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The empirical similarity approach for volatility prediction

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1. Introduction

Consider the task to forecast a process of interest. There are often several competing models available for this purpose with their strong and weak sides. How should forecasts from these different models be combined? Starting from the seminal contribution of Bates and Granger (1969) there has been suggested a large number of approaches to determine weights for model combination. These weights are usually related to the model prediction success probabilities (cf. Elliott and Timmermann, 2004) which can be interpreted as occurrence probabilities for the states in the coming period. The ability to evaluate probabilities is crucial for the classical decision theory in spirit of von Neumann–Morgenstern. However, such probabilistic approach is not always possible or desired.

By making decisions in situations under uncertainty or ignorance a decision maker can be unable or unwilling to evaluate probabilities but prefers to rely on *thinking by analogy* for learning from the past about the future. The analogical (case based) reasoning is widely applied for decision making in medicine, law, business, politics, or artificial intelligence (cf. Gilboa and Schmeidler, 2001). The case based decision theory presumes analogous thinking of human beings in cases where the current situation is evaluated by considering its similarity to previously experienced (past) situations (cf. Gilboa and Schmeidler, 2001). Cases which are more similar to the current situations obtain larger weights compared to those which are less similar. The concept of *empirical similarity* (Gilboa et al., 2006; Gilboa et al., 2011) provides the econometric framework for estimation of the similarity function from the data (Gilboa and Schmeidler, 2012). It allows to measure distances between cases (problems, situations) as they are perceived by decision makers.

In this paper we suggest and apply a methodology how to use the empirical similarity (ES) concept in order to combine forecasts from different models in a non-probabilistic manner. In our setting alternative forecasts originating from competing models could be evaluated as cases, which are to some extent similar to the currently observed state or realization. A model which recently provides more precise point forecasts should obtain a larger current weight compared to alternatives. The core idea of our approach is to measure the empirical similarity distance between the current observation and the last one-period-ahead forecasts from different models. This similarity distance determines model weights for the next period forecasts. Thus, our approach exploits the information about the recent performance of different models in order to determine the weights of the forecasting model combination. The advantages of such ES combination approach compared to the probabilistic alternatives are that (i) it does not require knowledge of model success probabilities; (ii) it relates the weights of the forecasting models to the preferences of economic agents; and (iii) it reveals from the data how decision makers evaluate the similarity between forecasts and realizations.

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We illustrate the application of the proposed ES forecast combination approach by modeling the daily process of realized volatilities. For this purpose we evaluate empirical similarities for combining volatility models which can be treated as approaches reflecting different investment horizons (cf. Ghysels et al., 2006; Corsi, 2009). In particular, we model series of daily realized volatilities of the leading world financial indices for about 13 recent years characterized by both high and low volatility periods. The parameters of the ES approach are estimated with the maximum likelihood methodology (cf. Lieberman, 2010) for both full sample and mowing windows of 250 daily observations. We compare forecasting performance of the ES approach with a set of popular volatility models by conducting both in-sample and out-of-sample predictions. The obtained estimation results reveal how forecasts from various volatility models are aggregated via the empirical similarities in the perception of decision makers. A special attention is drawn to the analysis of volatility patterns during and immediately after the recent subprime crisis with a highly complex volatility dynamics. The proposed empirical similarity model appears to provide the most suitable description of the volatility process during that period.

The rest of the paper is organized as follows. In Section 2 we propose a novel empirical similarity approach which allows to combine forecasts from different models. The ES methodology for combining volatility forecasts or components is presented in Section 3. The empirical study in Section 4 is devoted to the estimation and forecast comparison of competing volatility models. Moreover, we draw a special attention to the recent subprime crisis period which is characterized by a highly nonlinear volatility dynamics. Section 5 concludes the paper.

2. Empirical similarity for model combination

Assume that there are p models (forecasts, recommendations) which could be combined in order to forecast the variable of interest y_{t+1}. Define a finite set of distinct forecasts from different models as $\{x_{1,t},\dots x_{p,t}\}$ and consider the task of combining them in a parsimonious manner. A family of linear forecast combinations remains popular starting from the seminal paper of Bates and Granger (1969). A linear forecast combination is given as

$$\hat{y}_{t+1} = \sum_{i=1}^{p} a_{i,t} x_{i,t}, \tag{1}$$

where non-negative $a_{i,t}$s are the proportions of the ith model with $\sum_{i=1}^{p} a_{i,t} \equiv 1$. There is a straightforward probabilistic interpretation for the weights $a_{i,t}$, which are in general related to model success probabilities (cf. Elliott and Timmermann, 2004). Weighting models as in (1) presumes *the ability* to choose the weights $a_{i,t}$ appropriately by considering some given objective functions. Various probabilistic approaches are proposed for the choice of the proportions $a_{i,t}$, however, there is no dominating methodology up to now.

Now let us consider situations under uncertainty or ignorance where economic agents do not have specific (probabilistic) beliefs about model weights in future but simply prefer models which performed well in similar cases in the past. In these situations the agents should form their decisions relying on analogical case based reasoning (Gilboa and Schmeidler, 2001). The case based decision theory (cf. Gilboa and Schmeidler, 2001) is developed for situations where decision makers refrain from evaluating probabilities but relies on their experience in order to evaluate distances (similarities) between past cases (situations) and the current state of nature.

The empirical similarity (ES) approach of Gilboa et al. (2006) provides the econometric framework for estimation of the similarity functions from the data. In order to describe their concept assume that there is a vector of variables $\mathbf{z}_t$ characterizing the current situation, which is followed by the realization y_{t+1} in the next period. The ES postulates that the model combination weights $a_{i,t}$ should be replaced by non-negative similarity-based frequencies $\phi[\mathbf{z}_s, \mathbf{z}_t]$, which sum up to unity and serve as weights for the experienced realizations y_{s+1}. In this setting the DGP is driven directly by its historical observations weighted by $\phi[\mathbf{z}_s, \mathbf{z}_t]$'s. Then the corresponding ES model equation is given as

$$y_{t+1} = \sum_{s<t} \phi[\mathbf{z}_s, \mathbf{z}_t] y_{s+1} + \varepsilon_{t+1}, \quad \varepsilon_t \sim (0, \sigma^2), \tag{2}$$

where $\mathbf{z}_s$ is a vector characterizing the situation at time s, y_{s+1} is the realization of the process of interest experienced in the next period. Thus, the similarity function measures the distance between the vectors $\mathbf{z}_t$ and $\mathbf{z}_s$ as it is assessed by a decision maker.

Relying on the ES concept of Gilboa et al. (2006), we suggest an ES approach for combining forecasting models. For this purpose we unite the ideas behind the forecasting Eq. (1) and the ES model in (2). The resulting ES forecast combination is given as

$$y_{t+1} = \sum_{i=1}^{p} \phi[y_t, x_{i,t-1}] x_{i,t} + \varepsilon_{t+1}, \quad \varepsilon_t \sim (0, \sigma^2). \tag{3}$$

The essential difference to Eq. (2) is that we replace the vector of characteristics $\mathbf{z}_s$ by the forecast from the ith model $x_{i,t-1}$, so that we now measure the distance between the previous forecast $x_{i,t-1}$ and the corresponding realization y_t in order to obtain the weights $\phi[y_t, x_{i,t-1}]$. Then the forecast combination which is a weighted sum of the forecasts $\{x_{1,t},\dots,x_{p,t}\}$ is given as

$$\hat{y}_{t+1} = \sum_{i=1}^{p} \phi[y_t, x_{i,t-1}] x_{i,t}.$$

In our ES combination setting the process of interest y_{t+1} is driven directly by the alternative forecasts $x_{i,t}$s, allowing to interpret (3) as a proxy for the true DGP as it is perceived by decision makers.

The model in (3) incorporates nonlinear *autoregressive* features due to the fact that y_t enters the similarity function $\phi[\cdot,\cdot]$ which determines the DGP of y_{t+1}. Moreover, it has a *spatial* property by measuring distances between the forecasts and the realization, which are used for weighting $x_{i,t}$s in order to assess y_{t+1}. This point corresponds to the suggestion of Gilboa et al. (2006, pp. 437–438) that for time series the current observation could be compared not with a history but with a profile (cross-section) of components.

The weights $\phi[\cdot,\cdot]$ depend on the previous experience of decision makers. The distance between the proxy of the current realization and the ith model forecast is measured in our case as

$$\phi[y_t, x_{i,t-1}] = \frac{\theta[y_t, x_{i,t-1}]}{\sum_{j=1}^{p} \theta[y_t, x_{j,t-1}]}. \tag{4}$$

The weights $\phi[y_t, x_{i,t-1}] \in [0,1]$ can be interpreted as normalized relative empirical similarities with the property $\sum_{i=1}^{p} \phi[y_t, x_{i,t-1}] \equiv 1$, whereas $\theta[y_t, x_{i,t-1}]$ is the similarity (distance) function parameterized below. The interpretation of the similarity measures $\theta[y_t, x_{i,t-1}]$ is straightforward, namely a small distance between y_t and $x_{i,t-1}$ implies a high similarity value of $\theta[y_t, x_{i,t-1}]$, while a large distance indicates on low similarity.

There are several possibilities to specify the similarity function $\theta[y_t, x_{i,t-1}] \geqslant 0$ (cf. Golosnoy and Okhrin, 2008; Guerdjikova, 2008; Lieberman, 2010). In this paper we exploit a flexible specification of the exponential similarity function of Billot et al. (2008), which is given as

$$\theta[y_t, x_{i,t-1}] = \exp\left(-\omega_i (y_t - x_{i,t-1})^2\right), \quad \text{with} \quad \omega_i \in \mathbb{R}. \tag{5}$$

Thus, the empirical similarity function is known up a p-dimensional vector of parameters $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_p)'$ which reflects the opinion of decision makers concerning the similarities of forecasts and realizations. The distance in (5) is symmetric and not directional, however, asymmetric specifications can be applied as well (cf. Lieberman, 2010). Eq. (3) implies that y_t is a weighted sum (linear combination) of the model forecasts with an additive noise component. This feature makes estimation of this model quite straightforward. Lieberman (2010, 2012) provides the estimation theory for a broad family of similarity models.

The empirical similarity concept for combining forecasts introduced in (3)–(5) allows to estimate from the data how decision makers form their believes concerning performance of particular forecasting models. This is the essential difference between the weights $a_{i,t}$ from Eq. (1) and ES weights $\phi_{i,t}$, because the latter are actually inferred from the experience of decision makers, while the former are related to some exogenous performance criteria. Moreover, by estimating the parameters ω_is, we can test whether ω_i is significantly different from zero, i.e. whether the distance to the ith model is of importance for the formation of the process y_t.

3. Empirical similarity for volatility forecasts

To illustrate our ES approach we consider prediction of daily volatility in financial markets, which is of much importance in contemporary finance. First we briefly describe the stylized features frequently found in volatility time series and discuss some popular models for volatility forecasting purpose. Then we apply the suggested ES approach for combining volatility forecasting models.

3.1. Measuring and forecasting volatility

As the true daily volatility v_t is not directly observable it should be replaced by proper measures. The availability of ultra high-frequency (intraday) data enables much more precise volatility measures than those based on the daily data. The daily realized volatility estimator rv_t is constructed from the intraday log asset returns denoted by $r_{t,j} = p(t-1+j/j^*) - p(t-1+(j-1)/j^*)$, $j = 1, 2, \ldots, j^*$, where $p(\cdot)$ is a log price of the risky asset and j^* is the number of intraday periods. These realized volatility measures, popularized by Andersen and Bollerslev (1998), Barndorff-Nielsen and Shephard (2002), provide consistent estimators of daily volatility. The simplest realized volatility measure is given as

$$rv_t(j^*) = \sum_{j=1}^{j^*} r_{t,j}^2, \tag{6}$$

with $\lim_{j^* \to \infty} rv_t(j^*) \xrightarrow{L} v_t$ so that $rv_t(j^*)$ is a consistent estimator of the daily quadratic variation v_t. Hereafter we differentiate among the true (unobservable) volatility v_t, its forecast $\hat{v}_t$ and the realized volatility measure rv_t. The unobservable true value v_t is replaced by its measure rv_t for the further analysis, i.e. we consider the estimation risk as negligible. A more recent discussion of these issues can be found e.g. in Andersen et al. (2011). The availability of intraday data allows to construct some alternative volatility measures, such as the bipower variation (Barndorff-Nielsen and Shephard, 2004) or realized absolute values (Forsberg and Ghysels, 2007). The same set of volatility models could be estimated with these measures. However, since our empirical similarity approach is focused on revealing decisions of representative investors we concentrate on the realized volatility measure which is mostly applied in practice. Note that trying alternative volatility estimators leads to the forecasting results which are very similar to those found for the realized volatility measure.

Forecasting volatility remains a challenging task of the financial economics and econometrics. Dynamic properties of the volatility process are of crucial importance for proper volatility forecasts. Alternating periods of high and low volatility, denoted as volatility clusters, motivate the famous GARCH model family. A long memory feature with slowly decaying autocorrelation function is an essential characteristic of the volatility time series. In order to understand the origins of this phenomenon, Andersen and Bollerslev (1997) demonstrate that a mixture of numerous heterogenous short-run information arrivals may lead to a long-run dependence in the volatility process. Differences in investment time horizons for various types of trader allows to consider both long memory and volatility clustering from the volatility component perspective (cf. Müller et al., 1997). The idea to separate volatility components roots in economic interpretations of the volatility process. E.g., for the two component volatility model of Engle and Lee (1999), the secular component can be seen as driven by fundamental economic factors, whereas the short-term component reflects uncertainty in response to recent news. The informational cascade approach (cf. Bikhchandani et al., 1998) or multifractal volatility models (cf. Calvet et al., 2006) suggest alternative interpretations for these issues.

The recently proposed heterogeneous autoregressive (HAR) model of Corsi (2009) appears to be a very successful attempt to introduce a parsimonious component approach in order to forecast daily volatilities. It can be seen as an approximation of more sophisticated MIDAS (cf. Ghysels et al., 2006) and ARFIMA volatility models (cf. Andersen et al., 2011). The HAR combines volatility measures (components, aggregates) sampled at different frequencies in a simple linear regression framework. The HAR approach allows to mimic dynamic features of volatility time series so that it provides a reasonable out-of-sample forecasting results in applications. The standard HAR model for the daily volatility process v_t is given as

$$v_t = \alpha_0 + \omega_1 v_{t-1}^{(d)} + \omega_2 v_{t-1}^{(w)} + \omega_3 v_{t-1}^{(m)} + \varepsilon_t, \quad \varepsilon_t \overset{iid}{\sim} (0, \sigma^2), \tag{7}$$

where $v_{t-1}^{(d)} = v_{t-1}$ is daily, $v_{t-1}^{(w)}$ and $v_{t-1}^{(m)}$ are the average weekly and monthly volatility measures, respectively. They are defined as $v_t^{(w)} = 5^{-1}\sum_{i=1}^{5} v_{t-i+1}$ and $v_t^{(m)} = 22^{-1}\sum_{i=1}^{22} v_{t-i+1}$. The HAR model can be estimated as a standard OLS regression framework by replacing the true unobservable v_t with the realized volatilities rv_t. The economic interpretation of volatility components relate the long-term component $v^{(m)}$ to the fundamental macroeconomic uncertainty factors. The medium-term component $v^{(w)}$ reflects the current market uncertainty concerning processing of news, and the short-term component $v^{(d)}$ accounts for the speculative momentum uncertainty.

3.2. Empirical similarity for volatility models

The empirical similarity concept allows to estimate the forecasting weights of volatility models (components) directly from the experience of decision makers. Denote the competing volatility forecasts by $v_t^{(h)}$, which are the elements of a set of various models (components) $\mathcal{H}$. Then the empirical similarity model for volatility forecasts in line with Eqs. (3)–(5) is given as

$$\begin{aligned} v_t &= \sum_{h \in \mathcal{H}} \phi[v_{t-1}, v_{t-2}^{(h)}] \cdot v_{t-1}^{(h)} + \varepsilon_t \\ &= \frac{\sum_{h \in \mathcal{H}} \theta[v_{t-1}, v_{t-2}^{(h)}] \cdot v_{t-1}^{(h)}}{\sum_{h \in \mathcal{H}} \theta[v_{t-1}, v_{t-2}^{(h)}]} + \varepsilon_t. \qquad \varepsilon_t \overset{iid}{\sim} (0, \sigma^2). \end{aligned} \tag{8}$$

The similarity function, defined as $\theta\left[v_t, v_{t-1}^{(h)}\right] = \exp\left(-w_h\left(v_t - v_{t-1}^{(h)}\right)^2\right)$, measures a distance between the current volatility state v_t and the hth model forecast $v_{t-1}^{(h)}$. The non-negative weight $\phi\left[v_t, v_{t-1}^{(h)}\right]$ is assigned to the current component $v_t^{(h)}$ in order to forecast v_{t+1}. Recall that $\sum_{h \in \mathcal{H}} \phi\left[v_{t-1}, v_{t-2}^{(h)}\right] \equiv 1$.

As we use the HAR model as a benchmark, we concentrate on combining its three components with the ES approach. Our aim is to evaluate how decision makers assess relative distances between the current volatility state (realized measure) and volatility aggregates sampled at different horizons, which are transferred to weights of competing forecasts. Thus, we want to explore by means of the ES toolkit how economic agents with heterogenous investment horizons combine their views about the volatility process. The ES model with the HAR components, further denoted by ES1 approach, is given as

$$v_t = \frac{\theta[v_{t-1}, v_{t-2}]v_{t-1} + \theta[v_{t-1}, v^{(w)}_{t-2}]v^{(w)}_{t-1} + \theta[v_{t-1}, v^{(m)}_{t-2}]v^{(m)}_{t-1}}{\theta[v_{t-1}, v_{t-2}] + \theta[v_{t-1}, v^{(w)}_{t-2}] + \theta[v_{t-1}, v^{(m)}_{t-2}]} + \epsilon_t, \epsilon_t \sim (0, \sigma^2), \tag{9}$$

with $\theta[v_{t-1}, v_{t-2}] = \exp(-\omega_1(v_{t-1} - v_{t-2})^2), \theta[v_{t-1}, v^{(w)}_{t-2}] = \exp\left(-\omega_2\left(v_{t-1} - v^{w}_{t-2}\right)^2\right)$ and $\theta\left[v_{t-1}, v^{(m)}_{t-2}\right] = \exp\left(-\omega_3\left(v_{t-1} - v^{(m)}_{t-2}\right)^2\right)$.

The ES1 approach can be interpreted as a combination of forecasting models which suggests a simple weighting average of the different volatility aggregates. The component v_{t-1} is a forecast from a random walk model, while $v^{(w)}_{t-1}$ and $v^{(m)}_{t-1}$ are nothing else but moving window forecasts with different window sizes. Then the daily volatility v_t in (9) is just a weighted sum of the previous daily realized volatilities. Note the ES1 model has one parameter less than the HAR, namely there is no constant. Since there is no a priori knowledge about the ES1 weights, we also consider a naive 1/3 combination of these three components in the empirical study in the next section.

Of course, the proposed ES approach can also be used for combining any volatility predictors. In the empirical study we additionally consider the ES2 model which combines the previous day volatility v_{t-1}, the popular RiskMetrics forecast $v^{(rm)}_{t-1}$ as well as the HAR predictor $v^{(har)}_{t-1}$ in order to reflect the short run, medium run and long run volatility components, respectively. The equation for the ES2 model is given as

$$v_t = \frac{\theta[v_{t-1}, v_{t-2}]v_{t-1} + \theta\left[v_{t-1}, v^{(rm)}_{t-2}\right]v^{(rm)}_{t-1} + \theta\left[v_{t-1}, v^{(har)}_{t-2}\right]v^{(har)}_{t-1}}{\theta[v_{t-1}, v_{t-2}] + \theta\left[v_{t-1}, v^{(rm)}_{t-2}\right] + \theta\left[v_{t-1}, v^{(har)}_{t-2}\right]} + \epsilon_t, \quad \epsilon_t \sim (0, \sigma^2). \tag{10}$$

As earlier, we define $\theta[v_{t-1}, v_{t-2}] = \exp(-\omega_1(v_{t-1} - v_{t-2})^2)$, etc.

4. Empirical illustration

4.1. Data and preliminary analysis

The aim of the empirical analysis is to illustrate the ES forecast combination approach by predicting daily volatility of major financial indices. For this purpose we use the data provided by Oxford-Man Institute particularly the daily realized variance series based on 5 min returns with subsampling. Further details on data preparation and data cleaning can be obtained from the documentation of the Oxford-Man Institute under www.oxford-man.ox.ac.uk. We multiply the time series of realized volatilities with 10^2 for the numerical stability of the optimization algorithms. The chosen dataset contains the realized volatilities for NASDAQ, S&P500, and Nikkei225 indices covering the period from over 03.01.2000 till 25.02.2013. This period is characterized by both calm and highly volatile periods. Moreover, we draw a special attention to the subperiod from November 2007 till February 2011 which corresponds to the time during and after the recent subprime crisis (cf. Bekaert et al., 2012).

The basic statistical properties of the considered time series are summarized in Table 1 for both full sample and subprime crisis subsample. In general, the evidence from all time series supports the common stylized facts about the realized volatilities. They exhibit strong and persistent memory, the full-sample distribution is heavily right-skewed. Fig. 1 provides the plots and autocorrelation functions of S&P500, NASDAQ and Nikkei daily realized volatility series. Note that Nikkei exhibits an exponential decay of autocorrelation, whereas the decay of S&P500 and NASDAQ indices appears to be hyperbolic.

The volatility characteristics (e.g., mean, standard deviation, median) during the subprime crisis period are in general higher than those during the full sample. The autocorrelations during the subprime period decay much quicker as shown in the autocorrelation plots in Fig. 1 (right column). They appear to be insignificant already after about 2 months of daily observations pointing on a much quicker transfer of information during the crisis.

4.2. Full sample estimation results

The HAR model given in (7) and two ES models in (9) and (10) are estimated from the full sample with the maximum likelihood methodology by assuming normality for the process innovations. The parameter estimates, the corresponding standard errors in parenthesis and the residual variances are provided in Table 2. Moreover, we report residual autocorrelations which are much lower compared to the original data in Table 1. This finding implies that all three models in general succeed in removing autocorrelation from the daily volatility processes.

Next we characterize the parameter estimates for the empirical similarity models. The parameters of the ES1 model with the HAR components are economically reasonable and consistent with our expectations. The parameter ω_1 which determines the daily similarity $\theta[v_{t-1}, v_{t-2}]$ is positive and significant for all indices. This implies that a decreasing distance $\phi[v_{t-1}, v_{t-2}]$ (more similarity) increase the role of daily random walk forecast v_{t-1} as a part of the DGP v_t. The parameters of the second (weekly) component is positive and significant only for Nikkei but remains insignificantly different from zero for two other indices. Note that insignificance of ω_2 does not imply that this component is irrelevant for the volatility process as this component transmits its influence via the other ES components due to normalization as in Eq. (4). The third

Table 1
Basis statistics for the realized volatility series.

	Min	Max	Mean	Sd	Skewness	Kurtosis	Med	ρ_1	ρ_2	Length
Full sample statistics for the realized volatility series										
S&P500	4.264×10^{-4}	0.8545	0.0117	0.0258	14.34	372.5	5.811×10^{-3}	0.623	0.616	3276
NASDAQ	4.466×10^{-4}	0.4804	0.0156	0.0262	7.079	83.93	7.659×10^{-3}	0.659	0.598	3281
Nikkei	6.857×10^{-4}	0.3153	0.0113	0.0174	9.127	120.6	7.482×10^{-3}	0.741	0.651	3180
Subprime period statistics for the realized volatility series										
S&P500	4.264×10^{-4}	0.8545	0.0227	0.0476	8.827	126.1	9.596×10^{-3}	0.577	0.586	773
NASDAQ	4.466×10^{-4}	0.4804	0.0196	0.0339	6.258	58.34	10.31×10^{-3}	0.611	0.613	773
Nikkei	7.138×10^{-4}	0.3153	0.0163	0.0294	5.964	45.11	8.369×10^{-3}	0.784	0.712	773

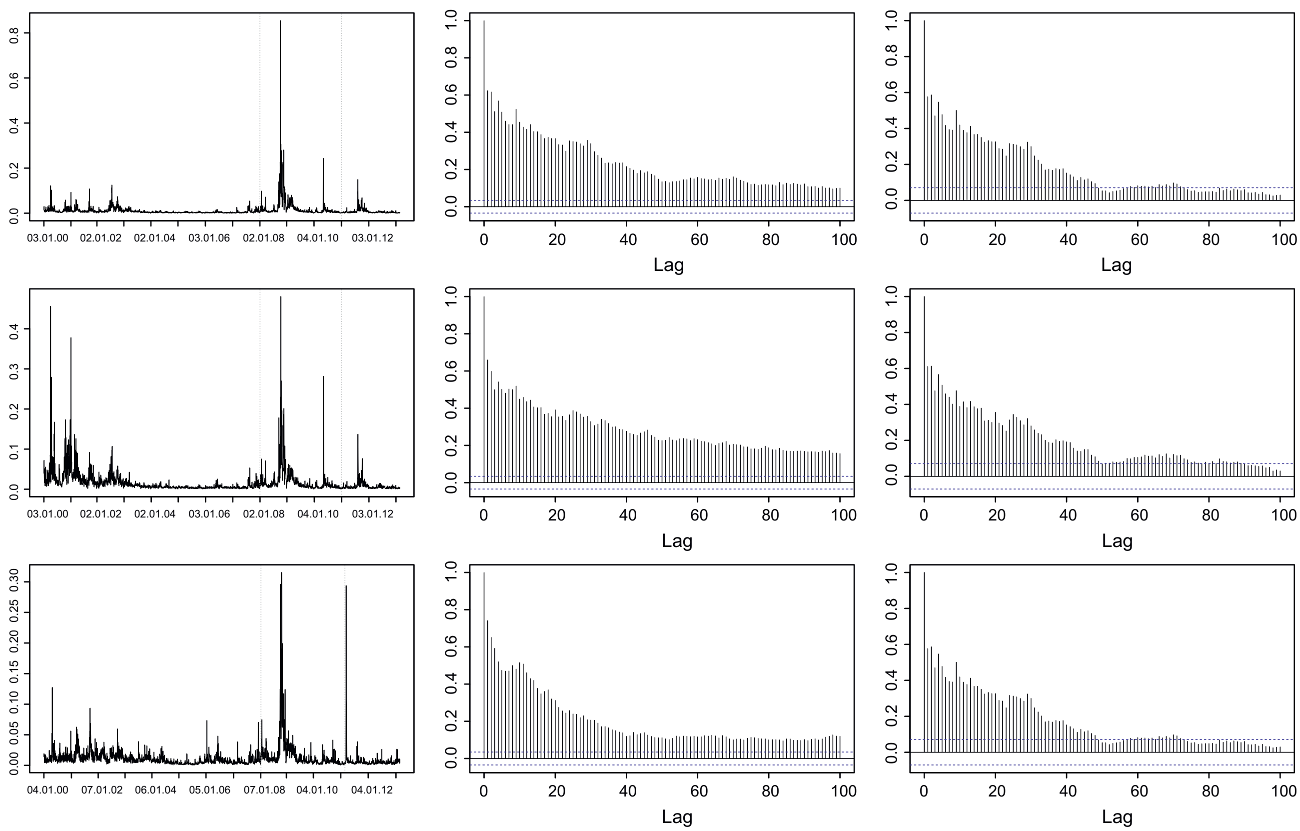

Fig. 1. The time series of the realized volatilities (left) and the corresponding ACFs for the full sample (middle) and the ACFs for the crisis subperiod (right) S&P500, NASDAQ, and Nikkei (from top to bottom).

(monthly) component is significant and positive for SP500 and Nikkei, and significantly negative for NASDAQ. For negative ω values a large distance between v_{t-1} and $v_{t-2}^{(m)}$ increases the weight of the component $v_{t-1}^{(m)}$. This result points on a type of mean-reverting behavior. For example, if the current realized volatility heavily deviates from the monthly volatility average, then the ES model with a negative weight ω_3 implies that in future the process should revert to this monthly average.

Concerning the ES2 approach which combines previous day volatility, RiskMetrics and HAR forecasts, the previous day coefficient ω_1 is significantly positive for all indices. This finding indicates that a large distance implies decreasing weight of the daily forecast as we also found for the ES1 approach. The HAR coefficient ω_3 is always positive but not always significant. The evidence concerning the RiskMetrics coefficient ω_2 is mixed, it is strongly significantly positive for NASDAQ series.

The normalized ES1 weights ϕ appear to be close to 1/3 during quite long period of time. For this reason we additionally investigate the 1/3 approach which can be seen as a naive combination of volatility components. It is mathematically equivalent to a simplified version of exponentially weighted moving average with step-wise decreasing weights which are constant over time. In particular, the daily measure v_{t-1} gets the largest weight, $v_{t-2},\ldots,v_{t-5}$ get the same smaller weight and $v_{t-3},\ldots,v_{t-22}$ obtain the same smallest weight. In order to analyze the performance of the 1/3-rule more precisely, we sum up absolute deviations between the ES1 component weights and fixed weight of 1/3 by $\sum_h|\phi\left[v_t, v_{t-1}^{(h)}\right] - 1/3|$. The resulting measure is plotted in the left column of Fig. 2. We observe that the periods with bursting absolute deviation sum strongly overlap with high volatility periods. The scatter plots of the deviations from 1/3 vs. realized volatilities are provided in the right column of Fig. 2. There is a positive dependence with moderate R^2 values of about 0.1–0.3. The simple 1/3 averaging appears to be close to the ES weights in calm or stable phases, when all approaches produce reasonable forecasts and market participants are relatively certain about future volatility dynamics. However, during and after volatility shocks the ES weights strongly deviate from the 1/3-benchmark.

4.3. Full sample forecast comparisons

In this subsection we compare both in-sample and out-of-sample forecasting performance of several models for daily volatility. In particular, we estimate the HAR, ES1 and ES2 models. Additionally, we consider the rule assigning 1/3-weights for all three HAR components as well as the RiskMetrics forecasting approach with the smoothing parameter $\lambda = 0.94$. To run a fair forecast comparison we employ the family of homogenous loss functions advocated by Patton (2011). These functions are robust to the presence of noise in realized volatility proxies and can be used for ranking alternative forecasting models. Parameterized by b, the family of loss function is defined as:

$$\mathcal{L}(rv,\widehat{v},b) = \begin{cases} \frac{1}{(b+1)(b+2)}(rv^{b+2} - \widehat{v}^{b+2}) - \frac{1}{b+1}\widehat{v}^{b+1}(rv - \widehat{v}) & \text{for } b \neg\in \{-1,-2\} \\ \widehat{v} - rv + rv\cdot\log(rv/\widehat{v}) & \text{for } b = -1 \\ \frac{rv}{\widehat{v}} - \log\frac{rv}{\widehat{v}} - 1 & \text{for } b = -2 \end{cases}$$

where rv is a volatility measure and $\hat{v}$ is a corresponding forecast. Note that for $b = -2$ the loss function corresponds to the ML criteria, while for $b = 0$ we obtain the mean squared error (MSE)

Table 2
The full sample parameter estimates, the corresponding standard deviation, the variance of the residuals for the considered models and the first two autocorrelations of the residuals. The autocorrelations significant at 5% level are marked with $^{(*)}$.

Model	$\alpha_0 \times 10^{-3}$	ω_1	ω_2	ω_3	$\sigma^2 \times 10^{-4}$	ρ_1	ρ_2
Estimation results for S&P 500							
HAR	1.159	0.200	0.489	0.208	3.412	−0.019	0.167*
	(0.385)	(0.021)	(0.036)	(0.032)	–		
1/3	–	–	–	–	3.484	−0.128*	0.175*
	–	–	–	–	–		
RM	–	–	–	–	4.288	0.369*	0.359*
	–	–	–	–	–		
ES1	–	227.329	−3.833	31.358	3.056	0.0902*	0.123*
	–	(38.679)	(7.887)	(12.836)	–		
ES2	–	215.525	−17.901	18.802	3.055	0.083*	0.166*
	–	(41.665)	(11.575)	(14.155)	–		
Estimation results for NASDAQ							
HAR	1.397	0.329	0.289	0.279	2.855	−0.031	0.159*
	(0.384)	(0.020)	(0.033)	(0.030)			
1/3	–	–	–	–	2.892	−0.035*	0.153*
	–	–	–	–			
RM	–	–	–	–	6.966	0.662*	0.586*
	–	–	–	–			
ES1	–	14.761	5.959	−73.988	2.749	0.041*	0.165*
	–	(6.817)	(11.264)	(15.417)			
ES2	–	56.750	37.489	2.559	3.005	−0.141*	0.186*
	–	(9.356)	(5.763)	(1.172)			
Estimation results for Nikkei							
HAR	1.175	0.524	0.173	0.199	1.293	−0.043*	0.062*
	(0.272)	(0.020)	(0.032)	(0.028)			
1/3	–	–	–	–	1.351	0.143*	0.122*
	–	–	–	–			
RM	–	–	–	–	1.875	0.550*	0.397*
	–	–	–	–			
ES1	–	69.239	194.108	23.309	1.295	0.062*	0.097*
	–	(20.466)	(47.724)	(8.873)			
ES2	–	51.507	531.992	16.143	1.228	−0.052*	0.084*
	–	(21.365)	(235.619)	(8.865)			

measure. For high positive values of b the loss function penalizes overestimation of the true value more heavily, while for negative values of b underestimation of the true value has higher impact on losses (cf. Patton, 2011). In the study we use the values $b \in \{1, 0, -1, -2\}$. We concentrate here on comparing average losses because the models appear to be rather close in forecasting performance during the long calm period in the middle of the sample so that statistical significance could be hardly established. The results on statistical significance are provided in the next subsection where we focus on volatility prediction during the recent subprime crisis period.

First we report the in-sample forecasting evidence which is based on the full sample model estimates. The in-sample model losses averaged over the full sample for all analyzed models in Table 3. The ES1 approach shows superior performance for all values of b. The HAR model is quite close to the ES2 approach, while the performance of the 1/3 model improves with lower values of b. The RiskMetrics approach is uniformly the worst one.

Next, in order to assess the out-of-sample forecasting ability of the similarity-based models we conduct a one-step-ahead forecasting exercise. In a moving window setup we reestimate each model using the recent 250 observations and compute one-step-ahead forecasting errors. The difference between the predicted values and the true observed realized volatility is measured using the robust loss functions as above. The corresponding losses averaged over the full sample are summarized in Table 4. We observe the performance of the ES1 model is still superior over the HAR for all values of the parameter b. However, the 1/3-approach should be slightly preferred for the out-of-sample forecasting purpose. This evidence is not surprising, because 250 daily observations (one year) is a rather short for estimation of the ES models with such complex dynamics. Thus, the success of 1/3-component weighting can be seen as an equivalent to the finding that it is rather hard to beat the uninformed $1/N$ approach of selecting the optimal portfolio composition (cf. DeMiguel et al., 2009).

Summarizing, we report that the ES1 (HAR-component) approach outperforms the HAR model both in sample and out of sample although it has one parameter less. Moreover, we find that a naive weighting with 1/3 proportion for each of three volatility components provides a decent out-of-sample performance. This evidence corresponds to the intuition that uninformed decision makers tend to forecasting the next day volatility by keeping equal weights for the recent daily, weekly and monthly volatility aggregates. These results remain stable for all considered volatility series.

4.4. Forecasting volatility during the crisis

Analyzing volatility behavior during the recent subprime crisis is for sure of immense importance for both practice and research due to extreme turbulence of global markets. For this reason in this subsection we investigate the subperiod covering time during and immediately after the crisis with respect to the success of the empirical similarity approach in volatility forecasting. We select November 2007 as the starting point of the subperiod and February 2011 as the end point, which appears to be in line with the crisis dating of Bekaert et al. (2012). The model comparison is conducted in line with those in the previous subsection. The average losses for daily out-of-sample volatility forecasts during this subperiod are summarized in Table 5, whereas the ordered/sorted models and the corresponding p-values from the model confidence set (MCS) of Hansen et al. (2011) are provided in Table 6.

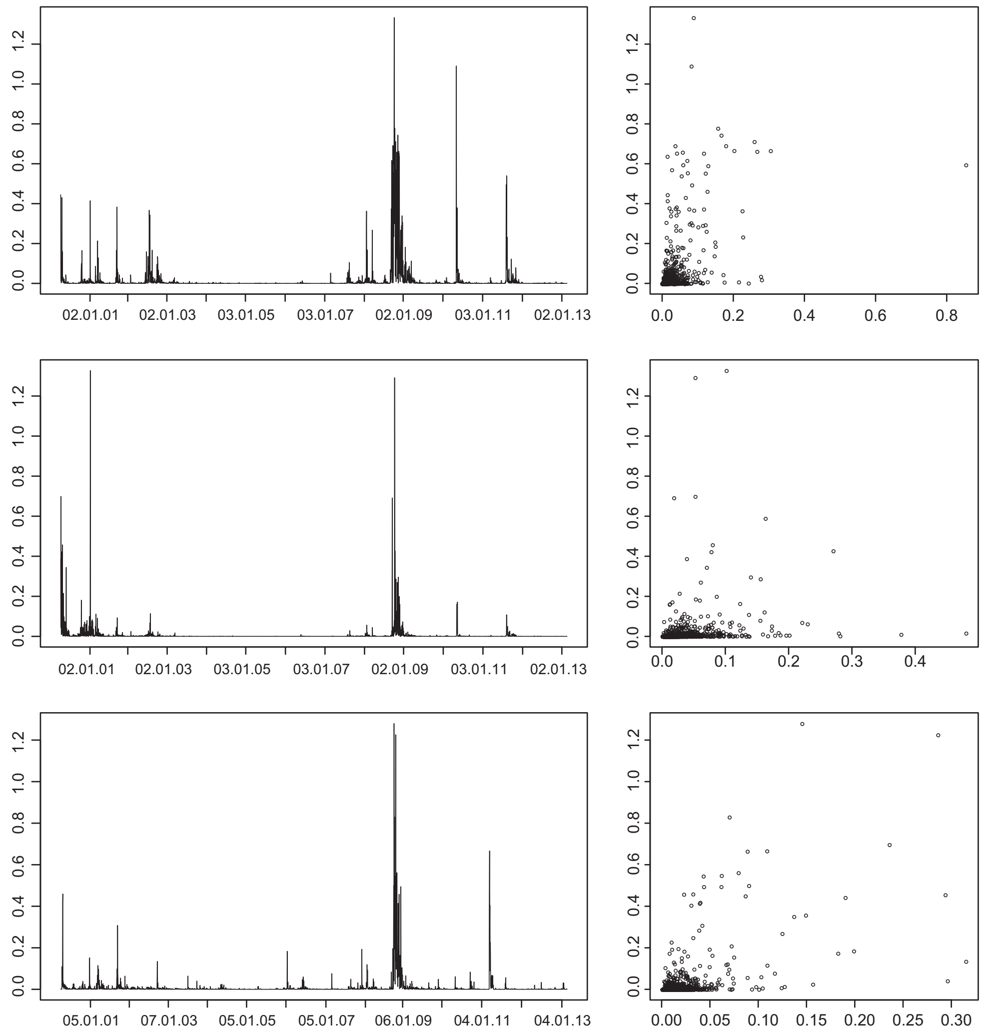

Fig. 2. The deviation measure $\sum_h |\phi(v_t, v_{t-1}^{(h)}) - 1/3|$ (right) and scatterplots of realized volatility vs. the deviation measure (left) of the ES1 model for the full sample estimation for S&P500, NASDAQ and Nikkei. R^2 measures equal to 0.29, 0.06 and 0.32 respectively.

Table 3
Average values of the in-sample losses for the considered models estimated based on the full sample information.

Model	$b = 1$	$b = 0$	$b = -1$	$b = -2$
Average losses for S&P500				
HAR	4.055×10^{-11}	1.704×10^{-8}	2.133×10^{-5}	1.422×10^{-1}
1/3	4.310×10^{-11}	1.742×10^{-8}	2.076×10^{-5}	1.330×10^{-1}
RM	4.184×10^{-11}	2.144×10^{-8}	3.248×10^{-5}	1.960×10^{-1}
ES1	3.706×10^{-11}	1.528×10^{-8}	1.986×10^{-5}	1.325×10^{-1}
ES2	3.679×10^{-11}	1.527×10^{-8}	2.010×10^{-5}	1.343×10^{-1}
Average losses for NASDAQ				
HAR	1.712×10^{-11}	1.426×10^{-8}	2.378×10^{-5}	1.315×10^{-1}
1/3	1.754×10^{-11}	1.446×10^{-8}	2.366×10^{-5}	1.252×10^{-1}
RM	6.017×10^{-11}	3.483×10^{-8}	4.115×10^{-5}	1.769×10^{-1}
ES1	1.612×10^{-11}	1.375×10^{-8}	2.331×10^{-5}	1.252×10^{-1}
ES2	1.881×10^{-11}	1.502×10^{-8}	2.394×10^{-5}	1.256×10^{-1}
	$b = 1$	$b = 0$	$b = -1$	$b = -2$
Average losses for Nikkei				
HAR	6.249×10^{-12}	6.445×10^{-9}	1.684×10^{-5}	1.295×10^{-1}
1/3	6.550×10^{-12}	6.753×10^{-9}	1.726×10^{-5}	1.299×10^{-1}
RM	8.160×10^{-12}	9.374×10^{-9}	2.484×10^{-5}	1.758×10^{-1}
ES1	6.176×10^{-12}	6.475×10^{-9}	1.690×10^{-5}	1.292×10^{-1}
ES2	5.696×10^{-12}	6.139×10^{-9}	1.670×10^{-5}	1.290×10^{-1}

Table 4
Average losses for one-step-ahead forecasts. The models are reestimated using the last 250 observations.

Model	$b = 1$	$b = 0$	$b = -1$	$b = -2$
Average losses for S&P500				
HAR	11.496×10^{-11}	3.082×10^{-8}	2.372×10^{-5}	1.440×10^{-1}
1/3	4.654×10^{-11}	1.834×10^{-8}	2.030×10^{-5}	1.322×10^{-1}
RM	4.432×10^{-11}	2.131×10^{-8}	3.002×10^{-5}	1.896×10^{-1}
ES1	4.694×10^{-11}	1.842×10^{-8}	2.032×10^{-5}	1.322×10^{-1}
ES2	6.229×10^{-11}	2.130×10^{-8}	2.085×10^{-5}	1.343×10^{-1}
Average losses for NASDAQ				
HAR	2.106×10^{-11}	1.284×10^{-8}	2.043×10^{-5}	1.340×10^{-1}
1/3	1.219×10^{-11}	0.969×10^{-8}	1.851×10^{-5}	1.238×10^{-1}
RM	1.236×10^{-11}	1.169×10^{-8}	2.553×10^{-5}	1.675×10^{-1}
ES1	1.223×10^{-11}	0.970×10^{-8}	1.851×10^{-5}	1.238×10^{-1}
ES2	1.426×10^{-11}	1.037×10^{-8}	1.864×10^{-5}	1.243×10^{-1}
	$b = 1$	$b = 0$	$b = -1$	$b = -2$
Average losses for Nikkei				
HAR	25.620×10^{-12}	11.813×10^{-9}	1.888×10^{-5}	1.373×10^{-1}
1/3	6.919×10^{-12}	6.800×10^{-9}	1.662×10^{-5}	1.291×10^{-1}
RM	8.654×10^{-12}	9.543×10^{-9}	2.423×10^{-5}	1.752×10^{-1}
ES1	6.916×10^{-12}	6.798×10^{-9}	1.662×10^{-5}	1.291×10^{-1}
ES2	8.390×10^{-12}	7.368×10^{-9}	1.683×10^{-5}	1.298×10^{-1}

Table 5
Average losses for one-step-ahead forecasts in the subprime crisis subperiod. The models are reestimated using the last 250 observations.

Model	$b = 1$	$b = 0$	$b = -1$	$b = -2$
Average losses for S&P500				
HAR	8.590×10^{-12}	10.033×10^{-9}	2.670×10^{-5}	1.899×10^{-1}
1/3	5.683×10^{-12}	7.962×10^{-9}	2.224×10^{-5}	1.496×10^{-1}
RM	5.983×10^{-12}	9.125×10^{-9}	2.940×10^{-5}	2.257×10^{-1}
ES1	5.650×10^{-12}	7.875×10^{-9}	2.201×10^{-5}	1.489×10^{-1}
ES2	6.759×10^{-12}	8.570×10^{-9}	2.283×10^{-5}	1.602×10^{-1}
	$b = 1$	$b = 0$	$b = -1$	$b = -2$
Average losses for NASDAQ				
HAR	13.616×10^{-12}	1.233×10^{-8}	2.761×10^{-5}	1.857×10^{-1}
1/3	8.085×10^{-12}	0.953×10^{-8}	2.474×10^{-5}	1.597×10^{-1}
RM	8.154×10^{-12}	1.018×10^{-8}	3.050×10^{-5}	2.318×10^{-1}
ES1	8.038×10^{-12}	0.945×10^{-8}	2.460×10^{-5}	1.595×10^{-1}
ES2	10.023×10^{-12}	1.031×10^{-8}	2.446×10^{-5}	1.632×10^{-1}
	$b = 1$	$b = 0$	$b = -1$	$b = -2$
Average losses for Nikkei				
HAR	2.429×10^{-13}	1.452×10^{-9}	1.185×10^{-5}	1.399×10^{-1}
1/3	2.203×10^{-13}	1.314×10^{-9}	1.065×10^{-5}	1.250×10^{-1}
RM	2.601×10^{-13}	1.544×10^{-9}	1.271×10^{-5}	1.525×10^{-1}
ES1	2.207×10^{-13}	1.314×10^{-9}	1.065×10^{-5}	1.249×10^{-1}
ES2	2.368×10^{-13}	1.397×10^{-9}	1.119×10^{-5}	1.303×10^{-1}

The results in Tables 5 and 6 indicate that the ES1 approach with the HAR components provides rather sound performance which is in many situations significantly better than the alternatives. The ES1 approach is almost everywhere the best one with the 1/3-strategy as the closest competitor. Furthermore, the MCS results state that for almost all cases only the ES1 and 1/3 strategies remain in the MCS, i.e. are statistically superior at 10% significance level. Note that extremely high volatility during the subprime crisis complicates establishing further statistical significance results during this subperiod.

Predicting volatility is of practical interest not only for daily data frequency but also for other, e.g. weekly, time horizons (cf. Andersen et al., 2007; Byun and Kim, 2013). For these reasons we also consider forecasting weekly volatility measure during the chosen subperiod by contrasting the HAR, 1/3 and ES1 approaches. The models are reestimated with the moving window of 250 days which is shifted for one day ahead. The averages losses for the weekly volatility forecast are presented in Table 7.

As for the one day ahead volatility forecasts, the ES1 model is overall the best one in forecasting the weekly measure. It appears to be rather close to 1/3 for Nikkei volatility, but is much better for S&P500 and NASDAQ indices. Thus, the empirical similarity concept appears to be applicable for weekly forecasting horizon as well. Note that volatility forecast comparisons for longer (monthly, quarterly) horizons are hardly reasonable with the similarity approach because the long term component is driven mostly by changes in (macro)economic fundamentals but not by short run daily fluctuations (cf. Andersen et al., 2005). Thus, the models for daily volatility are not very suitable for long term forecasting.

5. Conclusions

In this paper we propose the empirical similarity (ES) approach for combining forecasts of different models. The ES concept should be applied in situations where probabilistic model combinations are undesired or hardly possible. The ES concept of Gilboa et al. (2006) provides the econometric framework for analogy-based reasoning, which grounds on the case based decision theory (cf. Gilboa and Schmeidler, 2001). The ES allows to estimate how

Table 6
The ordered models using the out-of-sample model confidence sets for the crisis subperiod. The first models in the MCS can be dropped up to the corresponding significance level. The p-values are given in parentheses and the last model with $p = 1$ is the best.

	$b = 1$	$b = 0$	$b = -1$	$b = -2$
S&P 500	HAR, ES2, RM, 1/3, ES1 (0.060,0.060,0.060,0.068,1)	HAR, RM, ES2, 1/3, ES1 (0.039,0.039,0.114,0.114,1)	RM, HAR, ES2, 1/3, ES1 (0.009,0.009,0.145,0.145,1)	RM, HAR, ES2, 1/3, ES1 (0.020,0.020,0.285,0.636,1)
NASDAQ	HAR, ES2, RM, 1/3, ES1 (0.330,0.330,0.330,0.373,1)	HAR, RM, ES2, 1/3, ES1 (0.010,0.010,0.425,0.425,1)	RM, HAR, 1/3, ES1, ES2 (0.045,0.045,0.499,0.875,1)	RM, HAR, ES2, 1/3, ES1 (0.022,0.022,0.435,0.435,1)
NIKKEI	RM, ES2, HAR, ES1, 1/3 (0.099,0.099,0.099,0.369,1)	RM, HAR, ES2, ES1, 1/3 (0.045,0.045,0.045,0.712,1)	RM, HAR, ES2, 1/3, ES1 (0.047,0.047,0.047,0.771,1)	RM, HAR, ES2, 1/3, ES1 (0.014,0.057,0.117,0.482,1)

Table 7
Average losses for the weekly volatility forecasts during the subprime crisis subperiod. The models are reestimated using the last 250 observations.

Model	$b = 1$	$b = 0$	$b = -1$	$b = -2$
Average losses for S&P500				
HAR	2.521×10^{-12}	6.104×10^{-9}	2.618×10^{-5}	2.018×10^{-1}
1/3	1.765×10^{-12}	4.663×10^{-9}	1.943×10^{-5}	1.488×10^{-1}
ES1	1.687×10^{-12}	4.469×10^{-9}	1.893×10^{-5}	1.474×10^{-1}
Average losses for NASDAQ				
HAR	3.000×10^{-12}	5.942×10^{-9}	2.451×10^{-5}	1.945×10^{-1}
1/3	2.015×10^{-12}	4.902×10^{-9}	2.071×10^{-5}	1.592×10^{-1}
ES1	1.863×10^{-12}	4.558×10^{-9}	1.987×10^{-5}	1.569×10^{-1}
	$b = 1$	$b = 0$	$b = -1$	$b = -2$
Average losses for Nikkei				
HAR	1.615×10^{-13}	10.659×10^{-10}	9.081×10^{-6}	10.221×10^{-1}
1/3	1.045×10^{-13}	6.675×10^{-10}	5.913×10^{-6}	7.369×10^{-1}
ES1	1.045×10^{-13}	6.676×10^{-10}	5.914×10^{-6}	7.370×10^{-1}

economic agents evaluate the current situation with respect to their experience. In our approach the model combination weights are assigned by evaluating the similarity of the current realization to the recent model forecasts. The proposed ES forecast combination approach incorporates both time series and cross-section similarity features.

The suggested ES methodology is used for the purpose of daily volatility forecasting. We estimate the ES-based models in order to determine the weights of the volatility components sampled at different frequencies (daily, weekly, monthly) as in the HAR approach of Corsi (2009). We find that the ES model with the HAR components is systematically better than the original HAR both in sample and out of sample for various forecasting performance measures. Moreover, a naive ES strategy of assigning 1/3-weights to all three components is quite successful for the out-of-sample volatility prediction purpose. The volatility forecasting exercise for the subperiod during and after the subprime crisis supports the usefulness of the empirical similarity concept for explaining complex volatility dynamics.

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