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# Tax Bracket Creep and its Effects on Income Distribution \*

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## Abstract:

We quantitatively analyze the way inflation alters the inequality of the income distribution in the U.S. economy. The main mechanism emphasized in this paper is the “bracket creep” effect according to which inflation pushes income into higher tax brackets. Governments adjust the nominal income tax brackets slowly and incompletely due to the rise in prices. In the U.S. postwar history, this typically happens less often than once every other tax year. We develop a general equilibrium monetary model with income heterogeneity. In line with our time series evidence, it is rather the frequency of income tax schedule adjustments than the overall level of inflation that has a perceptible impact on the distribution of income. We find that a longer duration between two successive adjustments of the tax schedule reduces employment, savings, and output.

JEL classification: D31, E31, E44, E52, E62

Key Words: Bracket Creep, Progressive Income Taxation, Inflation, Income Distribution

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# 1 Introduction

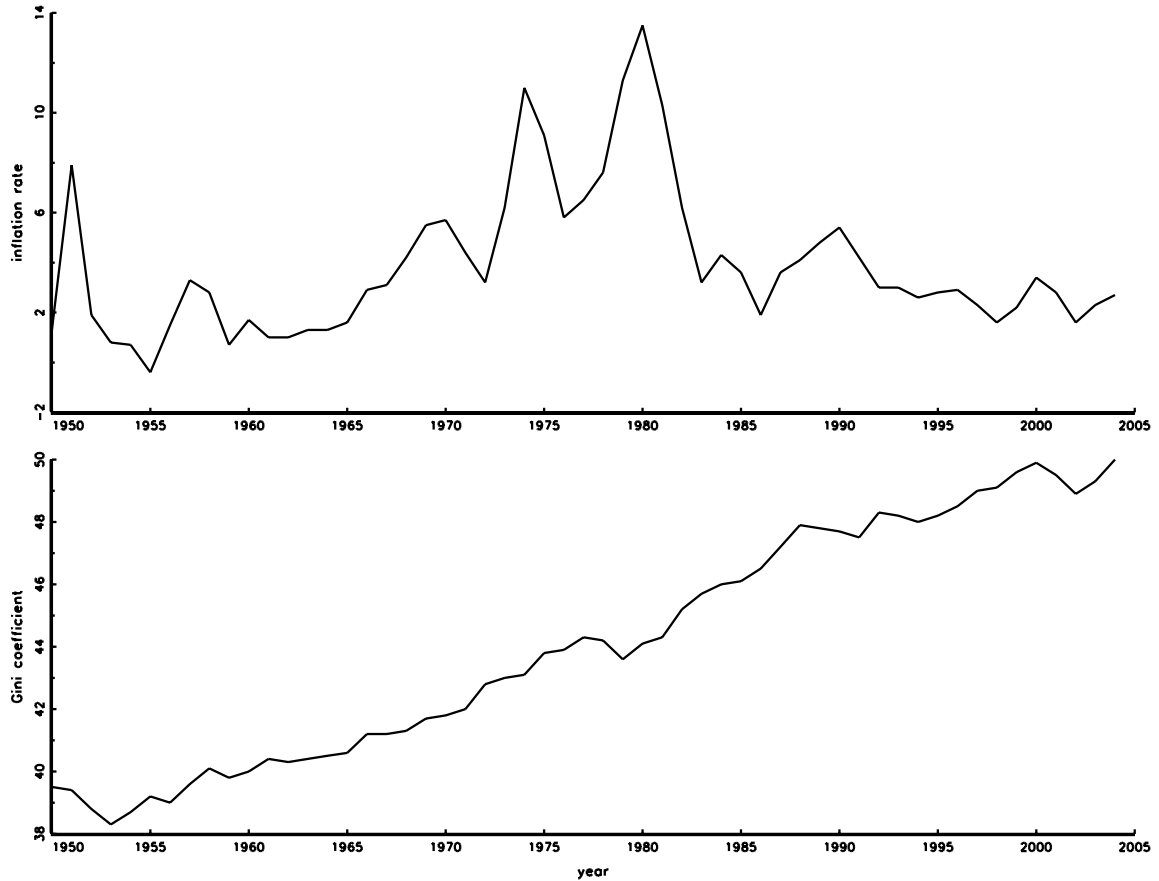
The “bracket creep” defines a shift of personal income into a higher tax bracket when taxable income grows over time. It occurs due to inflation. Higher inflation possibly increases tax burdens under a progressive personal income tax as taxpayers near the top-end of a tax bracket are more likely to “creep” to a higher bracket. Clearly, this effect alters inequality in after-tax income. Whether it increases or decreases inequality depends, among others, on the level and duration of inflation, the top income tax rate, and the initial distribution of income. The purpose of this paper is to assess the impact of the “bracket creep,” or rather its attenuation **through inflation indexation of the tax schedule**, on the distribution of income both empirically and in a dynamic stochastic general equilibrium (DSGE) model for the U.S. economy.

Like in the U.S. most personal income tax systems are progressive, i.e. structured with marginal tax rates exceeding average rates and increasing with the base. Taxpayers who receive only nominal increases in wages to offset higher inflation tend to be pushed into higher brackets. This effect is considered to be particularly severe (“the cruelest tax”) in times of high inflation as was seen during the last half of the 1970s when U.S. inflation rates averaged 8.9 percent annually (Blinder and Esaki, 1978). To combat bracket creep in the U.S. the Reagan Administration implemented an indexation of the personal exemptions and the tax brackets based on a cost-of-living index derived from the Consumer Price Index for All Urban Consumers (CPI-U). These provisions were actually enacted in 1981 as part of Economic Recovery Tax Act (ERTA), but delayed in their implementation and did not become effective until 1985; see Altig and Carlstrom (1991, 1993), Auerbach and Feenberg (2000).

In inflationary environments, with unchanged or loosely adjusted rate schedules and brackets, tax collections tend to rise. This raises the claim that bracket creep is strategically used by some governments to maintain tax revenues. A loose or strategically implemented “pure one-year-lag” index system can be shown to cause taxable income to be overstated by the current rate of inflation (Altig and Carlstrom, 1991, 1993). Apart from its (mis-)use as revenue instrument, the omission of inflation adjustment of marginal tax rates is also very likely to have a considerable effect on the distribution of income.

Figure 1 shows the annual time series of inflation and the Gini coefficient of market income

Figure 1: Inflation rate and Gini coefficient, 1948-2004



(before taxes) for the period from 1948 to 2004. It highlights the relationship between the two series. Both series are coined by an upward trend up to the 1980s. While this trend continues for the Gini, the inflation rate calms down and follows a slight downward trend as of the early 1980s. Overall, the two series seem to comove –sometimes more, sometimes less in phase– at business cycle frequencies. A close inspection reveals that the series get more entrained after the mid-1980s, suggesting that indexation following ERTA has led to a more contemporaneous relationship.

These observations are in line with the empirical part of the present study which finds that inflation has a transitorily inequality reducing impact that leads aggregate measures by about two years. The central strategy of this part, however, is to take a stand on how the effective U.S. tax system was affected during the total postwar period and then to

investigate the consequences of infrequent indexation relative to the sort of system that has been in place since the mid 1980s. Methodologically, we focus on a bivariate study of the correlation structure of the inflation rate and Gini coefficient series at business cycle frequencies in the spirit of Sims (1980).<sup>1</sup> Our methods are primarily descriptive and as such imposing less assumptions than the more structural specifications used in the literature. Yet we seek to contribute to the literature by assessing whether the progressive bias of inflation is predominantly driven by the level and persistence of positive inflation or rather by an infrequent adjustment of the tax schedule. This task requires to go beyond descriptive time series analysis.

In the theoretical part of the paper, we develop a monetary DSGE model of progressive income taxation.<sup>2</sup> In our simulations, we compare both high inflation environments (1970s) with moderate inflation environments (rest of postwar U.S. history) and infrequent schedule adjustment regimes (before ERTA) with less infrequent schedule adjustment regimes (after ERTA). In response to higher inflation or a longer duration of the bracket creep, individuals face higher income taxes, both on average and marginally. As a consequence, agents adjust their labor supply and savings decisions. Our results support the view of Altig and Carlstrom (1991, 1993). Accordingly, the indexing scheme introduced by ERTA bounded the problem but issues of inflation and tax-system interactions are far from moot and being solved.

To summarize our empirical results in Section 2, we find, using correlation analysis, that the relationship between the Gini coefficient and inflation rate dynamics got both more contemporaneous and statistically robust after introducing the indexation scheme. The former is confirmed by studying bivariate spectral measures.

Our results from the general equilibrium model exercise are as follows: The level of inflation has a rather small effect on income. In particular, the inflation elasticities of the

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<sup>1</sup> Recent studies examining the effects of bracket creep on income use either panel data or large scale macro-models (Saez, 2003; Immervoll, 2005). Romer and Romer (1998) and Galli and van der Hoeven (2001) analyze inflation as one central explanatory of inequality in cross-sections.

<sup>2</sup>In a monetary DSGE model with sticky prices and a progressive income tax similar to ours, Heer and Maussner (2012) show that in the presence of a lagged adjustment of marginal tax rate and pensions higher unexpected inflation results in a more unequal distribution. However, they only consider an adjustment lag of one year and do neither study the effect of the adjustment frequency nor the bracket creep in isolation. In addition, they consider the effects of a temporary increase of inflation rather than a permanent one.

Gini coefficients of wage and total after tax income amount to 0.01 and 0.15 percent, respectively. However, if we consider a tax policy regime that adjusts the tax schedule for inflation more frequently, we find that agents increase their labor supply by 6.3 percent and savings by about 4 percent compared to a system with less frequent adjustments. The implied Gini coefficient elasticities amount to approximately 0.2 percent. **Hence, in this scenario inflation elasticity increased. This insight can be reconciled with our findings from the empirical part: A more significant and contemporaneous relationship between the Gini coefficient and inflation rate dynamics for a more frequent indexation (annual adjustment since 1985) implies a more immediate and concerted reaction of private households' behavior to inflation. A more infrequent adjustment dilutes this direct reaction due to the possibility of spreading it. The former results in a higher, the latter in a lower elasticity as found in our model's simulations.** We conclude that the inflation rate exceeding some threshold (e.g. 5 or 10 percent) is less problematic for the effects of the bracket creep compared to the duration of creeping up brackets.

The remainder of the paper is structured as follows. Section 2 presents empirical evidence for U.S. time series. Section 3 introduces the OLG model with two assets: money and equity. The model is calibrated with regard to the characteristics of the U.S. economy in Section 4. Our numerical results are presented in Section 5. Finally, Section 6 concludes.

## 2 Empirical analysis

**In the first section of the empirical part, we find** that inflation indexation introduced in 1985 offsets the redistributive effects of inflation with a lag of approximately 1.5 years, leading to the more transitory and contemporaneous relationship that we observe since the mid-1980s (Figure 1). Overall, prior to indexation of the U.S. brackets, the statistical significance is rather weak. Since 1985 the relationship got both more contemporaneous and robust in terms of statistical significance. The comovement can also be rationalized theoretically. First, the after-tax income of the income-poor households, remaining in a bottom-end bracket, increases other things being equal. Secondly, the incentives to supply labor increases for the low-productivity households as the after-tax wage rate increases.<sup>3</sup>

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<sup>3</sup>With the help of U.S. panel data on individual tax returns, Saez (2003) uses the bracket creep as

In the second section of the empirical part, spectral analysis confirms the finding from the time domain that the relationship between the Gini coefficient dynamics and the inflation rate got more contemporaneous since the introduction of the indexation in 1985. Bivariate spectral density estimates are particularly informative with regard to the lead-lag relationship of income inequality and inflation as they are computed for a continuous range of ordinary frequency. We estimate these measures for time series on income inequality and inflation in the U.S. for a longer sample period than has previously been available. In particular, we find that the average duration of association between the inflation rate and the Gini coefficient shrinks from more than three years in the pre-1985 period to less than two years in the post-1984 period.

## 2.1 Data

Our annual data on aggregate income inequality is drawn from Gini coefficient series that were recently made available by Kopczuk *et al.* (2010). The series date back to the late 1930s. Hence, our period of observation is considerably longer than the one of studies using data from the Current Population Survey that became available in the 1960s. A fact that makes this series particularly suited for our purposes is that it is based on individual rather than family-level data, which is more adequate in the context of income taxation. The series is available up to the year 2004.

For the inflation rate series, we rely on CPI-U based time series (base year is chained, 1982-1984 = 100) provided in annual frequency by the Federal Reserve Bank of Minneapolis. In total, our period of observation covers 57 years. It ranges from 1948 to 2004.

## 2.2 Correlation analysis

Our analysis in the time and frequency domain requires stationary time series. For the U.S. inflation series it has not yet been conclusively resolved whether it is best treated as stationary or non-stationary. To survey the voluminous literature on this issue is beyond the scope of this paper. As Ng and Perron (2001) show, unit root (U.R.) test outcomes

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source of tax variation in order to construct instrumental variable estimates of the sensitivity of income to changes in tax rates. He estimates a labor supply elasticity of taxable income of around 0.4.

crucially depend on the choice of test. Unreflective reliance on U.R. tests seems hazardous in the present context. Thus, we adopt another approach in the spirit of Canova (1998). We compare results with the known potential distortions induced by the detrending filter used (A'Hearn and Woitek, 2001, p. 327-328), and compare across filters to judge robustness. For the inflation rate series, we also compare it to findings treating the raw series as stationary. The filters we consider are the widely used highpass Hodrick-Prescott filter with a smoothing weight  $\lambda$  for annual series equal to 100 (HP), the log-difference filter (logD) that would be ideal for a difference stationary process, and the recently proposed bandpass Baxter-King filter (BK) and Christiano-Fitzgerald filter (CF) both with a cut-off frequency of 15 years; see Hodrick and Prescott (1997), Baxter and King (1999), and Christiano and Fitzgerald (2003). Additionally, we use two recent modifications of the HP and BK filter suggested by Ravn and Uhlig (2002) and A'Hearn and Woitek (2001), respectively. The modified HP filter (MHP) sets the smoothing parameter  $\lambda = 6.25$  for annual series. The modified BK filter (MBK) takes care of the undesirable sidelobes in the gain function by so-called Lanczos's r-factors. Overall and similar to Wälde and Woitek (2004), we rely on six different filter devices.

Figure 2: Significant correlations between Gini coefficient and inflation rate (1948-2004)

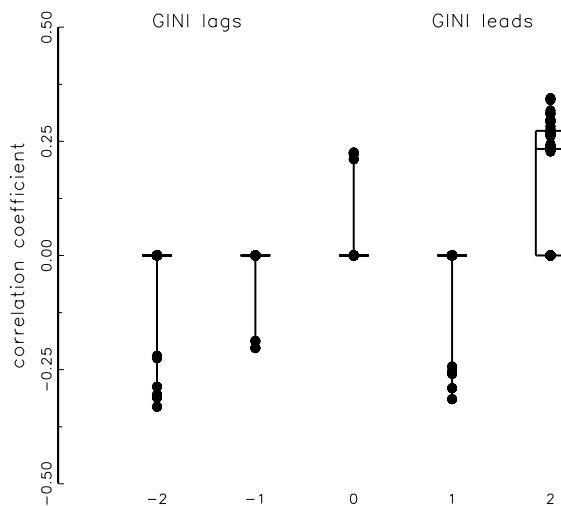
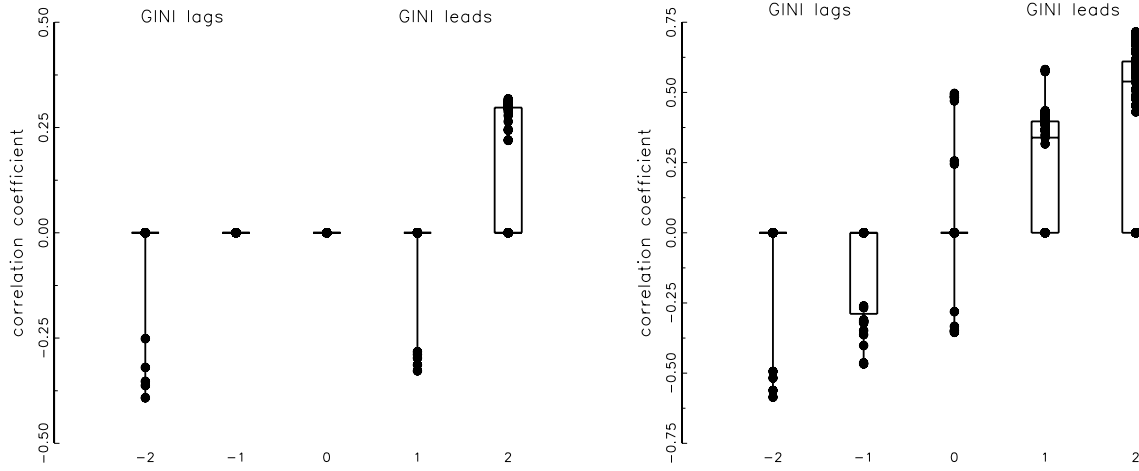


Figure 2 displays significant correlation coefficients between the inflation rate and Gini coefficient series for the total sample period, that is from 1948 to 2004. Correlation coefficients and corresponding standard errors are obtained by regressing cyclical components



Figure 3: Significant correlations between Gini and inflation (1948-1984 and 1985-2004)



of inflation rate data on cyclical components of the Gini coefficient series, where series were normalized to have zero mean and unity variance in advance. Standard errors are heteroskedasticity and autocorrelation consistent following Newey and West (1987). Figure 2 plots a dot corresponding to the estimated correlation coefficient for inflation rate leads (Gini coefficient lags) of two years and one year (starting from the left), for contemporaneous correlations, and for inflation rate lags (Gini coefficient leads) for one and two years.<sup>4</sup> Only coefficients significant at a 10% or lower level are displayed. They are shown as distributions for combinations of series filtered by different filtering techniques. As we take six different filters into account – and in the case of the inflation rate series also the raw series –, there is a maximum of  $7 \times 6 = 42$  significant correlation coefficients at each considered lag.

Overall, we estimate in about one fifth of considered possible correlations a significant coefficient. We find the most significant correlations at the two years lags and leads, respectively. For 30 combinations of differently filtered series, all correlation coefficients at the two years inflation rate lead (lag) are negative (positive). This suggests a counter-cyclical, though merely robust (only in 20 percent of considered detrending cases, we find

<sup>4</sup>As, on average, the U.S. income tax schedule has been subject to minor changes roughly every 1.6 years and to major changes about every second year over our total period of observation (see Appendix 7.1), we focus on first and second lags and leads, respectively.

significant correlations), relationship at the corresponding frequency. The effect that there are four positive contemporaneous estimates is a result one should expect: when two time series of the same length are countercyclical at a lag of two years, they need to be procyclical if one series is lagged by half the length of a cycle. Note that the finding of five negative correlations at a lead of one year can be interpreted as an indication for a more complex cyclical structure shared by the two underlying series. It is also noteworthy that the mean frequency of bracket adjustment in the postwar U.S. is 2.11 years (Appendix 7.1).

For the pre-1985 period, there is evidence for a significant correlation between inflation and income inequality in less than five percent of analyzed cases (left schedule of Figure 3). Both findings for the total and pre-1985 period somehow contrast with the ones of studies from the 1990s that find current inflation to be of progressive nature in the postwar U.S. (Bulir and Gulde, 1995 and Jäntti, 1994).<sup>5</sup>

For the post-1984 period, clearly significant results are found in nearly half of the considered correlations both for one year and two years lags (right schedule of Figure 3). They are the ones that are most in favor of an adjustment effect: Although, the U.S. income tax system has been effectively indexed for inflation as of 1985, a perfect indexation is extremely hard to realize in practice. The fact that it takes time to assess the exact inflation rate and to adjust tax-band limits and other nominally defined parameters of the tax code accordingly can be interpreted as responsible for the significant correlations between inflation and income inequality at first and second annual lag. It might be also due to a peculiarity of the cost-of-living index derived from CPI-U that is used to adjust bracket limits and personal exemption levels under ERTA. In this context Altig and Carlstrom (1991) note that ERTA defined the cost-of-living index as the average CPI-U for the 12-month period ending September 30 of the year prior to the tax year, divided by the average CPI-U for the analogous period. Thus, because tax years and “index years” are by definition not synchronized, ERTA mandates that inflation adjustments be made with an approximate lag of one year. The displacement of index inflation rate and actual inflation rate is not

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<sup>5</sup>Jäntti (1994, p. 373) notes that the ERTA of 1981 is among the tax policies that most likely have affected the U.S. income distribution. To control for these changes in policy his estimates include a dummy taking on a value of one from 1981 onward. However, given that the tax bracket indexation for inflation represents the crucial change introduced by the ERTA, a later dated structural break should have been used as indexing was delayed until 1985.

exactly one year as the former rate is constructed using the average of the CPI-U over the 12-month period ending 15 months (16 months since 1986) prior to the relevant tax year. Positive as well as negative correlations at a zero lag can be seen as suggesting a partial offsetting of a progressive tax effect of inflation induced by schedule adjustment measures.

We carefully interpret these findings from time domain techniques as lending support to a transitorily inequality reducing impact of inflation that leads aggregate measures of inequality by at least one to two periods (tax years). Since 1985 the relationship got both more contemporaneous and robust in terms of statistical significance.

## 2.3 Bivariate spectral analysis

An approach that characterizes the dynamics of multiple time series in an intuitive summary way and that is suited to describe and analyze contained cyclicalities at different frequencies is spectral analysis. Any  $n$ -dimensional stationary process  $X_t$  has a spectral representation at frequencies  $\omega \in [-\pi, \pi]$  in the form of a spectral density matrix  $\mathbf{F}(\omega)$ . It is given by the Fourier transform of the covariance function  $\gamma_{jk}(\tau)$ ,  $\tau = 0, \pm 1, \pm 2, \dots$ , for all  $j = 1, \dots, n$ ;  $k = 1, \dots, n$  of the process

$$\mathbf{F}(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{+\infty} \mathbf{\Gamma}(\tau) e^{-i\omega\tau}, \quad -\pi \leq \omega \leq \pi, \quad (1)$$

with

$$\mathbf{\Gamma}(\omega) = \begin{pmatrix} \gamma_{11}(\omega) & \cdots & \gamma_{1n}(\omega) \\ \vdots & \ddots & \vdots \\ \gamma_{n1}(\omega) & \cdots & \gamma_{nn}(\omega) \end{pmatrix} \text{ and } \mathbf{F}(\omega) = \begin{pmatrix} f_{11}(\omega) & \cdots & f_{1n}(\omega) \\ \vdots & \ddots & \vdots \\ f_{n1}(\omega) & \cdots & f_{nn}(\omega) \end{pmatrix}.$$

Because  $\mathbf{F}(\omega)$  is an even function, it is sufficient to examine it in the interval  $[0, \pi]$ . The diagonal elements  $f_{11}(\omega), \dots, f_{nn}(\omega)$  are the real-valued autospectra or power spectra. The off-diagonal elements represent cross spectra  $f_{jk}(\omega) = c_{jk}(\omega) - iq_{jk}(\omega)$ , consisting of  $c_{jk}(\omega)$  cospectra and  $q_{jk}(\omega)$  quadrature spectra.

Implementing (1) is problematic, for it requires autocovariances and covariances from  $-\infty$  to  $+\infty$ . The approach taken here follows A'Hearn and Woitek (2001). It consists in

estimating bivariate VAR models of order  $p$ ,<sup>6</sup> the lag length being determined by Akaike's information criterion, and letting the model parameters determine the covariance function. This allows estimation of the bivariate spectrum as follows.

$$\mathbf{F}(\omega) = \frac{1}{2\pi} \mathbf{A}(\omega)^{-1} \sum \mathbf{A}(\omega)^{-*}. \quad (2)$$

$\sum$  denotes the error variance-covariance matrix.  $\mathbf{A}(\omega)$  is the Fourier transform of the matrix lag polynomial  $\mathbf{A}(L) = I - A_1L - \dots A_pL^p$ , where  $L$  is the backshift operator. The superscript “\*” denotes complex conjugate transpose. As noted above, the cross-spectra are complex valued functions in  $\omega$ , but simple manipulations yield the more readily interpretable, real measures: phase shift  $ps(\omega)$  and squared coherency  $sc(\omega)$ .

$$ps(\omega) = \arctan \frac{-q_{jk}(\omega)}{c_{jk}(\omega)}, \quad (3)$$

$$sc(\omega) = \kappa_{jk}^2(\omega) = \frac{|f_{jk}(\omega)|^2}{f_{jj}(\omega) f_{kk}(\omega)}. \quad (4)$$

The phase shift ( $ps$ ) measures the phase lead ( $ps > 0$ ) or lag ( $ps < 0$ ) of a series  $j$  over the series  $k$  at a certain frequency  $\omega$ . The respective  $ps$  measure is computed at the maximum of squared coherency  $sc$ , i.e. at that frequency  $\omega$ , where the cyclic components contained in the two series at stake show the highest degree of linear relationship. The  $sc$  measure takes on values between 0 and 1. Precisely, it indicates the proportion of the variance of the component of frequency  $\omega$  of either series that can be explained by its linear regression on the other series; see Koopmans (1995, p. 142). Both spectral parameters  $ps$  and  $sc$  can be calculated and displayed for a range of different frequencies. This gives us the phase and coherence spectral densities.

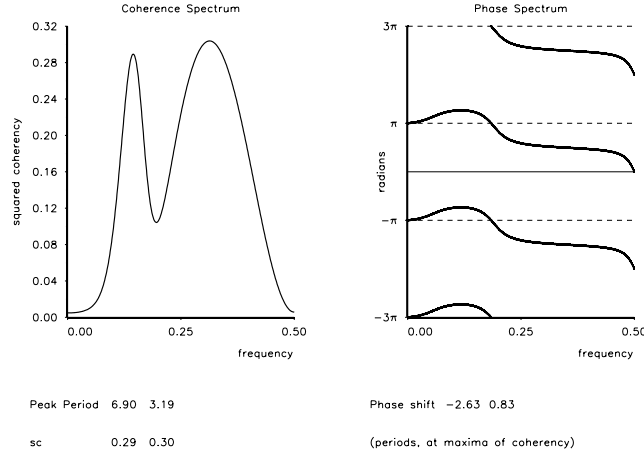
Figure 4 illustrates plots of the key bivariate measures coherence and phase over the total period for a sample detrending device combination case (cf. Section 2.1): It displays the results for the sample case, where the inflation rate has been filtered using a standard HP filter, the Gini coefficient using the CF filter, respectively.<sup>7</sup> Figure 5 is the corresponding one for the two considered subperiods. **In the following, we briefly interpret the shown sample cases before generalizing our results for all filters.**

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<sup>6</sup>We set the maximum order we allow for to  $p^{max} = 3$ . Our results are not sensitive to this choice.

<sup>7</sup>Note this assumes all of our considered filters to approximately show the properties of nonnegative definite filters that leave phase relationships contained in the series undisturbed. For our used filters, this assumption should mostly be justified; see Koopmans (1995, p. 138, pp. 207-208).

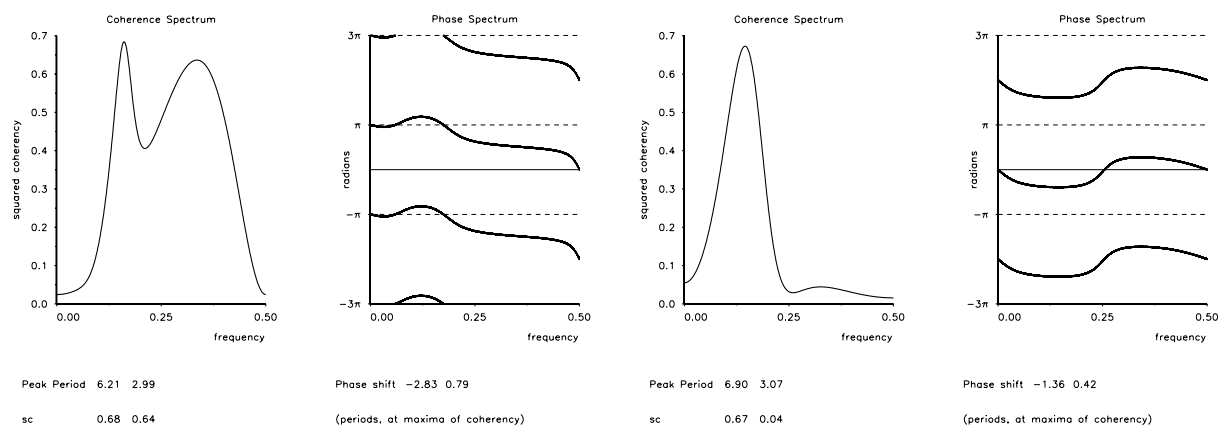
Figure 4: Sample spectra: Gini coefficient and inflation rate (1948-2004)



As can be seen from Figures 4 and 5, the coherence spectra are characterized by two peaks, corresponding to periodicities contained in the Gini coefficient series (see the coherence spectrum shapes – both in Figure 4 and the two plots in Figure 5).<sup>8</sup> The numbers shown below the left schedule of Figure 4 give the corresponding frequencies in years (“Peak Period”), where the linear association between the Gini series and the inflation series reaches its first and second maximum (“sc”). The same information is given at the bottom of the first and third diagram displayed in Figure 5. For the 1948-2004 and the 1948-1984 period there are nearly equally coherent periodicities contained in the two series corresponding to frequencies of 6 to 7 years and about 3 years, respectively. The squared coherency depicted on the ordinate is roughly doubled comparing the pre-1985 period with the total period. For the 1985-2004 period there is only one top coherent periodicity (with sc clearly different from zero) shared by the two series that shows an sc value of 0.67. It corresponds to the lower frequency or longer cycle of 6.9 years. For the total and pre-1985 period one of these cyclic components lags the corresponding cycle contained in the inflation rate series, the other one shows a contemporaneous coherency (implied  $|ps| \leq 1$  year) with the period in the inflation series. This can be seen in the phase spectra plots in Figures 4

<sup>8</sup>Note frequency denoted on the abscissa is in ordinary units. The corresponding period length is the inverse of these values. Hence, the highest frequency is 0.5, i.e. the Nyquist frequency, corresponding to a cycle with a period length of two ( $= 1/0.5$ ) years.

Figure 5: Sample spectra: Gini coefficient and inflation rate (1948-1984 and 1985-2004)



and 5.<sup>9</sup> The decisive point is that the phase spectrum for frequencies corresponding to the shared cyclic component of 6-7 years (i.e. the lower frequency, which is the closer to the origin on the abscissa of all diagrams) shows an approximately linear shape for this frequency interval with negative slope. This means that the Gini coefficient business cycle component lags the inflation rate series. For the post-1984 period the slope of the phase spectrum in the (0.15; 0.20) frequency interval (i.e. about 5 to 7 years cycles) is less steep, corresponding to a phase shift of -1.36 years as opposed to the -2.63 and -2.83 years in the total and pre-1985 years period (values shown at the bottom of the second diagram in Figure 4 and the second and fourth diagram in Figure 5), respectively. In the range of the higher frequency cycle the corresponding components of the two series fluctuate nearly in phase for all considered periods. The implied phase shift values range between 0.42 and 0.83 years. Note, for the total and pre-1985 period and for frequencies in-between the two marked phase constellations the corresponding *sc* estimates still take on considerable values. In our sample case illustration it equals about 10-28 percent (total period) and

<sup>9</sup>Instead of plotting the phase spectrum in the interval  $[-\pi, \pi]$ , we follow the suggestion in Priestly (1981, p. 709) and plot it in the intervals  $[-3\pi, \pi]$ ,  $[-\pi, \pi]$ , and  $[\pi, 3\pi]$ , in order to avoid discontinuities due to the fact that the phase is only defined mod  $2\pi$ . Readers not familiar with the interpretation of bivariate spectral measures are referred to Appendix 7.2 for a brief summary on how to read sample phase spectra.

about 40-70 percent (pre-1985 period), respectively. This does not apply to the post-1984 period that is characterized by a single-peaked coherence spectrum.

For all detrending device combinations, the corresponding  $sc$  values range from about ten to more than 90 percent, depending on observation period and filter.<sup>10</sup> A summary of the results is given in Table 1 (left part). It shows that the average absolute phase shift of a contained periodicity identified at maximum of squared coherency shrank from the pre-1985 to the post-1984 period by about 35 percent from 2.6 to 1.7 years. The characteristic double-peaking property of the coherence spectrum –see, for example, also the second coherence spectrum in Figure 5, although less pronounced– led us to also construct a measure of the duration  $D$  of association between the inflation rate and Gini coefficient dynamics, which might be interpreted as the dynamic (intertemporal) bias of bracket creep. It is defined as phase lead of inflation minus contemporaneous phase or more formally as follows:

$$D = |ps_0 - ps_1|,$$

where

$$ps_0(\omega^*) = \arctan \frac{-q_{jk}(\omega^*)}{c_{jk}(\omega^*)} \quad \text{if } -1 < ps < 1$$

$$ps_1(\tilde{\omega}) = \arctan \frac{-q_{jk}(\tilde{\omega})}{c_{jk}(\tilde{\omega})} \quad \text{if } ps \leq -1,$$

and  $\omega^*, \tilde{\omega}$  denote frequencies at local maximum of squared coherency, respectively. A summary of implied  $D$ -values for our estimates is given in the right part of Table 1.

Table 1: Average phase shift and dynamic bias for different detrending device combinations

Average phase shift (years)			Average dynamic bias (years)		
pre-1985	post-1984	1948-2004	pre-1985	post-1984	1948-2004
−2.60	−1.71	−2.26	3.27	1.89	3.02
(−2.65)*	(−1.76)	(−2.42)	(3.34)	(1.91)	(3.03)

\* medians in parentheses

Accordingly, the dynamic bias of bracket creep in years nearly halved since 1985. These findings suggest that the annual inflation indexation of the U.S. tax schedule as introduced

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<sup>10</sup>A detailed table of estimated bivariate spectral measures for all considered combinations of detrending devices is available on request from the authors.

in 1985 offsets the redistributive effects of inflation with a lag of approximately one and half to two years. According to Altig and Carlstrom (1991, 1993), this lag can be attributed, at least in parts, to an idiosyncratic definition of an “index year” as opposed to a tax year introduced by the automatic U.S. tax code indexation in the mid-1980s. They label the U.S. system a “pure one-year-lag index system.”

In combination with our findings from time domain techniques, we interpret the results from frequency domain techniques as lending support to an inequality reducing impact of inflation that last longer and represented a continuing effect over several years in the period before 1985. After the inflation indexation of the U.S. tax schedule became effective in 1985, there is still an impact from lagged inflation on income inequality; see, for example, also Altig and Carlstrom (1991, 1993). However, for the post-1984 regime it is clearly of more transitory nature.

Quantitatively assessing the distributional effects of changes of tax-band limits is a complex task for it requires to consider adjustment-induced changes in the behavior of the private sector. These changes can virtually not be controlled for in time series analysis or other econometric models. A candidate model that is able to account for these changes is an adequate DSGE model that is set up in the following.

### 3 Model

In this section, we develop a general equilibrium overlapping generations model with endogenous equity and money distribution. Four sectors can be depicted: households, production, the government, and the central bank. Households maximize discounted life-time utility. Agents can save either with money or with capital. Individuals are heterogeneous with regard to their productivity and cannot insure against idiosyncratic income risk. Firms maximize profits. Output is produced with the help of labor and capital. The government provides unfunded public pensions which are financed by a progressive tax on wage and capital income. The money growth rate is set by the central bank and seignorage is collected by the government.



### 3.1 Households

Every year, a generation of equal measure is born. A subscript  $j$  of a variable denotes the age of the generation. The total measure of all households is normalized to one.

Households live a maximum of  $T + T^R$  years. Lifetime is stochastic and agents face a probability  $s_j$  of surviving up to age  $j$  conditional on surviving up to age  $j - 1$ . During their first  $T$  years, agents supply labor  $l$  elastically. After  $T$  years, retirement is mandatory. Agent  $i$  maximizes her life-time utility:

$$E_0 \left[ \sum_{j=1}^{T+T^R} \beta^{j-1} (\Pi_{h=1}^j s_h) u(c_j^i, m_j^i, 1 - l_j^i), \right] \quad (5)$$

where  $\beta$ ,  $c_j^i$ , and  $m_j^i$  denote the discount factor, consumption and real money balances of agent  $i$  at age  $j$ , respectively. Instantaneous utility  $u(c, m, 1 - l)$  is given by:

$$u(c, m, 1 - l) = \ln c + (1 - \gamma) \ln m + B \ln(1 - l). \quad (6)$$

Workers are heterogeneous with regard to their labor earnings per working hour. The worker's labor productivity  $e(z, j)$  is stochastic and depends on his age  $j$  and an idiosyncratic labor productivity shock  $z$ . We assume that the idiosyncratic part of productivity follows a first order finite state Markov chain with conditional transition probabilities given by:

$$\pi(z'|z) = Pr\{z_{t+1} = z' | z_t = z\}, \quad (7)$$

where  $z, z' \in \mathcal{E}$ . Although the dynamics of productivity may be modeled slightly better by a second order Markov chain (Shorrocks, 1976) the improvement in accuracy is rather small and does not justify the considerable increase in the model's complexity.

Furthermore, agents are born without wealth,  $a_1 = 0$ , and cannot borrow,  $a_j \geq 0$  for all  $j$ . Wealth  $a$  is composed of real money  $m$  and capital  $k$ . Capital or, equally, equity  $k$  earns a real interest rate  $r$ . We further assume a short-sale constraint  $k \geq 0$ . Parents do not leave altruistic bequests to their children. All accidental bequests are confiscated by the state.

Agent  $i$  receives income from capital  $k^i$  and labor  $l^i$ . The budget constraint of the working agent at age  $j = 1, \dots, T$  in period  $t$  is given by

$$a_{j+1,t+1}^i = k_{j+1,t+1}^i + m_{j+1,t+1}^i = (1 + r_t)k_{jt}^i + \frac{m_{jt}^i}{1 + \pi_t} + w_t e(z, j)l_{jt}^i + tr_t - \frac{\tau_t(P_t y_{jt}^i)}{P_t} + c_{jt}^i, \quad (8)$$

where  $w_t$  and  $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$  denote the wage rate per efficiency unit labor and the inflation rate in period  $t$ , respectively.  $P_t$  is the price level in period  $t$ . Individual nominal income  $P_t y_{jt}^i \equiv P_t w_t e(z, j)l_{jt}^i + P_t r_t k_{jt}^i$  is taxed at the progressive rate  $\tau$ .<sup>11</sup> In addition, the households receive transfers  $tr_t$  from the government.

During retirement, agents receive public pensions  $pen_t$  in period  $t$  irrespective of their employment history and the budget constraint of the retired agent at age  $j = T+1, \dots, T+T^R$  is given by

$$a_{j+1,t+1}^i = k_{j+1,t+1}^i + m_{j+1,t+1}^i = (1 + r_t)k_{jt}^i + \frac{m_{jt}^i}{1 + \pi_t} + pen_t + tr_t - \frac{\tau_t(P_t y_{jt}^i)}{P_t} - c_{jt}^i. \quad (9)$$

The necessary conditions of the working households with regard to consumption  $c_{jt}^i$ , capital  $k_{j+1,t+1}^i$ , real money  $m_{j+1,t+1}^i$ , and labor  $l_{jt}^i$  are as follows:

$$\lambda_{jt}^i = u_c(c_{jt}^i, m_{jt}^i, 1 - l_{jt}^i) \quad (10)$$

$$\lambda_{jt}^i = \beta s_{j+1} E_t \left[ \lambda_{j+1,t+1}^i \left( 1 + r_{t+1} \left( 1 - \frac{\partial \tau}{\partial P_{t+1} y_{j+1,t+1}^i} \right) \right) \right] \quad (11)$$

$$\lambda_{jt}^i = \beta s_{j+1} E_t \left[ \lambda_{j+1,t+1}^i \frac{1}{1 + \pi_{t+1}} + u_m(c_{j+1,t+1}^i, m_{j+1,t+1}^i, 1 - l_{j+1,t+1}^i) \right] \quad (12)$$

$$u_l(c_{jt}^i, m_{jt}^i, 1 - l_{jt}^i) = \lambda_{jt}^i w_t e(j, z) \left[ 1 - \frac{\partial \tau}{\partial P_t y_{jt}^i} \right], \quad (13)$$

where  $u_x(\cdot)$  denotes the first partial derivative of the utility function with regard to the argument  $x = c, 1 - l, m$ . The first-order conditions of the retired household are given by (10)-(12) with  $l_{jt}^i \equiv 0$ .

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<sup>11</sup>In models similar to ours (but without a monetary sector), Ventura (1999) and Castañeda *et al.* (2003) study the effects of a flat rate tax reform on distribution and welfare. We follow Castañeda *et al.* who assume that both labor and interest income are taxed at the same rate.

### 3.2 Production

Firms are of measure one and produce output with effective labor  $N$  and capital  $K$ . Effective labor  $N_t$  is the product of working hours and individual productivity and is defined in more detail below.

Effective labor  $N_t$  is paid the wage  $w_t$ . Capital  $K_t$  is hired at rate  $r_t$  and depreciates at rate  $\delta$ . Production  $Y_t$  is characterized by constant returns to scale and assumed to be Cobb-Douglas:

$$Y_t = F(K_t, N_t) = K_t^\alpha N_t^{1-\alpha}. \quad (14)$$

In a factor market equilibrium, factors are rewarded with their marginal product:

$$w_t = (1 - \alpha)K_t^\alpha N_t^{-\alpha}, \quad (15)$$

$$r_t = \alpha K_t^{\alpha-1} N_t^{1-\alpha} - \delta. \quad (16)$$

### 3.3 Government

Government expenditures consists of government consumption  $G_t$ , government lump-sum transfers  $Tr_t$  to households, and social securities expenditures  $Pen_t$ . Government expenditures are financed by an income tax  $Tax_t$ , seignorage  $S_t$ , and confiscated accidental bequests  $Beq_t$ :

$$G_t + Pen_t + Tr_t = Tax_t + S_t + Beq_t. \quad (17)$$

We follow Castañeda *et al.* (2003) and characterize the U.S. income tax structure by a progressive tax function. In particular, we also adapt the following functional form for the income tax function that is based upon the estimates of Gouveia and Strauss (1994):

$$\tau(P_t y_t) = b_{0,t} \left( y - (y^{-b_{1,t}} + b_{2,t})^{-\frac{1}{b_{1,t}}} \right) \quad (18)$$

We further assume that the government adjusts the nominal income tax brackets every  $TB$  year. Without the loss of generality, we assume that the income tax rate schedule is adjusted in periods (=years)  $t \in \{0, TB, 2TB, 3TB, \dots\}$ . With regard to our tax function

(18), this is equivalent to assume that the tax parameters  $\{b_{0,t}, b_{1,t}, b_{2,t}\}$  are adjusted every  $TB$  years so that the real tax burden is the same as in the benchmark year  $t = 0$ . As a consequence, agents average and marginal income tax rates increase in the years between two successive tax rate adjustments as inflation increases the nominal income  $P_t y_{jt}^i$  ceteris paribus.

### 3.4 Monetary authority

Nominal money grows at the exogenous rate  $\theta$ :

$$\frac{M_t - M_{t-1}}{M_{t-1}} = \theta. \quad (19)$$

The seignorage is transferred lump-sum to the government:

$$S_t = \frac{M_t - M_{t-1}}{P_t}. \quad (20)$$

### 3.5 Stationary equilibrium

In the stationary equilibrium, all goods and factor markets are in equilibrium, the government adjusts the tax schedule every  $TB$  years, the government budget is balanced, money growth at the exogenous rate  $\theta$ , and individual and aggregate behavior are consistent. A full description of the stationary equilibrium together with the computational methods is provided in the Technical Appendix that is available from the authors upon request.

## 4 Calibration

Periods correspond to years. We assume that agents are born at real lifetime age 20 which corresponds to  $j = 1$ . Agents work  $T = 40$  years corresponding to a real lifetime age of 60. They live a maximum life of 60 years ( $T^R = 20$ ) so that agents do not become older than real lifetime age 80. The sequence of conditional survival probabilities  $\{s_j\}_{j=1}^{59}$  is set

equal to the Social Security Administration's survival probabilities for men aged 20-78 for the year 1994.<sup>12</sup> The survival probabilities decrease with age, and  $s_{60}$  is set equal to zero.

The calibration of the parameters  $\alpha$ ,  $\delta$ ,  $pen$ , and  $\theta$  and the Markov process  $e(z, j)$  is chosen in accordance with existing general equilibrium studies. Following Prescott (1986), the capital income share  $\alpha$  is set equal to 0.36. The annual rate of depreciation is set equal to  $\delta = 0.08$ . Pensions are distributed lump-sum to the retired agents. The replacement ratio of pensions to net average earnings amounts to 50% in every period  $t$ . Hence, pensions are a function of the distribution  $\mu_t$  and, hence,  $K_t$  and  $N_t$ , and are the same every  $TB$  periods. The income tax rate is adjusted every  $TB = 3$  years in accordance with the average of adjustment frequencies in the pre-85 period and the total U.S. postwar history: This can be seen by calculating the average of means reported in the fifth line from bottom and the last line in Table 6 (Appendix).<sup>13</sup> The model parameters are summarized in Table 2.

The tax function (18) is calibrated with the help of the estimates from Gouveia and Strauss (1994).<sup>14</sup> In particular, we set the income tax parameters in period  $t = 0$  (where we normalized the price level to one,  $P_0 = 1$ ) equal to  $b_0 = 0.258$ ,  $b_1 = 0.768$ ,  $b_2 = 0.031$ . The income tax parameters  $b_0$  and  $b_1$  are taken from Gouveia and Strauss for the tax year 1989, while  $b_2$  has been adjusted so that the tax rate of the average income in our model is equal to the tax rate of the average U.S. income. Every  $TB$  years, these parameters are adjusted so that the average and marginal tax rates of the real income are unchanged between period  $TB$  and  $p \cdot TB$ ,  $p = 1, 2, \dots$

The labor endowment process is given by  $e(z, j) = e^{z_j + \bar{y}_j}$ , where  $\bar{y}_j$  is the mean lognormal income of the  $j$ -year old. The mean efficiency index  $\bar{y}_j$  of the  $j$ -year-old worker is taken from Hansen (1993), and interpolated to in-between years. As a consequence, the model is able to replicate the cross-section age distribution of earnings of the U.S. economy. Following İmrohoroglu *et al.* (1998), we normalize the average efficiency index to one. The age-productivity profile is hump-shaped and earnings peak at age 50.

The idiosyncratic productivity shock  $z_j$  follows a Markov process. The Markov process is

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<sup>12</sup>We thank Mark Huggett and Gustavo Ventura for providing us with the data.

<sup>13</sup>Our qualitative results are the same in the cases  $TB = 2$  and  $TB = 4$ . The results for the case  $TB = 4$  are also presented in section 5.

<sup>14</sup>These parameter values have also been applied by Castañeda *et al.* (2003).

Table 2: Calibration of parameter values for the U.S. economy

Description	Function	Parameter
utility function	$U = \gamma \ln c + (1 - \gamma) \ln m + B \ln(1 - l)$	$\gamma = 0.974, B = 1.72$
discount factor	$\beta$	$\beta = 0.969$
production function	$Y = K^\alpha N^{1-\alpha}$	$\alpha = 0.36$
depreciation	$\delta$	$\delta = 0.08$
money growth rate	$\theta$	$\theta = 0.05$
pension replacement rate		0.50
periods between tax schedule adjustments	$TB$	$TB = 3$
income tax function in $t = 0$ with $P_0 = 1$	$\tau(y) = b_0 \left( y - (y^{-b_1} + b_2)^{-\frac{1}{b_1}} \right)$	$b_0 = 0.258, b_1 = 0.768,$ $b_2 = 0.031$
labor endowment process	$z_t = \rho z_{t-1} + \epsilon_t, \epsilon_t \sim N(0, \sigma_\epsilon)$ $\ln e(z, 1) \sim N(\bar{y}_1, \sigma_{y_1})$	$\rho = 0.96, \sigma_\epsilon = 0.045$ $\sigma_{y_1} = 0.38$

given by:

$$z_j = \rho z_{j-1} + \epsilon_j, \quad (21)$$

where  $\epsilon_j \sim N(0, \sigma_\epsilon)$ . Huggett (1996) uses  $\rho = 0.96$  and  $\sigma_\epsilon = 0.045$ . Furthermore, we follow Huggett and choose a lognormal distribution of earnings for the 20-year old with  $\sigma_{y_1} = 0.38$  and mean  $\overline{y_1}$ . As the log endowment of the initial generation of agents is normally distributed, the log efficiency of subsequent agents will continue to be normally distributed. This is a useful property of the earnings process, which has often be described as log normally in the literature.

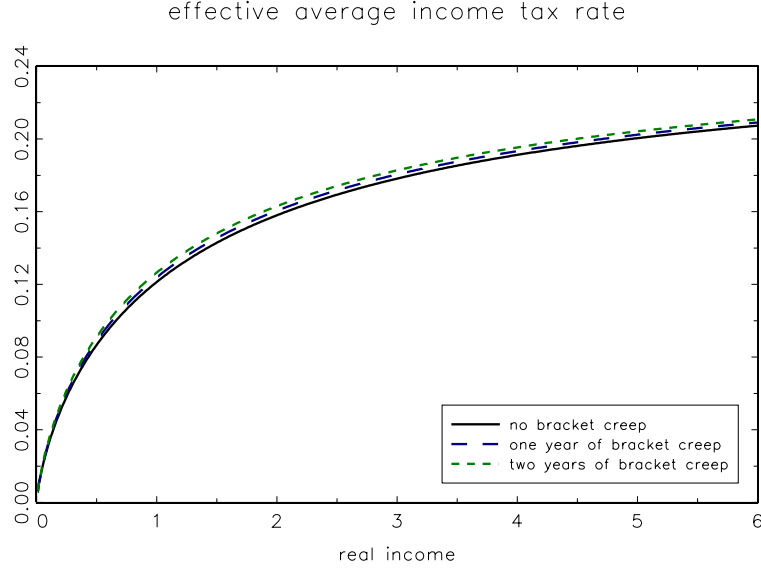
The remaining three parameters  $\beta$ ,  $B$ , and  $\gamma$  from the utility function are chosen to match the following characteristics of the U.S. economy as closely as possible: i) the capital-output ratio  $K/Y$  amounts to 3.0 as found by Auerbach and Kotlikoff (1995), ii) the average labor supply of the working households amounts to approximately one third of available time, and iii) the average velocity of money  $PY/M$  is equal to the annual velocity of M1 during 1960-2001, which is equal to 5.18. Our calibration  $\beta = 0.969$ ,  $B = 1.72$ , and  $\gamma = 0.974$  implies a capital-output ratio equal to 2.98, an average labor supply  $\bar{l} = 0.326$ , and an annual velocity of money equal to 5.12.

## 5 Results

In this section, we study the effects of a change of the money growth rate  $\theta$  or, equally, the inflation rate  $\pi$  on the stationary distribution of income. Remember that, in our benchmark case, the inflation rate is equal to 5% (in the U.S. the average CPI-U for the 1970s, 1980s, and 1990s equals 5.2%), and the tax schedule is adjusted every 3 years. The effect of the “bracket creep” on the average tax rates is illustrated in Figure 6 (where the average real income of the economy is normalized to one). The average income tax and the marginal tax rate hardly change after one or two years of bracket creep. Hence, we would expect only small effects from the bracket creep on the individual’s savings and labor supply.

Table 3 summarizes our results for the benchmark case. The first column gives the number of periods that have been elapsed since the last tax schedule adjustment. The remaining columns present the aggregate capital stock  $K_t$ , average labor supply  $\bar{l}_t$ , aggregate effective labor  $N_t$ , aggregate production  $Y_t$ , average real money balances  $\bar{m}_t$ , government transfers

Figure 6: Average income tax rate



$tr_t$ , and total income taxes  $Tax_t$ , and the Gini coefficients of the income distribution. Notice that the increase in the marginal and average income tax rates between two successive periods of the tax schedule adjustment results in an increase of total income taxes of approximately 2.5% each year. Similarly, transfers to the households also increase in order to keep the government budget balanced. Surprisingly, aggregate savings,  $K_t + \bar{m}_t$ , and average labor supply  $\bar{l}_t$  even increase with higher marginal tax rates. However, quantitative effects are of relatively small order. As a consequence, the pre-tax wage income remains almost unchanged during the course of bracket creep and is characterized by a Gini coefficient equal to approximately 0.56. Notice that this value is close to values observed empirically. Díaz-Giménez *et al.* (1997) find a value of 0.51 for households aged 36-50. The Gini coefficient of the total net income is smaller and amounts to only 0.49 as income is taxed progressively and transfers are distributed lump-sum. During the course of no adjustment, the Gini coefficient of total net income falls by about 0.4%. Given that the Gini coefficient in the U.S. increased roughly by 2% per decade from the 1970s to 1990s (based on figures by Kopczuk *et al.* 2010), the effect can be made responsible for offsetting about one fifth of the decadal rise in income inequality. In absolute terms, therefore, this effect approximately corresponds to the effect the decline in union membership had on earnings inequality in the U.S. as calculated by Freeman (1993). According to figures provided by



Table 3: “Bracket creep”, aggregate values and distribution for  $\pi = 5\%$ 

year $t$ after tax code adjustment								Gini coefficient	
	$K_t$	$\bar{l}_t$	$N_t$	$Y_t$	$\bar{m}_t$	$tr_t$	$Tax_t$	Wage Income	Total Net Income
0	2.063	0.2963	0.3604	0.6754	0.1425	0.03532	0.06514	0.561	0.496
1	2.065	0.2992	0.3638	0.6797	0.1419	0.03676	0.06693	0.560	0.494
2	2.066	0.3027	0.3671	0.6838	0.1412	0.03820	0.06871	0.559	0.492

Table 4: “Bracket creep”, aggregate values and distribution for  $\pi = 10\%$ 

year $t$ after tax code adjustment								Gini coefficient	
	$K_t$	$\bar{l}_t$	$N_t$	$Y_t$	$\bar{m}_t$	$tr_t$	$Tax_t$	Wage Income	Total Net Income
0	2.065	0.2943	0.3583	0.6731	0.0900	0.0348	0.0647	0.561	0.495
1	2.068	0.2972	0.3616	0.8303	0.0898	0.6774	0.0676	0.560	0.492
2	2.069	0.3002	0.3649	0.8353	0.0895	0.6815	0.0705	0.559	0.489

the U.S. Census Bureau’s Current Population Survey (Annual Social and Economic Supplement) the Gini coefficient of the white, not Hispanic, U.S. population increased from 39.2% in 1972 to 44.9% in 1999. In this case, the mean increase per decade is even lower equaling 1.9%, and the distributional bias of the bracket creep corresponds to about 21% of this effect in absolute terms.

In the U.S. postwar history, double-digit inflation rates of 10-13.5% have been observed in the mid and late 1970s and early 1980s. In this sense, our strategy turns to counterfactual simulations combining the average adjustment practice of the pre-85 and total period with the high levels of inflation witnessed in the last half of the 1970s. In order to consider the effects of a higher inflation rate, we recompute the model for the money growth rate  $\theta = \pi = 10\%$ . The results for the high-inflation economy are summarized in Table 4.

Following an increase of inflation from 5% to 10%, agents reduce their stationary real money balances. The average real money balances  $\bar{m}_t$  drops from 0.142 to 0.090. Furthermore,

Table 5: “Bracket creep”, aggregate values and distribution for  $\pi = 5\%$   
Four-annual tax adjustments,  $TB = 4$

year $t$ after tax code adjustment								Gini coefficient	
	$K_t$	$\bar{l}_t$	$N_t$	$Y_t$	$\bar{m}_t$	$tr_t$	$Tax_t$	Wage Income	Total Net Income
0	1.983	0.2774	0.3333	0.6334	0.1451	0.03148	0.05909	0.560	0.495
1	1.986	0.2803	0.3370	0.6382	0.1445	0.03292	0.06089	0.560	0.494
2	1.988	0.2834	0.3407	0.6429	0.1436	0.03438	0.06271	0.559	0.492
3	1.989	0.2868	0.3446	0.6477	0.1426	0.03585	0.06453	0.557	0.490

agents are subject to a more severe bracket creep and governmental tax receipts increase by a higher percentage between period 0, 1, and 2. Again, our finding for an inflation rate  $\pi = 5\%$  is confirmed that aggregate savings and average labor supply increase during the course of bracket creep. In addition, higher inflation reduces both the average labor supply and the inequality of the after-tax income distribution in the presence of bracket creep. The Gini coefficient of total net income two periods after the most recent tax schedule adjustment drops from 49.2% to 48.9%. In absolute terms, this effect equals nearly one third of the mean rise in income inequality of the 1970s, 1980s, and 1990s as measured by the Gini coefficient (in particular, for the white, not Hispanic, population).

The U.S. government used to adjust its income tax schedule less frequently in the years prior to 1985 than in recent years. In order to analyze the effects of a less frequent income tax schedule adjustment and, hence, a longer duration of the bracket creep, we extend the duration of the bracket creep to  $TB = 4$  years (keeping the inflation rate at  $\pi = 5\%$ ); see the corresponding mean frequency line for the pre-85 period in Appendix 7.1. As can be seen by inspection of Table 4 and Table 5, agents decrease aggregate savings in this case by approximately 4.0% (compare the dimension of entries in the respective table). The fall in the average labor supply  $\bar{l}_t$  is even more pronounced and amounts to approximately 6.3%. Accordingly, the duration of the “bracket creep” seems to be more important for the individual’s labor supply and savings decision than the yearly increase in the marginal and average income tax rates. With regard to the progressive bias on income inequality, a longer period of bracket creep results in an effect which equals one fourth of the absolute

value of the mean decadal change in the Gini coefficient.

The implied profound role of the frequency with which the income tax schedule is adjusted for inflation confirms our empirical findings in the time and frequency domain. In line with our bivariate spectral analytic results, a more frequent inflation adjustment (shorter periods of “creep”) implies a lower dynamic bias in the inflation-inequality relationship (Table 1).

## 6 Conclusion

“Bracket creep” has often been cited in the literature as one of the major distortionary effects of inflation. However, whether moderate levels of inflation also affect income-inequality through bracket creep has virtually not been analyzed since the 1970s. Both our empirical and theoretical analysis suggest a progressive effect. In terms of size, it amounts to about one fifth to one third of the absolute value of the mean change in the Gini coefficient for one decade. The quantitative effects depend not only on the level of inflation but, in particular, also on the indexation system that is in place. In sum, we find that the duration of the bracket creep, i.e. the time period between two successive income tax schedule adjustments, is more important for equilibrium values of aggregate savings and average labor supply than the annual change in the tax rates due to bracket creep. A shorter duration of bracket creep results in higher equilibrium labor supply and output. In this sense, our results suggest the change in U.S. tax policy after 1985 and the inflation-indexation under ERTA to represent a successful change.

## 7 Appendix

### 7.1 U.S. income tax: changes of brackets and rates 1948-2004

The post-war changes of the U.S. income tax schedule are summarized in Table 6 below. The main source of the entries in this table is the IRS (2003) along with volumes of the U.S. Major Tax Guide and the ‘Individual Tax Statistics: Complete Report Publications’ of the IRS, where these volumes were available.

In the first column of the table, the second date is the decisive one and gives the year of implementation of either a change of tax bracket boundaries ( $TBC_t$ ) or of regular income tax rates for fixed boundaries ( $TRC_t$ ) or of, at least, one of the former:  $TCC_t = \max\{TBC_t; TRC_t\}$ .  $TCC_t$  captures any change in a nominally defined parameter of the tax code. Index  $t$  denotes the specific year of change. The strength of adjustment is classified ‘substantial’ (‘partial’) in case of at least two (at most one) changing brackets (bracket) and/or at least two (at most one) adjusted tax rates (rate) for fixed boundaries. In this context, it is noteworthy that the partial changes for tax years 1968, 1969, and 1970 refer to the highest bracket’s tax rate which was additionally burdened with a Vietnam War surcharge equal to 7.5% of tax for 1968, 10% of tax for 1969, and 2.5% of tax for 1970. This surcharge did not alter any other than the highest bracket’s rate.

For more detail on the major legislative changes enacted and realized during the period of observation the reader is referred to the outline in Auerbach and Feenberg (2000). The changes of tax brackets reported in the following Table 6 are based on figures of boundaries for statutory taxable net income, i.e. income after subtracting deductions but before subtracting personal exemptions. Income in this definition still is the tax base for regular income tax, applicable to U.S. citizens and residents. Deductions and provisions unique to nonresident aliens are not considered. The same holds for the tax rates underlying variable  $TRC_t$ . They also exclude the effect of tax liability reducing tax credits and refer to regular income tax, consisting in normal tax and surtax.<sup>15</sup>

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<sup>15</sup>in effect, combined into a single rate structure; see IRS (2003), p. 325.

Table 6. U.S. individual income tax: changes of brackets and rates 1948-2004

Consecutive tax years	Adjustment strength	Major legislative change	Variables		
			$TBC_t$	$TRC_t$	$TCC_t$
49 - 50	substantial	—	0	1	1
50 - 51	substantial	—	0	1	1
51 - 52	substantial	—	0	1	1
53 - 54	substantial	—	0	1	1
63 - 64	substantial	Revenue Act	1	1	1
64 - 65	substantial	—	1	1	1
67 - 68	partial	—	0	1	1
68 - 69	partial	Reform Act	0	1	1
69 - 70	partial	—	0	1	1
76 - 77	substantial	—	1	0	1
78 - 79	substantial	—	1	0	1
80 - 81	substantial	Recovery Tax Act (I)	0	1	1
81 - 82	substantial	—	1	1	1
82 - 83	substantial	—	1	1	1
83 - 84	partial	—	1	0	1
84 - 85	substantial	Recovery Tax Act (II)	1	0	1
85 - 86	substantial	Reform Act	1	0	1
86 - 87	substantial	—	1	1	1
87 - 88	substantial	—	1	1	1
88 - 89	substantial	—	1	0	1
89 - 90	substantial	—	1	0	1
90 - 91	substantial	OB Reconciliation Act	1	1	1
91 - 92	substantial	—	1	0	1
92 - 93	substantial	—	1	1	1
93 - 94	substantial	—	1	0	1
94 - 95	substantial	—	1	0	1
95 - 96	substantial	—	1	0	1
96 - 97	substantial	—	1	0	1

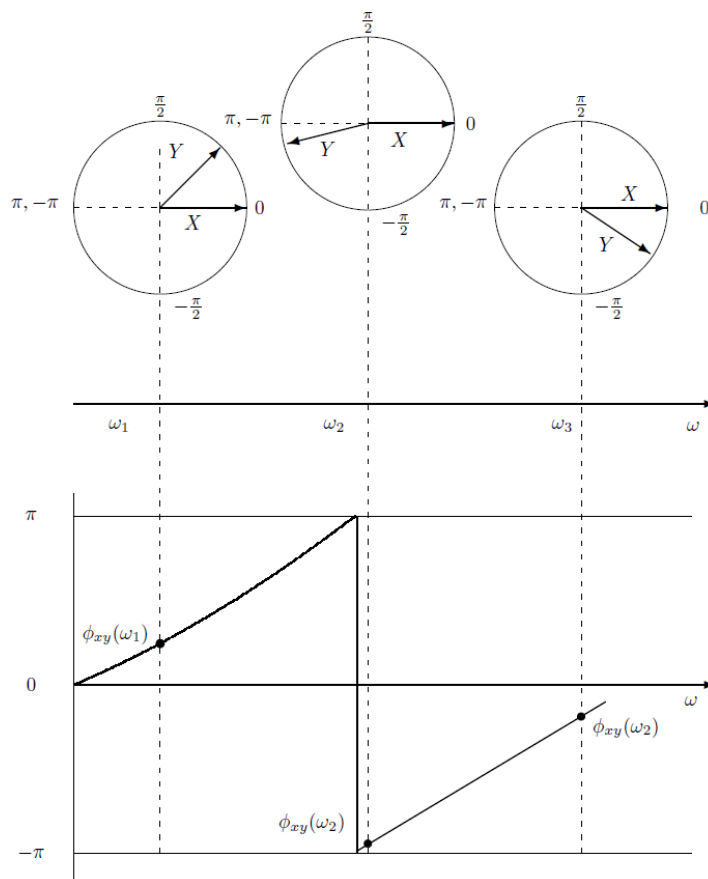
Table 6 (continued). U.S.income tax: changes of brackets and rates 1948-2004

Consecutive tax years	Adjustment strength	Major legislative change	Variables		
			$TBC_t$	$TRC_t$	$TCC_t$
97 - 98	substantial	—	1	0	1
98 - 99	substantial	—	1	0	1
99 - 00	substantial	—	1	0	1
00 - 01	substantial	—	1	1	1
01 - 02	substantial	—	1	1	1
02 - 03	substantial	—	1	1	1
03 - 04	substantial	—	1	0	1
pre-85: (i) sum			7	12	15
(ii) mean frequency (yrs)			5.29	3.08	2.46
post-84: (i) sum			20	7	20
(ii) mean frequency (yrs)			1.00	2.86	1.00
total period: (i) sum			27	19	35
(ii) mean frequency (yrs)			2.11	3.00	1.63

In general, there are four different (historical) sets of rates and brackets depending on the respective tax paying person(s): First, “income splitters”, i.e. married taxpayers who “use the joint return filling status” and split their income for tax purposes in an effort to effectively double the width of their taxable (or net income) size brackets. Figures underlying the chronological categorization of Table 6 above are based on this set. Second, starting with 1952, a set of rates was introduced for “heads of households”, i.e., for unmarried individuals who paid over half of the cost of maintaining a home for a qualifying person (e.g., a child or parent), or for certain married individuals who had lived apart from their spouses for the last six months of the tax year. This filling status was liberalized in 1970 and provides approximately half the advantages of the income-splitting. Third, the so-called “surviving spouse”-set of rates and brackets for which both, rates and taxable income brackets, are designed analogously to the ones of income-splitters. Finally, the remaining taxpayer-set is given for single persons. Since the late 1960s there has been an effort of convergence of this set with the one of married couples filling jointly.

## 7.2 Interpretation of phase spectra

Figure 7: The phase shift on circular and linear scale



In Figure 7, we assume two stationary time series  $X$  and  $Y$ . The phase shift between these series can be visualized either on circular scale (top diagram) or linear scale (bottom diagram):

In the upper schedule, there are three phase angles for the cyclical components corresponding to the frequencies  $\omega_1$ ,  $\omega_2$ , and  $\omega_3$ . At frequency  $\omega_1$ ,  $Y$  obviously leads  $X$  with a phase shift of about  $\pi/4$ . In this example, the phase shift rises with increasing  $\omega$  (second circle, top row). There is a frequency where the phase shift reaches value  $\pi$ . However, this phase shift cannot be distinguished from  $\phi_{xy}(\omega) = -\pi$ ; i.e. there is no difference between the statement “Series  $Y$  leads series  $X$  with half a cycle length” and the statement “Series  $Y$  lags series  $X$  with half a cycle length.” A discontinuity in the linear representation (bottom

diagram) between  $\omega_1$  and  $\omega_2$  results. A practical solution to overcome this discontinuity for interpretational ease is given in the text.

A positive (negative) slope in the linear representation of the phase spectrum suggests that cyclic components  $Y$  lead (lag) cyclic components  $X$  in a particular frequency band.



### 7.3 Bivariate spectral analysis: Gini/inflation<sup>a b</sup> (not for publication)

filter		BK		CF		HP	
None	<i>sc</i> :	0.42	0.15	0.28	0.32 <sup>†</sup>	0.30	0.28
	<i>ps</i> :	−2.48	0.84	−2.72	0.87 <sup>†</sup>	−2.24	0.64
BK	<i>sc</i> :	0.49	0.16	0.43	0.17	0.44	0.17
	<i>ps</i> :	−2.46	0.56	−2.48	0.64	−2.48	0.64
CF	<i>sc</i> :	0.33	0.26	0.30	0.29	0.30	0.30 <sup>†</sup>
	<i>ps</i> :	−2.66	1.78	−2.62	0.83	−2.60	0.83 <sup>†</sup>
HP	<i>sc</i> :	0.35	0.25	0.29	0.30 <sup>†</sup>	0.34	0.13
	<i>ps</i> :	−2.63	1.58	−2.63	0.83 <sup>†</sup>	−2.39	0.67
MHP	<i>sc</i> :	0.46	0.22	0.29	0.35 <sup>†</sup>	0.26	0.15
	<i>ps</i> :	−2.40	0.58	−2.53	0.70 <sup>†</sup>	−2.26	0.65
logD	<i>sc</i> :	0.25	0.23	0.20	0.24 <sup>†</sup>	0.19	0.28 <sup>†</sup>
	<i>ps</i> :	−1.23	0.79	−1.40	0.75 <sup>†</sup>	−1.28	0.76 <sup>†</sup>
MBK	<i>sc</i> :	0.49	0.19	0.42	0.22	0.43	0.21
	<i>ps</i> :	−2.42	0.75	−2.42	0.64	−2.43	0.63
		MHP		logD		MBK	
None	<i>sc</i> :	0.32	0.38 <sup>†</sup>	0.32	0.36 <sup>†</sup>	0.40	0.18
	<i>ps</i> :	−1.97	0.64 <sup>†</sup>	−2.88	0.36 <sup>†</sup>	−2.45	0.79
BK	<i>sc</i> :	0.44	0.23	0.47	0.25	0.47	0.19
	<i>ps</i> :	−2.43	0.59	+2.15	0.41	−2.45	0.58
CF	<i>sc</i> :	0.35	0.32	0.25	0.42 <sup>†</sup>	0.33	0.27
	<i>ps</i> :	−2.39	0.71	+3.15	0.59 <sup>†</sup>	−2.61	1.38
HP	<i>sc</i> :	0.30	0.39 <sup>†</sup>	0.22	0.38 <sup>†</sup>	0.36	0.27
	<i>ps</i> :	−2.03	0.62 <sup>†</sup>	+3.08	0.39 <sup>†</sup>	−2.58	1.27
MHP	<i>sc</i> :	0.28	0.46 <sup>†</sup>	0.20	0.45 <sup>†</sup>	0.43	0.25
	<i>ps</i> :	−2.03	0.61 <sup>†</sup>	+2.66	0.38 <sup>†</sup>	−2.40	0.59
logD	<i>sc</i> :	0.17	0.31 <sup>†</sup>	0.28	0.51 <sup>†</sup>	0.24	0.27 <sup>†</sup>
	<i>ps</i> :	−1.09	0.75 <sup>†</sup>	−1.63	0.32 <sup>†</sup>	−1.21	0.78 <sup>†</sup>
MBK	<i>sc</i> :	0.44	0.27	0.46	0.28	0.46	0.22
	<i>ps</i> :	−2.38	0.60	+2.13	0.42	−2.39	0.59

Continued: Bivariate spectral analysis: Pre-1985 period 1948-1984<sup>c</sup>

filter		BK		CF		HP	
None	<i>sc</i> :	0.74	–	0.52	0.66 <sup>†</sup>	0.44	0.47 <sup>†</sup>
	<i>ps</i> :	–1.97	–	–3.22	0.92 <sup>†</sup>	–2.51	0.69 <sup>†</sup>
BK	<i>sc</i> :	0.85	0.41	0.76	0.51	0.77	0.49
	<i>ps</i> :	–2.69	0.68	–2.65	0.70	–2.66	0.70
CF	<i>sc</i> :	0.82	0.46	0.69	0.62	0.70	0.63
	<i>ps</i> :	–2.86	0.83	–2.82	0.80	–2.83	0.81
HP	<i>sc</i> :	0.81	0.46	0.68	0.64	0.55	0.53
	<i>ps</i> :	–2.83	0.81	–2.83	0.79	–2.43	0.63
MHP	<i>sc</i> :	0.82	0.52	0.71	0.67	0.54	0.62 <sup>†</sup>
	<i>ps</i> :	–2.65	0.68	–2.69	0.70	–2.25	0.61 <sup>†</sup>
logD	<i>sc</i> :	0.40	0.42 <sup>†</sup>	0.26	0.50 <sup>†</sup>	0.27	0.48 <sup>†</sup>
	<i>ps</i> :	–0.42	0.65 <sup>†</sup>	–2.19	0.67 <sup>†</sup>	–2.13	0.66 <sup>†</sup>
MBK	<i>sc</i> :	0.81	0.45	0.74	0.56	0.74	0.55
	<i>ps</i> :	–2.71	0.70	–2.60	0.69	–2.61	0.70
		MHP		logD		MBK	
None	<i>sc</i> :	0.52	0.58 <sup>†</sup>	0.41	0.54 <sup>†</sup>	0.74	0.47
	<i>ps</i> :	–2.40	0.64 <sup>†</sup>	–3.28	0.38 <sup>†</sup>	–2.72	0.94
BK	<i>sc</i> :	0.81	0.45	0.79	0.51	0.84	0.42
	<i>ps</i> :	–2.71	0.70	+1.97	0.45	–2.70	0.69
CF	<i>sc</i> :	0.79	0.61	0.68	0.70 <sup>†</sup>	0.80	0.47
	<i>ps</i> :	–2.88	0.77	+2.47	0.56 <sup>†</sup>	–2.85	0.83
HP	<i>sc</i> :	0.59	0.62 <sup>†</sup>	0.46	0.62 <sup>†</sup>	0.79	0.47
	<i>ps</i> :	–2.41	0.61 <sup>†</sup>	+2.68	0.38 <sup>†</sup>	–2.82	0.82
MHP	<i>sc</i> :	0.55	0.67 <sup>†</sup>	0.40	0.69 <sup>†</sup>	0.80	0.52
	<i>ps</i> :	–2.31	0.61 <sup>†</sup>	+2.37	0.38 <sup>†</sup>	–2.65	0.69
logD	<i>sc</i> :	0.32	0.44 <sup>†</sup>	0.32	0.52 <sup>†</sup>	0.35	0.42 <sup>†</sup>
	<i>ps</i> :	–1.68	0.81 <sup>†</sup>	–2.81	0.47 <sup>†</sup>	–1.65	0.83 <sup>†</sup>
MBK	<i>sc</i> :	0.79	0.50	0.76	0.55	0.82	0.47
	<i>ps</i> :	–2.65	0.70	+1.95	0.46	–2.64	0.69

Continued: Bivariate spectral analysis: Post-1984 period 1985-2004<sup>d</sup>

filter		BK		CF		HP	
None	<i>sc</i> :	0.75	0.10	0.71	–	0.67	0.33
	<i>ps</i> :	–2.13	–0.10	–1.54	–	–2.08	0.19
BK	<i>sc</i> :	0.93	0.55	0.71	0.10	0.86	0.36
	<i>ps</i> :	–1.79	0.21	–1.60	0.18	–1.78	0.21
CF	<i>sc</i> :	0.50	0.13	0.53	–	0.60	–
	<i>ps</i> :	–1.87	0.81	–1.28	–	–1.29	–
HP	<i>sc</i> :	0.93	0.42	0.67	0.04	0.51	0.15
	<i>ps</i> :	–1.98	–0.17	–1.36	0.42	–1.41	0.45
MHP	<i>sc</i> :	0.90	0.43	0.68 <sup>†</sup>	0.07	0.60	0.42
	<i>ps</i> :	–1.71	0.11	–0.98 <sup>†</sup>	0.30	–1.03	0.38
logD	<i>sc</i> :	0.81 <sup>†</sup>	0.24	0.75 <sup>†</sup>	0.26	0.63 <sup>†</sup>	0.46
	<i>ps</i> :	–0.69 <sup>†</sup>	0.49	–0.24 <sup>†</sup>	0.55	–0.02 <sup>†</sup>	0.57
MBK	<i>sc</i> :	0.93	0.45	0.81	0.10	0.88	0.62
	<i>ps</i> :	–1.77	0.16	–1.50	0.61	–1.89	0.43
		MHP		logD		MBK	
None	<i>sc</i> :	0.59	0.34	0.66	0.65	0.77	0.13
	<i>ps</i> :	–2.05	0.18	–2.67	–0.07	–2.20	–0.10
BK	<i>sc</i> :	0.91	0.50	0.81	0.41	0.92	0.56
	<i>ps</i> :	–1.82	0.14	–2.65	–0.13	–1.77	0.21
CF	<i>sc</i> :	0.43 <sup>†</sup>	–	0.94	0.43	0.85	0.39
	<i>ps</i> :	–0.97 <sup>†</sup>	–	–2.78	–0.18	–2.33	–0.33
HP	<i>sc</i> :	0.37	0.15	0.30	0.21	0.92	0.44
	<i>ps</i> :	–1.20	0.49	–4.69	–0.59	–1.96	–0.21
MHP	<i>sc</i> :	0.78	0.64	0.66	0.62	0.90	0.45
	<i>ps</i> :	–1.64	0.33	–2.37	0.01	–1.69	0.09
logD	<i>sc</i> :	0.71	0.74 <sup>†</sup>	0.61	0.84 <sup>†</sup>	0.81 <sup>†</sup>	0.27
	<i>ps</i> :	–0.74	0.87 <sup>†</sup>	–1.56	0.35 <sup>†</sup>	–0.69 <sup>†</sup>	0.54
MBK	<i>sc</i> :	0.91	0.48	0.79	0.39	0.92	0.53
	<i>ps</i> :	–1.79	0.09	–2.64	–0.11	–1.75	0.16

Note: <sup>a</sup>Columns: Gini coefficient series; Rows: inflation rate; † relative higher *sc* at  $|ps| \leq 1$ ;

<sup>b</sup> BK: 1951-2001, CF: 1950-2002, logD: 1951-2004

<sup>c</sup> BK: 1951-1984, CF: 1950-1984, logD: 1951-1984

<sup>d</sup> BK: 1985-2001, CF: 1985-2002, logD: 1985-2004

## 7.4 Stationary Equilibrium of the Model in Section 3 (not for publication)

The concept of equilibrium applied in this paper uses a recursive representation of the consumer's problem following Stokey et al. (1989). Let  $V_{jt}^i(k_{jt}^i, m_{jt}^i, z, \mu)$  be the value of the objective function of the  $j$ -year old agent with equity  $k_{jt}^i$ , real money  $m_{jt}^i$ , and idiosyncratic productivity level  $z$  in period  $t$ . The distribution of money and capital is denoted by  $\mu(\cdot)$ . As the tax schedule is adjusted every  $TB$  periods, the period  $t$  is also an argument of the value function.  $V_{jt}^i(k_{jt}^i, m_{jt}^i, z, \mu)$  is defined as the solution to the dynamic program:

$$V_{jt}^i(k_{jt}^i, m_{jt}^i, z, \mu) = \max_{k_{j+1,t+1}^i, m_{j+1,t+1}^i, c_{jt}^i, l_{jt}^i} \left\{ u(c_{jt}^i, m_{jt}^i, 1 - l_{jt}^i) + \beta s_{j+1} E_t [V_{j+1,t+1}^i(k_{j+1,t+1}^i, m_{j+1,t+1}^i, z', \mu')] \right\} \quad (22)$$

subject to (7), (8) or (9) and  $k, m \geq 0$ . Optimal decision rules of the agent  $i$  in period  $t$  at age  $j$  are a function of the individual state variables  $k_{jt}^i$ ,  $m_{jt}^i$ , and  $z$ , the distribution of money and capital,  $\mu$ , and the period  $t$ . Let  $c_t(k, m, z, j, \mu)$ ,  $l_t(k, m, z, j, \mu)$ ,  $k'_t(k, m, z, j, \mu)$ , and  $m'_t(k, m, z, j, \mu)$  denote the optimal consumption, labor supply, next-period capital stock, and next-period real money balances for a  $j$ -year aged individual with productivity  $z$ , capital stock  $k$ , and real money balances  $m$ , and distribution of capital  $k$  and money  $m$  in period  $t$ . Furthermore, let  $\mu_t(k, m, z, j)$  denote the measure of  $j$ -year old agents with productivity  $z$  in period  $t$  that hold capital  $k$  and real money balances  $m$ .

We will consider a stationary equilibrium where the inflation rate is constant in every period  $t$ . Furthermore, government consumption is assumed to be constant,  $G_t = G$ , and pensions  $pen_t$  are assumed to be of equal magnitude every  $TB$  periods, respectively. As a consequence, the factor prices, aggregate capital and labor, and the distribution  $\mu_t$  are also the same every  $TB$  periods, respectively.

A stationary equilibrium for a given government policy  $\{b_{0,t}, b_{1,t}, b_{2,t}, G_t, pen_t\}$  and central bank policy  $\theta_t = \theta$  is a collection of value functions  $V_{jt}^i(k, m, z, \mu)$ , individual policy rules  $c_t(k, m, z, j, \mu)$ ,  $l_t(k, m, z, j, \mu)$ ,  $k'_t(k, m, z, j, \mu)$ ,  $m'_t(k, m, z, j, \mu)$ , relative prices of labor and capital  $\{w_t, r_t\}$ , and a law of motion for the distribution  $\mu_{t+1} = g(\mu_t)$  such that:

1. Money grows at the exogenous rate  $\theta$  and the seignorage (20) is transferred lump-sum to the government.
2. The inflation rate  $\pi_t$  is constant and equal to the money growth rate  $\theta$ .
3. The government adjusts the tax schedule in the years  $\{0, TB, 2TB, \dots\}$ . In the years  $t \in \{p + TB, p + 2TB, p + 3TB, \dots\}$ ,  $p = 0, \dots, TB - 1$ , government consumption  $G_t$  and individual pensions  $pen_t$  are the same, respectively so that all exogenous variables in the economy are the same every  $TB$  periods.
4. The government budget (17) is balanced.
5. Individual and aggregate behavior are consistent:

$$K_t = \sum_{j=1}^{T+T^R} \int_k \int_m \int_z k \mu_t(k, m, z, j) dz dm dk, \quad (23)$$

$$N_t = \sum_{j=1}^T \int_k \int_m \int_z l_t(k, m, z, j, \mu) e(z, j) \mu_t(k, m, z, j) dz dm dk \quad (24)$$

$$C_t = \sum_{j=1}^{T+T^R} \int_k \int_m \int_z c_t(k, m, z, j, \mu) \mu_t(k, m, z, j) dz dm dk, \quad (25)$$

$$Pen_t = \sum_{j=T+1}^{T+T^R} \int_k \int_m \int_z pen_t \mu_t(k, m, z, j) dz dm dk, \quad (26)$$

$$Tax_t = \sum_{j=T+1}^{T+T^R} \int_k \int_m \int_z \frac{\tau(P_t y_t(k, m, z, j))}{P_t} \mu_t(k, m, z, j) dz dm dk, \quad (27)$$

$$Beq_t = \sum_{j=1}^{T+T^R} \int_k \int_m \int_z (1 - s_{j+1}) a'_{t-1}(k, m, z, j, \mu) \mu_{t-1}(k, m, z, j) dz dm dk \quad (28)$$

$$\frac{M}{P} = \sum_{j=1}^{T+T^R} \int_k \int_m \int_z m \mu_t(k, m, z, j) dz dm dk, \quad (29)$$

where  $a'_t(.) \equiv k'_t(.) + m'_t(.)$  are the optimal next-period assets and  $y_t(k, m, z, j)$  denotes the real income of a  $j$ -year old agent with productivity  $z$ , capital  $k$ , and money  $m$  in period  $t$ . Furthermore,  $tr_t = Tr_t$ .

6. Relative prices  $\{w_t, r_t\}$  solve the firm's optimization problem by satisfying (15) and (16).
7. Given the government policy  $\{b_{0,t}, b_{1,t}, b_{2,t}, G_t, pen_t\}$  and the distribution  $\mu_t$ , the individual policy rules  $c_t(.)$ ,  $k'_{t+1}(.)$ ,  $m'_{t+1}(.)$ , and  $l_t(.)$  solve the consumer's dynamic program (22).
8. The goods market clears in every period  $t$ :

$$K_t^\alpha N_t^{1-\alpha} = C_t + \delta K_t + K_{t+1} - K_t. \quad (30)$$

9. The dynamics of the distribution  $\mu_{t+1} = g(\mu_t)$  are consistent with individual behavior:

$$\mu_{t+1}(k', m', z', j+1) = \int_k \int_m \int_z 1_{k'=k'_t(k,m,z,j,\mu)} \cdot 1_{m'=m'_t(k,m,z,j,\mu)} \cdot Pr(z'|z) \cdot \mu_t(k, m, z, j) dz dm dk, \quad (31)$$

where  $1_{k'=k'_t(.)}$  is an indicator function that takes the value one if  $k' = k'_t(.)$  and zero otherwise.  $1_{m'=m'_t(.)}$  is defined in an analogous way. Furthermore, the new-born generation has zero wealth,  $k = 0$  and  $m = 0$ .<sup>16</sup> Notice further that, in particular,  $\mu_t = \mu_{t+TB}$  in a stationary equilibrium.

## 7.5 Computation (not for publication)

The model of section 3 cannot be solved analytically, but only numerically. The solution algorithm is described by the following steps:

1. Parameterize the model. Let  $TB$  denote the number of years between two adjustments of the nominal income tax schedule.

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<sup>16</sup>For computational purpose, agents of the first-year generation are endowed with small money balances so that the utility function does not take the value of infinity.

2. Make initial guesses of the law of motion for the aggregate capital stock  $\{K_0, K_1, K_2, \dots, K_{TB-1}\}$ , aggregate effective labor  $\{N_0, N_1, N_2, \dots, N_{TB-1}\}$ , aggregate real money  $\{M/P_0, M/P_1, M/P_2, \dots, M/P_{TB-1}\}$  and aggregate (=individual) transfers  $\{tr_0, tr_1, \dots, tr_{TB-1}\}$ .
3. Compute the values of  $w_t$  and  $r_t$  for  $t = 0, 1, \dots, TB - 1$  that solve the firm's Euler equations. Compute the pension  $pen_t$  so that the replacement rate of pensions with regard to net average labor income is equal to the empirical value.
4. Compute the household's decision functions by solving the Euler equations.
5. Compute the distribution  $\mu_t$  of the individual state variable  $\{k, m, z, j\}$  by forward induction over age  $j = 1, \dots, T + T^R$  for  $t = 0, 1, \dots, TB - 1$ .
6. Compute the aggregate capital stock  $\{K_0, K_1, \dots, K_{TB-1}\}$ , aggregate effective labor  $\{N_0, N_1, N_2, \dots, N_{TB-1}\}$ , aggregate real money  $\{M/P_0, M/P_1, M/P_2, \dots, M/P_{TB-1}\}$  and aggregate transfers  $\{tr_0, tr_1, \dots, tr_{TB-1}\}$ . Update  $\{K_0, K_1, \dots, K_{TB-1}\}$ ,  $\{N_0, N_1, N_2, \dots, N_{TB-1}\}$ ,  $\{M/P_0, M/P_1, M/P_2, \dots, M/P_{TB-1}\}$  and  $\{tr_0, tr_1, \dots, tr_{TB-1}\}$  and return to step 2 until convergence.

We discretize the state space  $(k, m, z)$  using an equispaced grid over the capital stock  $k$ , the money balances  $m$ , and the individual productivity  $z$ . The upper grid points  $k_{max} = 20.0$  and  $m_{max} = 0.4$  are found to be non-binding. For the productivity  $z$ , the (five-point) grid ranges from  $-2\sigma_{y_1}$  to  $2\sigma_{y_1}$ . The probability of having productivity shock  $z_1$  in the first period of life is computed by integrating the area under the normal distribution. The transition probabilities are computed using the method of Tauchen (1986). As a consequence, the efficiency index  $e(z, j)$  follows a finite Markov chain.

In step 4, a finite-time dynamic programming problem is to be solved. We use piecewise linear functions in order to approximate the policy functions  $c_t(k, m, z, j)$ ,  $k'_t(k, m, z, j)$ ,  $m'_t(k, m, z, j)$ , and  $l_t(k, m, z, j)$  between grid points. In particular, we solve the Euler functions (10)-(13) for given sequence of the aggregate capital stock  $K_t$ , aggregate effective employment  $N_t$ , and transfers  $tr_t$ . The methods for the computation of the policy functions and the aggregate variables are described in detail in Heer and Maußner (2009).

As the household is born without any assets, his first-period wealth and his real money balances are zero. As a consequence, the value function would take the value  $-\infty$  as

$m_{1t} = 0$ . For computational purposes, therefore, we slightly change the utility function and introduce a small constant  $\psi$  into (6),  $\tilde{u} = u(c, m + \psi, 1 - l)$ .

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