavior of banks: An analysis of open-end leverage certificates on the German market

Oliver Entrop, Hendrik Scholz \*, Marco Wilkens

*Ingolstadt School of Management, Catholic University of Eichstätt-Ingolstadt, Auf der Schanz 49, D – 85049 Ingolstadt, Germany*

## A B S T R A C T

This paper presents the first analysis of open-end leverage certificates on the German market. The major innovations of these certificates are twofold. First, issuers announce a price-setting formula according to which they are willing to buy and sell the certificates over time. Second, the product's lifetime is potentially endless. Our main findings are that the price-setting formula is (i) designed to strongly favor the issuer and (ii) is consistent with the main outcome of the 'life cycle hypothesis' for structured financial products [Stoimenov, P.A., Wilkens, S., 2005. Are structured products 'fairly' priced? An analysis of the German market for equity-linked instruments. *Journal of Banking and Finance* 29, 2971–2993]. (iii) This holds for different product features and also in the presence of issuers' credit risk and jump risk in the underlying.

### JEL classification:

G13  
G21  
G24

### Keywords:

Structured products  
Certificates  
Hedging  
German market  
Pricing

## 1. Introduction

In many countries since the mid-90s, exchange-traded innovative financial products (IFPs) have become increasingly important in the retail market. IFPs are basically debt instruments issued by financial institutions which combine contingent claim features of standard options or futures with the characteristics of equities or fixed income products. Popular examples are discount certificates and equity-linked bonds which can be constructed from zero-coupon bonds and put options.<sup>1</sup> Off-exchange trades of IFPs are usually settled by issuers, whereas exchange trades are primarily conducted by market makers. As issuing banks normally handle the market making by themselves, they de facto dominate not only the primary but also the secondary market. Since short selling of IFPs is impossible, market makers can systematically quote prices that do not match fair theoretical values but favor themselves. Furthermore, they usually keep the price-setting mechanism hidden from inves-

tors. Since IFPs are often complex, it is frequently difficult for private investors to evaluate the 'fairness' of the quotes.

Due to this intransparency, a large body of empirical work has been carried out to analyze the price-setting behavior of issuers by comparing quoted prices and theoretical fair values of IFPs. Chen and Kensinger (1990), Chen and Sears (1990), Baubonis et al. (1993) and Benet et al. (2006) report significant deviations for equity-linked products on the US market. Brown and Davis (2004) detected significant price deviations for endowment warrants on the Australian market. Analogue results were found for the Swiss market by Wasserfallen and Schenk (1996), Burth et al. (2001) and Grünbichler and Wohlwend (2005), and for the German market by Wilkens et al. (2003), Stoimenov and Wilkens (2005) and Baule et al. (2008). All these empirical studies reveal the pricing behavior of issuers: At issuance, they regularly sell IFPs for their theoretical value plus a positive premium, and later they buy them back paying the theoretical value plus a decreased premium. As a result, issuers gain by diminishing overpricing in the course of product lifetimes. Wilkens et al. (2003) and Stoimenov and Wilkens (2005) analyze this behavior in detail for IFPs and subsume decreasing premiums over time under 'life cycle hypothesis'.

In recent years, several banks have issued a new type of IFP, namely leverage products with a knock-out feature. The leverage effect of these products on price changes of the respective underlying results because a clearly lower amount of capital has to be invested compared to a direct investment in the underlying.

\* Corresponding author. Tel.: +49 841 937 1878; fax: +49 841 937 2878.

E-mail addresses: [oliver.entrop@ku-eichstaett.de](mailto:oliver.entrop@ku-eichstaett.de) (O. Entrop), [hendrik.scholz@ku-eichstaett.de](mailto:hendrik.scholz@ku-eichstaett.de) (H. Scholz), [marco.wilkens@ku-eichstaett.de](mailto:marco.wilkens@ku-eichstaett.de) (M. Wilkens).

<sup>1</sup> Due to the immense number of product forms, markets for IFPs worldwide are still fairly intransparent. To alleviate difficulties investors have in perceiving the structure and comparability of IFPs, New York's Structured Products Association (SPA) has, for example, suggested a standardized nomenclature in a 'white paper' ([www.structuredproducts.org](http://www.structuredproducts.org)).

The knock-out characteristic describes the fact that these products cease to exist if the price of the underlying reaches a certain barrier.<sup>2</sup> The first generation of these leverage products in Germany analyzed by Muck (2006, 2007) and Wilkens and Stoimenov (2007) is basically equivalent to one-sided barrier options. Muck (2006) and Wilkens and Stoimenov (2007) report clear positive premiums that favor issuers. In mitigation, however, Muck (2007) finds that jump risk at least partially justifies these premiums.

This paper is the first to analyze a new generation of leverage products on the German retail market, namely open-end leverage certificates.<sup>3</sup> Compared to financial products analyzed by the studies mentioned above, and in particular in contrast to the first generation of leverage certificates, this new generation exhibits two main innovative features: (i) Issuers announce ex-ante a relatively simple price-setting formula, according to which they are willing to sell and repurchase these certificates over time. (ii) Open-end leverage certificates do not have a fixed product maturity, but a potentially perpetual lifetime. Feature (i) removes the ‘arbitrariness’ of the issuers’ quotes for IFPs, from the investors’ point of view. To the best of our knowledge, this study is the first that does not have to rely on quotes collected on the primary and secondary market for analyzing the price-setting behavior of issuers. Since we focus directly on the price-setting formula, we are able to fill a conspicuous gap in the present empirical literature.

The paper is organized as follows: Section 2 presents the construction of open-end leverage certificates and the price-setting formula. Section 3 describes and analyzes a simple hedging strategy used and communicated by issuers for open-end leverage certificates. Based on this we find that the price-setting formula implies a relatively high profit potential for issuers. Our results are consistent with the above-mentioned life cycle hypothesis – a finding that could not be shown for the first generation of leverage certificates. Section 4 contains a comparative static analysis of the ‘value of mispricing’ from the issuers’ point of view. This valuation is based on a Black and Scholes (1973) and Merton (1973) world with stochastic interest rates. In this context we discuss main factors influencing the theoretical value of open-end leverage certificates, in particular the volatility of the underlying and the so-called ‘funding rate spread’ set by the issuer. Section 5 discusses the impact of issuer’s credit risk, jump risk in the underlying price and differing product features on our main findings. None of these issues substantially changes the results of our previous analysis. Section 6 concludes.

## 2. Main characteristics of open-end leverage certificates

Open-end leverage certificates (OELCs) are issued in the German retail market into two basic forms: as long certificates which benefit from increasing prices of the underlying, and as short certificates which profit from decreasing prices. We focus on long certificates, which are more important considering the number of issues: 24,177 OELCs were issued in Germany in 2006, more than 60% as long certificates (14,635).<sup>4</sup> Nevertheless, our analysis can be transferred straightforwardly to open-end short certificates.<sup>5</sup> Stocks and stock indices are mainly used as underlyings, but also – more rarely – exchange rates, precious metals and others. Since stocks and stock indices are the most common underlyings, we only consider these here.

Depending on the issuer of OELCs, we find slightly varying product features in practice. However, the main characteristics of these

certificates match closely. An OELC is based on an underlying  $S$ , has a strike  $X$  and a knock-out barrier  $B$ . To keep the illustration general and intuitive, the following analysis is based on a stylized definition of OELCs. We analyze the influence of differing product features in Section 5.3.

OELCs have a potentially perpetual lifetime. Nevertheless, they become due when the underlying price hits or falls below the barrier for the first time. This first passage time  $\tau$  is given by

$$\tau = \inf\{t: S_t \leq B_t\}, \quad (1)$$

where  $S_t$  denotes the price of the underlying and  $B_t$  the barrier in  $t$ . In the case of a knock-out in  $t$ , the investor receives a settlement amount (rebate)  $P_t$ :

$$P_t = S_t - X_t. \quad (2)$$

The settlement amount is the difference between the price of the underlying  $S_t$  and the strike  $X_t$ .<sup>6</sup> It is limited to zero. As the central innovative feature of OELCs, issuers use Eq. (2) also as the price-setting formula on the secondary market. At any time during the lifetime of a certificate, they offer to sell and buy the certificate for a price according to (2).<sup>7</sup>

Barrier and strike are not constant over time. The initial strike  $X_0$  increases over time, related to the so-called ‘funding rate’. This funding rate consists of a short-term money market interest rate  $r'_t$  such as EONIA (Euro OverNight Index Average) and a ‘funding rate spread’  $z \geq 0$ . In practice, banks adjust the strike on a daily basis. Given a strike  $X_t$  in  $t$ , one day later the revised strike will be  $X_t(1 + (r'_t + z)/360)$ .<sup>8</sup>

The goal of the barrier is to make sure that, in general,  $S_t - X_t$  does not become negative. Without such a barrier, issuers would have to establish, e.g., a margin system to secure their possible claims on the investor. This would make the distribution of OELCs much more complex and would disqualify many customers from this market segment. More precisely, the barrier permanently exceeds the strike by the factor  $a > 0$ :  $B_t = (1 + a) X_t$ . This factor  $a$  is usually large enough to ensure that, at knock-out,  $S_t - X_t$  is positive, especially when discontinuous price changes occur such as overnight jumps, i.e., differences between closing and opening prices, or occasional illiquidity of the underlying after a knock-out event. Both values  $a$  and  $z$  are set by issuers and will mainly determine the attractiveness of OELCs.

Assuming  $r'_t$  and  $z$  are continuously compounded we have:

$$X_t = X_0 \exp\left(\int_0^t (r'_s + z) ds\right) = X_0 \exp\left(\int_0^t r'_s ds + zt\right), \quad (3)$$

$$B_t = (1 + a)X_t = (1 + a)X_0 \exp\left(\int_0^t r'_s ds + zt\right). \quad (4)$$

Inserting (3) into (2), the issuers’ price-setting formula can be written as

$$P_t = S_t - X_0 \exp\left(\int_0^t r'_s ds + zt\right). \quad (5)$$

Hence the price of the certificate in  $t$  only depends on the underlying price in  $t$ , the initial strike, past money market rates and the funding rate spread. We emphasize (5) being independent of the knock-out barrier and the volatility of the underlying.

<sup>6</sup> The conversion ratio is often not one, thus the investor receives a part or a multiple of this difference. Issuers usually determine the settlement amount of OELCs within, e.g., one hour following the knock-out based on the prices they get from terminating their hedging instruments (see, e.g., HSBC Trinkaus & Burkhardt, 2006). This characteristic transfers the liquidity risk in context with the hedging instruments to the investor. However, we will abstract from this in our analysis.

<sup>7</sup> In the following we abstract from bid-ask spreads existing in practice.

<sup>8</sup> Moreover, the strike will be reduced in the case of dividend payments of the underlying to offset a negative dividend effect on the settlement amount according to (2). Therefore, in the following we refrain from dividend payments.

<sup>2</sup> Applying the SPA’s standardized nomenclature, the main characteristics of leveraged products with knock-out feature can be described as ‘Principal at Risk’ and ‘Enhanced Participation’.

<sup>3</sup> These certificates are sold under different names, see Table 1.

<sup>4</sup> Deriva GmbH Financial IT and Consulting.

<sup>5</sup> We discuss the price-setting formula issuers apply for open-end short certificates in Footnote 11.

### 3. Simple superhedging strategy and life cycle hypothesis

What is the intuition behind the price-setting formula (5)? When the issuer sells the certificate for  $P_0 = S_0 - X_0$  in  $t = 0$ , he can at the same time purchase the underlying for  $S_0$  and issue revolving short-term debt for the notional amount  $X_0$  which comes out to a total payment of zero. This procedure is regularly communicated by issuers in their product brochures and sales prospectuses for OELCs.<sup>9</sup> As banks can refinance themselves at short-term money market interest rates  $r'_t$ , such as EONIA in the interbank market, the value of this hedge position, the 'leveraged underlying',  $LU_t$ , in  $t$  is

$$LU_t = S_t - X_0 \exp\left(\int_0^t r'_s ds\right), \quad (6)$$

where we assume that interest rate payments are accrued. A decomposition of the price-setting formula (5) clarifies the relation between this formula and the leveraged underlying:

$$P_t = \underbrace{S_t - X_0 \exp\left(\int_0^t r'_s ds\right)}_{\text{leveraged underlying } LU_t} - \underbrace{X_0 \left( \exp\left(\int_0^t r'_s ds + zt\right) - \exp\left(\int_0^t r'_s ds\right) \right)}_{\text{profit potential } PP_t}. \quad (7)$$

Hence the price of the OELC in  $t$  equals the value of the leveraged underlying  $LU_t$  minus a term  $PP_t$  which, given  $z > 0$ , is positive by definition at any  $t > 0$ . Thus buying the leveraged underlying in  $t = 0$  represents a simple (semi-static) superhedging strategy which has to be terminated in the case of a knock-out or a repurchase. If the investor returns the certificate in  $t$  or, alternatively, if the OELC is knocked-out in  $t$ , the bank can sell the leveraged underlying and settle the investor's claim.

Given that  $S_t - X_t$  is positive at knock-out, which usually is the case, the value of the leveraged underlying is higher than the quoted price of the OELC yielding a bank's positive (gross<sup>10</sup>) profit  $PP_t$ . Therefore, we denote  $PP_t$  as banks' 'profit potential'. The term 'potential' is used because the point in time of a knock-out or repurchase is ex-ante unknown. However, in the most unlikely event of a large negative jump below both the barrier and the strike, or surprisingly high illiquidity in the underlying,  $S_t - X_t$  could become negative at knock-out. In such rare cases, the simple hedging strategy can fail and the resulting gap risk is taken by the issuer. Note that this hedging strategy is based on spot market instruments (or alternatively on futures) and can easily be implemented, as it works without potentially illiquid derivatives.

The issuer's profit potential  $PP_t$  from OELCs equals zero at issuance and increases over time for  $z > 0$ .<sup>11</sup> This finding is clearly in

<sup>9</sup> Especially for OELCs on broader stock indices, it might be better for issuers to use futures contracts instead of buying the underlying stocks directly. The following considerations also hold for this modification.

<sup>10</sup> The costs of structuring, distribution, etc. have to be deducted from the gross profit (potential) to get to the net profit (potential). The addition 'gross' will be abandoned in the following.

<sup>11</sup> The decomposed price-setting formula for open-end short certificates can be written as:  $P_t^{\text{short}} = X_0 \exp\left(\int_0^t r'_s ds\right) - S_t - \exp(-zt)X_0 \left( \exp\left(\int_0^t r'_s ds + zt\right) - \exp\left(\int_0^t r'_s ds\right) \right)$ . Obviously, a simple hedging strategy for banks issuing open-end short certificates contains an investment of  $X_0$  in a short-term money market account and a short position in the underlying  $S$ . Since most issuers show equal funding rate spreads  $z$  for their long and short certificates, the profit potential of short certificates compared to long certificates increases slower over time by the factor  $\exp(-zt)$ , which is less attractive for issuers. On the other hand, strikes of short certificates are above today's value of the underlying. Hence, they are higher than strikes of long certificates, yielding higher profit potentials. Since these two effects countervail, one cannot distinguish in general whether short or long certificates generate a higher profit potential for issuers over time.

line with the main outcome of the life cycle hypothesis for IFPs, quoting systematically rising gains for issuers in the course of product lifetimes. However, the above-mentioned studies dealing with the first generation of leverage certificates could not confirm the life cycle hypothesis. Additionally, in contrast to papers analyzing the life cycle hypothesis for other IFPs, our finding can be derived analytically based on the price-setting formula rather than indirectly by observing quoted prices and calculating fair theoretical values relying on option valuation models.

### 4. Valuation from the bank's perspective

#### 4.1. Valuation algorithm

To derive the valuation algorithm we have to use a set of model-assumptions. Based on the valuation algorithm and the following comparative static analysis, issuers can assess how attractive OELCs are depending on different underlyings, product features and market conditions.

Like all studies mentioned in Section 1 analyzing IFPs, we apply arbitrage-free risk-neutral valuation techniques. It is evident that all IFPs, and OELCs in particular, offer arbitrage opportunities to banks as already discussed in Sections 1 and 3. This seeming conflict can be resolved by a market segmentation hypothesis similar to that of Jarrow and van Deventer (1998): There are a number of banks with access to capital markets, whereas this access is limited for individual investors for several reasons, such as legal restrictions, large entry barriers or excessive transaction costs. It can be assumed that markets are (nearly) arbitrage-free for issuers when they hedge such IFPs, but private investors cannot buy the replicating portfolio or the payoff profile of IFPs, at least not without extra costs. Additionally, no one can take short-positions in IFPs to benefit from unfair quotes, i.e. to make arbitrage gains. Hence, when risk-neutral valuation techniques are applied, the resulting values are fair theoretical values for banks. The analysis is based on the following assumptions.

**Assumption 1.** The default-free short rate  $r_t$  is stochastic. Under the risk-neutral measure  $Q$ , the stock (index) price follows a geometric Brownian motion, i.e. satisfies

$$dS_t/S_t = r_t dt + \sigma dW_t, \quad (8)$$

where  $\sigma > 0$  denotes the constant volatility and  $W_t$  a standard Brownian motion.<sup>12</sup>

Since geometric Brownian motion has continuous paths, the simple superhedge from Section 3 cannot fail. We discuss the impact of jump risk in the stock (index) price on the valuation of OELCs separately in Section 5.2.

**Assumption 2.** Investors plan to hold the OELC for a finite holding period  $T$ .

Since the realized profit potential of banks increases with the length of time investors hold these certificates, this time period also affects the theoretical fair values of OELCs. The above-mentioned studies deal with IFPs exhibiting a fixed maturity and implicitly assume that investors hold these products until maturity, although in general they have the opportunity to sell them back to the issuer at any time. To obtain comparable results to these studies, we analogously assume that investors plan to hold OELCs for a certain time period  $T$ . Therefore, a payment prior to the expiration of this holding period only occurs in the case of a knock-out. To show the impact of different assumed holding periods on theoretical values, we report the results for various

<sup>12</sup> Since OELCs are dividend-protected we do not have to consider dividend payments, see Footnote 8.

choices of  $T$  in a comparative static analysis in Section 4.2 and discuss further extensions in Section 6.

**Assumption 3.** The issuer of the certificate is default-free. The bank's short-term refinance rate  $r_t^r$  equals the default-free short rate  $r_t$ .

The discussion of the impact of the issuers' credit risk on the valuation of OELCs in Section 5.1 shows that this assumption is not crucial.

According to the risk-neutral valuation technique, the present value of a security results from the expected value of its discounted payoffs. Taking a possible knock-out into account, the point in time when the investor receives a payment is  $\tau^T = \min(\tau, T)$  given a planned holding period  $T$ . Based on (7), the today's fair theoretical value  $OELC_0^T$  of open-end long certificates can be calculated as:<sup>13</sup>

$$\begin{aligned} OELC_0^T &= E_Q \left( \exp \left( - \int_0^{\tau^T} r_s ds \right) P_{\tau^T} \right) = S_0 - X_0 - X_0 (E_Q(\exp(z\tau^T)) - 1) \\ &= \underbrace{S_0 - X_0}_{LU_0} - \underbrace{X_0 (\exp(zT)(1 - Q(\tau \leq T)) + E_Q(1_{\{\tau \leq T\}} \exp(z\tau)) - 1)}_{VPP_0^T}, \end{aligned} \quad (9)$$

where  $E_Q(\cdot)$  denotes expectation with respect to  $Q$ . In the Appendix we derive closed-form solutions for the risk-neutral cumulative knock-out probability  $Q(\tau \leq t)$  (see (A.1)) and the expression  $E_Q(1_{\{\tau \leq t\}} \exp(z\tau))$  (set  $c=0$  in (A.3)) for every  $t > 0$ . Since we have  $\sigma^2/2 + z > 0$ ,  $Q(\tau \leq t)$  converges to 1 for large  $t$ . The valuation of OELCs according to (9) discloses that their values do not depend on the short rate and its dynamics. This is a natural result, since the short rate enters both the drift of the stock price process (8) and the time-varying barrier (4).

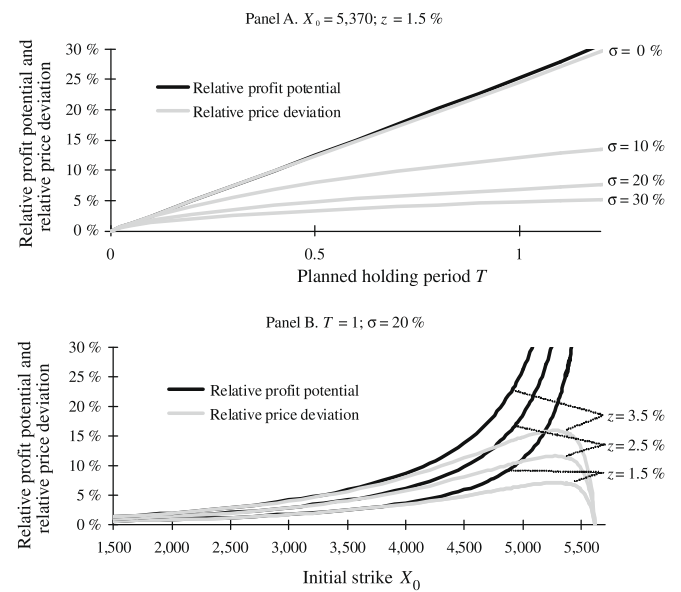
The representation (9) allows for an economic interpretation of the theoretical value of OELCs. Today's certificate value  $OELC_0^T$  consists of the value of the leveraged underlying  $LU_0 (= P_0 - X_0)$  minus today's theoretical value of the profit potential  $VPP_0^T$ . In other words,  $VPP_0^T$  represents today's value of the difference between the simple superhedging strategy and the price of the OELC. Clearly, banks are most interested in increasing this difference, as it presents the value of their arbitrage gains.

The essential target analyzed in earlier studies is the relative price deviation between the price set by the issuer and the fair theoretical value of the IFP. This deviation can be interpreted as issuers' percentage profit under the standard assumption of investors remaining invested in the product until maturity. Applying an analogue proceeding, we determine the relative price deviations  $RPD_0^T$  of OELCs as

$$RPD_0^T = \frac{P_0 - OELC_0^T}{P_0} = \frac{VPP_0^T}{P_0}. \quad (10)$$

In contrast to earlier studies, here we relate the difference between the current price  $P_0$  and the value of the certificate  $OELC_0^T$  to its price and not to its theoretical value. The price of an OELC in  $t=0$  equals the value of the hedging instruments, whereas the value of common hedge positions of classical IFPs (e.g., discount certificates) matches the theoretical value of the product. With this in view, we relate the price deviation to the value of the hedge position, like in other empirical studies.

<sup>13</sup> Note that this representation does not depend on the assumption of a geometric Brownian motion, but holds for other dynamics of the underlying with continuous paths. Furthermore, most of the sensitivities presented in Section 4.2 could, similar to Merton (1973), also be derived with a weaker assumption than a geometric Brownian motion.



**Fig. 1.** Relative profit potential of banks and relative price deviation of an open-end long leverage certificate, depending on the investor's planned holding period and the initial strike. For an open-end long leverage certificate on the DAX, this figure shows the relative profit potential, i.e. the profit potential  $PP_T$  (see (7)) divided by the actual certificate price  $P_0$ , and the relative price deviation  $RPD_0^T$  (see (10)) as a function of the investor's planned holding period  $T$  (Panel A) and as a function of the initial strike  $X_0$  (Panel B), respectively. In both panels, the relative difference between barrier and strike of the certificate is  $a = 1.5\%$ , the short rate is constantly  $r_t = 3\%$  and the actual price of the DAX is  $S_0 = 5,700.00$ . Under the risk-neutral measure, the DAX is assumed to follow a geometric Brownian motion according to (8). In Panel A, the certificate shows an initial strike of  $X_0 = 5,370.00$ , an initial barrier  $B_0 = 5,450.55$  and a funding rate spread  $z = 1.5\%$ . The relative profit potential is presented as well as relative price deviations for different volatilities  $\sigma$  of the DAX of 0%, 10%, 20% and 30%. In Panel B, the planned holding period of the investor is  $T = 1$  and the volatility of the DAX  $\sigma = 20\%$ . The relative profit potential and the relative price deviations are presented for funding rate spreads  $z$  of 1.5%, 2.5% and 3.5%.

## 4.2. Comparative static analysis

In this section, we analyze the impact of different product designs and market conditions on the theoretical values of OELCs. This shows which product design is especially attractive for issuers. As a starting point, we examine an OELC on the German blue-chip stock index DAX. The main characteristics of the certificate match real OELCs offered by HSBC Trinkaus & Burkhardt. Initially, the certificate has a strike  $X_0$  of 5,370.00 and a barrier exceeding the strike by  $a = 1.5\%$ . Thus at issuance, the barrier amounts to  $B_0 = 5,450.55$ . According to (3), the certificate's strike is continuously compounded based on the funding rate  $r_t + z = r_t + 1.5\%$ . For a current DAX of  $S_0 = 5,700.00$ , the price of the certificate at issuance is  $P_0 = 330.00$ . Given a volatility of the DAX of  $\sigma = 20\%$  and a holding period of  $T = 1$ , the value of the OELC is  $OELC_0^1 = 307.03$ .

For illustration purposes, let us assume that the short rate  $r_t$  is constant at 3%.<sup>14</sup> The black-labeled line in Fig. 1, Panel A, shows the resulting issuer's relative profit potential  $PP_T/P_0$  depending on the investor's planned holding period  $T$ . Evidently, the relative profit potential almost linearly increases in  $T$ . For example, given a 1-year holding period, the relative profit potential is 25.34% – much more than for other IFPs analyzed in the studies mentioned above.

The grey plotted lines show the relative price deviation  $RPD_0^T$  according to (10) for various volatilities  $\sigma$ . Panel A shows that for higher volatilities the relative price deviation also increases in

<sup>14</sup> In general, the short rate and thus the future (relative) profit potential are stochastic.

dependence on the holding period, but much more flatly than the relative profit potential. Obviously, the risk-neutral probability of a premature knock-out of the certificate strongly impacts the value of the profit potential. Since higher volatilities increase the knock-out probability,<sup>15</sup> the value of the profit potential and thus the relative price deviation are lower for higher volatilities. This is because the investor is more likely to be 'forced out' of the certificate. Consequently, a higher volatility is disadvantageous for the bank.

Besides the volatility of the underlying  $\sigma$ , the essential factor determining the risk-neutral knock-out probability is the ratio of the initial strike and the underlying price at issuance. Assuming an investor's holding period of  $T = 1$  and a DAX volatility of  $\sigma = 20\%$ , Panel B in Fig. 1 shows the relative profit potential and the relative price deviation depending on the initial strike for OELCs with funding rate spreads  $z$  of 1.5%, 2.5% and 3.5%, respectively. First, higher funding rate spreads yield higher relative profit potentials  $PP_1/P_0$  (black lines) and thus higher relative price deviations  $RPD_0^1$  (grey lines). Second, the relative profit potential increases with higher initial strikes. On the other hand, the relative price deviations rise at first, reach a peak at about 7%, 11% and 15%, respectively, and decrease to zero at an initial strike of 5,615.76. Given this strike, the certificate is instantly knocked-out as the barrier just matches the current DAX:  $B_0 = (1 + a)X_0 = 1.015 \cdot 5,615.76 = 5,700.00 = S_0$ . The difference between the relative profit potential and the relative price deviation is again mainly determined by the knock-out probabilities. Certificates with a higher strike create a higher profit potential for banks. But the knock-out probability increases as well, which counteracts the positive impact of the higher profit potential on the relative price deviation.

Since the profit potential is zero for an initial strike of  $X_0 = 0$  and the certificate is instantly knock-out for  $X_0 = S_0/(1 + a)$ , the relative price deviation is zero for these strikes and thus an initial strike exists yielding a maximum relative price deviation. However, issuing only certificates with this 'most profitable' initial strike would not take into account that various investors will experience different possible utility gains from buying OELCs with different initial strikes. Therefore, to maximize their absolute profit, issuers additionally have to consider demand-side aspects (see, e.g., Breuer and Perst, 2007). This could explain why in practice banks regularly offer a large variety of strikes to attract a large number of investors. For example, BNP Paribas at once offered 25 certificates on the DAX with a wide range of initial strikes from 2,000 to 5,300 when entering the market for OELCs in June 2006 (BNP Paribas, 2006).

## 5. Impact of issuers' credit risk, jump risk and differing product features

### 5.1. Impact of the issuer's credit risk

From a legal point of view IFPs are unsecured bonds and thus potentially affected by an issuer's default. Recent papers that incorporate credit risk when valuing IFPs rely on Hull and White (1995).<sup>16</sup> Transferring this approach to the stylized OELC from Section 4.2 and assuming a realistic issuer's credit spread of 0.5% (0.3%, 0.7%) lowers its value from 307.03 (default-free) to 305.79 (306.28, 305.29) (see the Appendix for the valuation algorithm).

However, a closer look at the features of OELCs results in a different conclusion and reveals once more the striking particularities

<sup>15</sup> The sensitivities of the risk-neutral knock-out probability can be derived analytically. The positive dependence of the risk-neutral knock-out probability on the volatility of the underlying holds for  $\ln(B_0/S_0) + tz < 0$ . It can be shown that this condition holds if and only if the probability of a knock-out until time  $t$  converges to zero for  $\sigma \rightarrow 0$ .

<sup>16</sup> See, e.g., Stoimenov and Wilkens (2005), Wilkens and Stoimenov (2007). An exception is Baule et al. (2008) who apply a structural model.

of this category of new IFPs in comparison to other IFPs and other securities such as bonds. The price-setting formula (5) defines the price at which the issuer is willing to buy and sell OELCs at any time, also in the case of a decline of its credit quality. Consequently, the investor will not be harmed by the issuer's credit risk, as long as he sells the OELC before the issuer legally goes bankrupt.<sup>17</sup> As all issuers in the German market are high-quality borrowers, the likelihood of a bankruptcy event occurring within such a short time as to not allow the investor to react seems negligible. Therefore, taking credit risk into account, for example via the Hull/White approach, is only appropriate if we assume investors not reacting on a deterioration of the issuer's credit quality. In this sense, the resulting value can be interpreted as a lower bound for the OELCs' value from an issuer's perspective. However, assuming the more realistic case of investors selling OELCs in response to a severe decline of the issuer's credit quality, the value of an OELC in the case of default risk approximately equals that without default risk.<sup>18</sup>

### 5.2. Impact of jump risk in the underlying price

As already discussed in Section 2, OELCs exhibit a barrier that permanently exceeds the strike by the factor  $a$ , which should be usually large enough to cover, among others, overnight jumps and occasional illiquidity in the underlying. However, there still is the risk of large random jumps, e.g., serious market crashes, which may cause the price of the underlying to undershoot both the barrier and the strike at once. If the underlying jumps below both thresholds, investors do not fully suffer from this decrease in the underlying because the settlement amount becomes zero regardless of the amount the strike is undershot. This gap risk is left to the issuer.

To analyze how the presence of gap risk affects our findings on the valuation of OELCs in Section 4, we replace here the dynamics of the underlying under the risk-neutral measure (see (8)) with a jump-diffusion process which incorporates random and overnight jumps separately according to Boes et al. (2007):<sup>19</sup>

$$dS_t/S_{t-} = (r - \lambda\mu_{Rf})dt + \sigma dW_t + d \sum_{i=1}^{N_t} (Y_i - 1) + d \sum_{i=1}^{\lfloor 252t \rfloor} (V_i - 1). \quad (11)$$

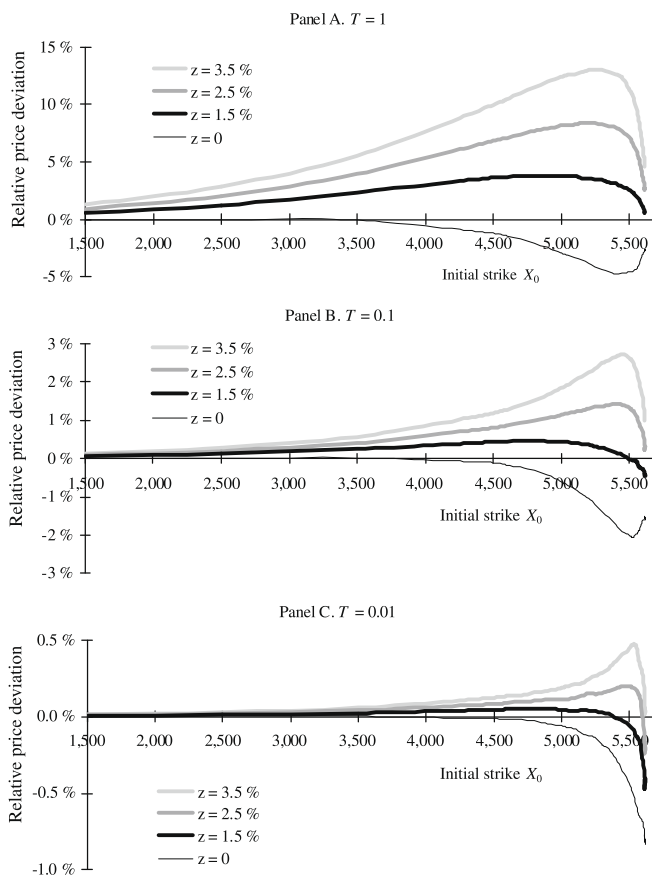
$N_t$  is a Poisson process with intensity  $\lambda$  that counts the number of random jumps until  $t$  and is independent of the Brownian motion  $W_t$ . The last term in (11) represents overnight jumps that are assumed to occur at each of the 252 trading days per year where  $\lfloor \cdot \rfloor$  denotes the floor function. The size of the random jumps and the size of the overnight jumps are independently lognormally distributed:  $\log(Y_i) \sim N(\log(1 + \mu_{Rf}) - \sigma_{Rf}^2/2, \sigma_{Rf}^2)$  and  $\log(V_i) \sim N(-\sigma_{OJ}^2/2, \sigma_{OJ}^2)$ , respectively, and also independent of  $N_t$  and  $W_t$ .<sup>20</sup>

<sup>17</sup> Therefore, these certificates can be interpreted as a kind of preferred claim from an economic point of view. For similar considerations see Elkamhi et al. (2007) discussing – among other things – the value of puttable bonds in the presence of issuers' credit risk.

<sup>18</sup> From the issuer's perspective, this shortening of the initially assumed investor's holding period  $T$  in the rare case of a severe deterioration of the issuer's solvency will result in a slightly higher value of the OELC compared to the value derived in Section 4.1. This implies a slightly lower value of the issuer's profit potential. From the investor's perspective (which is not the focus of this paper), default-free OELCs are certainly more valuable to them than OELCs with default risk if we assume investors expect benefits from investing in and holding OELCs.

<sup>19</sup> Boes et al. (2007) additionally allow for stochastic volatility which we here ignore to reduce the calculation burden when calibrating the process. Câmara (2008) proposes a model with two counters of random jumps. Taylor (2007) shows how to incorporate overnight information to better estimate conditional volatilities.

<sup>20</sup> This specification implies that, given a random jump occurs, the expected (jump) stock return is  $\mu_{Rf}$ , whereas daily jumps do not change the stock price in expectation, see Boes et al. (2007).



**Fig. 2.** Relative price deviation of an open-end long leverage certificate assuming a jump-diffusion process of the underlying, depending on the initial strike. For an open-end long leverage certificate on the DAX, this figure shows the relative price deviation  $RPD_0^l$  (see (10)) as a function of the initial strike  $X_0$  for funding rate spreads  $z$  of 0, 1.5%, 2.5% and 3.5%. The planned holding period of the investor is  $T = 1$  in Panel A,  $T = 0.1$  in Panel B and  $T = 0.01$  in Panel C. The relative difference between barrier and strike is  $a = 1.5\%$  and today's DAX is  $S_0 = 5,700.00$ . Under the risk-neutral measure, the DAX is assumed to follow a jump-diffusion process according to (11). Parameters are given by:  $\sigma = 16\%$ ,  $\lambda = 18.3\%$ ,  $\mu_{RJ} = -8.3\%$ ,  $\sigma_{RJ} = 16.6\%$ ,  $\sigma_{OJ} = 0.7\%$ . The valuation is based on a Monte Carlo simulation with 5,000,000 runs.

To base our analysis on a realistic parameterization of (11) we calibrate the process to prices of plain vanilla puts on the DAX, traded on the EUREX in August 2006. This is a natural approach since there is evidence in the literature that jump risk, besides stochastic volatility, causes the volatility smile (e.g., Bakshi et al., 1997; Pan, 2002; Boes et al., 2007). At each trading day we consider the price of the last trade of each put option with a remaining time to maturity of up to one year. We basically follow the procedure described in Boes et al. (2007): For each trading day, we determine those parameters  $\sigma$ ,  $\lambda$ ,  $\mu_{RJ}$ ,  $\sigma_{RJ}$ , and  $\sigma_{OJ}$  minimizing the relative pricing error, i.e. the sum of the quadratic relative deviations between model put prices<sup>21</sup> and observed put prices. For illustration purposes, our following analysis is based on the respective averages of those parameters in August 2006:  $\sigma = 16\%$ ,  $\lambda = 18.3\%$ ,  $\mu_{RJ} = -8.3\%$ ,  $\sigma_{RJ} = 16.6\%$  and  $\sigma_{OJ} = 0.7\%$ .

<sup>21</sup> To calculate model put prices we apply a Monte Carlo simulation of  $\ln(S_t)$  with 500,000 runs. We basically follow Glasserman (2003, pp. 137–139) and add overnight jumps.

Based on these parameters, we use a Monte Carlo simulation to determine the value of OELCs.<sup>22</sup> Analogous to Fig. 1, panels in Fig. 2 show the relative price deviation for various choices of the funding rate spread  $z$  and holding periods  $T$  of 1, 0.1 and 0.01, depending on the initial strike. Given a funding rate spread  $z$  of zero, OELCs generate no profit potential for the issuer according to (7). Therefore, the negative price deviations for higher initial strikes in Fig. 2 reflect the impact of the gap risk. However, for funding rate spreads  $z$  of 1.5%, 2.5% and 3.5%, the economic behavior of the relative price deviation remains almost unchanged compared to Fig. 1. This implies that  $z$  is large enough to (over)compensate the issuer for taking the gap risk resulting in a positive price deviation. Therefore, the size of  $z$  still reveals the large value of the profit potential even after considering gap risk. Only for very short holding periods and for certificates that are very close to knock-out may the funding rate spread  $z$  be too low to compensate for the gap risk (see  $z = 1.5\%$  in Panel B and  $z = 1.5\%$  or  $z = 2.5\%$  in Panel C). However, bid-ask spreads existing in practice<sup>23</sup> regularly more than offset these negative relative price deviations.

### 5.3. Impact of differing product features

OELCs analyzed in the previous sections are closely related to the product design by HSBC Trinkaus & Burkhardt, but exist in very similar forms for other issuers. Table 1 provides an overview of the characteristics of OELCs on the DAX for different banks.

The table shows that all banks adjust the strike daily, but most banks adjust the barrier according to (4) only once a month. This does not affect the simple hedge portfolio from Section 3 and thus also not our considerations concerning the life cycle hypothesis. However, it leads to a slightly lower barrier between the monthly adjustment dates and hence to somewhat lower risk-neutral knock-out probabilities. Additionally, it implies that interest rates influence the knock-out probabilities, as the short rate no longer vanishes in the formula for the knock-out probabilities (see the derivation of the knock-out probability in the Appendix). However, the effect on the value of OELCs is negligible. Analogue considerations hold for the different kinds of refinancing rates issuers use to adjust the strikes, as these rates are very similar.

Factor  $a$  and funding rate spread  $z$  need not be fixed over the product's lifetime. Some banks state they might change  $a$  in extraordinary market conditions (see, e.g., Sal. Oppenheim, 2006). This is sensible since, for example, higher jump risk can result in a much higher probability of the underlying undershooting both the barrier and the strike, which harms the issuer. Raising  $a$  decreases this probability whereas increasing  $z$  gives the issuer the opportunity to increase the profit potential and, by this, the compensation for increased gap risk. Changing  $a$  and  $z$  does not affect our considerations concerning the simple hedge portfolio. However, the valuation model had to be modified, for which more detailed information about the issuer's behavior would be necessary.

As discussed above, the different time intervals concerning the adjustment of the barrier are negligible for judging OELCs. Given a strike  $X_0$ , a holding period  $T$  chosen by the investor, and given a

<sup>22</sup> We simulate 5,000,000 paths of  $\ln(S_t)$  using the Euler discretization with step size  $\Delta t = 1/1008$  which implies that an overnight jump occurs at every fourth time step:

<sup>23</sup> For the notional certificate on the DAX the (absolute) bid-ask spread is about 1 to 2 regardless of the initial strike.

**Table 1**  
Specifications of open-end long leverage certificates on the DAX, listed according to the issuing bank in August 2006.

Issuer	Product name	Adjustment StrikeBarrier	Reference interest rate	Factor <i>a</i>	Funding rate spread <i>z</i>
ABN Amro	Mini Future Certificate; DAX Index Mini Long Certificate	Daily Monthly	Money market rate	At issuance: 1.5%, possible changes over time, min. 1.5%, max. 5.0%	At issuance: 3.0%, possible changes over time, max. 3.0%
BNP Paribas	Open-end Turbo Long Warrant	Daily Daily	1-month EURIBOR	Normally: 1.5%, for certain certificates: 2.0% or 3.0%	At issuance: 2.5%, possible changes over time, min. 0%, max. 5.0%
Citigroup	Open-end Stop Loss Bull Turbo; Open-end Turbo Stop Loss Knock-out Warrant	Daily Monthly	1-month EURIBOR	At issuance: about 1.5%, possible changes over time	Currently about 2.0%, possible changes over time
Commerzbank	Unlimited Turbo Bull Certificate	Daily Monthly	1-month EURIBOR	Currently about 1.5%, possible changes over time	Currently about 3.0%, possible changes over time
Deutsche Bank	Wave XXL; Call Wave XXL Knock-out Warrant	Daily Monthly	EONIA	Currently 2.0%, possible changes over time, min. 2.0%, max. 10.0%	At issuance: 3.25%, possible changes over time
Dresdner Bank	Call Open-end Knock-out Warrant	Daily Monthly	EONIA	Currently about 2.0%, possible changes over time, min. 2.0%, max. 10.0%	At issuance: 1.5%, possible changes over time
Goldman Sachs	Mini Future Turbo Warrant	Daily Monthly	EUR LIBOR Overnight	At issuance: 2.0%, possible changes over time, max. 5.0%	At issuance: 2.0%, possible changes over time, max. 4.0%
HSBC Trinkaus & Burkhardt	Mini Future Certificate	Daily Daily	EONIA	1.5% (older certificates: 3.0%)	1.5%
Lang & Schwarz	Open-end Turbo Call	Daily Monthly	1-month EURIBOR	About 1.75%	Currently 1.5%, possible changes over time
Sal. Oppenheim	Turbo Open-end Warrant	Daily Daily	1-month EURIBOR	At issuance: 3.0%, possible changes over time	At issuance: 2.0%, possible changes over time
Société Générale	Open-end Turbo Long Knock-out Warrant	Daily Monthly	EUR LIBOR Overnight	At issuance: 2.0%, possible changes over time, max. 7.0%	2.5%

The table shows the specification of open-end leverage products on the DAX, listed according to the issuer. German product names are translated into English. The data were collected from internet-published documents such as product brochures and sales prospectuses of issuers on August 20, 2006.

certain market environment, the key factors determining issuers' profits are the factor *a* and the funding rate spread *z*, both set by the issuers. A scenario analysis (not reported here) covering the *a/z*-combination applied by the issuers (see Table 1) reveals that *z* is the dominating factor for ranking OELCs according to their profits. Except in cases of a barrier very close to the current value of the DAX, *z* accounts for the lion's share of the differences in the  $RPD_0^T$ . Hence ranking issuers should be based on the funding rate spread.

## 6. Conclusion

This paper presents the first analysis of open-end leverage certificates. In contrast to earlier studies of innovative financial products, we do not have to rely on prices collected from primary and secondary markets since issuers communicate the price-setting formula for OELCs. The price-setting is designed to ensure systematically increasing profits for issuers over the product's lifetime which is consistent with the life cycle hypothesis. This 'mispricing by construction' does not cause arbitrage activities of market participants, because short-positions in the certificates are impossible. The barrier protects issuers against illiquidity and moderate jumps in the underlying. Our robustness analysis reveals that our findings apply even after taking possible large random jumps into account. The only exceptions are OELCs with low funding rate spreads very close to knock-out. However, our analysis shows that a respective underpricing is generally outweighed by bid-ask spreads set by the issuers.

We determined the value of OELCs from the issuers' perspective based on standard valuation techniques and fixed planned holding periods. This makes our results comparable to earlier studies. In contrast to other innovative financial products, we found the issuers' credit risk to be negligible for the value of OELCs. Due to the funding rate spread included in the price-setting formula, banks produce an enormous profit potential. Given a planned holding period of, for example, one year and realistic model parameters, the value of the banks' profit potential for open-end leverage long certificates on the DAX can be 5–10% of the certificate price – even after considering gap risk. Issuers can gain ex-post profits of about

20–30% p.a. related to the initial price if neither a knock-out nor a repurchase occurs. These high profits for issuers over time can also be realized if certificates that are knocked-out are substituted through new ones leaving the volume of outstanding OELCs constant. However, in determining the net profit of issuers, an adequate payment for the issuer's service to customers should be incorporated; and this should at least cover the costs of structuring, distribution, etc. In the future we do expect decreasing issuers' profits due to the rising competition in this segment of the retail market, which will probably yield lower funding rate spreads, one of the central drivers of the value of OELCs.

Of course, our analysis can be extended in several ways. In context with the investor's behavior, assumptions other than a specified ex-ante holding period could be considered. This would not affect our considerations concerning the simple hedging strategy and concerning the life cycle hypothesis, and could be integrated into our valuation approach. One example is to assume that investors sell their OELCs when a certain loss or gain has been reached. However, due to a lack of respective empirical data in context with OELCs, at present this would be fairly speculative.

As we concentrate on the issuers' perspective in this paper, another challenging topic is focusing on the demand side to better understand the great market success of OELCs. This could be based on prospect theory along the lines of, e.g., Shefrin and Statman (1993) and Breuer and Perst (2007) who analyze other innovative financial products. In this context possible utility gains from OELCs could be compared with direct stock investment or with other stock-related products such as discount certificates. Also challenging is the integration of the issuers' and the customers' perspectives when considering different product designs of OELCs and optimizing them with respect to different groups of customers and varying market conditions. Especially when empirical data about investors' behavior is available, this would be an interesting topic for a subsequent study.

## Acknowledgements

Parts of this research were done while Oliver Entrop was visiting the School of Banking and Finance, University of New South

Wales. He thanks Terry Walter and the academic and administrative staff for their hospitality and support. We are grateful to Rainier Jankowitsch, Klaus Röder and Hato Schmeiser for helpful comments and suggestions on an earlier draft of this paper. Furthermore, we thank the anonymous referees for their very helpful comments. All remaining errors are our own.

## Appendix A

### A.1. Derivation of $Q(\tau \leq t)$

Let the process  $S_t$ ,  $t \geq 0$ , satisfy  $dS_t = r_t S_t dt + \sigma S_t dW_t$  under the risk-neutral probability measure  $Q$ , where  $r_t$  denotes the short rate in  $t$ ,  $\sigma > 0$  is a constant and  $W_t$  denotes a standard Brownian motion.

By definition we have  $S_t = S_0 \exp\left(\int_0^t r_s ds - \sigma^2 t/2 + \sigma W_t\right)$ . Let the barrier in  $t$  be given by  $B_t = B_0 \exp\left(\int_0^t r_s ds + zt\right)$  for some constant  $z$  and  $0 < B_0 < S_0$ . The first passage time is defined by  $\tau = \inf\{t: S_t \leq B_t\}$ .

We have  $\{S_t \leq B_t\} = \{S_0 \exp\left(\int_0^t r_s ds - \sigma^2 t/2 + \sigma W_t\right) \leq B_0 \exp\left(\int_0^t r_s ds + zt\right)\} = \{S_0 \exp(-\sigma^2 t/2 + \sigma W_t) \leq B_0 \exp(zt)\} = \{\ln(S_0/B_0) + (-\sigma^2/2 - z)t + \sigma W_t \leq 0\} = \{Y_t \leq 0\}$ , where  $Y_t = \ln(S_0/B_0) + (-\sigma^2/2 - z)t + \sigma W_t$ . By applying Bielecki and Rutkowski (2002, Lemma 3.1.2, p. 67) we obtain for every  $t > 0$

$$Q(\tau \leq t) = N(h_1(t)) + \left(\frac{B_0}{S_0}\right)^{\frac{-2(\sigma^2/2+z)}{\sigma^2}} N(h_2(t)),$$

$$h_1(t) = \frac{\ln(B_0/S_0) + (\sigma^2/2 + z)t}{\sigma\sqrt{t}}, \quad h_2(t) = \frac{\ln(B_0/S_0) - (\sigma^2/2 + z)t}{\sigma\sqrt{t}}, \quad (\text{A.1})$$

where  $N(\cdot)$  denotes the standard Gaussian cumulative distribution function.

### A.2. Derivation of $E_Q(1_{\{\tau \leq t\}} \exp((z-c)\tau))$

Let  $c \geq 0$ . Based on the above representation of  $Q(\tau \leq t)$  we can conclude:

$$E_Q(1_{\{\tau \leq t\}} \exp((z-c)\tau)) = \int_0^t \exp((z-c)x) dQ(\tau \leq x)$$

$$= \int_0^t \exp((z-c)x) dN(h_1(x))$$

$$+ \left(\frac{B_0}{S_0}\right)^{\frac{-2(\sigma^2/2+z)}{\sigma^2}} \int_0^t \exp((z-c)x) dN(h_2(x)). \quad (\text{A.2})$$

By applying Bielecki and Rutkowski (2002, Lemma 3.2.1, p. 74) to each summand in (A.2) separately, we obtain for  $\sigma^2 \neq 2z$  after rearranging and collecting terms:

$$E_Q(1_{\{\tau \leq t\}} \exp((z-c)\tau)) = \left(\frac{B_0}{S_0}\right)^{\frac{-\sigma^2 - 2z + 2\sigma d}{2\sigma^2}} N(h_3(t)) + \left(\frac{B_0}{S_0}\right)^{\frac{-\sigma^2 - 2z - 2\sigma d}{2\sigma^2}} N(h_4(t)),$$

$$h_3(t) = \frac{\ln(B_0/S_0) + \sigma dt}{\sigma\sqrt{t}}, \quad h_4(t) = \frac{\ln(B_0/S_0) - \sigma dt}{\sigma\sqrt{t}},$$

$$d = \sqrt{\left(\frac{\sigma^2/2 + z}{\sigma}\right)^2 - 2(z-c)}. \quad (\text{A.3})$$

The same formula holds for  $\sigma^2 = 2z$  as  $E_Q(1_{\{\tau \leq t\}} \exp((z-c)\tau))$  is a continuous bounded function of  $\sigma$ .

## Appendix B. Valuation of OELCs with credit risk according to Hull and White (1995)<sup>24</sup>

In the framework of Hull and White (1995), assuming independence of market and credit risk, the value of a vulnerable European claim equals the value of an analogue default-free claim discounted by the spread  $c$  of an issuer's zero-coupon bond that ranks pari passu with the vulnerable claim and has the same maturity. The payoff of the OELC can be approximated by a family of European derivatives of the form  $1_{\{t_{i-1} < \tau \leq t_i\}}(S_{t_i} - X_{t_i})$  with expiry date  $t_i$ . Therefore, we can transfer the Hull/White result to OELCs and obtain for any  $t$ :

$$OELC_{0,HW}^t = E_Q \exp(-c\tau^t) \exp\left(-\int_0^{\tau^t} r_s ds\right) P_{\tau^t}$$

$$= \exp(-ct) E_Q \left(1_{\{\tau > t\}} \exp\left(-\int_0^t r_s ds\right) S_t\right) \quad (\text{A.4})$$

$$+ E_Q(1_{\{\tau \leq t\}} \exp((z-c)\tau)) B_0$$

$$- X_0(\exp((z-c)t)(1 - Q(\tau \leq t)))$$

$$+ E_Q(1_{\{\tau \leq t\}} \exp((z-c)\tau)),$$

where we assume the spread structure to be flat, which is common practice (see, e.g., Stoimenov and Wilkens, 2005). The last three summands can be calculated with (A.1) and (A.3). The calculation of the first summand is based on the relation

$$S_t^* := \exp\left(-\int_0^t r_s ds\right) S_t = B_0 \exp(zt + Y_t).$$

Bielecki and Rutkowski (2002, Corollary 3.1.2, p. 69) provide a formula for  $Q(Y_t \geq y, \tau > t)$  which we use to obtain

$$Q(S_t^* > y, \tau > t)$$

$$= N\left(\frac{\ln(S_0/y) - \sigma^2/2t}{\sigma\sqrt{t}}\right)$$

$$- \left(\frac{B_0}{S_0}\right)^{\frac{-2(\sigma^2/2+z)}{\sigma^2}} N\left(\frac{\ln(B_0^2/(S_0 y)) - \sigma^2/2t}{\sigma\sqrt{t}}\right). \quad (\text{A.5})$$

As  $Q(\tau > t)$  does not depend on  $y$ , we have  $dQ(S_t^* \leq y, \tau > t) = dQ(\tau > t) - dQ(S_t^* > y, \tau > t) = -dQ(S_t^* > y, \tau > t)$ . Thus, the equation

$$E_Q(1_{\{\tau > t\}} S_t^*) = \int_{B_0 \exp(zt)}^{\infty} y - dQ(S_t^* > y, \tau > t)$$

holds. Based on (A.5), this integral can, like (A.2), be expanded into two summands. Applying Bielecki and Rutkowski (2002, Lemma 3.2.1, p. 74) yields:

$$E_Q\left(1_{\{\tau > t\}} \exp\left(-\int_0^t r_s ds\right) S_t\right) = S_0 N(-h_4^*(t)) - \left(\frac{B_0}{S_0}\right)^{\frac{-2z}{\sigma^2}} B_0 N(h_3^*(t)),$$

where  $h_3^*(t)$  and  $h_4^*(t)$  denote  $h_3(t)$  and  $h_4(t)$ , respectively, with  $d = (\sigma^2/2 - z)/\sigma$ .

## References

- Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. *Journal of Finance* 52, 2003–2049.
- Baubonis, C., Gastineau, G., Purcell, D., 1993. The banker's guide to equity-linked certificates of deposit. *Journal of Derivatives* 1 (Winter), 87–95.
- Baule, R., Entrop, O., Wilkens, M., 2008. Credit risk and bank margins in structured financial products: Evidence from the German secondary market for discount certificates. *Journal of Futures Markets* 24, 376–397.

<sup>24</sup> A more detailed description of the derivations is available upon request from the authors.



- BNP Paribas, 2006. Open-End Long Warrants on the DAX® Performance Index (in German). Sales Prospectus, June 22, 2006, Frankfurt/Main and Paris.
- Benet, B.A., Giannetti, A., Pissaris, S., 2006. Gains from structured product markets: the case of reverse-exchangeable securities (RES). *Journal of Banking and Finance* 30, 111–132.
- Bielecki, T.R., Rutkowski, M., 2002. *Credit Risk: Modeling, Valuation and Hedging*. Springer, Berlin.
- Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81, 637–654.
- Boes, M.-J., Drost, F.C., Werker, B.J.M., 2007. The impact of overnight periods on option pricing. *Journal of Financial and Quantitative Analysis* 42, 517–534.
- Breuer, W., Perst, A., 2007. Retail banking and behavioral financial engineering: the case of structured products. *Journal of Banking and Finance* 31, 827–844.
- Brown, C., Davis, K., 2004. Dividend protection at a price. *Journal of Derivatives* 12 (Winter), 62–68.
- Burth, S., Kraus, T., Wohlwend, H., 2001. The pricing of structured products in the Swiss market. *Journal of Derivatives* 9 (Winter), 30–40.
- Câmara, A., 2008. Two counters of jumps. *Journal of Banking and Finance*. doi:10.1016/j.jbankfin.2008.08.017.
- Chen, A.H., Kensinger, J.W., 1990. An analysis of market-index certificates of deposit. *Journal of Financial Services Research* 4 (2), 93–110.
- Chen, K.C., Sears, R.S., 1990. Pricing the SPIN. *Financial Management* 19 (Summer), 36–47.
- Elkamhi, R., Ericsson, J., Wang, H., 2007. What Risks Do Corporate Bond Put Features Insure Against? Working Paper. McGill University.
- Glasserman, P., 2003. *Monte Carlo Methods in Financial Engineering*. Springer, New York.
- Grünbichler, A., Wohlwend, H., 2005. The valuation of structured products: empirical findings for the Swiss market. *Financial Markets and Portfolio Management* 19, 361–380.
- HSBC Trinkaus & Burkhardt, 2006. DAX® Mini Future Certificates (in German). Sales Prospectus, August 14, 2006, Düsseldorf.
- Hull, J.C., White, A., 1995. The impact of default risk on the prices of options and other derivative securities. *Journal of Banking and Finance* 19, 299–322.
- Jarrow, R.A., van Deventer, D.R., 1998. The arbitrage-free valuation and hedging of demand deposits and credit card loans. *Journal of Banking and Finance* 22, 259–272.
- Merton, R.C., 1973. Theory of rational option pricing. *Bell Journal of Economics and Management Science* 4, 141–183.
- Muck, M., 2006. Where should you buy your options? Limited arbitrage and imperfect competition in the options market. *Journal of Derivatives* 14 (Fall), 82–96.
- Muck, M., 2007. Pricing turbo certificates in the presence of stochastic jumps, interest rates, and volatility. *Die Betriebswirtschaft* 66, 224–240.
- Pan, J., 2002. The jump-risk premia implicit in options: evidence from an integrated time-series study. *Journal of Financial Economics* 63, 3–50.
- Sal. Oppenheim, 2006. Index Turbo Open-End Warrant (in German). Sales Prospectus, August 16, 2006, Frankfurt/Main.
- Shefrin, H., Statman, M., 1993. Behavioral aspects of the design and marketing of financial products. *Financial Management* 22 (Summer), 123–134.
- Stoimenov, P.A., Wilkens, S., 2005. Are structured products 'fairly' priced? An analysis of the German market for equity-linked instruments. *Journal of Banking and Finance* 29, 2971–2993.
- Taylor, N., 2007. A note on the importance of overnight information in risk management models. *Journal of Banking and Finance* 31, 161–180.
- Wasserfallen, W., Schenk, C., 1996. Portfolio insurance for the small investor in Switzerland. *Journal of Derivatives* 3 (Spring), 37–43.
- Wilkens, S., Erner, C., Röder, K., 2003. The pricing of structured products in Germany. *Journal of Derivatives* 11 (Fall), 55–69.
- Wilkens, S., Stoimenov, P.A., 2007. The pricing of leverage products: an empirical investigation of the German market for 'long' and 'short' stock index certificates. *Journal of Banking and Finance* 31, 735–750.