Employment and Welfare Effects of a Two-Tier Unemployment Compensation System

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Abstract

In Germany, as in many OECD countries, such as the United Kingdom, unemployment compensation consists of unemployment insurance and unemployment assistance. Unemployment assistance is provided subsequent to the expiration of entitlement to unemployment insurance and is lower. The effects of this two-tier unemployment compensation system are studied in a general equilibrium job search model with endogenous distributions of income, wealth, and employment which is calibrated with regard to the characteristics of the German economy. Our results are as follows: (i) employment is a decreasing function of both unemployment insurance and unemployment assistance. (ii) Aggregate savings are (not) a monotone decreasing function of unemployment assistance (unemployment insurance) payments. (iii) Optimal unemployment compensation payments are found to be a decreasing function over time.

Keywords: unemployment compensation, search unemployment, general equilibrium, overlapping generations

JEL Code: J65, J64, E60, E62

1. Introduction

In Germany, like in many other OECD countries such as e.g., in the United Kingdom or France, the unemployment compensation system consists of two tiers: unemployment insurance (*Arbeitslosengeld,* henceforth UI) and unemployment assistance (*Arbeitslosenhilfe,* henceforth UA). Unemployed workers receive unemployment insurance if they have been employed for at least 12 months during the last 3 years. Furthermore, unemployment insurance is granted up to one year for agents aged 42 and younger, while it may last up to 3 years for agents aged above 42. After the expiration of unemployment insurance, unemployed workers are entitled to unemployment assistance, which is basically granted without any time limit. As an essential feature of the German unemployment compensation system, UA payments are below UI payments. Consequently, unemployment compensation of the typical unemployed worker is declining over time in Germany.

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The existing literature does not provide a unanimous answer to the question as to whether unemployment compensation should decline over time. In their seminal paper, Shavell and Weiss (1979) show that optimal unemployment compensation decreases monotonically over the unemployment spell. In their principal-agent model, the government provides unemployment insurance to the unemployed worker. The government, however, is unable to observe the worker's search effort, which determines the unemployed worker's probability to find a job and also causes disutility to the unemployed worker.[1] Fredriksson and Holmlund (2001) study the optimal sequence of unemployment compensation payments over the spell of unemployment in a general equilibrium model of search unemployment drawing on Pissarides (1990). Employment depends on the search effort of unemployed agents and on wages, which are determined in decentralized Nash bargains. Unemployment compensation consists of two tiers: unemployment insurance and unemployment assistance. Workers who lose their job are entitled to unemployment insurance; however, unemployed workers may lose their benefits with an exogenous probability subsequently and may have to rely on unemployment assistance which is lower. Fredriksson and Holmlund (2001) show that a two-tier system is superior to a system with infinite constant UI payments. In their numerical example, the percentage difference between UI payments and UA payments is substantial. In a model very similar to the one of Fredriksson and Holmlund (2001), Cahuc and Lehmann (2000) study the importance of a two-tier unemployment compensation system on wage formation.[2] As UI payments rise relative to UA payments, unemployment may rise if wages are renegotiable and, hence, if the relevant fall-back position for the worker is the state of insured unemployment.

The present paper is closely related to the work by Fredriksson and Holmlund (2001). Unemployment compensation also consists of two tiers. Both unobservable search effort and the wage are affected by the parameters of the unemployment compensation system. Similar to Fredriksson and Holmlund (2001), there are two opposing effects of a rise in UI payments relative to UA payments on employment: (i) unemployed workers not entitled to UI payments increase their search effort in order to find a job and be entitled to UI payments in the future. This effect is also known as the 'entitlement effect' in the literature (see Mortenson, 1977; and Hammermesh, 1979). (ii) The reservation wage of the worker increases and, consequently, employment falls as firms post fewer vacancies. However, unemployment compensation affects wage formation differently in our model than in the model of Fredriksson and Holmlund (2001) as we also introduce intertemporal consumption smoothing of the workers. In particular, the short-term unemployed worker is much more likely to remain unemployed in the next period than the employed worker is likely to lose his job. As a consequence, the short-term unemployed worker also saves a relatively large proportion of his current income for precautionary reasons. Therefore, the utility of the short-term unemployed worker and hence his reservation wage is less sensitive with regard to the level of UI payments (if accompanied by an offsetting decline in UA payments in order to keep government expenditures on unemployment compensation constant) as soon as one allows for consumption smoothing over time.[3]

The organisation of the paper is as follows. Section 2 introduces the model. Our model is an extension of Costain (1997). In Section 3, the model is calibrated with regard to characteristics of the German economy. In Section 4, our numerical results are presented. Section 5 concludes.

2. The Model

The model follows Costain (1997) with some modifications and extensions. Three sectors can be depicted: the household sector, the production sector, and the government. Households live for 60 years and maximize discounted lifetime utility. Agents can either be employed or unemployed during their working life. If unemployed, agents search for a job. Firms maximize the discounted sum of profits. They post vacancies, hire labor and capital, and pay out dividends to their shareholders. The government provides social insurance which it finances by a tax on wage income. In addition to Costain (1997), we adapt the model to the characteristics of the German unemployment compensation system.[4] In particular, if agents lose their job, they are entitled to unemployment insurance during the next two periods (corresponding to one year); after the expiration of unemployment insurance benefits, unemployed agents have to rely on unemployment assistance. Since we will only analyze steady-state allocations, the time index is omitted from stationary variables like the interest rate r or the wage rate w.

2.1. Households

Agents live for $T + T^R = 120$ periods (60 years). The first $T = 80$ periods (40 years), they are workers. They are either employed supplying one unit of labor and receiving wage w, which is taxed at the rate τ, or they are unemployed and search for a job with intensity s. The last $T^R = 40$ periods of their life, they retire and receive pension payments w_R. Households are of measure one and each generation is of equal measure 1/120. The household maximizes his intertemporal utility:

$$E \sum_{j=1}^{T+T^R} \beta^{j-1} \left[\frac{c_j^{1-\sigma} - 1}{1-\sigma} - D^W 1_{\epsilon_j=1} - D^S s_j \right], \tag{1}$$

where c_j and E denote the consumption of the j-period old and the expectation operator conditional on information at the beginning of age j, which contains the agent's employment status and his wealth. Instantaneous utility is discounted with the factor β, and σ denotes the coefficient of relative risk aversion. Working causes disutility D^W to the agent and $1_{\epsilon_j=1}$ is an index function which takes the value one if the employment status ϵ_j is equal to one at age j and zero otherwise. The employment status can take six different values $\epsilon \in \{1, 2, 3, 4, 5, 6\}$: 1) agents are employed and receive after-tax wage $(1 - \tau)w$, 2) agents are unemployed for the first period and were employed in the previous period and, hence, are entitled to unemployment insurance w_{UI}, 3) agents are unemployed for half a year and entitled to unemployment insurance this period and, in case of continuing unemployment, unemployment assistance w_{UA} next period, 4) agents are unemployed for at least one year, but have been employed before and hence receive unemployment assistance w_{UA}, 5) workers who have never found employment during their life have to rely on welfare payments w_W, and 6) retired agents who receive pensions w_R. Finally, D^S denotes the marginal disutility from searching. If agents are employed or retired, they do not search ($s_j = 0$ for $\epsilon_j = 1$ or $j > T$).

Agents are born without any assets. Furthermore, agents face a borrowing constraint, $a_j \geq -b_j$, where b_j is the discounted sum of the individual's minimum discounted lifetime income (consisting of welfare payments during the working life and pensions during retirement). Furthermore, we assume that private insurance markets are absent.[5] Depending on his employment status ϵ, an agent at age j receives labor income $y(\epsilon_j)$ and earns interest income at rate r:

$$a_{j+1} + c_j = (1+r)a_j + y(\epsilon_j) + tr. \tag{2}$$

where a_{j+1} and tr denote next period's asset holdings and government transfers, respectively. The labor income $y(\epsilon)$ of a household with employment status ϵ is given by:

$$y(\epsilon) = \begin{cases} (1-\tau)w & \epsilon = 1 \\ w_{UI} & \epsilon = 2, 3 \\ w_{UA} & \epsilon = 4 \\ w_W & \epsilon = 5 \\ w_R & \epsilon = 6. \end{cases} \tag{3}$$

The probability of employment in the first period is given by p_0. During their working life, agents lose employment with an exogenous probability δ. Unemployed agents looking for a job with search intensity s will find a job with probability $1 - e^{-\pi s^z}$, $z \in (0, 1)$.[6] For the individual agent, the probability parameter π and the probability of finding a job in the first period of life, p_0, are exogenous while they are determined endogenously in the labor market. The parameter z influences the response of the individual search effort to a change in policy. For low values of π and s, the individual probability to find a job is approximately equal to πs^z and z can be identified with the elasticity of the individual job finding probability with regard to the search effort s.

Let u, n, n_{UI}, n_{UA}, n_W, n_R, and $\phi(a, \epsilon, j)$ denote the unemployment rate, the number of people who are employed, entitled to unemployment insurance, unemployment assistance and welfare payments, the number of retired agents, and the measure of the j-period old people with wealth a and employment status ϵ, respectively, implying:

$$u \equiv \frac{n_{UI} + n_{UA} + n_W}{n + n_{UI} + n_{UA} + n_W} \tag{4}$$

$$n \equiv \sum_j \int_a \phi(a, 1, j)\, da \tag{5}$$

$$n_{UI} \equiv \sum_j \int_a (\phi(a, 2, j) + \phi(a, 3, j))\, da \tag{6}$$

$$n_{UA} \equiv \sum_j \int_a \phi(a, 4, j)\, da \tag{7}$$

$$n_W \equiv \sum_j \int_a \phi(a, 5, j)\, da \tag{8}$$

$$1 = \sum_j \sum_\epsilon \int_a \phi(a, \epsilon, j)\, da \tag{9}$$

$$\frac{T^R}{T + T^R} = \sum_j \int_a \phi(a, 6, j)\, da \equiv n_R. \tag{10}$$

In this paper, the 'short-term' unemployed workers are taken to mean the workers who are unemployed for less or equal to one year and who are entitled to unemployment insurance. Similarly, the 'long-term' unemployed workers are taken to mean the workers who are unemployed for more than one year and who are entitled to unemployment assistance. The number of short-term and long-term unemployed workers is equal to n_{UI} and n_{UA}, respectively.

2.2. Government

The government uses the revenues from taxing labor in order to finance its expenditures on social security and lump-sum transfers tr:

$$\tau w n = w_{UI} n_{UI} + w_{UA} n_{UA} + w_W n_W + w_R \frac{T^R}{T + T^R} + tr. \tag{11}$$

The government policy is characterized by the set $\Omega = \{\vartheta_{UI}, \vartheta_{UA}, \vartheta_W, \vartheta_R, \tau, tr\}$, where $\vartheta_x = \frac{x}{(1-\tau)w}$ denotes the replacement ratio of $x \in \{w_{UI}, w_{UA}, w_W, w_R\}$.

2.3. Firms

Firms are of measure one. They hire labor n and capital k taking the interest rate r as given. Let $V(n_t, k_t)$ denote the value function of the firm in period t:

$$V(n_t, k_t) = \max_{i_t, h_t} \left[A_0 k_t^\alpha n_t^{1-\alpha} - w_t n_t - h_t - i_t + \frac{1}{1+r} V(n_{t+1}, k_{t+1}), \right], \tag{12}$$

where i_t and h_t denote investment and the number of vacancies in period t, respectively. Every period, workers separate from employment with the exogenous probability δ:

$$n_{t+1} = q_t h_t + (1 - \delta) n_t. \tag{13}$$

The firm takes the hiring probability q_t as exogenous. The capital stock accumulates according to

$$k_{t+1} = i_t + (1 - \delta_k) k_t, \tag{14}$$

where the depreciation rate of capital is given by δ_k. In steady state, employment n, capital k, investment i, and hiring probability q are all constant, and the first-order conditions of the firm are described by:

$$r + \delta_k = A_0 \alpha k^{\alpha-1} n^{1-\alpha} \tag{15}$$
$$r + \delta = q(A_0(1-\alpha)k^{\alpha} n^{-\alpha} - w) \tag{16}$$
$$qh = \delta n \tag{17}$$
$$i = \delta_k k. \tag{18}$$

Furthermore, firms pay dividends $d = A_0 k^{\alpha} n^{1-\alpha} - wn - i - h$ to the asset owners.

2.4. Matching and Wage Determination

Labor markets are subject to frictions and are characterized by two-sided search. Unemployed agents search with intensity s, and firms post vacancies h. The steady state matching coefficients π and q are given by:[7]

$$\pi = \mu \frac{h}{\left(h^{1/\rho} + (n_{UI} + n_{UA} + n_W)^{1/\rho}\right)^{\rho}} \tag{19}$$
$$q = \frac{1}{h} \sum_{\epsilon, j} \int \phi(a, \epsilon, j)\left(1 - e^{-\pi s^{z}}\right) da. \tag{20}$$

Wages result from collective bargaining at the firm level. In particular, insiders who have been employed for at least one period and, therefore, are eligible for unemployment insurance bargain for all the workers belonging to the firm. The insiders are represented by an employed worker whose consumption is equal to the average consumption of all employed workers. Both the firm and the employed workers receive a rent from a successful match. The wage which results from a bargaining process depends both on the fall-back position of the firms and the fall-back position of the representative insider. The fall-back of the firm is given by zero production, while the fall-back of the representative insider is the state of insured short-term unemployment. Under these simplifying assumptions, the wage is an additively separable function of the marginal product of labor, $A_0(1-\alpha)k^{\alpha}n^{-\alpha}$, unemployment insurance w_{UI}, unemployment assistance w_{UA}, the disutility from working, D^W, and the marginal disutility from searching, D^S:

$$w = \psi_1 A_0(1-\alpha)k^{\alpha}n^{-\alpha} + \psi_2 w_{UI} + \psi_3 w_{UA} + \psi_4 D^W - \psi_5 D^S \tag{21}$$

where the parameters $\psi_i > 0$, $i = 1, \ldots, 5$, are functions of the endogenous variables π, τ, r, c, and s and are derived in the appendix. The partial equilibrium effect of labor market tightness on the wage behaves as expected. In particular, for our calibration, an exogenous rise in the ratio of vacancies to unemployed agents, $h/(n_{UI} + n_{UA} + n_W)$, results in an increase of the matching coefficient π and the insiders bargain for a higher wage w.

2.5. Stationary Equilibrium

The concept of equilbrium applied in this paper uses a recursive representation of the consumer's problem following Stokey, Lucas and Prescott (1989). Let $W(a, \epsilon, j)$ be the value of the objective function of a j-period old agent with beginning-of-period asset holdings a and employment status ϵ. $W(a, \epsilon, j)$ is defined as the solution to the dynamic program:

$$W(a, \epsilon, j) = \max_{c,a',s} \left[\frac{c^{1-\sigma} - 1}{1 - \sigma} - D^W 1_{\epsilon=1} - D^S s + \beta E\{W(a', \epsilon', j + 1)\} \right], \tag{22}$$

subject to the budget constraint (2). E, again, denotes the expectation operator conditional on information at age j, while a' and ϵ' are the next-period wealth and employment status, respectively.

Definition. A Stationary Equilibrium for a given set of government policy parameters $\Omega = \{\vartheta_{UI}, \vartheta_{UA}, \vartheta_W, \vartheta_R, tr, \tau\}$ is a collection of value functions $W(a, \epsilon, j)$ of the households and $V(n, k)$ of the firms, individual policy rules $c(a, \epsilon, j)$, $s(a, \epsilon, j)$, and $a'(a, \epsilon, j)$, age-dependent, time-invariant measures of agent types $\phi(a, \epsilon, j)$ for each age $j = 1, 2, \ldots, T + T^R$, relative prices of labor and capital $\{w, r\}$, such that:

1. Given relative prices $\{w, r\}$ and the government policy Ω, the individual policy rules $c(.)$, $s(.)$, and $a'(.)$ solve the consumer's dynamic progam (22) and firms maximize profits (12) with respect to investment i and vacancies h.
2. The goods market clears:

$$A_0 k^\alpha n^{1-\alpha} = \sum_{\epsilon,j} \int_a c(a, \epsilon, j)\phi(a, \epsilon, j)\, da + i + h. \tag{23}$$

3. Households hold equity of the firms. The interest earnings by the households on the assets are equal to the dividend payments d by the firms:

$$r \sum_{\epsilon,j} \int_a \phi(a, \epsilon, j) a\, da = d = A_0 k^\alpha n^{1-\alpha} - wn - h - i. \tag{24}$$

4. In each period, $\int_a \phi_T(a, 1, T)\, da$ agents retire from work and the fraction $1 - \delta$ of these jobs is inherited by the newborn generation, implying:

$$p_0 = \frac{(1 - \delta) \int_a \phi(a, 1, T)\, da}{1/120}. \tag{25}$$

5. Wages w result from decentralized bargains according to equation (21).

6. The number of total matches formed in the labor market is equal to the number of job outflows:

$$qh = \sum_{\epsilon,j} \int_a \phi(a,\epsilon,j)\left(1 - e^{-\pi s(a,\epsilon,j)^{\varsigma}}\right) da = \delta n. \tag{26}$$

7. Finally, the government budget (11) is balanced.

3. Calibration

The steady state distribution of wealth, search effort, and employment and the effects of a change in the unemployment compensation system on welfare and distribution cannot be studied analytically but only numerically. For this reason, the model is calibrated in order to match characteristics of the German economy after unification. If not mentioned otherwise, the time series data refer to the period 1991–1997.[8] The following parameters need to be calibrated: σ, β, D^W, D^S, the replacement ratios of welfare payments ϑ_W and pensions ϑ_R, tr, μ, δ, δ_k, α, z, ρ, and λ.[9] We also choose the replacement ratios of unemployment insurance and unemployment assistance, ϑ_{UI} and ϑ_{UA}, for the benchmark case. The remaining variables are computed endogenously in our model.

3.1. Households

In our benchmark case, we will assume a coefficient of relative risk aversion $\sigma = 2$.[10] Targeting an annual real interest rate of approximately $r = 4\%$, the household's discount factor is set equal to $\beta = 0.996$. The calibration of the model's parameters is summarized in Table 1.

3.2. Government

The government provides unemployment compensation. Previous estimates of German replacement ratios, henceforth defined as the ratio of net benefits received in unemployment to net earnings received at work, vary considerably. According to a Centre d'Etude des Revenues et des Coûts (CERC) study of European unemployment programs cited by Burtless (1987), replacement ratios drop from 66% to 56% from the first to the second year of unemployment for the agent who earned the average wage or two-thirds of the average wage. For the agents who earned twice the average wage, these numbers fall to 51% and 44%, respectively.[11] Steiner (1997) computes the replacement ratio with respect to expected net wage yielding a mean value of 50% for those receiving unemployment insurance between 1983 and 1994. In accordance with the latter two studies, the replacement ratios of unemployment insurance and unemployment assistance are set equal to 50% and 40% in the benchmark case, respectively, but a sensitivity analysis of w_{UI} and w_{UA} is

Table 1. Calibration of parameter values for Germany 1991–1997.

Description	Function	Parameter
Utility function	$U_t = \frac{c^{1-\sigma}-1}{1-\sigma} - D^W 1_{\epsilon=1} - D^S s$	$\sigma = 2$, $D^W = 0.15$, $D^S = 0.10$
Discount factor	β	$\beta = 0.996$
Production function	$y = A_0 k^\alpha n^{1-\alpha}$	$\alpha = 0.35$, $A_0 = 1$
Depreciation	δ_k	$\delta_k = 0.02$
Job separation	δ	$\delta = 0.05$
Matching function	$\pi = \mu \frac{h}{(h^{1/\rho}+(n_{UI}+n_{UA}+n_W)^{1/\rho})^\rho}$	$\rho = 0.78$, $\mu = 1.0$
	$1 - e^{-\pi s^z}$	$z = 0.4$
Bargaining power	λ	$\lambda = 0.5$
Unemployment insurance	w_{UI}	$\vartheta_{UI} \equiv w_{UI}/(1-\tau)w = 50\%$
Unemployment assistance	w_{UA}	$\vartheta_{UA} \equiv w_{UA}/(1-\tau)w = 40\%$
Welfare payments	w_W	$\vartheta_W \equiv w_W/(1-\tau)w = 30\%$
Pension payments	w_R	$\vartheta_R \equiv w_R/(1-\tau)w = 50\%$
Transfers	tr	$tr = 0$

performed in Section 4 as well. Welfare payments are computed from the ratio of average current expenditures on welfare, multiplied by a factor of 1.5 in order to correct for rent and health insurance payments provided by the local government, to net average income of a production worker. The welfare payments replacement ratio is set equal to 30%.[12]

Public pension payments are equal to 50% of after-tax wages. This value is in line with previous studies from Fehr (1997) and Chauveau and Loufir (1997). Transfers *tr* are set equal to zero in the benchmark case.[13] The income tax rate τ is calculated endogenously from the government budget (11) and amounts to 24.8%.

3.3. Production

The production parameters are taken from Heer and Linnemann (1998). The production elasticity of capital is set equal to $\alpha = 0.35$. A_0 is normalized to one. The half-annual depreciation in Germany amounts to $\delta_k = 0.02$.

3.4. Labor Market and Disutility from Labor and Searching

The paramaters of the matching functions $z = 0.4$ is taken from Costain (1997), while $\rho = 0.78$ is taken from den Haan, Ramey and Watson (1997). The bargaining power of workers λ is set equal to 0.5. The parameters describing the disutility from working and searching, D^W and D^S, together with the separation probability δ and the matching parameter μ are chosen simultaneously in order to match the following empirical regularities as closely as possible: (i) the unemployment rate amounts to 10.1% on average during 1991–1997, (ii) the unemployed workers, on average, find a new employment within half a year (one period) with a probability equal to 42%,[14] (iii) a labor income share equal to 64%, and (iv) the number of short-term unemployed workers relative to the number of long-term unemployed workers, who are unemployed for more than one year, is approximately equal to 2.30.[15] Furthermore, we demand that the average disutility from searching does not exceed the disutility from working. For our calibration reported in Table 1, we get (i) an unemployment rate u equal to 10.2%, (ii) an average annual job finding probability of the unemployed workers equal to 45.5%, (iii) a labor income share of 60.7%, and (iv) a ratio $n_{UI}/n_{UA} = 2.38$. The average disutility from searching is slightly less than the disutility from working. The calibrated half-annual job separation probability $\delta = 0.05$ matches the empirical annual flow from employment into unemployment relative to total employment quite closely: the empirical value of the annual probability over the period 1991–1997 is equal to 0.099.[16]

4. Results

In this section, the effects of a two-tier unemployment compensation system on employment and welfare are presented. First, equilibrium properties of the benchmark case are illustrated. Second, a change in the benefit level of both unemployment insurance and unemployment assistance is analyzed. Employment and welfare effects are demonstrated to be considerable.

4.1. Equilibrium Properties

In this subsection, we study the properties of the benchmark equilibrium which is characterized by the parameterization as presented in Table 1. The behavior of the households depends on wealth, the employment status, and age. In particular, consumption of the workers is a monotone increasing function of asset holdings (not illustrated) and labor income. Average consumption within cohorts is an increasing function of age (not illustrated) as the subjective half-annual discount rate of the households $1/\beta - 1 = 0.4\%$ is lower than the half-annual interest $r = 2.05\%$.

The half-annual wealth-income ratio is $k/y = 8.71$, which is in the proximity of empirical values of the capital-output ratio observed in Germany. In the years 1991–1997, the ratio of the capital stock to annual GDP was equal to 5.0 (2.6) in Germany for the total economy (producing sector). The distribution of wealth, however, is more equal than the one observed empirically in the German economy; in our model, the Gini coefficient is equal to 32.7% and falls short of values close to 0.59–0.89 as reported by Bomsdorf (1989). There are three important reasons why our model underestimates the degree of asset dispersion: first, agents are homogenous with regard to their skills, second, we neglect self-employment, and third, agents do not leave bequests.[17]

The optimal search effort of the 30-year old is illustrated in Figure 1. The search effort is a monotone declining function of wealth a. In addition, the search effort of the newly unemployed ($\epsilon = 2$) is smaller than the search effort of the workers who are unemployed for half a year or even longer ($\epsilon = 3, 4$). The reason for this behavior is that the individual's search effort depends negatively on the unemployment compensation of next period (if the

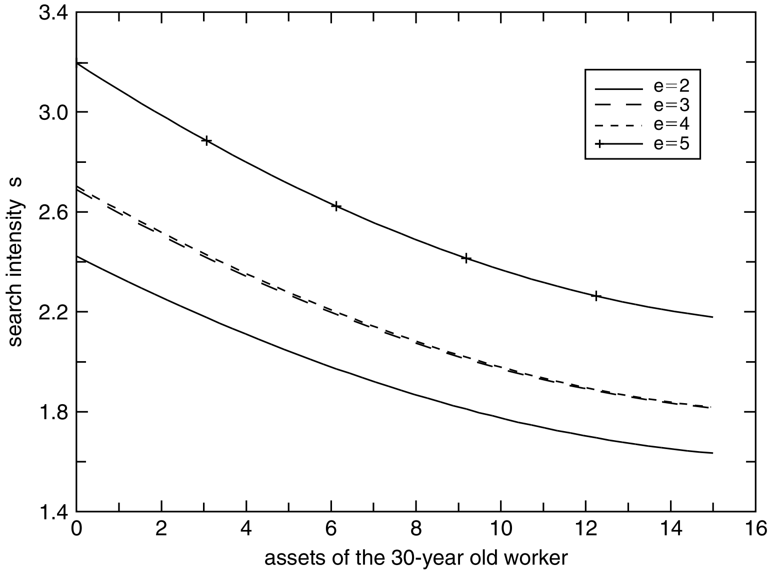

Figure 1. Optimal search effort of the 30-year old worker.

worker remains unemployed) rather than on unemployment compensation in the current period. The incentives for the newly employed workers ($\epsilon = 2$) to search for a job are smaller compared to the ones of the other unemployed workers because, in case of continuing unemployment, they receive unemployment insurance w_{UI} rather than unemployment assistance w_{UA}.

Furthermore, average search effort declines with age for two reasons (compare Figure 2): (i) average wealth of the old unemployed workers exceeds average wealth of the young unemployed workers, and (ii) expected discounted labor earnings decrease with age as the potential employment period is getting shorter. At age 60, finally, unemployed workers do not search any more as they face mandatory retirement in the following period.

In our benchmark, the equilibrium unemployment rate amounts to 10.2%. The number of short-term unemployed workers exceeds the number of long-term unemployed workers by a factor of 2.38. In our model, the number of employed workers is declining over time after an initial rise at age 21 as presented in Figure 3. In the first period of their life at age 21, agents have a probability of $p_0 = 80.6\%$[18] to find a job and replace the old employed workers who retire at the end of age 60. The behavior of the employment rate in each cohort, of course, closely mimics the search behavior of the unemployed workers as presented in Figure 2. At age 20 (20–21), unemployed workers are not entitled to unemployment insurance (assistance) as none among them has worked (received unemployment insurance) before. Over time, the number of unemployment assistance recipients is increasing and amounts to approximately 7.3% of all workers at age 60. The number of welfare recipients, those who have never found a job in their life, falls to almost zero within four years.

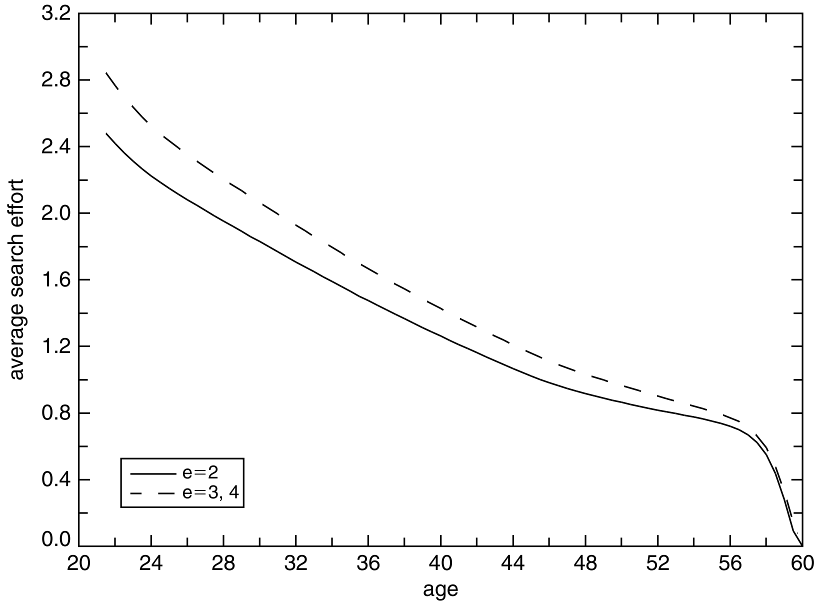

Figure 2. Search effort-age profiles.

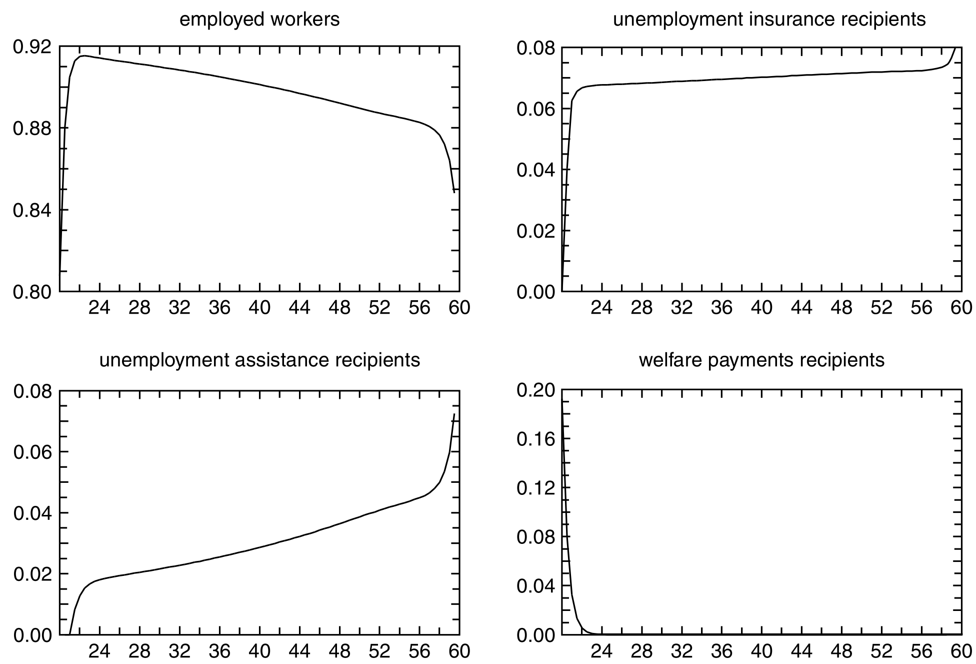

Figure 3. Employment status-age profiles.

Table 2. Effects of unemployment insurance w_{UI}.

ϑ_{UI}	k	u	$\frac{n_{UI}}{n_{UA}}$	w	τ	$\bar{s}_{UI1}$	$\bar{s}_{UA}$	Gini	Δ_c
0.1	16.03	8.61%	3.52	1.87	22.5%	2.08	1.19	33.3%	−0.91%
0.2	16.16	8.92%	3.23	1.88	23.0%	1.92	1.22	32.9%	−0.72%
0.3	16.35	9.24%	2.97	1.91	23.6%	1.76	1.24	32.8%	−0.50%
0.4	16.55	9.68%	2.67	1.92	24.2%	1.61	1.28	32.7%	−0.21%
0.5	16.60	10.2%	2.38	1.94	24.8%	1.48	1.34	32.7%	0
0.6	16.66	10.9%	2.04	1.96	25.6%	1.38	1.42	32.6%	0.13%
0.7	16.72	11.8%	1.72	1.98	26.5%	1.29	1.52	32.7%	0.17%
0.8	16.81	12.5%	1.47	2.00	27.3%	1.21	1.61	32.9%	0.14%
0.9	16.73	13.4%	1.22	2.02	28.2%	1.17	1.67	33.0%	0.02%
1.0	16.61	14.5%	1.01	2.03	28.9%	1.15	1.69	33.3%	−0.15%

4.2. *Optimal Replacement Ratios*

In this subsection, a change in the benefit level of unemployment insurance and unemployment assistance is analyzed. We will first discuss equilibrium effects of different unemployment compensation schemes, before we turn to distribution and welfare effects of unemployment insurance and unemployment assistance.

4.2.1. Equilibrium Effects Table 2 presents our results for unemployment insurance w_{UI}. Each row in Table 2 represents a different unemployment compensation arrangement. The first column gives the replacement ratio of unemployment insurance $\vartheta_{UI} = \frac{w_{UI}}{(1-\tau)w}$, and the remaining columns contain the equilibrium values of aggregate variables, the average search effort of the newly unemployed ($\epsilon = 2$) and the UA recipient ($\epsilon = 4$), s_{UI1} and s_{UA}, the Gini coefficient, and the change in welfare Δ_c associated with the unemployment compensation policy. Our benchmark case is given by the replacement ratio of $\vartheta_{UI} = 50\%$.

First observe that wages w increase with higher unemployment insurance payments as the worker's fall-back position in the wage bargain improves. Consequently, firms post less vacancies and unemployment increases. Increasing UI payments from current levels, $\vartheta_{UI} = 50\%$, to full insurance, $\vartheta_{UI} = 100\%$, increases unemployment from 10.2% to 14.5%. Following an increase of both the number of unemployed workers and the wage, the income tax rate τ increases from 24.8% to 28.9% as well. There are multiple opposing effects of a rise in the replacement ratio ϑ_{UI} on the search behavior of the agents. First, firms post less vacancies and, consequently, the job finding probability and the matching coefficient π decrease. Second, net wages fall. And third, the entitlement effect is more pronounced. As a consequence of the first two effects, the search effort of the workers who are newly unemployed, s_{UI1}, decreases. For the long-term unemployed agents, the entitlement effect predominates and they increase their search effort s_{UA} in order to become reemployed and, in case they lose employment once again, to receive higher unemployment insurance benefits w_{UI} in the future. The quantitative effect of unemployment insurance on search effort is quite pronounced. In fact, increasing the unemployment insurance level from 50% to 100% of net wage income results in a change of search effort by approximately 25% for both the UI and the UA recipients, even though in opposite directions, respectively.

Notice further that the aggregate capital stock k and aggregate household savings in the form of equity (not presented) rise with an increase in UI payments w_{UI} for low values of the replacement ratio ϑ_{UI}.[19] At first glance, this result is surprising as most theoretical studies concede that unemployment compensation payments decrease earnings uncertainty and hence precautionary savings, as demonstrated by Drèze and Modigliani (1972), Miller (1976), Sandmo (1970), or Levhari, Mirman and Zilcha (1980) to name but a few. Contrary to these studies, however, we consider a two-tier unemployment compensation system in a general equilibrium model where the employed agents' probability to be employed in the next period is different from the one of the unemployed workers. Following a rise in unemployment insurance benefits w_{UI}, employed workers decrease their savings. This effect, however, is rather minor as the probability to lose one's job $\delta = 0.05$ is rather low for employed workers. Short-term unemployed workers, however, will increase their precautionary savings as they are very likely to remain unemployed the following period as well and will have to rely on unemployment insurance w_{UI} or even unemployment assistance w_{UA}. In fact, the average reemployment probability of short-term unemployed workers only amounts to approximately 44.3% in the benchmark case with $\vartheta_{UI} = 50\%$. As a consequence, the drop of the employed agents' savings is overcompensated by the increase of precautionary savings among the short-term unemployed workers. For replacement ratios ϑ_{UI} exceeding 80%, however, the aggregate capital stock decreases as the fall in aggregate employment and, hence, aggregate income and savings becomes more pronounced.

The effects of unemployment assistence payments w_{UA} on equilibrium values are presented in Table 3. An increase in unemployment assistance payments $w_{UA} = \vartheta_{UA}(1-\tau)w$ has strong disincentive effects on the unemployed worker's search effort. Contrary to unemployment insurance, higher unemployment assistance decreases the search effort of both the short-term and the long-term unemployed workers. As can be seen from the inspection of Table 3, a 20% rise of ϑ_{UA} from 40% to 60%, for example, results in a fall of the average search effort of the newly unemployed workers, $\bar{s}_{UI1}$, and the UA recipients, $\bar{s}_{UA}$, by 22% and 45%, respectively. As a consequence, unemployment increase from 10.2% to 13.2%. As both the number of unemployed agents and the level of UA benefits increase, the government has to raise the income tax rate τ in order to keep the government budget (11) balanced.

The quantitative effect of an increase in UA benefits on wages is rather modest and does not exceed 1% following a 10% change in the UA replacement ratio. According to the wage

Table 3. Effects of unemployment assistance w_{UA}.

ϑ_{UA}	k	u	$\frac{n_{UI}}{n_{UA}}$	w	τ	$\bar{s}_{UI1}$	$\bar{s}_{UA}$	Gini	Δ_c
0	16.80	8.52%	4.11	1.92	23.7%	1.94	2.33	33.1%	0.44%
0.1	16.75	8.85%	3.74	1.93	23.9%	1.84	2.15	32.9%	0.72%
0.2	16.71	9.19%	3.28	1.93	24.1%	1.74	1.89	32.7%	0.67%
0.3	16.64	9.64%	2.81	1.94	24.5%	1.63	1.63	32.5%	0.31%
0.4	16.60	10.2%	2.38	1.94	24.9%	1.48	1.34	32.7%	0
0.5	16.55	11.2%	1.84	1.94	25.6%	1.33	1.05	32.9%	−0.42%
0.6	16.41	13.2%	1.26	1.94	27.0%	1.16	0.74	33.3%	−1.01%

equation (21), the partial equilibrium effect of a rise in unemployment assistance w_{UA} on negotiated wages is positive, (in the benchmark equilibrium, $\psi_3 = 0.041$). However, there are additional general equilibrium effects as (i) the income tax rate τ increases, (ii) the average search effort of the unemployed workers decreases, and (iii) average consumption of unemployed agents falls. While the last two effects depress the reservation wages, the first effect pushes wages upward. The net effect of UA payments w_{UA} on wages w is positive in our economy.

While aggregate savings and the aggregate capital stock k are not a montone decreasing function of UI benefits w_{UI}, UA benefits w_{UA} have an unambiguous effect on the aggregate capital stock and savings. The aggregate capital stock and, similarly, savings decrease unanimously with a rise in ϑ_{UA}. Following a rise of ϑ_{UA} from 10% to 40%, for example, the aggregate capital stock k declines from 16.75 to 16.60. There are multiple effects of higher UA benefits on savings: (i) precautionary savings of all agents are reduced, (ii) income is redistributed to the long-term unemployed workers, and (iii) the decline in net wages $(1-\tau)w$ also reduces welfare payments w_W and, in particular, pensions w_R so that workers accumulate higher savings for old age. In our economy, the first effect predominates.

4.2.2. Distribution and Welfare Effects The effects of both unemployment insurance and unemployment assistance on the distribution of wealth are rather small. For low values of the replacement ratio $\vartheta_{UI} < 60\%$, a rise in unemployment insurance w_{UI} decreases wealth heterogenity as income is redistributed from employed workers with high income to short-term unemployed workers with low income. Increasing the replacement ratio ϑ_{UI} from 10% to 50% only results in a change of the Gini coefficient from 33.3% to 32.7% (compare Table 2). However, for a replacement ratio exceeding $\vartheta_{UI} = 60\%$, the effect of a rising share of long-term unemployed workers with low labor income dominates and wealth inequality as measured by the Gini coefficient increases.

Similarly, higher UA payments also have an ambiguous effect on wealth heterogenity. With rising ϑ_{UA}, income is redistributed to the long-term unemployed workers who, on average, are characterized by low wealth. On the other hand, the number of long-term unemployed workers increases as the disincentive effect of higher UA benefits is rather pronounced. For the latter reason, wealth heterogeneity even increases for replacement ratio exceeding $\vartheta_{UA} = 30\%$.

In order to compare the welfare effects of alternative unemployment compensation arrangements, we need a measure of average utility. For government policy Ω, we measure welfare by the expected discounted life-time utility of the newborn generation which is simply equal to $W(\Omega) = p_0 W(0, 1, 1) + (1 - p_0) W(0, 5, 1)$. In order to quantify the welfare benefits, we take the benchmark equilibrium as presented in Table 1 as our reference economy. The change in welfare Δ_c is computed as the compensation in consumption (relative to the reference economy) required in order to make the average newborn indifferent between the reference economy and an economy under an alternative unemployment compensation arrangement.

For given replacement ratio of unemployment assistance ϑ_{UA}, full insurance $\vartheta_{UI} = 100\%$ is not found to be optimal. The total consumption equivalent increase Δ_c following a rise of the replacement ratio ϑ_{UI} from 50% to the optimal level of 70% amounts to 0.17%.[20] Not surprisingly, it is also not optimal to provide high unemployment assistance as

(i) unemployment u increases, (ii) both the aggregate capital stock k and output y fall, and (iii) wealth inequality as measured by the Gini coefficient rises modestly only for low levels of unemployment assistance w_{UA}. For given unemployment insurance replacement ratio $\vartheta_{UI} = 50\%$, the optimal unemployment assistance replacement ratio ϑ_{UA} is only 10%.[21]

We also computed the optimal unemployment compensation policy $\{\vartheta_{UI}, \vartheta_{UA}\}$ for the case that the government is able to vary the two replacement ratios ϑ_{UI} and ϑ_{UA} simultaneously. The optimal unemployment compensation policy is found to amount to $(\vartheta_{UI}, \vartheta_{UA}) = (70\%, 20\%)$ implying a welfare gain equal to 1.21% of total consumption. Accordingly, the optimal level of unemployment insurance payments is found to exceed the present one in Germany, which is characterized by a replacement ratio of about $\vartheta_{UI} = 50\%$, while the optimal level of unemployment assistance is only half the present level in Germany.

5. Conclusion

We analyzed the effects of a two-tier unemployment compensation system on employment, savings, and welfare in a model of search unemployment with consumption smoothing. Unemployment compensation consists of unemployment insurance, which is paid to the short-term unemployed workers, and unemployment assistance, which is paid to the long-term unemployed workers. The effects of UI payments and UA payments on macroeconomic variables such as wages and savings are shown to be different. Unemployment insurance payments affect wages as wages can be renogotiated at any time and the relevant fall-back position of the employed worker is insured unemployment. An increase of UI payments relative to UA payments also results in a strong positive 'entitlement effect' as long-term unemployed workers are searching harder for a job in order to get reemployed. Unemployment assistance payments, on the contrary, mainly affect search behavior of unemployed workers. A rise in UA payments decreases search effort of all unemployed workers. The effect of UA payments on wages, however, is rather modest. Both unemployment insurance and unemployment assistance are demonstrated to reduce equilibrium employment significantly.

Unemployment insurance and unemployment assistance also have different effects on aggregate savings. While savings increase with higher unemployment insurance payments at low levels of UI payments, UA payments are demonstrated to decrease aggregate savings unanimously. The main reason for the latter result is the two-tier structure of the unemployment compensation system and the different probabilities to be employed in the next period faced by the employed and unemployed workers, respectively. On the one hand, precautionary savings of employed workers hardly decrease with higher UI benefits as the job loosing probability of employed workers is rather low. On the other hand, short-term unemployed workers accumulate high precautionary savings because they have a high probability to remain unemployed the next year as well in which case they might receive lower unemployment compensation in the form of UA payments. As the ratio of unemployment insurance payments to unemployment assistance payments rises, the short-term unemployed worker's incentives to build up precautionary savings for the next periods increase.

Not surprisingly, the optimal level of unemployment insurance and unemployment assistance are different from each other as these two forms of unemployment compensation affect the economic behavior of households and firms differently. Optimal unemployment compensation is shown to decline over time. Our normative result is related to previous contributions on the analysis of optimal unemployment compensation discussed in the introduction. We find that the results by Shavell and Weiss (1979) and Fredriksson and Holmlund (2001) extend to a general equilibrium setting with consumption smoothing. Furthermore, we also show in accordance with the findings of Fredriksson and Holmlund (2001) that the benefits from a switch to an optimal two-tired structure of unemployment compensation can be substantial. In our model which is calibrated with regard to the characteristics of the German economy full unemployment insurance is not optimal in the short-run, but the optimal level of unemployment insurance is still demonstrated to be higher than the current level of unemployment insurance in Germany. On the contrary, the current level of unemployment assistance is found to be too high in Germany and should be reduced by half according to the results from our welfare analysis.

6. Appendix

6.1. Deriving the Wage Equation

In order to derive the wage equation (21), we need the simplifying assumption that wages are bargained between the firms and the representative employed worker whose consumption is equal to the average consumption of all employed workers and whose fall-back is the value of the representative insured unemployed worker who has been employed in the previous period. In addition, the representative worker does not take into account that unemployment insurance w_{UI} will be affected by the outcome of the bargain. With these simplifying assumptions, the value of employment, W^e, the value of short-term insured unemployment in his first half year of unemployment, W^{UI1}, and second half year of unemployment, W^{UI2}, and the value of long-term unemployment with unemployment assistance, W^{UA}, are given by the following Bellman equations:

$$W^e = (1-\tau)wMU^e - D^W + \beta[(1-\delta)W^e + \delta W^{UI1}] \tag{27}$$

$$W^{UI1} = w_{UI}MU^{UI1} - D^S s_{UI1} + \beta\left[\left(1-e^{-\pi s_{UI1}{}^z}\right)W^e + e^{-\pi s_{UI1}{}^z}W^{UI2}\right] \tag{28}$$

$$W^{UI2} = w_{UI}MU^{UI2} - D^S s_{UI2} + \beta\left[\left(1-e^{-\pi s_{UI2}{}^z}\right)W^e + e^{-\pi s_{UI2}{}^z}W^{UA}\right] \tag{29}$$

$$W^{UA} = w_{UA}MU^{UA} - D^S s_{UA} + \beta\left[\left(1-e^{-\pi s_{UA}{}^z}\right)W^e + e^{-\pi s_{UA}{}^z}W^{UA}\right], \tag{30}$$

where MU^e, MU^{UI1}, MU^{UI2}, and MU^{UA} denote the marginal utility of the representative employed worker, the representative unemployed worker receiving w_{UI} during the first half and the second half year of unemployment, and the representative unemployed worker receiving w_{UA}, respectively.[22] Similarly, $s_x, x \in \{UI1, UI2, UA\}$, denotes the average search effort of the searching agent who has been employed in the previous period, is unemployed in his second half year, and unemployed for more than one year, respectively.

In the stationary equilibrium, the value of a filled vacancy to the firm is given by:

$$V^f = A_0(1-\alpha)n^{-\alpha}k^\alpha - w + \frac{1}{1+r}[(1-\delta)V^f + \delta V^v], \tag{31}$$

and the value of an unfilled vacancy is equal to zero:

$$V^v = 0. \tag{32}$$

Wages are bargained in every period according to the Nash bargaining solution where the fall-back positon of the insiders is the state of short-term insured unemployment, W^{UI1}. The wage rate satisfies:

$$w = \arg\max_w (W^e - W^{UI1})^\lambda (V^f - V^v)^{1-\lambda}, \tag{33}$$

where λ denotes the bargaining power of the workers. From (27), (28), (31), and (32), the first-order condition of program (33) reads as:

$$(1-\lambda)\frac{W^e - W^{UI1}}{(1-\tau)MU^e} = \lambda(V^F - V^v). \tag{34}$$

After some tedious algebra, the wage equation (21) can be derived with the help of the equations (27)–(34). The parameters ψ_i, $i = 1, \ldots, 5$ are given by:

$$
\begin{aligned}
(\psi_1)^{-1} &= 1 + \zeta_0\zeta_1 \frac{1-\lambda}{\lambda}\frac{r+\delta}{1+r},\\
\psi_2 &= \psi_1 \frac{1-\lambda}{\lambda}\frac{r+\delta}{1+r}\frac{1}{(1-\tau)MU^e}(1-\zeta_0\zeta_1\beta\delta)\big(MU^{UI1} + \beta e^{-\pi s^z_{UI1}} MU^{UI2}\big),\\
\psi_3 &= \psi_1 \frac{1-\lambda}{\lambda}\frac{r+\delta}{1+r}\frac{1}{(1-\tau)MU^e}\,(1-\zeta_0\zeta_1\beta\delta)\,\beta^2 MU^{UA}\frac{e^{-\pi s^z_{UI1}} e^{-\pi s^z_{UA}}}{1-\beta e^{-\pi s^z_{UA}}},\\
\psi_4 &= \frac{1-\lambda}{\lambda}\frac{r+\delta}{1+r}\frac{1}{(1-\tau)MU^e}\zeta_0\zeta_1\psi_1,\\
\psi_5 &= \psi_1 \frac{1-\lambda}{\lambda}\frac{r+\delta}{1+r}\frac{1}{(1-\tau)MU^e}\,(1-\zeta_0\zeta_1\beta\delta)\\
&\quad\times\left(s_{UI1} + \beta e^{-\pi s^z_{UI1}} s_{UI2} + \frac{\beta^2 e^{-\pi s^z_{UI1}} e^{-\pi s^z_{UI2}}}{1-\beta e^{-\pi s^z_{UA}}} s_{UA}\right),
\end{aligned}
$$

and

$$
\begin{aligned}
(\zeta_0)^{-1} &= 1 - \beta(1-\delta) - \beta^2\delta\left(1 - e^{-\pi s_{UI1}{}^z}\right) - \beta^3\delta e^{-\pi s_{UI1}{}^z}\left(1 - e^{-\pi s_{UI2}{}^z}\right)\\
&\quad - \beta^4\delta e^{-\pi s_{UI1}{}^z} e^{-\pi s_{UI2}{}^z}\frac{1-e^{-\pi s_{UA}{}^z}}{1-\beta e^{-\pi s_{UA}{}^z}},\\
\zeta_1 &= 1 - \beta\left(1 - e^{-\pi s_{UI1}{}^z}\right) - \beta^2 e^{-\pi s_{UI1}{}^z}\left(1 - e^{-\pi s_{UI2}{}^z}\right)\\
&\quad - \beta^3 e^{-\pi s_{UI1}{}^z} e^{-\pi s_{UI2}{}^z}\frac{1-e^{-\pi s_{UA}{}^z}}{1-\beta e^{-\pi s_{UA}{}^z}}.
\end{aligned}
$$

In our benchmark case, $\psi_1 = 0.882$, $\psi_2 = 0.113$, $\psi_3 = 0.041$, $\psi_4 = 0.232$, and $\psi_5 = 0.340$.

6.2. The Solution Algorithm

The model has no analytical solution. Algorithms to solve heterogenous-agent models with an endogenous distribution have only recently been introduced in the economic literature. Notable studies in this area are Aiyagari (1994), Costain (1997), den Haan (1996), Huggett (1993), and İmrohoroğlu, İmrohoroğlu and Joines (1995). Like most of these studies, we will only focus on the steady state of the model. Our algorithm is described by the following steps:

1. Choose the policy parameters tr, ϑ_{UI}, ϑ_{UA}, ϑ_W, and ϑ_R.
2. Make initial guesses of r, π, and w.
3. Compute the household's decision function by backwards induction.
4. Compute the steady-state distribution of assets, employment, consumption, and search effort.
5. Compute n, n_{UI}, n_{UA}, n_W, average consumption of the employed workers, average consumption and search effort of the unemployed workers, and the average asset holdings of all households.
6. Compute the values τ, k, q, i, w, r, and π that solves the firm's Euler equations, the wage equation, the government budget, and the aggregate consistency conditions.
7. Update r, π, and w, and return to step 3 if necessary.

In step 3, a simple finite-time dynamic programming problem is solved by the use of projection methods.[23] The decision rules of the j-year old generation can be found by a single recursion working backwards from the last period of life. The value functions and the policy functions are approximated by a Chebyshev polynomial of order three.[24] We computed the coefficients with orthogonal collocation by solving a system of nonlinear equations consisting of the Bellman equation and the first-order conditions, evaluated at the zeros of the Chebyshev polynomials. In step 4, the steady-state distribution is computed by forward iteration starting with the 21-year old (corresponding to the 1-period old generation in the model) who has no wealth and given employment probability p_0.[25]

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Notes

1. Further notable studies on the effects of unemployment insurance on employment and welfare in the case of unobservable stochastic employment opportunities include the general equilibrium models by Hansen and İmrohoroğlu (1992) and Atkeson and Lucas (1993).

2. In the model of Cahuc and Lehmann (2000), unemployed workers lose entitlement to UI payments if they are unemployed for more than one period. Besides, Cahuc and Lehmann apply the Rawls criterion of welfare rather than the utilitarian approach applied by Fredriksson and Holmlund (2001).
3. Neglecting intertemporal consumption smoothing may also result in a biased welfare anaylsis of unemployment compensation as agents cannot build up precautionary savings for times of unemployment. In addition to Fredriksson and Holmlund (2001), we are able to study the effects of alternative unemployment schemes on the distribution of wealth.
4. The model is further different from the one of Costain with regard to the modelling of the wage determination.
5. Chiu and Karni (1998) show that the presence of private information about individual's work effort help explain the failure of the private sector to provide unemployment insurance. We refrained from the introduction of work effort in our model in order to keep the model tractable. For a consideration of voluntary quits by employed workers see also Wang and Williamson (1996).
6. By this choice of its functional form, the employment probability is bounded between zero and one.
7. By this choice, which follows den Haan, Ramey and Watson (1997) and Costain (1997), the equilibrium number of aggregate matches is approximately equal to $qh \approx \int_a \phi(a, \epsilon, j)\pi s^z \, da \approx \frac{\mu \bar{s}^z (n_{UI}+n_{UA}+n_W)h}{(h^{1/\rho}+(n_{UI}+n_{UA}+n_W)^{1/\rho})^\rho}$, which depends positively on average search effort $\bar{s}$. The aggregate matching function is increasing in its arguments h, $n_{UI} + n_{UA} + n_W$, and $\bar{s}$, and has constant returns to scale in its first two arguments.
8. The annual data on the unemployment rate, the flow of employed workers into unemployment, and the number of unemployment insurance and unemployment assistance recipients are taken from the yearbooks of the *German Statistical Office (Statistisches Bundesamt)*. The data on labor income of workers, GDP, the capital-output ratio, and the numbers of self-employed and workers are taken from *Statistisches Bundesamt, Fachserie 18*.
9. The bargaining power λ is introduced in the appendix in the derivation of the wage equation.
10. All our qualitative results also hold for the case $\sigma \in \{1, 4\}$.
11. At the time of the CERC study, the maximum benefit of unemployment insurance and unemployment assistance amounted to 68% and 58% of net earnings, respectively, while they declined to 67% and 57% during the calibration period, respectively.
12. Almost the same number results from a computation using average expenditures of local governments for welfare payments relative to average expenditures of the federal government for unemployment insurance, after correcting the former for payments to disabled people (health risk is not considered in our model).
13. We also computed the equilibrium values for different values of the replacement ratio of welfare payments ($\vartheta_W = 20\%$), the replacement ratio of pensions ($\vartheta_R = 40\%$), and transfers (equal to 1% and 5% of GDP). Our qualitative results are the same as in the benchmark case.
14. This number is based on the average survival rate in unemployment after 6 months estimated by Steiner (1997). The average survival rate depends on general labor market conditions. During 1987–1992, Steiner estimates an average survival rate of 50.6%, while this number drops to 35.9% during 1993–1994 as Germany is subject to a severe recession.
15. The number of unemployment insurance recipients relative to the number of unemployment assistance recipients is a little higher amounting to 2.88.
16. Total flows in and out of employment are about twice as much, as many agents enter employment from out of the labor force. Since the model, however, does not account for this phenomenon, these flows are neglected. Burda and Wyplosz (1994) provide a detailed analysis of the flows in and out of employment in European countries.
17. Quadrini and Ríos-Rull (1997) review recent studies of endogenous wealth inequality in models of heterogenous agents with uninsurable idiosyncratic exogenous shocks to earnings, including attempts to include business ownership, higher rates of return on high asset levels, and changes in health and marital status, among others. Heer (2001) also considers bequests in his analysis of a life-cycle model.
18. This number is in good accordance with empirical findings by Steiner (1997) who provides estimates of survival rates for different ages during 1993–1994 using data from the Socio-Economic Panel for West Germany (GSOEP). For the 21-year old, he finds a job finding probability of 79.1%.
19. In equilibrium, aggregate savings A and the capital stock k can be shown to be related by $k = A - n/q$, which can be obtained from inserting equations (15)–(18) in (24).
20. As the intertemporal elasticity of substitution, $1/\sigma$, decreases, individuals prefer to increase consumption smoothing. As a consequence, the welfare gain following a change in ϑ_{UI} increases with the coefficient of risk aversion σ. Furthermore, the quantitative welfare changes increase if a stricter constraint is imposed on the individual's borrowing.

21. In the presence of stricter binding borrowing constraints, $a \geq 0$, for example, the optimal level of unemployment assistance is higher by approximately 10 to 20 percentage points depending on the level of w_{UI}. In our benchmark economy, 3.6% of the agents hold negative wealth.
22. In our numerical computations, the marginal utility of the representative employed worker is calculated using the average consumption of employed workers $\bar{c}^e$, $MU^e = (\bar{c}^e)^{-\sigma}$. MU^{UI1}, MU^{UI2}, and MU^{UA} are computed using the average consumption levels of the unemployment insurance and unemployment assistance recipients, respectively.
23. A more detailed description of this method is contained in Judd (1992, 1998).
24. We also approximated the functions by a Chebyshev polynomial of higher order, however, results did not improve as the Chebyshev coefficients drop off rapidly.
25. A more detailed descripton of the numerical computation of the stationary distribution can be found in Huggett (1993).

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