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Angaben zur Veröffentlichung / Publication details:

Heer, Burkhard, Alfred Maußner, and Paul D. McNelis. 2011. "The money-age distribution: empirical facts and the limits of three monetary models." *Journal of Macroeconomics* 33 (3): 390–405. https://doi.org/10.1016/j.jmacro.2011.03.003.

The Money-Age Distribution: Empirical Facts and the Limits of Three Monetary Models *

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January 28, 2011

Abstract

The money-age distribution is hump-shaped for the US post-war economy. There is no clear cut relation between the variation of money holdings within generations and age. Furthermore, money is found to be only weakly correlated with both income and wealth. We analyze three motives for money demand in an overlapping generations setup in order to explain these observations: 1) money in the utility, 2) an economy with costly credit service, and 3) limited participation. All three models are consistent with the hump-shaped relation between average money holdings and age, yet they predict a much closer association between money holdings, income, wealth, and age than we find in the data. Only the limited-participation model partly replicates the low bivariate correlation between money and income as well as between money and interest bearing assets. None of the three models satisfactorily explains these stylized facts.

JEL Classification: E41, E31, D30

Key Words: Money-Age Distribution, Money Demand, OLG Model

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*Preliminary versions of this paper have been presented at the CESifo area conference on Money, Macro and International Finance in Munich, the CEF conference in Cyprus, the Annual Meeting of the German Economic Association in Bayreuth, and at seminars at the Catholic University of Leuven, and the universities of Bilbao and Hamburg. We would like to thank Kenneth Judd and Kenneth Kletzer for helpful comments. All remaining errors are ours. Burkhard Heer kindly acknowledges support from the German Science Foundation (Deutsche Forschungsgemeinschaft DFG) during his stays at Georgetown University, University of Pennsylvania, and Stanford University.

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1 Introduction

The dynamic macroeconomic general equilibrium literature has discussed the distribution of income and wealth, but has mostly ignored the distribution of money. Díaz-Giménez, Quadrini and Ríos-Rull (1997) document the facts on the U.S. distribution of earnings, income, and wealth. Earnings and income are much less concentrated than wealth, and are only weakly correlated with it. Huggett (1996) shows that these facts can be replicated in a satisfactory manner in an overlapping generations (OLG) model where agents are characterized by heterogeneous productivity and receive social security. Huggett and Ventura (2000) also explain the consumption behavior over the life-cycle and explain why low-income households do not save.

In more recent literature on the modeling of the distribution of assets, attention has also been directed to housing wealth. In this vein, Coco (2005) and Nakijama (2005) study the life-cycle distribution of housing wealth. To the best of our knowledge, however, there is no comparable study available for the life-cycle distribution of money.

In this paper, we address this issue. We use empirical evidence from the United States to document the following stylized facts:

1. money holdings are hump-shaped over the life-cycle,
2. there is no clear-cut relation between the variation of money holdings and age,
3. income, wealth, and age explain only a small fraction of the variation of money holdings.

In addition, this empirical evidence is found to be stable through the past two decades.

We develop three alternative widely-used monetary general equilibrium models to explain the heterogeneity of money holdings across individuals. They differ in the way money is introduced. We compare the following approaches:

1. Money in the utility function, in which households save in the form of money or capital.
2. Costly credit, in which households consume a continuum of commodities that can be purchased with either money or credit. Credit, however, is costly, as in Dotsey and Ireland (1996). Money is a poor store of value since it is dominated in return by capital.

3. Limited participation, in which firms need to finance wage expenditures with a loan, while households deposit part of their money at a bank. The central bank injects the money into the banking sector after the households have made the deposits, but before the firms ask for a loan.

There are, of course, many other monetary models, but money-in-utility, costly credit and limited participation frameworks are widely used by empirical researchers involved in medium-term macroeconomic policy analysis. For this reason, we believe it is important to evaluate the properties of the stationary distributions of these widely-used models. Berensten (2002) has shown that in search theoretic models, in the tradition of Kiyotaki and Wright (1989), any initial distribution of the optimal quantity of money holdings, if production costs are no too high, converges asymptotically to a uniform distribution.

Ich persoenlich finde diesen neuen Absatz von Paul eher schlampig geschrieben; so ist der Bezug zu den search-theoretic models sehr abrupt. Sei's drum, lassen wir ihn halt. Der referee scheint ihn gemocht zu haben.

The models we use are of course very different from the search-theoretic approach. We find that all three popular models explain the hump-shaped pattern between average money holdings and age but fail to produce the low predictive power of income, wealth, and age for the distribution of money holdings. The limited-participation model, however, can account for the low bivariate correlations between income and money and between money and household's holdings of interest-bearing assets. Therefore, our results suggest that a cash-in-advance constraint should be specified so that the households can use wage income in order to finance consumption.

Section 2 documents the empirical facts of the money-age distribution for the US economy. Section 3 introduces the overlapping-generations model with two assets, money and capital. The model is calibrated with regard to the characteristics of the US economy in Section 4. Our numerical results are presented in Section 5, followed by the conclusion.[1]

2 Empirical Observations

We use data from the 1994, 1999, 2001, and 2005 University of Michigan Personal Survey of Income Dynamics (PSID) family, income, and wealth files. These are the only four data sets for which we are able to match data on income, age, money and

[1] Appendix 1 covers additional empirical evidence. Appendix 2 performs a sensitivity analysis with regard to our calibration. A more detailed technical explanation of the solution methods for each of the models appears in a Technical Appendix that is available upon request from the authors.

capital.[2] Our data set includes families with strictly positive money holdings where the head of household is of age between 20 and 80. This gives us 21,574 observations.[3]

To analyze the money holding behavior depending on age, we group the households in the following age categories: 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74, 75-80. Money, M, is defined as money in checking or savings accounts, money market funds, certificates of deposit, government savings bonds, and treasury bills.[4] Capital, K, consists of shares of stock in publicly held corporations, mutual funds, and investment funds and other savings or assets, such as bond funds and life insurance policies. Total family income is made of taxable and transfer income of head, wife, and other family unit members and Social Security Income. In addition to the PSID data we use data for income, earnings, and wealth from the Survey of Consumer Finances 1992.

k durch K ersetzt

Figures 2.1 pictures the distributions of income, earning, money and wealth, while Figure 2.2 pictures the distribution of money balances over the life-cycle. Table 2.1 shows empirical correlations between money/income, money/capital, and capital/income for 1994, 1999, 2001, and 2005, represented by $\rho(M,Y)$, $\rho(M,K)$, and $\rho(Y,K)$, while Table 2.2 reports the results of fixed-effect panel regressions of money holdings on income, wealth and age.

Figures und Tables muessen noch von 1 bis 10 numerisch durchnu-meriert werdern.

Bitte Figure 2 ueberpruefen, war falsch bei Paul.

Table 2.1

Empirical Correlations

Year	$\rho(M,Y)$	$\rho(M,W)$	$\rho(Y,W)$
1994	.21	.18	.21
1999	.20	.28	.12
2001	.26	.29	.11
2005	.29	.07	.04

Notes: **to be added**

The behavior of money/net worth ratio in Table 2.2 is worth noting. We see that the ratio jumped in 2004, as asset holders transferred their wealth out of stocks into shorter-term more liquid assets.

[2]For this reason we do not control for cohort effects in the computation of the inequality of money holdings as is done by Storesletten, Telmer, and Yaron (2004).

[3]While data on household consumption would be desirable, the PSID survey only includes expenditure on food consumption.

[4]This definition of money in the PSID data does not strictly match the definition of money as a purely non-interest bearing asset as it appears in our models. The PSID wealth files do not make this distinction in money holdings of households.

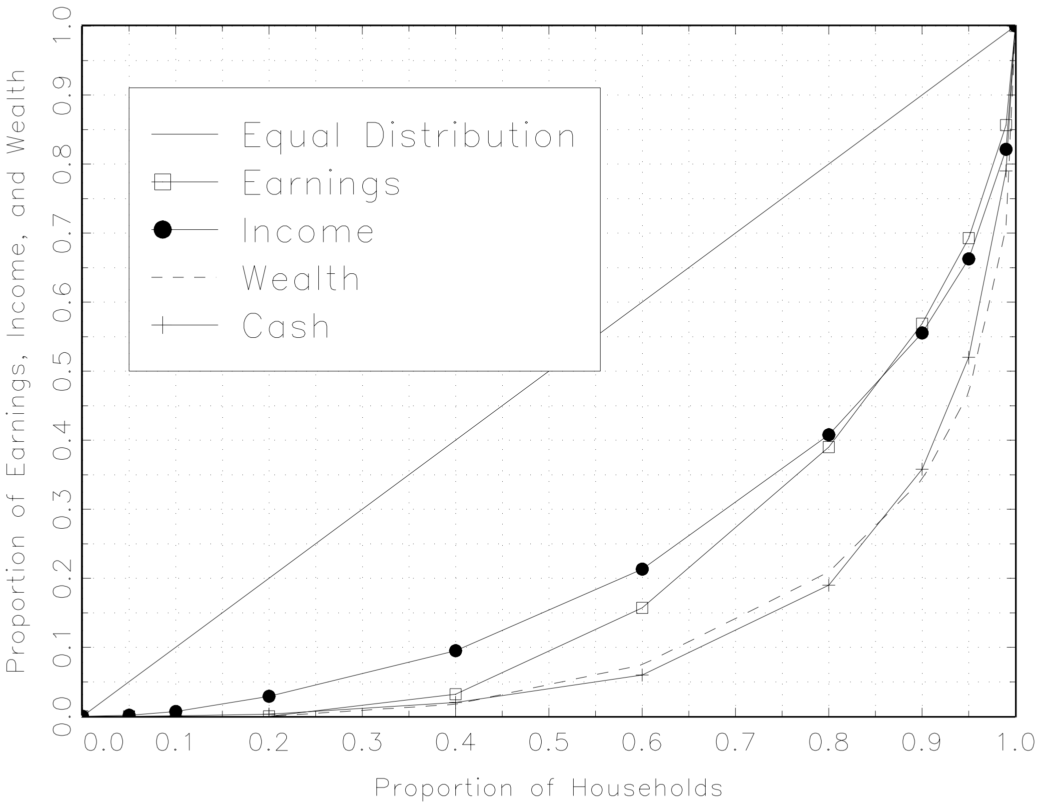

Figure 2.1: Distribution of Income, Earnings, Money and Wealth

Table 2.3 contains fixed-effects panel regressions of money holdings M on income Y, wealth W, and age T, as well as the square values of these variables. The symbol R^2 is the multiple correlation coefficient, while the values in parentheses are the robust t-ratios. The coefficient estimates in the first row use capital as the indicator of wealth, while the coefficients appearing in the third row use total wealth less money holdings as the indicator of household wealth. Capital in the first regression is defined as household-owned shares of stocks in publicly held corporations, mutual funds or investment trusts, not including stocks in employer-based pensions or individual retirement accounts. We

Table 2.2

Money/Net Worth Ratios

Year	1994	1999	2001	2005
Mean	.30	.21	.21	.73
Std.Dev	1.63	.94	.98	2.88

Notes: to be added

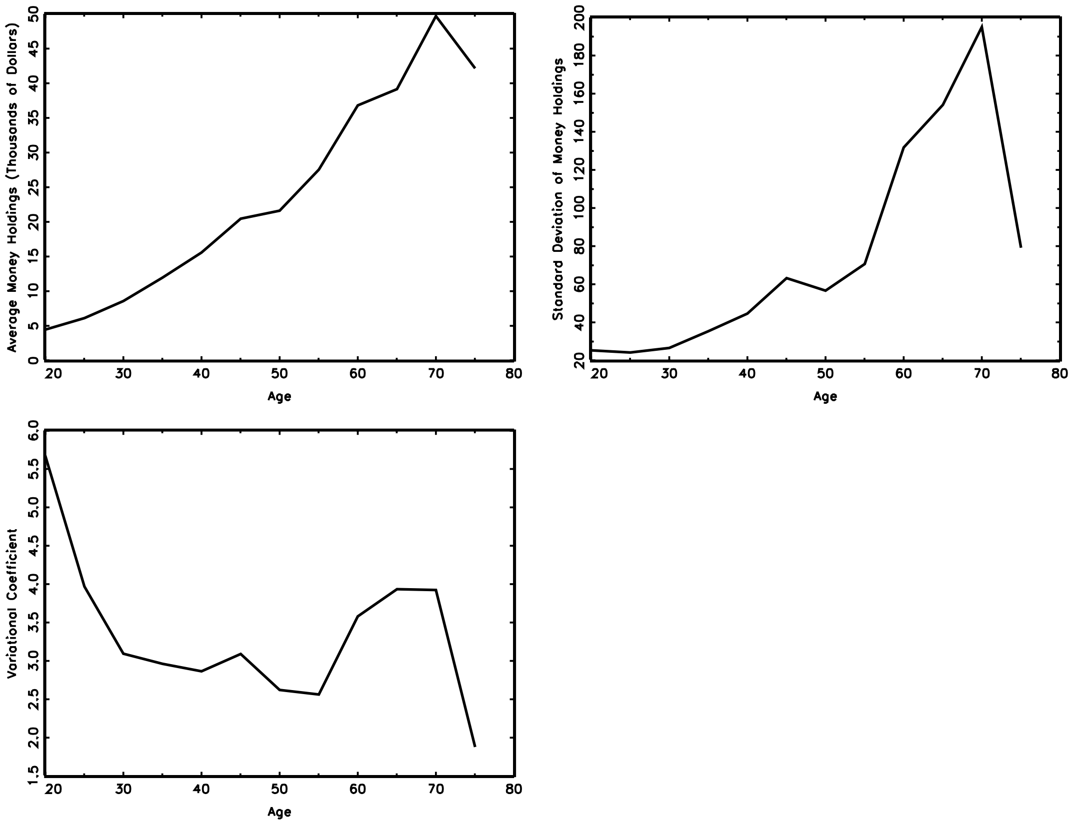

Figure 2.2: Money Balances over the Life Cycle

see that the results are quite robust to either definition of wealth.

We observe the following regularities:[5]

1. Money is much more concentrated than income or earnings and almost as unequally distributed as wealth, as shown in Figure 2.1.
2. Money M is only weakly correlated with income and capital.
3. Money holdings increase steadily over most of the life-cycle and only decrease at ages 75-80 so that a hump-shaped pattern emerges.
4. The standard deviation of money is hump-shaped as well. See the upper right panel in Figure 2.2.
5. The dispersion as measured by the coefficient of variation of money holdings has no obvious relation to age. See the lower left panel in Figure 2.2.

[5]The Appendix in Section 7 demonstrates that most of the findings reported below are not a feature of pooling but also emerge in the individual data sets.

Table 2.3

Regressions of Money on Income, Wealth and Age

Argument:	Y	Y^2	W	W^2	T	T^2	R^2
Coeff:	0.32	-0.00	0.01	0.00	-1.66	0.02	.14
	(6.72)	(-5.72)	(1.27)	(2.64)	(-4.58)	(6.06)	
Coeff:	0.33	-0.00	-0.00	0.00	-1.67	0.03	.12
	(6.16)	(-5.16)	(0.38)	(1.82)	(-4.58)	(5.99)	

Notes: to be added

6. When we regress money on income, income squared, capital, capital squared, age, and age squared, we find that money holdings increase with income (this relation is significantly hump-shaped) and decrease with age (this relation is u-shaped). This also holds, when we use total family wealth less money holdings as our definition of wealth. Yet, Table 2.3 also shows that these variables explain only a small share of the variation of money holdings over income and age groups.[6]

Fehler bei Paul: Table 3 statt 2

3 The Model

As mentioned above, we use a general equilibrium overlapping generations model with three different frameworks for money demand: the use of money as an argument in the household utility function, the device of differentiating cash and credit goods, and limited participation by households in the financial system (restricting their savings to deposits in the banking system).

Four sectors appear in the model: households, production, banking, and the government. Households maximize discounted life-time utility. Agents can save either with money or with capital. Individuals are heterogeneous with regard to their productivity and cannot insure against idiosyncratic income risk. Firms maximize profits. Output is produced with the help of labor and capital. The government collects taxes from labor and interest income in order to finance its expenditures on government consumption. The government also provides social security and controls the money supply. In the limited-participation model, banks receive deposits from households and lend them to firms. We restrict our analysis to steady-state behavior. For simplicity of notation we drop the time indices of our variables whenever appropriate.

[6]We also included the number of children in each household in the regressions but this variable was insignificant in the fixed effect method as well as year-by-year estimation.

3.1 Households

Every year a generation of equal measure is born. The total measure of all generations is normalized to one. As we only study steady-state behavior, we concentrate on the behavior of an individual h born in period 0. The first period of life is period $s = 1$. We use s to refer to the age of agent h. The total measure of all households is normalized to one.

Households live a maximum of T years. Lifetime is stochastic and agents face a probability ϕ_s of surviving up to age s conditional on surviving up to age $s-1$. During their first $R-1$ years, agents supply one unit of labor inelastically. After R years, retirement is mandatory. Workers are heterogeneous with regard to their labor earnings. Labor earnings $e(s, z_h)w$ are stochastic and depend on individual age s, idiosyncratic labor productivity z_h, and the wage rate w. After retirement, households receive pensions $b(\bar{e}_h)$ which depend on the average lifetime earnings $\bar{e}_h$ of the individual h. Furthermore, agents hold two kinds of assets, real money $m = M/P$ and capital k, where M and P denote nominal money and the price level, respectively. The household h is born without any capital: $k_{h1} \equiv 0$. In the money-in the utility function model, the first generation is endowed with a strictly positive amount of nominal money, $M_{h1} = \bar{M}_{h0}$.[7] Capital or, equally, equity k earns a real interest rate r. Parents do not leave altruistic bequests to their children. All accidental bequests are confiscated by the state.

The household h maximizes life-time utility:

$$\left[\sum_{s=1}^{T} \beta^{s-1} \left(\Pi_{j=1}^{s} \phi_s\right) u(\cdot),\right] \tag{3.1}$$

where β denotes the discount factor.

In our first case, we simply consider money in the utility function:

$$\text{case 1: } u(c, m) = \frac{\left(c^{\gamma} m^{1-\gamma}\right)^{1-\sigma}}{1-\sigma} \tag{3.2}$$

where c, m, and $\sigma > 0$ denote consumption, real-money balances, and the coefficient of relative risk aversion, respectively.[8]

In our second specification, consumers can purchase consumption with cash or credit as in Schreft (1992), Gillman (1993), or Dotsey and Ireland (1996). The consumption goods are indexed by $i \in [0, 1]$, and the consumption aggregate is given by

[7]Otherwise, the level of utility at age 1 is not well-defined. The calibration of $\bar{M}_{h0}$ is discussed in Section 4.

[8]We also considered a CES-index in consumption and real money balances, but found the results not superior to those implied by the Cobb-Douglas case considered in equation (3.2).

$c = \inf_i \{c(i)\}$. Therefore, the individuals will consume the same amount of all goods as in Schreft (1992). Utility $u(\cdot)$ is of the form

$$\text{case 2 and 3: } u(c) = \frac{c^{1-\sigma}}{1-\sigma}. \tag{3.3}$$

In order to buy an amount c of good i with credit, the household must purchase $\kappa(i)$ units of financial services. The function $\kappa(i)$ is strictly increasing in i, and satisfies $\lim_{i \to 1} \kappa(i) = \infty$. According to the latter assumption, some goods will be purchased with cash, and the demand for money is well defined. In particular, we follow Dotsey and Ireland (1996) and specify the transaction technology as:

$$\kappa(i) = \kappa_0 \left(\frac{i}{1-i} \right)^{\chi}. \tag{3.4}$$

According to this specification transactions costs are independent of the level of consumption $c(i)$ so that households with high consumption have relatively low transaction costs and will buy more goods with the help of credit than households with low consumption.

Intermediation of credit services is subject to perfect competition, and in order to produce one unit of service one efficiency unit of labor is used. In equilibrium, the financial service companies make zero profit, and the fees per unit of financial service sold is equal to the wage rate w.

The household of age s and type h will purchase a fraction $\zeta_{hs} \in [0, 1)$ of consumption goods with credit and faces the following cash-in-advance constraint on the remaining purchases:

$$\text{case 2:} \quad c_{hs}(1 - \zeta_{hs}) \leq m_{hs}. \tag{3.5}$$

In the third specification, households deposit part of the financial wealth at banks at the gross nominal interest Q. The firms pay wages to the households before they sell their output. To finance the wage bill, firms borrow money from the banking sector. The government injects the money into the banking sector. Crucially, banks receive the monetary transfer after households have made their deposits in the banking system.

Households hold nominal financial wealth $M_{hs} = D_{hs} + X_{hs}$ where D_{hs} is the amount deposited at banks and X_{hs} are money balances kept for the purchase of consumption goods. Since households receive wages before they go shopping, their cash-in-advance constraint is

$$c_{hs} \leq \begin{cases} x_{hs} + (1 - \tau_w - \theta) w_t e(s, z_h) & s < R, \\ x_{hs} + b(\bar{e}_{hs}), & s \geq R. \end{cases} \tag{3.6}$$

where $x_{hs} := X_{hs}/P$, τ_w, and θ denote real money balances, labor income taxes, and social security contributions, respectively. Furthermore, cash holdings cannot be negative, $x_{hs} \geq 0$.

The s-year old agent h receives income from capital k_{hs} and labor $e(s, z_h)w$ in each period s of his life. After retirement agents do not work, $e(s, z_h) = 0$ for $s \geq R$. The real budget constraint of the s-year old household h is given by:[9]

$$\begin{aligned}&(1-\tau_r)rk_{hs} + (1-\tau_w-\theta)we(s,z_h) + b(\bar{e}_{hs}) + tr + k_{hs} + m_{hs} \\ &= \begin{cases} c_{hs} + k_{hs+1} + \pi m_{hs+1} - Seign & \text{case 1} \\ c_{hs} + w\int_0^\zeta \kappa(c,i)\, di \quad + k_{hs+1} + \pi m_{hs+1} - Seign & \text{case 2} \\ c_{hs} - (1-\tau_r)(Q-1)d_{hs} + \Omega^B + k_{hs+1} + \pi m_{hs+1} & \text{case 3} \end{cases}\end{aligned} \tag{3.7}$$

where $Seign$ and $\pi = P_t/P_{t-1}$ denote seigniorage and the inflation factor between two successive periods $t-1$ and t, respectively.

Note that in the stationary equilibrium π is a constant and equals the money growth factor. In cases 1 and 2, households receive the seigniorage. In the limited participation model, the central bank injects the increase in the money supply into the banking sector, while households receive lump-sum profits from banks, Ω^B, and earn interest $Q-1$ on their real deposits d_{hs}. Real interest income is taxed at the rate τ_r.

In addition, the households receive transfers tr from the government. Social security benefits $b(s, \bar{e}_h)$ depend on the agent's age s as well as on an average of past earnings $\bar{e}_h$ of the household h. Following Huggett and Ventura (2000), social security benefits are composed of a lump-sum component and an earnings-related benefit:

$$b(s, \bar{e}_h) = \begin{matrix} 0 & s < R \\ b_0 + b_1(\bar{e}_{hs}) & s \geq R \end{matrix} \tag{3.8}$$

The function $b_1(\bar{e}_{hs})$ is described in more detail in Section 4.

3.2 Production

Firms are of measure one and produce output with effective labor N and capital K. Effective labor N is paid the wage w. In the case of the limited participation model, firms have to pay workers in advance and have to borrow wN at the nominal interest rate $Q-1$ in advance. Capital K is hired at rate r and depreciates at rate δ. Production Y is characterized by constant returns to scale and assumed to be Cobb-Douglas:

$$Y = F(K, N) = K^\alpha N^{1-\alpha}. \tag{3.9}$$

[9] At the end of the final period, $k_{hT+1} = M^h_{hT+1} \equiv 0$.

In a factor market equilibrium, factors are rewarded with their marginal product:

$$(1-\alpha)K^{\alpha}N^{-\alpha} = \begin{cases} w & \text{cases 1 and 2} \\ Qw & \text{case 3} \end{cases} \tag{3.10}$$

$$\alpha K^{\alpha-1}N^{1-\alpha} - \delta = r. \tag{3.11}$$

Consequently, profits are zero.

3.3 Banking Sector

In the limited participation model we also model a banking sector. At the beginning of period t banks receive deposits of size D_t from households. Government transfers the amount $M_{t+1} - M_t$ to the banks that are able to lend $D_t + M_{t+1} - M_t$ to firms. At the end of the period t they pay interest and principal QD_t to their creditors and distribute the remaining real profits Ω^B to the households:

$$\Omega_t^B = \frac{Q\left(D_t + M_{t+1} - M_t\right)}{P_t} - \frac{QD_t}{P_t} = Q\frac{M_{t+1} - M_t}{P_t}. \tag{3.12}$$

In a credit market equilibrium the supply of credit is equal to its demand:

$$w_t N_t = \frac{D_t + M_{t+1} - M_t}{P_t}. \tag{3.13}$$

3.4 Government

The government consists of the fiscal and monetary authority. Nominal money grows at the exogenous rate μ:

$$\frac{M_{t+1} - M_t}{M_t} = \mu. \tag{3.14}$$

In cases 1 and 2, seigniorage $Seign = \mu M/P$ is transferred lump-sum. In case 3, money is injected into the banking sector.

The government uses the revenues from taxing income and aggregate accidental bequests Beq in order to finance its expenditures on government consumption G, government transfers tr, and transfers to the one-year old households $\tilde{m}$:[10]

$$G + tr + \tilde{m} = \begin{cases} \tau_r rk + \tau_w N + Beq + Seign & \text{cases 1 and 2} \\ \tau_r rk + \tau_w N + Beq & \text{case 3} \end{cases} \tag{3.15}$$

[10]Following Heer and Süssmuth (2007), we assume that in case 1 the first-period money balances are financed by the government.

We assume that transfers tr are distributed lump-sum to all households. Furthermore, the government provides social security benefits $Pens$ that are financed by taxes on labor income:

$$Pens = \theta w N. \tag{3.16}$$

3.5 Stationary Equilibrium

The concept of equilibrium applied in this paper uses a recursive representation of the consumer's problem following Stokey, Lucas, and Prescott (1989). Let $\varphi_s(k, m, d, \bar{e}, z)$ and $V_s(k, m, d, \bar{e}, z)$ denote the measure and the value of the objective function of the s-year old agent with equity k, real money m, deposits d, average earnings $\bar{e}$, and idiosyncratic productivity level z, respectively. $V_s(k, m, d, \bar{e}, z)$ is defined as the solution to the dynamic program:

$$V_s(k, m, d, \bar{e}, z) = \max_{k', m', d', c} \{u + \beta \phi_{s+1} E \left[V_{s+1}(k', m', d', \bar{e}', z')\right]\} \tag{3.17}$$

subject to (3.7), (3.5) and (3.7), (3.6) and (3.7) in cases 1, 2, and 3, respectively. k', m', d', $\bar{e}'$, and z' denote the next-period value of k, m, d, $\bar{e}$, and z, respectively. Optimal decision rules at age s are functions of k, m, d, $\bar{e}$, and z, i.e. consumption $c_s(k, m, d, \bar{e}, z)$, next period deposits $d_{s+1}(k, m, d, \bar{e}, z)$, next-period capital stock $k_{s+1}(k, m, d, \bar{e}, z)$, and next-period real money balances $m_{s+1}(k, m, d, \bar{e}, z)$. In cases 1 and 2, deposits are zero, $d \equiv 0$. In case 2, the optimal share of cash goods also depends on the individual state variables, $\zeta_s = \zeta_s(k, m, \bar{e}, z)$.

We will consider a stationary equilibrium where factor prices, aggregate capital, and labor are constant and the distribution of wealth is stationary.

Definition
A stationary equilibrium for a given government policy $\{\tau_r, \tau_w, \theta, G, tr, b(\cdot), \mu\}$ is a collection of value functions $V_s(k, m, d, \bar{e}, z)$, individual policy rules $c_s(k, m, d, \bar{e}, z)$, $k' = k_{s+1}(k, m, d, \bar{e}, z)$, $m' = m_{s+1}(k, m, d, \bar{e}, z)$, $d' = d_{s+1}(k, m, d, \bar{e}, z)$, and $\zeta(k, m, \bar{e}, z)$, relative prices of labor and capital $\{w, r\}$, and distributions $(\varphi_1(.), \ldots, \varphi_T(.))$, such that:

1. Individual and aggregate behavior are consistent:

$$N = \sum_{s=1}^{T} \int_k \int_m \int_d \int_{\bar{e}} \int_z e(z,j)\varphi_s(k,m,d,\bar{e},z)\ dz\ d\bar{e}\ dd\ dm\ dk,$$

$$K = \sum_{s=1}^{T} \int_k \int_m \int_d \int_{\bar{e}} \int_z k\ \varphi_s(k,m,d,\bar{e},z)\ dz\ d\bar{e}\ dd\ dm\ dk,$$

$$C = \sum_{s=1}^{T} \int_k \int_m \int_d \int_{\bar{e}} \int_z c_s(k,m,d,\bar{e},z)\ \varphi_s(k,m,d,\bar{e},z)\ dz\ d\bar{e}\ dd\ dm\ dk,$$

$$Beq = \sum_{s=1}^{T} \int_k \int_m \int_d \int_{\bar{e}} \int_z (1-\phi_{s+1})\, a_{s+1}(k,m,d,\bar{e},z)\varphi_s(k,m,d,\bar{e},z)dz$$
$$d\bar{e}\ dd\ dm\ dk,$$

$$\frac{M}{P} = \sum_{s=1}^{T} \int_k \int_m \int_d \int_{\bar{e}} \int_z m\ \varphi_s(k,m,d,\bar{e},z)\ dz\ d\bar{e}\ dd\ dm\ dk,$$

$$\tilde{m} = \int_m \int_{\bar{e}} \int_z m\ \varphi_1(0,m,0,\bar{e},z)\ dz\ d\bar{e}\ dm,$$

where $a_{s+1}(k,m,d,\bar{e},z) \equiv k_{s+1}(k,m,d,\bar{e},z) + m_{s+1}(k,m,d,\bar{e},z) + d_{s+1}(k,m,d,\bar{e},z)$.

2. Relative prices $\{w,r\}$ solve the firm's optimization problem by satisfying (3.11) and (3.10).

3. Given relative prices $\{w,r\}$ and government policy $\{\tau_r, \tau_w, \theta, b(.), G, tr, \mu\}$, individual policy rules $c_s(\cdot)$, $k_{s+1}(\cdot)$, $m_{s+1}(\cdot)$, and $d_{s+1}(\cdot)$ solve the consumer's dynamic program (3.17).

4. The government budget (3.15) is balanced.

5. Social security benefits equal taxes:

$$\theta w N = Pens := \sum_{s=R}^{T} \int_k \int_m \int_d \int_{\bar{e}} \int_z b(\bar{e},j)\varphi_s(k,m,d,\bar{e},z)\ dz\ d\bar{e}\ dd\ dm\ dk. \tag{3.18}$$

6. Money grows at the exogenous rate μ.

7. The goods market clears:

$$K^\alpha N^{1-\alpha} = C + G + \delta K + TC. \tag{3.19}$$

In particular, transaction costs in the case 2 are a social cost:

$$TC = \sum_{s=1}^{T} \int_k \int_m \int_{\bar{e}} \int_z \left(\int_0^{\zeta(k,m\bar{e},z)} w\kappa\left(c_s(k,m,\bar{e},z),i\right) di \right) \varphi_s(k,m,\bar{e},z)\, dz\, d\bar{e}\, dm\, dk. \tag{3.20}$$

4 Calibration

Periods correspond to years. We assume that agents are born at the real lifetime age 20 which corresponds to $s = 1$. Agents work $R - 1 = 40$ years corresponding to a real lifetime age of 60. They live a maximum life of 60 years ($T = 60$) so that agents do not become older than the real lifetime age 79. The sequence of conditional survival probabilities $\{\phi_s\}_{s=1}^{59}$ is set in accordance with the age-specific death rates in the US in the year 2000. The data is taken from the United States Life Tables 2000 provided by the National Center of Health.[11] The survival probabilities almost monotonously decrease with age. For the final period of our model, we set the survival probability ϕ_{60} equal to zero.

Our numerical specification appears in Table 4.1. The calibration of the production parameters α and δ and the Markov process $e(s, z_h)$ is chosen in accordance with existing general equilibrium studies: Following Prescott (1986), the capital income share α is set equal to 0.36. The annual rate of depreciation is set at $\delta = 0.08$. Earnings are the product of real wage per efficiency unit times the labor endowment $e(s, z_h)$. The labor endowment process is given by $e(s, z_h) = e^{z_h + \bar{y}_s}$, where $\bar{y}_s$ is the mean lognormal income of the s-year old. The mean efficiency index $\bar{y}_s$ of the s-year-old worker is taken from Hansen (1993) and interpolated to in-between years. As a consequence, we are able to replicate the cross-section age distribution of earnings of the US economy. We also normalize the average efficiency index to one. The age-productivity profile is hump-shaped and earnings peak at age 50. Agents differ in log labor endowments at birth and there is no income mobility within an age cohort so that z_h is constant for all $s = 1, \ldots, R - 1$. We follow Huggett (1996) and choose a lognormal distribution of earnings for the 20-year old with $\sigma_{y_1} = 0.38$ and mean $\bar{y}_1$. As the log endowment of the initial generation of agents is normally distributed, the log

[11]See Table 1 in Arias (2002).

efficiency of subsequent agents will continue to be normally distributed. This is a useful property of the earnings process, which has often been described as lognormal in the literature. With our earnings specification, we come close to the earnings heterogeneity that is observed in US data. Henle and Ryscavage (1980) compute an earnings Gini coefficient for men of 0.42 in the period 1958-77. In our model the Gini coefficient is 0.36.

The social security payment $b(s, \bar{e}_h)$ is calibrated and parameterized in order to match the US Social Security System and exactly follows Huggett and Ventura (2000).[12] Average earnings $\bar{e}_{s,t}$ of the s-year old in period t accumulate according to:

$$\bar{e}_{s,t} = \begin{cases} \left(\bar{e}_{s-1,t-1}(j-1) + \min\{e(s, z_{ht})w_t, e_{max}\}\right)/j & \text{for } s < R-1 \\ \bar{e}_{s-1,t-1} & \text{for } s \geq R-1 \end{cases} \tag{4.1}$$

We note that in the US benefits depend on mean earnings that are indexed so that later contributions in life are not discounted. Furthermore, average earnings are only calculated for up to some maximum earnings level e_{max} which amounts to 2.47 times average earnings $\bar{E}$.[13]

Following Huggett and Ventura, we set the lump-sum benefit b_0 equal to 12.42% of GDP per capita in the model economy. Finally, benefits are regressive and a concave function of average earnings. Let $\bar{e}_h$ and $\bar{E}$ denote the average earnings of individual h and the average earnings of all workers, respectively. Depending on which earnings bracket the retired agent's average earnings $\bar{e}_h$ were situated, he received 90% of the first $20\% \cdot \bar{E}$, 32% of the next 104% of $\bar{E}$, and 15% of the remaining earnings $(\bar{e}_h - 1.24\bar{E})$ in 1994. Therefore, the marginal benefit rate declines with average earnings. The social security contribution rate θ is calibrated so that the budget of the social security balances. The remaining parameters of the government policy that we need to calibrate are the two tax rates τ_r and τ_w and government expenditures G. The two tax rates $\tau_r = 42.9\%$ and $\tau_w = 24.8\%$ are computed as the average values of the effective US tax rates over the time period 1965-88 that are reported by Mendoza, Razin, and Tesar (1994). The share of government consumption in GDP is $G/Y = 19.5\%$, which is equal to the average ratio of G/Y in the US during 1959-93 according to the Economic Report of the President (1994). The model parameters are presented in Table 4.1.

We choose the coefficient of risk aversion $\sigma = 2$.[14] The discount factor $\beta = 1.011$ is set

[12]For a more detailed description of this procedure please see Huggett and Ventura (2000).

[13]In the US Social Security System, only the 35 highest earnings payments are considered in the calculation of the average earnings. We simplify the analysis by using all 40 working years in our model.

[14]All our qualitative results also hold for the case $\sigma \in \{1, 4\}$.

equal to the estimate of Hurd (1989).[15] In case 1, the remaining parameter γ from the utility function is chosen to match the average velocity of money PY/M. During 1960-2001, the average annual velocity of M1 amounted to 6.0, while the average inflation rate was equal to 4.32%. We set $\gamma = 0.9787$ implying a velocity of money in our benchmark model without productivity mobility equal to 6.0 (for $\pi = 4.32\%$). The initial endowment with money is chosen so that $\bar{M}_{h0}/P_t$ is close to $\bar{M}_{h2}/P_t$, the optimal stock of money accumulated by the $s = 1$ year old households for their next period of life $s = 2$. In case 2, we follow Dotsey and Ireland (1996) and choose κ_0 and χ to match two statistics: the share of cash goods in total consumption expenditures and the semi interest rate elasticity of money demand. In particular, the stationary equilibrium of our model replicates a share of cash goods of 0.82 found by Avery et al. (1987) for the US in 1984 and the semi interest rate elasticity computed as in Dotsey and Ireland (1996), p. 38, equals 5.95 percent, which is the estimate of these authors from US data between 1959 and 1991. The computation of the model is described in the Appendix that is available from the authors upon request.

5 Findings

Figures 5.1, 5.2, and 5.3 display the age profile of assets (capital and money balances), consumption, and gross income generated by the money in the utility function model (MIUF for further reference), the costly credit model (CC), and the limited participation model (LP).

The consumption smoothing behavior is clearly discernible and common to all three models. Irrespective of the level of income – as governed by the exogenously specified time paths of productivity – the time path of consumption is hump-shaped, despite the sudden decline of gross income taking place at the age of retirement (see the lower left and lower right panels of Figures 5.1 through 5.3). Corresponding to the time path of consumption is the hump-shaped time path of interest bearing assets (capital in the MIUF and CC models, capital and bank deposits in the LP model) for the richer households ($j = 3, 4, 5$).

The upper right panels of Figures 5.1-5.3 reveal the consequences of the different motives to store money on the time profile of real money holdings. In the MIUF model, real money holdings are proportional to consumption and, thus, their time profile is also humped-shaped.

[15] Appendix 2 performs a sensitivity analysis for $\beta = 0.99$.

Table 4.1

Calibration of Parameters for the U.S. Economy

Description	Function/Parameter	Parameter Value
Utility Function	$u = \frac{\left(c^{\gamma} m^{1-\gamma}\right)^{1-\sigma}}{1-\sigma}$	$\sigma = 2.0$
		$\gamma = .9787$ (case 1)
		$\gamma = 1$ (cases 2,3)
Discount Rate	β	$\beta = 1.011$
Production	$Y = K^{\alpha} N^{1-\alpha}$.	$\alpha = .36$
Depreciation	δ	$\delta = 0.08$
Financial Services	$\kappa_0 \left(\frac{i}{1-i}\right)^{\chi}$.	$\kappa_0 = .1428, \chi = .1587$
Money Growth	μ	$\mu = .0432$
Income Tax	τ_r, τ_w	$\tau_r = .429, \tau_w = .248$
Government	G	$G/Y = .195$
Social Security:		
Max Earnings	$e_{\max}$	$e_{\max} = .2.47\overline{E}$
Lump Sum Benefit	b_0	$b_0 = .1241Y$
$b_1(\overline{\epsilon})$	Earning bracket	Marginal benefit
	$[0..2\overline{E}]$	.90
	$[.2\overline{E}, 1.24\overline{E}]$	.32
	$[1.24\overline{E} < e < e_{\max}]$	.15

Notes: to be added

In the CC model this only holds for the poorer households $j \in \{1, 2, 3\}$, since there are two opposing effects. For a given share of credit goods ζ_{hs}, the cash in advance constraint implies that real money balances mimic the age-profile of consumption and, thus, also display a humped-shaped pattern. However, as credit costs per unit of consumption decline with consumption, the share of credit goods increases so that less money is needed to facilitate a given level of consumption. This can be seen in Figure 5.2 for the richer households, but it holds also for the poorer households though it is hardly discernible in the plot. For the very rich households $j = 5$ the second effect dominates almost from the beginning of their life and real money balances decline with age. For the poorer households $j \in \{1, 2, 3\}$ this effect never dominates.

Bitte meine neuen references noch einmal ueberpruefen

In the LP model the time profile of cash balances is the mirror image of the time profile

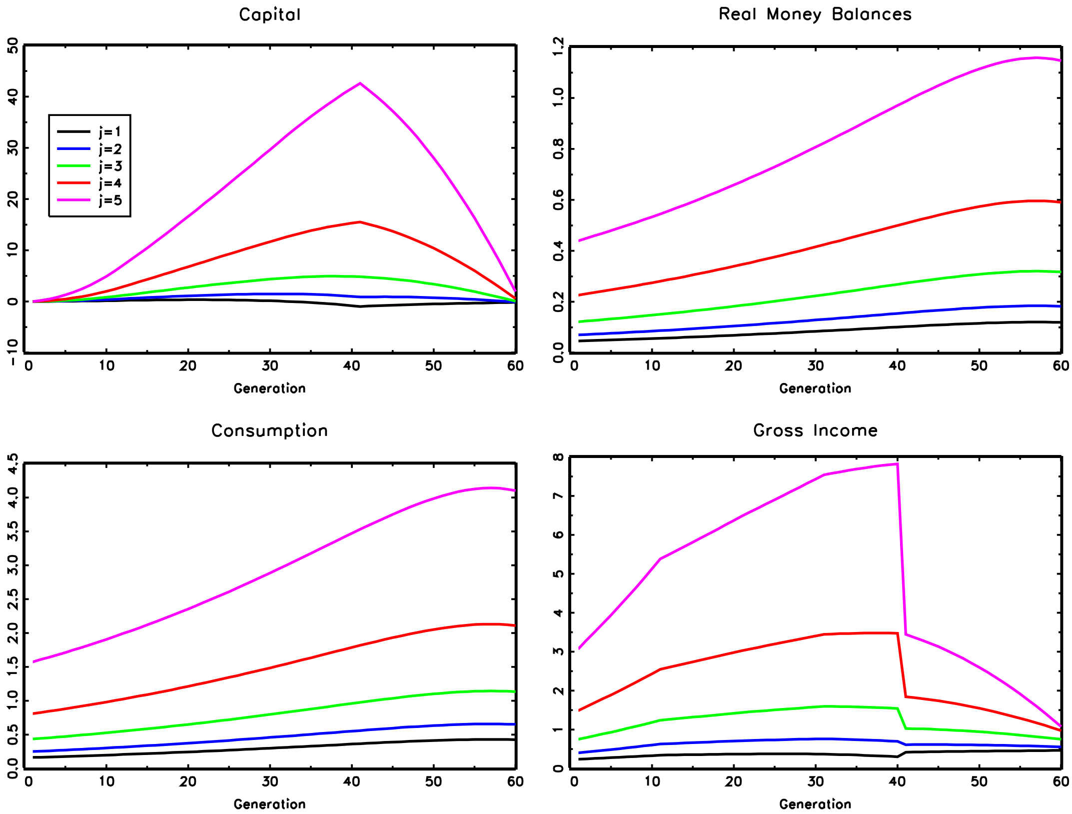

Figure 5.1: Assets, Consumption, and Gross Income in the MIUF Model

of gross income.[16] Households face a cash-in-advance constraint and can use money and labor income (or labor replacement income) in order to purchase consumption goods. At the age of retirement, labor income falls for all agents as pensions are much lower than labor income. The decline in labor income, however, is more pronounced for the high-income households because pensions also contain a considerable lump-sum component. In order to sustain consumption, especially the rich households must build up considerable cash balances at the age of retirement. On the other hand, the households with high productivity do not hold any cash balances at young age at all. Households that receive high wage income at young age save part of this income. The cash-in-advance constraint does not bind for this group in their youth, and, consequently, they do not hold any cash balances.

Table 5.1 presents the correlations between money holdings, gross income, and interest bearing assets implied by the three different models. The LP model comes close to

[16]Notice that we consider cash balances x rather than money balances $m = x + d$ in the LP model. In this model, the deposit holdings d of the individual households are indeterminate because, in equilibrium, the households are indifferent between the holdings of the two interest-bearing assets, deposits d and capital k.

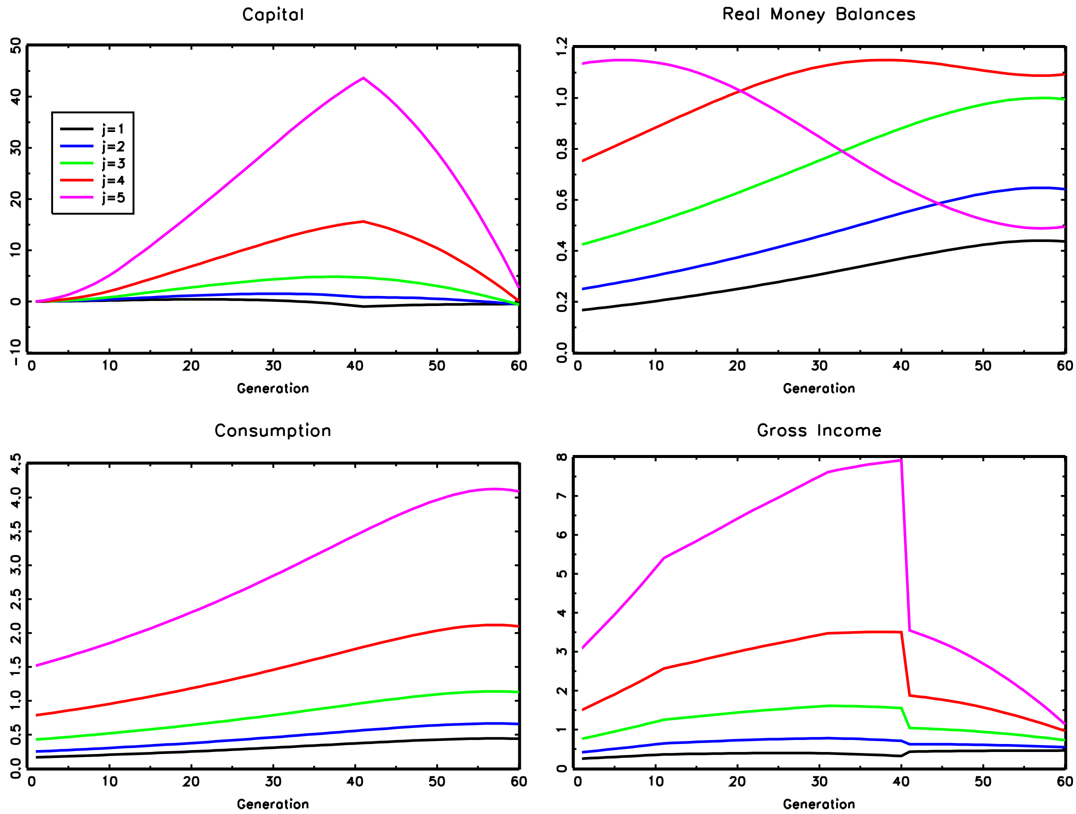

Figure 5.2: Assets, Consumption and Gross Income in the CC Model

reproduce the low correlation between income and money as well as between money and interest bearing assets found in the data.

In our three models, the households with the highest income (wealth) are the high-productivity agents at age s between 30 and 40 just prior to retirement (35 and 45 just prior and after retirement). As can be seen from Figure 5.6, these households with the highest income almost hold no cash in the LP model which reduces the correlation of income and money almost to zero (0.09) while we observe empirical correlations between 0.20-0.29. The strong correlation of money and wealth that we observe in the MIUF case (0.84) is also not present in the LP model (0.43) and is in much better accordance with empirical observations (0.07-0.29). Not only do the wealth-rich households (those with high productivity) at age 30-40 hold little cash, but also do the wealth poor households (those aged 50 and above) hold relatively large cash balances in the LP model. In the CC model, the rich households use credit rather than cash to finance consumption so that the money income correlation is also much lower, even though to a smaller extent than in the LP model. In addition, the model implies a correlation between money and capital stocks that is in line with the empirical

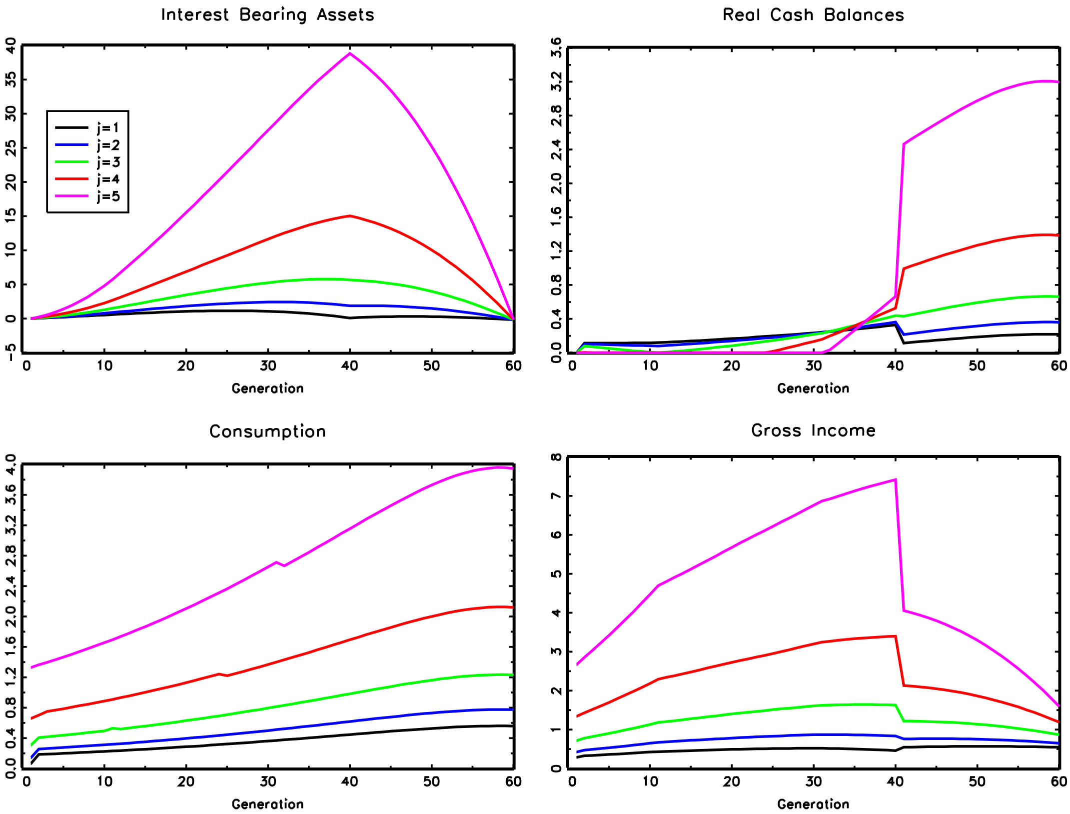

Figure 5.3: Assets, Consumption and Gross Income in the LP Model

correlations reported in Table 2.1. Yet, all three models predicts a much too strong association between gross income and interest bearing assets.

Figures 5.4-5.6 shed light on the intra-generational distribution of money holdings. With respect to average money holdings the MIUF and the LP model predict the hump-shaped profile found in the data while the CC model comes close to that pattern. The CC and the LP model imply no clear cut relation between the variational coefficient of money holdings and age, similar to the one observed in the PSID data sets for the years 1999, 2001, and 2005.

Table 5.2 pictures regression results with the model data, again with robust t-ratios in parentheses below each of the coefficients. The overall association between money holdings, gross income, interest bearing assets, and age predicted by our three models is much stronger than we observe for US households. The multiple correlation coefficient R^2 obtained from all three models is four to five times larger than the empirical magnitudes shown in Table 2.3. All three models are consistent with the empirically observed signs of the coefficients of income. The MIUF model also predicts the correct signs of the coefficients of age. Noch ein falscher Verweis bei Paul. Bitte pruefen.

Table 5.1
Correlations Implied by Simulated Models

Year	$\rho(M,Y)$	$\rho(M,W)$	$\rho(Y,W)$
MIUF	.67	.84	.77
CC	.42	.29	.76
LP	.09	.43	.83

Notes: to be added

Table 5.2
Regression Results with Simulated Data

Arg:	*cons*	Y	Y^2	W	W^2	T	T^2	R^2
MIUF	0.15	0.10	-.01	0.03	-.00	-0.02	0.00	.88
	(6.64)	(7.77)	(-8.84)	(8.81)	(-4.69)	(-8.20)	(8.71)	
CC	-.15	0.58	-0.06	-0.04	0.00	0.02	0.00	.71
	(3.61)	(24.04)	(-25.83)	(-3.93)	(0.06)	(4.52)	(1.25)	
LP	0.29	0.13	-0.07	0.11	-0.00	-.0.05	-0.00	.75
	(3.53)	(1.84)	(-9.88)	(5.47)	(-1.31)	(-7.30)	(-2.06)	

Notes: to be added

We offer two different explanations for this result. First, we might have neglected further characteristics besides age, income and capital in our model that help to explain the correlation of wealth and money. For example, we do not model housing. Second, due to the lack of data, we had to include assets like T-Bills in our empirical measure of money. In old age, agents might decumulate their savings in the form of shares and investment funds in order to switch their asset holdings to the relatively safe asset in the form of T-Bills.[17]

6 Conclusion

When we extend the infinitely-lived representative-agent framework with the money-in-the-utility, costly credit or limited participation models, we encounter many counterfactual implications for the money-age and cross-sectional money distribution. None of

[17]Campbell et al. (2001) derive this kind of optimal portfolio allocation on stocks and bonds over the life-cycle in an OLG model.

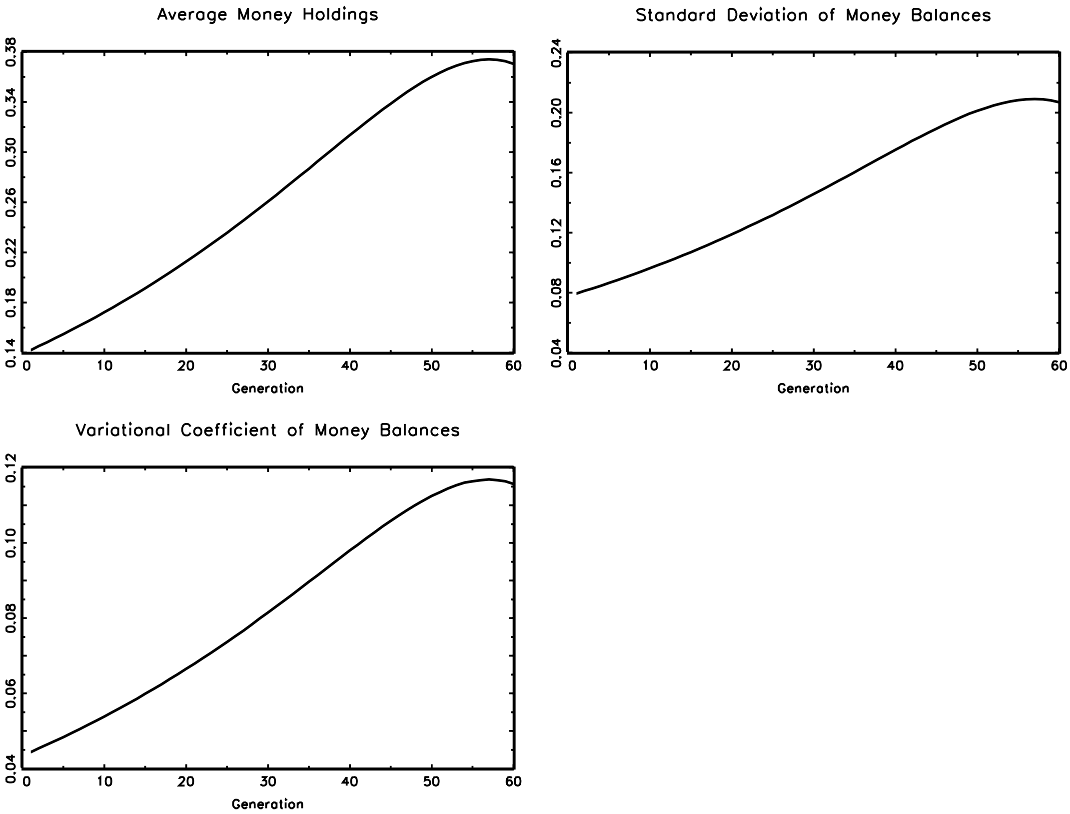

Figure 5.4: Distribution of Money in the MIUF Model

these models can reconcile their implications with empirical facts about the dispersion of money holdings as well as with cross-section correlations of money with income and wealth.

We conclude that our knowledge of the cross-section distribution of money is limited. Newer approaches are needed to explain the dispersion of money holdings over the life-cycle. One possible promising avenue of future research is the consideration of both idiosyncratic and aggregate risk. In a life-cycle model without money, Storesletten, Telmer, and Yaron (2007) show that non-tradable idiosyncratic risk has a substantial effect on asst market risk premiums and on the portfolio composition over the life-cycle holdings. In order to introduce aggregate risk and to model its effect on the risk-return properties of money adequately, nominal frictions have to be considered as well in our general-equilibrium model. An example for such a kind of a life-cycle model is Heer and Maußner (2011) who analyze the redistributive effects of unanticipated inflation in response to both a technology and a monetary shock. To extend their model by idiosyncratic income risk is likely to help us to better understand the money-holdings of individuals over the life-cycle.

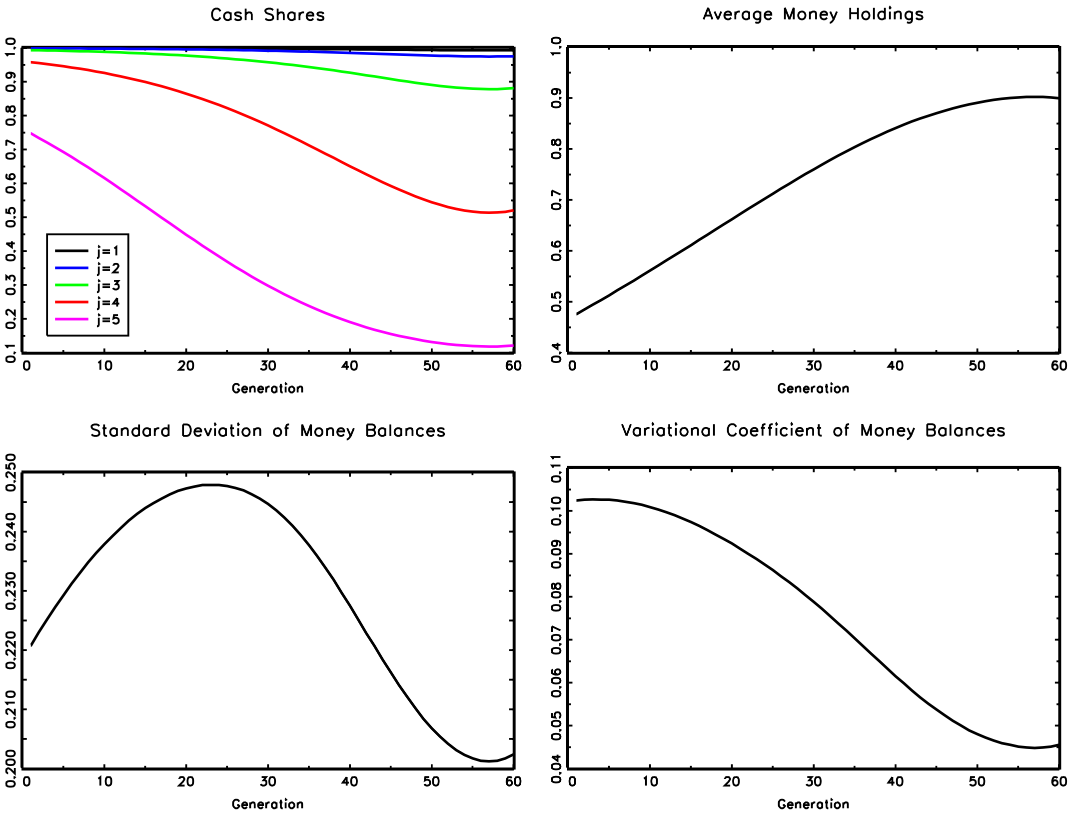

Figure 5.5: Distribution of Money in the CC Model

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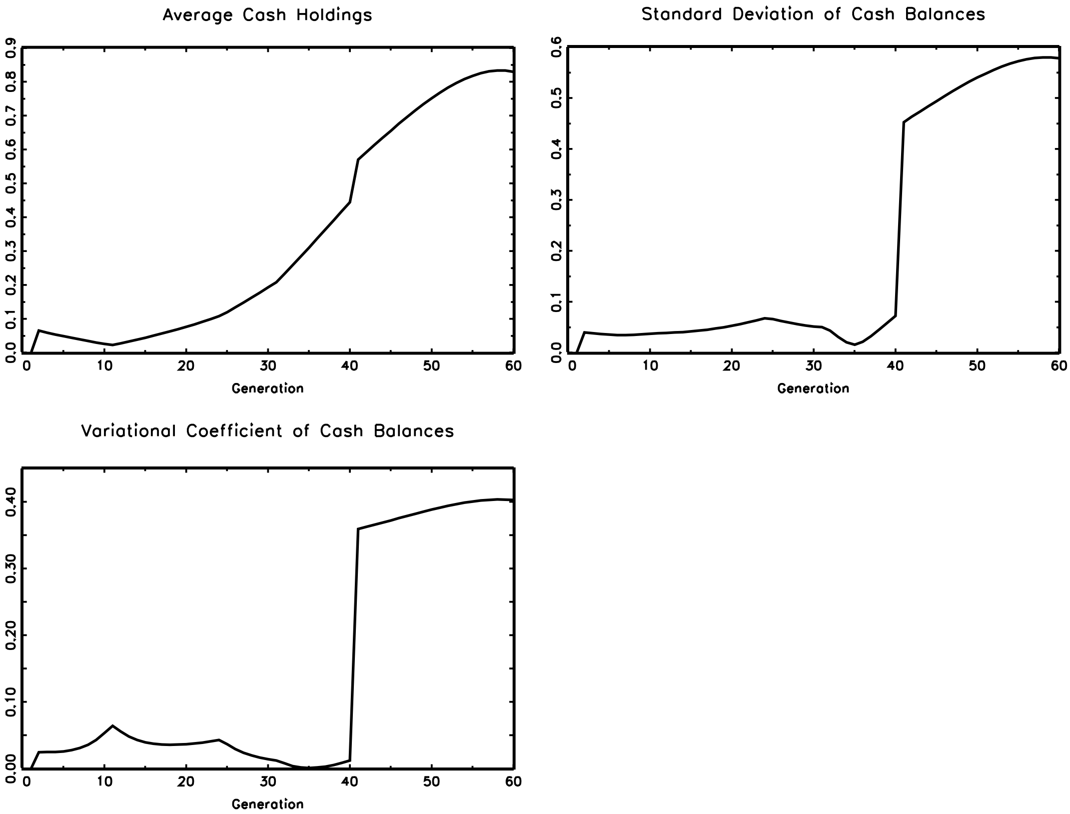

Figure 5.6: Distribution of Money in the LP Model

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7 Appendix 1

In Appendix 1, we provide further empirical evidence on the individual data sets.

7.1 Analysis of the PSID Data Sets: 1994, 1999, 2001, and 2005

Figures 7.1 through 7.4 display the relation between money holdings and age. The smooth curves are cubic polynomials fitted to the data to highlight possible trends.

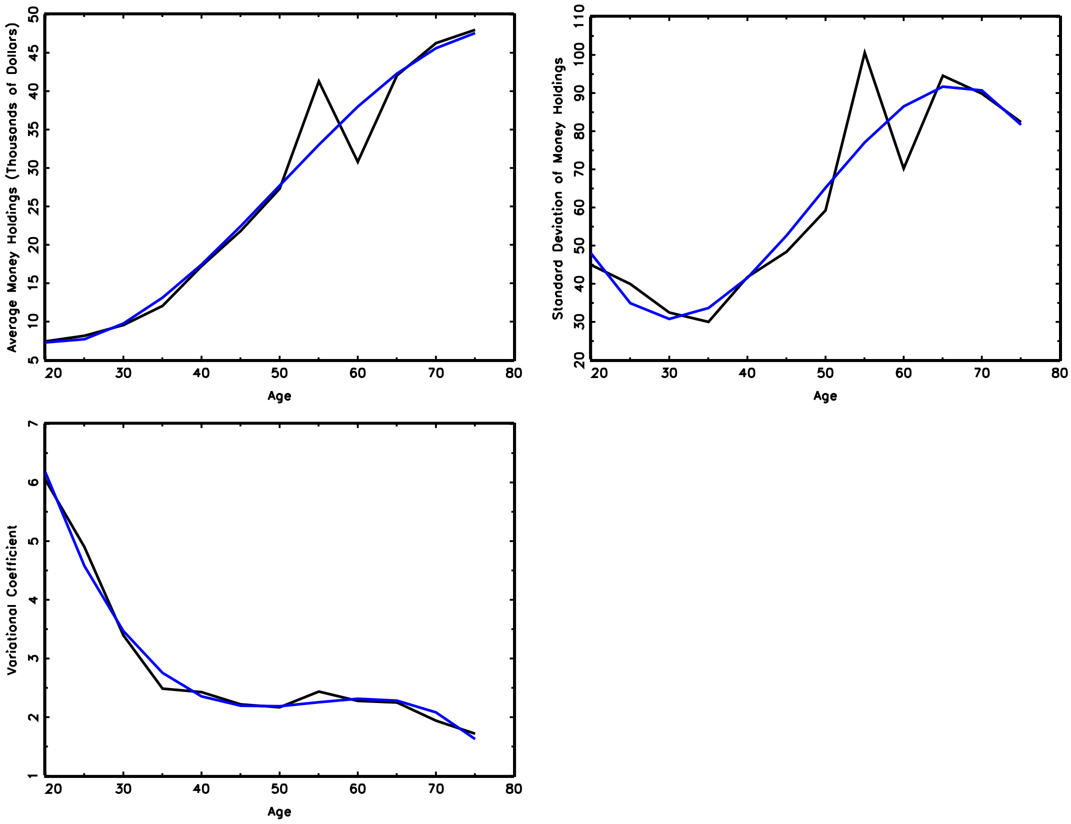

Figure 7.1: Money Balances over the Life Cycle, 1994

Table 7.1 contains the regression results using the individual data sets for each year, with robust t-ratios in parentheses.

8 Appendix 2: Sensitivity Analysis

$\beta = 0.99$

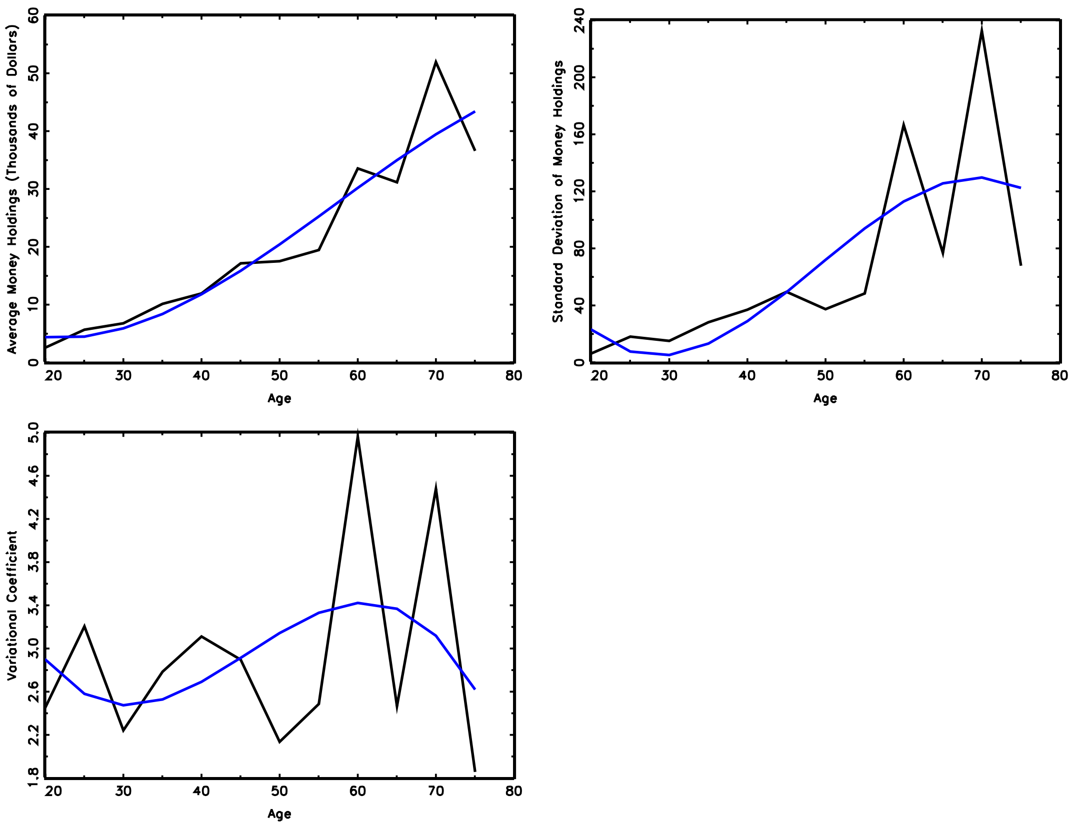

Figure 7.2: Money Balances over the Life Cycle, 1999

9 Appendix 3 (not for publication)

In Appendix 2 we provide more technical details on the solution and computational strategy we used to obtain our results.

9.1 Individual Productivity and Aggregate Labor

Let $\bar{y}_s$ denote the mean efficiency index of the s-year old worker. We approximate the productivity distribution among the members of generation $s = 1$ by the distribution of earnings for the 20-year old used by Huggett (1996). We discretize his distribution at $l = 5$ points $y_{h1}, h = 1, 2, \ldots, l$. Since there is no income mobility, we are able to index households with the index h. Thus, the productivity of household h at age s is given by $e_s z_h$, where $e_s = e^{\bar{y}_s}$ and $z_h = e^{y_{h1}}$. Let ψ_s denote the mass of generation s. We normalize the total mass of all generations to one, $\sum_{s=1}^{T} \psi_s \equiv 1$. Note that

$$\psi_{s+1} = \phi_s \psi_s.$$

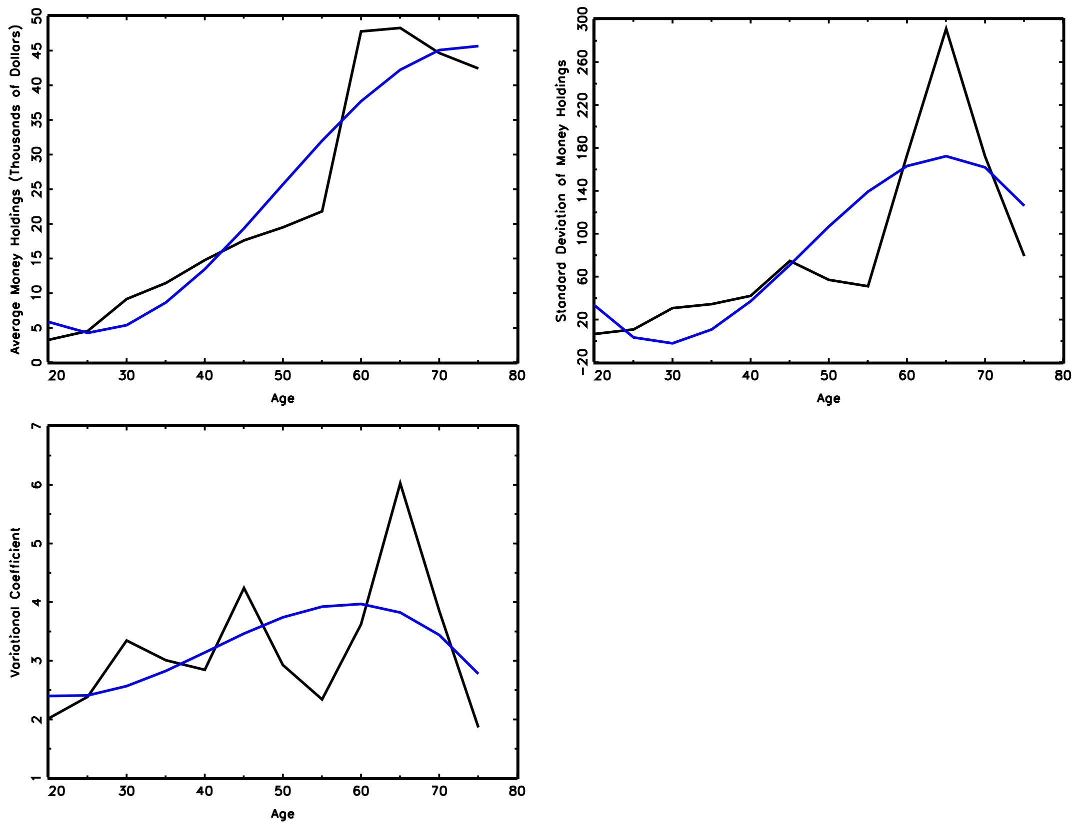

Figure 7.3: Money Balances over the Life Cycle, 2001

We use ν_h, $\sum_{h=1}^{l} \nu_h \equiv 1$ to denote the mass of households with productivity z_h. Since individual labor supply is exogenous and equal to $n_{hs} = 1$ for all $h = 1, \dots, l$ and $s = 1, 2, \dots R-1$, aggregate effective labor input N equals

$$N = \sum_{s=1}^{T} \sum_{h=1}^{l} \psi_s \nu_h e_s z_h. \tag{9.1}$$

Given the aggregate wage rate w, which is a constant in the stationary equilibrium of the model, we are able to compute the social security benefits of retired households. These benefits depend on the household's productivity parameter z_h but not on his age:

$$b_{hs} = \begin{cases} 0 & \text{for } s = 1, 2, \dots, R-1, \\ \bar{b}_h > 0 & \text{for } s = R, R+1, \dots, T. \end{cases} \tag{9.2}$$

This allows us to calibrate the social security tax rate θ from the knowledge of w alone:

$$\theta = \frac{Pens}{wN}, \quad Pens = \sum_{s=R}^{T} \sum_{h=1}^{l} \psi_s \nu_h \bar{b}_h. \tag{9.3}$$

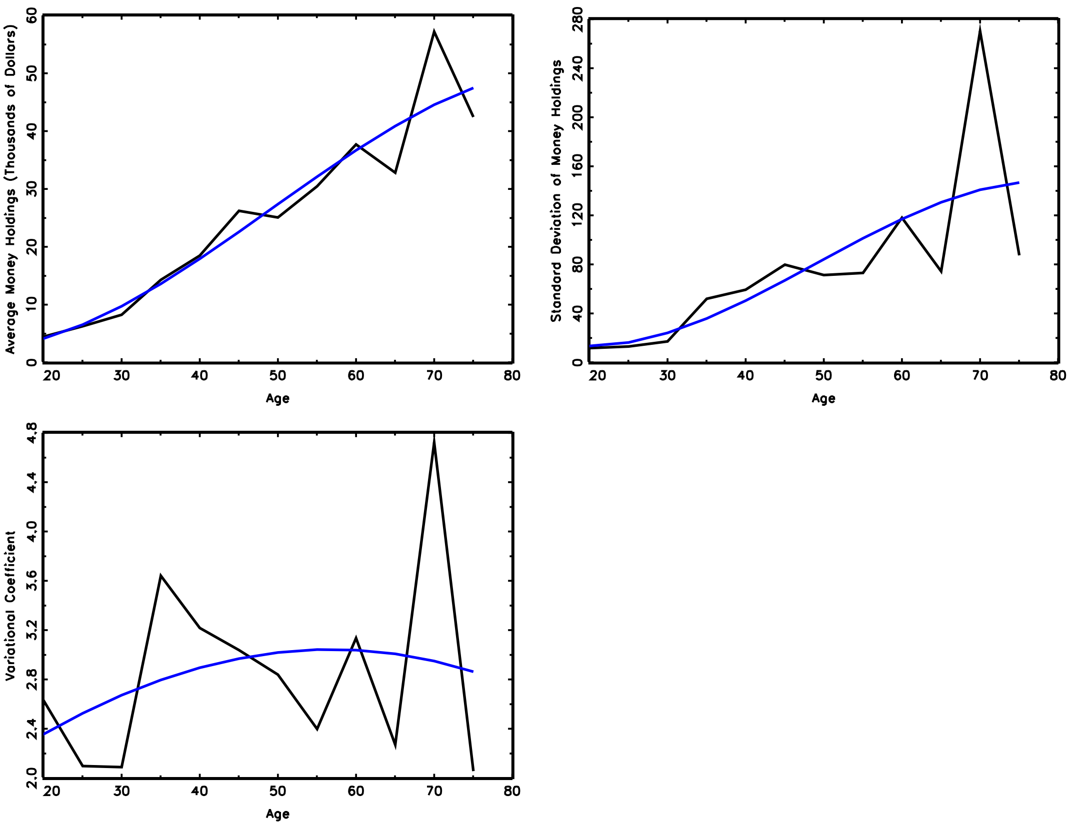

Figure 7.4: Money Balances over the Life Cycle, 2005

9.2 Money in the Utility Function

9.2.1 First order conditions

In the stationary solution the wage rate w, the real interest rate r, the inflation factor $\pi = 1 + \mu$, household labor supply $n \equiv 1$, government transfers tr, and social security payments $\bar{b}_h$ are independent of calendar time and exogenously given to household $h \in \{1, 2, \ldots l\}$. Since transfers are distributed lump sum and since the mass of all agents is one, aggregate equal individual transfers. The Lagrangian of the household's

Table 7.1
Regression of Money with PSID Data

Argument:	*cons*	Y	Y^2	W	W^2	T	T^2	R^2
1994	10.84	0.18	0.00	0.06	0.00	-0.91	0.02	.13
T-Ratio	(1.37)	(3.24)	(-0.88)	(5.54)	(-4.35)	(-2.31)	(3.75)	
1999	34.05	0.39	0.00	0.01	0.00	-2.61	0.03	.12
T-Ratio	(2.90)	(3.15)	(-3.77)	(1.09)	(-0.04)	(-2.92)	(3.30)	
2001	23.13	0.34	0.00	0.00	0.00	-.2.02	0.03	.14
T-Ratio	(2.83)	(4.68)	(-0.60)	(-0.36)	(1.91)	(-3.50)	(4.05)	
2005	11.29	0.41	0.00	-0.14	0.00	-1.55	0.02	.28
T-Ratio	(1.48)	(5.70)	(-4.99)	(-2.25)	(2.63)	(-2.91)	(3.78)	

Notes: to be added

decision problem at age $s = 1$ is given by

$$\pounds = \sum_{s=1}^{T} \beta^{s-1} \prod_{j=1}^{s} \phi_j \frac{c_{hs}^{\gamma(1-\sigma)} m_{hs}^{(1-\gamma)(1-\sigma)}}{1-\sigma} \tag{9.4}$$

$$+ \sum_{s=1}^{R-1} \beta^{s-1} \prod_{j=1}^{s} \phi_j \lambda_{hs} \begin{bmatrix} (1-\tau_w-\theta) w e_s z_h \\ +(1-(1-\tau_r)r) k_{hs} + Seign \\ +tr + m_{hs} - c_{hs} - \pi m_{hs+1} - k_{hs+1} \end{bmatrix}$$

$$+ \sum_{s=R}^{T} \beta^{s-1} \prod_{j=1}^{s} \phi_j \lambda_{hs} \begin{bmatrix} \bar{b}_h + (1-(1-\tau_r)r) k_{hs} \\ +tr + m_{hs} + Seign \\ -c_{hs} - \pi m_{hs+1} - k_{hs+1} \end{bmatrix},$$

where λ_{hs} denotes the Lagrange multiplier of the budget constraint effective at age s.

The first order conditions with respect to c_{hs}, k_{hs+1}, and m_{hs+1} are:

$$\frac{\partial \pounds}{\partial c_{hs}} = \beta^{s-1} \prod_{j=1}^{s} \phi_j \left[\gamma c_{hs}^{\gamma(1-\sigma)-1} m_{hs}^{(1-\gamma)(1-\sigma)} - \lambda_{hs} \right] = 0 \tag{9.5}$$

$$\frac{\partial \pounds}{\partial k_{hs+1}} = \beta^{s-1} \prod_{j=1}^{s} \phi_j \begin{bmatrix} -\lambda_{hs} + \beta \phi_{s+1} \lambda_{hs+1} (1 \\ +(1-\tau_r) r) \end{bmatrix} = 0 \tag{9.6}$$

$$\frac{\partial \pounds}{\partial m_{hs+1}} = \beta^{s-1} \prod_{j=1}^{s} \phi_j \begin{bmatrix} -\pi \lambda_{hs} \\ +\beta \phi_{s+1} \begin{pmatrix} (1-\gamma) c_{hs+1}^{\gamma(1-\sigma)} m_{hs+1}^{(1-\gamma)(1-\sigma)-1} \\ +\lambda_{hs+1} \end{pmatrix} \end{bmatrix} = 0 \tag{9.7}$$

These three equations can be reduced to Together with the household's budget constraint

$$m_{hs+1} = \frac{(1-\gamma)/\gamma}{\pi(1+(1-\tau_r)r)-1} c_{hs+1} \tag{9.8}$$

$$1 = \beta\phi_{s+1} \left(\frac{c_{hs+1}}{c_{hs}}\right)^{\gamma(1-\sigma)-1} \left(\frac{m_{hs+1}}{m_{hs}}\right)^{(1-\gamma)(1-\sigma)} (1+(1-\tau_r)r) \tag{9.9}$$

$$c_{hs} = (1-\tau_w-\theta)we_s z_h + b_{hs} + (1+(1-\tau_r)r)k_{hs} + tr + m_{hs} + Seign - \pi m_{hs+1} - k_{hs+1} \tag{9.10}$$

They form a system in $2(T-1)+T$ equations in the unknowns c_{hs}, $s=1,\ldots,T$, k_{hs+1}, and m_{hs+1}, $s=1,\ldots,T-1$.

9.2.2 Computational strategy

Suppose we are given individual capital stocks k_{hs}^0 and real money holdings m_{hs}^0, $h = 1,2,\ldots,l$, $s=2,\ldots,T$ as well as money transfers from the government to the newborn m_{h1}, $h=1,\ldots,l$. We compute new values in the following steps:

Step 1: N is given from (9.1). The aggregate stock of capital is

$$K = \sum_{s=2}^{T}\sum_{h=1}^{l} \psi_s \nu_h k_{hs}. \tag{9.11}$$

This allows us to compute the average wage rate w and the real interest rate r via equations (3.10) and (3.11). Furthermore, aggregate output is

$$Y = N^{1-\alpha}K^{\alpha}, \tag{9.12}$$

so that government's purchases of goods equal $G = 0.195Y$.

Step 2: Given w, equation (9.3) delivers θ and equations (4.1) imply $\bar{b}_h$.

Step 3: We compute government transfers. Before we are able to do so, we need to know seigniorage and aggregate bequests. The latter are given by

$$Beq = \sum_{s=1}^{T-1}(1-\phi_{s+1})\psi_s \sum_{h=1}^{l} \nu_h (k_{hs+1} + \pi m_{hs+1}), \tag{9.13}$$

and the former by

$$Seign = (\pi - 1) \sum_{s=2}^{T} \sum_{h=1}^{l} \psi_s \nu_h m_{hs}. \tag{9.14}$$

Thus, the government's budget constraint implies

$$tr = \tau_r r K + \tau_w w N + Beq + Seign - G - \psi_1 \sum_{h=1}^{l} \nu_h m_{h1}. \tag{9.15}$$

Step 4: Given this information we can solve equations (**??**) for new values of k_{hs+1} and m_{hs+1}. The fixed point of this mapping is the solution to our model.

We supply starting values for a non-linear equations solver from a simpler model without money. The individual capital stocks in this model can be found from solving a system of linear equations. We use individual consumption implied by this solution and equation (9.8) to compute m_{h1} and to initialize m_{hs+1}.[18]

9.3 Costly Credit

9.3.1 First order conditions

The credit costs of household h are given by

$$TC_{hs}(\zeta_{hs}) = w \int_0^{\zeta_{hs}} \kappa_0 \left(\frac{i}{1-i} \right)^{\chi} di \tag{9.16}$$

The derivation of this function with respect to ζ_{hs} is:

$$TC'_{hs}(\zeta_{hs}) = w\kappa_0 \left(\frac{\zeta_{hs}}{1-\zeta_{hs}} \right)^{\chi}. \tag{9.17}$$

It is obvious from the specification of (9.16) that the agent will never choose $\zeta_{hs} = 1$. Therefore, we only need to consider the case $\zeta_{hs} \in [0, 1)$. The Lagrangian of the

[18]Techniques to solve overlapping generations models are discussed in more detail in Heer and Maußner (2009), Chapter 9.

household is

$$\begin{aligned}
\pounds_h &= \sum_{s=1}^{T} \beta^{s-1} \prod_{j=1}^{s} \phi_j \frac{c_{hs}^{1-\sigma}}{1-\sigma} \\
&+ \sum_{s=1}^{R-1} \beta^{s-1} \prod_{j=1}^{s} \phi_j \lambda_{hs} \begin{bmatrix} (1-\tau_w-\theta) e_s z_h w + (1+(1-\tau_r)r) k_{hs} \\ +tr + m_{hs} + \\ Seign - TC_{hs}(\zeta_{hs}) - c_{hs} \\ -k_{hs+1} - \pi m_{hs+1} \end{bmatrix} \\
&+ \sum_{s=R}^{T} \beta^{s-1} \prod_{j=1}^{s} \phi_j \lambda_{hs} \begin{bmatrix} \bar{b}_h + (1+(1-\tau_r)r) k_{hs} \\ +tr + m_{hs} + \\ Seign - TC_{hs}(\zeta_{hs}) \\ -c_{hs} - k_{hs+1} - \pi m_{hs+1} \end{bmatrix} \\
&+ \sum_{s=1}^{T} \beta^{s-1} \prod_{j=1}^{s} \phi_j \left[\Gamma_{hs} \left(m_{hs} - (1-\zeta_{hs}) c_{hs} \right) + \Psi_{hs} \zeta_{hs} \right],
\end{aligned} \tag{9.18}$$

where Γ_{hs} and Ψ_{hs} denote the Lagrange multiplier of the cash-in-advance constraint and the non-negativity constraint on ζ_{hs}, respectively.

The first-order conditions with respect to the share of credit-goods are:

$$\begin{aligned}
\frac{\partial \pounds}{\partial \zeta_{hs}} &= \beta^{s-1} \prod_{j=1}^{s} \phi_j \left[-\lambda_{hs} TC'_{hs}(\zeta_{hs}) + \Gamma_{hs} c_{hs} + \Psi_{hs} \right] = 0, \\
0 &= \Psi_{hs} \zeta_{hs}, \\
0 &\le \Psi_{hs}, \\
0 &\le \zeta_{hs}.
\end{aligned} \tag{9.19}$$

Therefore, if $0 < \zeta_{hs} < 1, \lambda_{hs} TC'_{hs}(\zeta_{hs}) = \Gamma_{hs} c_{hs}$ and if $\zeta_{hs} = 0, \lambda_{hs} TC'_{hs}(\zeta_{hs}) \ge \Gamma_{hs} c_{hs}$.

The first-order condition for consumption is:

$$\frac{\partial}{\partial c_{hs}} = \beta^{s-1} \prod_{j=1}^{s} \phi_j \left[c_{hs}^{-\sigma} - \lambda_{hs} - (1-\zeta_{hs}) \Gamma_{hs} \right] = 0, \tag{9.20}$$

implying

$$c_{hs}^{-\sigma} = \lambda_{hs} + (1-\zeta_{hs}) \Gamma_{hs}. \tag{9.21}$$

The first-order condition with respect to k_{hs+1} implies

$$\lambda_{hs} = \beta \phi_{s+1} \lambda_{hs+1} (1 + (1-\tau_r) r), \tag{9.22}$$

and the first-order conditions with respect to m_{hs+1} derives from

$$\frac{\partial \pounds}{\partial m_{hs+1}} = \beta^{s-1} \prod_{j=1}^{s} \phi_j \left[-\pi\lambda_{hs} + \beta\phi_{s+1} \left(\lambda_{hs+1} + \Gamma_{hs+1}\right)\right] = 0, \tag{9.23}$$

which implies

$$\lambda_{hs} = (\beta/\pi)\phi_{s+1}(\lambda_{hs+1} + \Gamma_{hs+1}). \tag{9.24}$$

Combining (9.22) and (9.24) gives the following expression:

$$Q := \pi(1 + (1 - \tau_r)r) = \frac{\lambda_{hs+1} + \Gamma_{hs+1}}{\lambda_{hs+1}}. \tag{9.25}$$

Since the nominal interest factor Q is constant in the stationary solution this also implies

$$\Gamma_{hs} = (Q - 1)\lambda_{hs}, \quad s = 1, 2, \ldots, T. \tag{9.26}$$

Furthermore, if $Q > 1$ the cash-in-advance constraint binds for all $s = 1, 2, \ldots, T$. This allows us to write the following:

$$m_{hs} = (1 - \zeta_{hs})c_{hs}. \tag{9.27}$$

(9.21) and (9.26) can be combined to yield

$$c_{hs}^{-\sigma} = \lambda_{hs}(1 + (1 - \zeta_{hs})(Q - 1)). \tag{9.28}$$

Substituting for Γ_{hs} in (**??**) from (9.26) in turn yields

$$TC'_{hs}(\zeta_{hs}) = (Q - 1)c_{hs}. \tag{9.29}$$

In addition to (9.27), (9.28), and (9.29), the budget constraint must be satisfied:

$$\begin{aligned} c_{hs} &= (1 - \tau_w - \theta)e_s z_h w + b_{hs} \\ &\quad + (1 + (1 - \tau_r)r)k_{hs} + m_{hs} + tr + Seign \\ &\quad - TC_{hs}(\zeta_{hs}) - k_{hs+1} - \pi m_{hs+1}. \end{aligned} \tag{9.30}$$

9.3.2 Computational strategy

Suppose we have initial values for the households' stock of capital k_{hs}, $h = 1, 2, \ldots, m$, $s = 2, 3, \ldots, T$ and consumption c_{hs}, $h = 1, 2, \ldots, m$, $s = 1, 2, \ldots, T$. Our purpose is to set up a non-linear system of equations that can be solved numerically.

Step 1: This step is equivalent to Step 1 in the MIUF model. In addition to the variables computed there, we solve for Q from

$$Q = \pi(1 + (1 - \tau_r)r).$$

Step 2: Given w, Q and c_{hs} we can solve for ζ_{hs} from (9.29):

$$\frac{(Q-1)c_{hs}}{\kappa_0 w} = \left(\frac{\zeta_{hs}}{1-\zeta_{hs}}\right)^{\chi}. \tag{9.31}$$

Given this solution we can compute

$$m_{hs} = (1 - \zeta_{hs})c_{hs}. \tag{9.32}$$

Step 3: Since we now know m_{hs} we are able to compute seigniorage and aggregate bequests. From these magnitudes we derive the transfer payments via (9.15).

Step 4: We compute individual consumption from the agents' budget constraints and subtract the result from the given initial c_{hs}. This supplies mT equations in the unknown consumption vector. In doing so we use Gauss-Chebyshev integration to compute TC_{hs}.

Step 5: The further $m(T-1)$ equations in the unknown individual capital stocks are derived from (9.28):

$$\left(\frac{c_{hs+1}}{c_{hs}}\right)^{-\sigma} = \frac{\lambda_{hs+1}}{\lambda_{hs}} \frac{1 + (1-\zeta_{hs+1})(Q-1)}{1 + (1-\zeta_{hs})(Q-1)} 0 \tag{9.33}$$

Since

$$\frac{\lambda_{hs+1}}{\lambda_{hs}} = \frac{1}{\beta\phi_{s+1}(1 + (1-\tau_r)r)}$$

we get:

$$\beta\phi_{s+1}(1 + (1-\tau_r)r)\left(\frac{c_{hs+1}}{c_{hs}}\right)^{-\sigma} = \frac{1 + (1-\zeta_{hs+1})(Q-1)}{1 + (1-\zeta_{hs})(Q-1)}. \tag{9.34}$$

As starting values for consumption and capital we use the solution of the same model that we use to initialize the non-linear equations solver in the case of the MIUF-model.

9.4 Limited Participation

9.4.1 Aggregate relations

Money supply M_t grows at the constant rate μ. In the stationary equilibrium the price level P_t evolves according to $P_{t+1}/P_t = \pi = 1 + \mu$. Let Q_t denote the nominal interest factor, $w_t N_t$ the aggregate real wage bill, D_t the nominal aggregate amount of bank deposits and X_t the nominal aggregate level of money holdings. Total nominal lending of banks to firms is $D_t + (\pi - 1)M_t$ so that

$$w_t N_t = \frac{D_t + (\pi - 1)M_t}{P_t}. \tag{9.35}$$

The profits of banks amount to

$$\Omega_t = Q_t(\pi - 1)\frac{M_t}{P_t}. \tag{9.36}$$

Profit maximization of producers implies

$$Q_t w_t = (1 - \alpha)N_t^{-\alpha} K_t^{\alpha}, \tag{9.37a}$$

$$r_t = \alpha N_t^{1-\alpha} K_t^{\alpha-1} - \delta. \tag{9.37b}$$

In the stationary equilibrium of the model we can drop all time indices from the above equations. For further reference we define

$$\tilde{r} := (1 + (1 - \tau_r)r) > 1. \tag{9.38}$$

9.4.2 First order conditions

In the following we omit the index of the productivity type h as well as the index of calendar time t and consider the problem faced by an agent of age $s = 1$ who is born into a stationary environment. Lower case letters denote individual as opposed to aggregate variables. d_s and x_s are the agent's real bank deposits and real money holdings, respectively. Both are measured in terms of the current period price level so that πd_{s+1} and πx_{s+1} are bank deposits and money holdings acquired at age s and put aside for age $s + 1$.

The Lagrangian of the agent is:

$$
\begin{aligned}
\pounds =& \sum_{s=1}^{T} \beta^{s-1} \prod_{j=1}^{s} \phi_j \frac{c_s^{1-\sigma}}{1-\sigma} \\
&+ \sum_{s=1}^{R-1} \beta^{s-1} \prod_{j=1}^{s} \phi_j \begin{bmatrix} (1-\tau_w-\theta)e_s zw + \tilde{r}k_s \\ +tr + \Omega^B + x_s + \\ d_s(1-\tau_r)(Q-1) - c_s \\ -k_{s+1} - \pi(x_{s+1} + d_{s+1}) \end{bmatrix} \lambda_s \\
&+ \sum_{s=R}^{T} \beta^{s-1} \prod_{j=1}^{s} \phi_j \begin{bmatrix} \bar{b} + \tilde{r}k_s + tr \\ +\Omega^B + x_s + d_s + \\ d_s(1-\tau_r)(Q-1) - c_s \\ -k_{s+1} - \pi(x_{s+1} + d_{s+1}) \end{bmatrix} \lambda_s \\
&+ \sum_{s=1}^{R-1} \beta^{s-1} \prod_{j=1}^{s} \phi_j [\Gamma_s \begin{pmatrix} x_s + (1-\tau_w \\ -\theta)e_s zw - c_s \end{pmatrix}] \\
&+ \sum_{s=R}^{T} \beta^{s-1} \prod_{j=1}^{s} \phi_j \left[\Gamma_s \left(x_s + \bar{b} - c_s\right)\right] \\
&+ \sum_{s=1}^{T-1} \beta^{s-1} \prod_{j=1}^{s} \phi_j \xi_{s+1} x_{s+1},
\end{aligned}
\tag{9.39}
$$

where Γ_s and ξ_s denote the Lagrange multiplier of the cash-in-advance constraint and the non-negativity constraint on cash balances, respectively.

The first-order condition for consumption is:

$$
\frac{\partial \pounds}{\partial c_s} = \beta^{s-1} \prod_{j=1}^{s} \phi_j \left[c_s^{-\sigma} - \lambda_s - \Gamma_s\right] = 0 \tag{9.40}
$$

implying

$$
c_s^{-\sigma} = \lambda_s + \Gamma_s
$$

The first-order condition with respect to k_{s+1} implies

$$
\lambda_s = \beta \phi_{s+1} \lambda_{s+1} \tilde{r} \tag{9.41}
$$

Setting to zero the derivatives the Lagrangean with respect to x_{s+1} and d_{s+1} delivers:

$$\lambda_s = (\beta/\pi)\phi_{s+1}(\lambda_{s+1} + \Gamma_{s+1}) + (1/\pi)\xi_{s+1}$$
$$\lambda_s = (\beta/\pi)\phi_{s+1}\lambda_{s+1}\left(1 + (Q-1)(1-\tau_r)\right)$$

In addition, there are the slackness conditions of the cash-in-advance constraint,

$$0 \leq \Gamma_s \tag{9.42}$$
$$0 = \Gamma_s(x_s + (1 - \tau_w - \theta)e_s zw - c_s), \tag{9.43}$$
$$s = 1, ...R - 1 \tag{9.44}$$
$$0 = \Gamma_s(x_s + b - c_s), \tag{9.45}$$
$$s = R, \ldots, T \tag{9.46}$$

and of the non-negativity constraint on cash balances:

$$0 \leq \xi_{s+1}.s = 1, 2, \ldots, T - 1 \tag{9.47}$$
$$0 = \xi_{s+1}x_{s+1}, s = 1, 2, \ldots, T - 1 \tag{9.48}$$

Implications. Combining the first-order conditions gives

$$\tilde{r} = (1 + (1 - \tau_r)r) = (1 + (Q - 1)(1 - \tau_r))/\pi. \tag{9.49}$$

This condition implies that we are not able to solve for d_s. Therefore, we define the households interest bearing assets as

$$a_s := k_s + d_s/\pi.$$

This allows us to write the budget constraint as

$$
\begin{aligned}
a_{s+1} &= (1-\tau_w-\theta)e_s z w + \tilde{r} a_s + \Omega^B \\
&\quad + tr + x_s - \pi x_{s+1} - c_s, \\
&\quad s = 1,\ldots,R-1 \\
a_{s+1} &= \bar{b} + \tilde{r} a_s + \Omega^B + tr \\
&\quad + x_s - \pi x_{s+1} - c_s, \\
&\quad s = R,\ldots T
\end{aligned} \tag{9.50}
$$

Note that whenever the cash-in-advance constraint does not bind real cash balances will be zero. To see this, assume $x_{s+1} > 0$ and $c_{s+1} < (1-\tau_w-\theta)e_{s+1}zwn$ so that $\xi_{s+1} = \Gamma_{s+1} = 0$. This yields $\lambda_s = (\beta/\pi)\phi_{s+1}\lambda_{s+1}$ from (**??**). Yet, since $\pi\tilde{r} > 1$ this contradicts condition (**??**).

9.4.3 Government budget constraint

Let

$$
A = \sum_{s=1}^{T} \sum_{h=1}^{m} \psi_s \nu_h a_{sh} \tag{9.51}
$$

denote aggregate interest bearing assets. Aggregate income from capital taxation is given by

$$
Tax = \tau_r r K + \tau_r (Q-1)(D/P) + \tau_w w N \tag{9.52}
$$

where

$$
K = A - (D/P)/\pi \tag{9.53}
$$

Note, that we are not able to write the capital income taxes of this equation as a function of rA only since $\pi r \neq (Q-1)$. Aggregate bequests are given by

$$Beq = \sum_{s=1}^{T-1}(1-\phi_{s+1})\sum_{h=1}^{m}\psi_s \nu_h (a_{hs+1} + \pi x_{hs+1}) \tag{9.54}$$

Thus, aggregate transfers (which equal individual transfers tr since the total mass of all living agents is normalized to unity) are derived from

$$tr = Tax + Beq - G \tag{9.55}$$

9.4.4 Computational strategy

Suppose we have initial values for the aggregate stock of capital K_0, the aggregate level of real money balances $(M/P)_0$, the aggregate level of bequests Beq_0, the consumption of generation $s=1$, $c_{1h}, h=1,2,\ldots,m$, as well as the Lagrange multiplier of the first-year budget constraint $\lambda_{1h}, h=1,2,\ldots,m$. We derive new values for these variables in the following steps.

Step 1: Since N is only a function of given parameters, we are able to compute

$$Y = N^{1-\alpha}K^{\alpha}$$
$$G = gY$$

(where $g = 0.195$) as well as

- r via (**??**),
- Q via (9.19),
- w via (9.37a),
- D/P via (9.35),
- $\bar{b}_h$ via the pension scheme (4.1).

Given these variables we are in the position to compute tr as well as $\theta = Pens/(wN)$, where $Pens = \sum_{s=R}^{T}\sum_{h=1}^{l}\psi_s\nu_h\bar{b}_h$. Thus all variables that are exogenous to the individual budget constraint are known.

Step 2: We check the cash-in-advance constraint for generation $s = 1$: If $c_{1h} < (1-\tau_w-\theta)e_s z_h w$, $\Gamma_{1h} = 0$, else the cash-in-advance constraint applies: $c_{1h}-(1-\tau_w-\theta)e_s z_h w = 0$. This delivers m conditions for our $2m+3$ unknowns.

Step 3: We compute consumption, cash balances, and interest bearing assets for all generations. Given λ_{1h} we compute the sequence of Lagrange multipliers from (**??**). For each s we first assume $x_{sh} > 0$ (and, thus, that the cash-in-advance constraint binds). In this case we get

$$c_{sh} = \left(\frac{\pi\lambda_{s-1}}{\beta\phi_s}\right)^{-1/\sigma}.$$

If

$$\begin{aligned} c_{sh} &> (1-\tau_w-\theta)e_s z_h w, s = 2,\ldots,R-1 \\ c_{sh} &> \bar{b}_h, s = R,\ldots,T \end{aligned} \tag{9.56}$$

we compute the cash balances of the s year old household from

$$\begin{aligned} x_{sh} &= c_{sh} - (1-\tau_w-\theta)e_s z_h w, s = 2,\ldots,R-1 \\ x_{sh} &= c_{sh} - \bar{b}_h, s = R,\ldots,T \end{aligned}$$

If condition (9.56) does not apply, $x_{sh} = 0$ and $\Gamma_{sh} = 0$, we have

$$c_{sh} = \left(\frac{\lambda_{s-1}}{\beta\phi_s\tilde{r}}\right)^{-1/\sigma}.$$

Given consumption and cash balances, we compute interest bearing assets from the following equations:

For $s = 2,\ldots,R-1$,

$$\begin{aligned} a_{sh} = {} & (1-\tau_w-\theta)e_{s-1}z_h w + \tilde{r}a_{s-1h} + \Omega^B \\ & + tr + x_{s-1h} - \pi x_{sh} - c_{s-1h} \end{aligned}$$

and for $s = R,\ldots,T$,

$$\begin{aligned} a_{sh} = {} & \bar{b}_h + \tilde{r}a_{s-1h} + \Omega^B \\ & + tr + x_{s-1h} - \pi x_{sh} - c_{s-1h} \end{aligned}$$

The budget constraints of the T year old households imply m further conditions:

$$0 = \bar{b}_h + \tilde{r} a_{Th} + \Omega^B + tr + x_{Th} - c_{Th}.$$

Step 4: Finally, we compute the aggregate variables: Via (**??**) we compute aggregate wealth and – since $D/P = wN - (\pi - 1)(M/P)_0$ – we get $K_1 = A - (D/P)/\pi$. From (**??**) we get Beq_1. Furthermore

$$(M/P)_1 = D/P + \sum_{s=2}^{T} \sum_{h=1}^{m} \psi_s \nu_h x_{hs}.$$

Thus, we have these additional three equations:

$$0 = K_1 - K_0,$$
$$0 = (M/P)_1 - (M/P)_0,$$
$$0 = Beq_1 - Beq_0.$$

We solve this system with a non-linear equations solver using initial values on K, Beq, c_{1h}, and $\lambda_{1h} = c_{1h}^{-1/\sigma}$ from our baseline model without money.

To compute the agent's gross income, for $s = 1, \ldots, R-1$,

$$y_{hs} = (1 - \tau_w - \theta) e_s z_h w + \tilde{r} k_{hs} + (Q - 1)(1 - \tau_r) d_{hs} + \Omega^B + tr$$

while for $s = R, \ldots, T$,

$$y_{hs} = \bar{b}_h + \tilde{r} k_{hs} + (Q - 1)(1 - \tau_r) d_{hs} + +\Omega^B + tr$$

we assume $d_{hs} = (D/X) x_{hs}$ for all s and h.