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On the stochastic behaviour of the run length of EWMA control schemes for the mean of correlated output in the presence of shifts in σ

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Summary: This paper discusses in detail the impact of shifts on the process variance (σ^2) on the run length (RL) of modified upper one-sided EWMA charts for the process mean (μ) when the output is correlated.

Quite apart from the relevance of a process variance change in its own right, a dilation in σ^2 can cause an undesirable stochastic decrease in the detection speed of some specific shifts in μ. This and other stochastic results are proved and illustrated with a few examples.

1 Background and aim

The industrial age was easing into its second century when a young engineer named Walter A. Shewhart came along and altered the course of industrial history by proposing the what is grandly termed as control scheme. Control schemes are used to monitor the distribution parameters (such as the mean) of random variables (i.e., the process output) over time and to identify the presence of special causes that may affect these parameters and therefore the quality of the output.

When control schemes for the mean μ are used, two standard assumptions are that the process output is *independent* and has *constant* variance σ^2 equal to the *known* target value σ_0^2.

The independence assumption can be far from reasonable for some processes of interest and can also have a severe impact on the performance of the standard control schemes, as reported by several authors in numerical studies, such as [9], [1], [13], [23], and proved and discussed in detail namely by [19], [20] and [18].

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Some attention has been given in the statistical process control (SPC) literature to the RL distribution of schemes for the expected value μ, when the target of the process variance is unknown (although constant), and therefore the control limits have to be estimated. For the case of independent output see, for instance, [2], [8], and [6]. The influence of the variance estimation on the RL is examined by [10] for correlated output. In both cases it may be quite misleading to assume that the behaviour of RL for σ^2 known can be carried over to draw conclusions about the case where σ^2 is unknown.

Another compelling question concerning schemes for μ is to know what happens to the RL distribution in the presence of shifts in the process variance σ^2, if the operator falsely assumes that σ^2 is constant and equal to a known target value σ_0^2, and designs the control scheme for μ accordingly.

Average run length (ARL) tables for two-sided (Shewhart, CUSUM and EWMA) schemes for μ are provided by [7], in such a setting. These tables cast some light on the performance of these schemes. For example, Tables 6–8 show that, for some large but fixed shifts in μ, the ARL of these two-sided schemes can be an increasing or even a nonmonotone function of σ. This behaviour – although not commented by the author – is apparent in the last lines of Tables 6–8, and it implies that these schemes become progressively less sensitive to some shifts in the process mean, as the process variance grows.

Moreover, [16] derived sufficient conditions for the RL of the upper one-sided $\bar{X}$ and EWMA schemes for the process mean of independent output to have a (stochastically) increasing or decreasing behaviour. These results were proved using the Markov approach ([5], [12], [15]) and were subsequently extended by [14] for the upper one-sided CUSUM, combined CUSUM-Shewhart and combined EWMA-Shewhart schemes, also for the process mean of independent output.

This paper gives analogue results concerning one-sided EWMA schemes for the process mean of correlated output. These results are proved using a different approach because the presence of correlation destroys the Markov property of the summary statistic, as noted by [24]. Moreover, the schemes considered here plot the original EWMA observations and have control limits adjusted to account for the autocorrelation inherent to the output. Thus, their summary statistic is different from the one used in [16] which immediately resets any value below the target mean, μ_0, to μ_0, turning the analysis of the RL distribution practically unfeasible in the case of correlated output.

2 Model

In what follows $\{Y_t\}$ denotes the target process and we assume it is a (weakly) stationary process with mean μ_0 and autocovariance function $\{\gamma_v\}$. Let the observations $x_1, x_2, \ldots$ denote a realization of the observed process $\{X_t\}$. We assume here that the processes are related as follows

$$X_t = \begin{cases} Y_t & \text{for } t \leq 0 \\ \mu_0 + \delta\sqrt{\gamma_0} + \Delta(Y_t - \mu_0) & \text{for } t = 1, 2, \ldots, \end{cases} \tag{2.1}$$

where $-\infty < \delta < \infty$ and $\Delta > 0$.

If $\delta \neq 0$ ($\Delta \neq 1$) then a sustained shift in the location parameter μ (scale parameter σ) has been observed at time $t = 1$. Clearly, $\{X_t\}$ is said to be in-control if $(\delta, \Delta) = (0, 1)$, and to be out-of-control otherwise.

In order to control the process mean behaviour EWMA charts are widely applied (see, e.g., [17], [12], [20] and [11]).

Modified EWMA charts for the mean of stationary processes were introduced by [20]. They are based on the following EWMA statistic

$$Z_t = \begin{cases} \mu_0 & \text{for } t = 0 \\ (1-\lambda)Z_{t-1} + \lambda X_t & \text{for } t = 1, 2, \ldots, \end{cases} \tag{2.2}$$

where $\lambda \in (0, 1]$ is a smoothing parameter that represents the weight given to the most recent observation. Following [20], the expectation and variance of this summary statistic – with respect to Model (2.1) – are equal to

$$\begin{aligned} E_{\delta,\Delta}(Z_t) &= E_{0,1}(Z_t) + \delta\sqrt{\gamma_0}\left[1-(1-\lambda)^t\right] \\ &= \mu_0 + \delta\sqrt{\gamma_0}\left[1-(1-\lambda)^t\right] \end{aligned} \tag{2.3}$$

$$\begin{aligned} \operatorname*{Var}_{\delta,\Delta}(Z_t) &= \Delta^2 \operatorname*{Var}_{0,1}(Z_t) \\ &= \Delta^2 \frac{\lambda}{2-\lambda}\gamma_0 \left[1-(1-\lambda)^{2t}\right] \\ &\quad + \Delta^2 \frac{2\lambda}{2-\lambda}\sum_{v=1}^{t-1}\gamma_v(1-\lambda)^v\left[1-(1-\lambda)^{2(t-v)}\right]. \end{aligned} \tag{2.4}$$

The indexes "δ, Δ" and "0, 1" mean, throughout the remainder of this paper, that the quantity (an expectation, a variance, a probability, etc.) is calculated with respect to Model (2.1) and to the in-control situation (that is, given an arbitrary (δ, Δ) and $(\delta, \Delta) = (0, 1)$, respectively).

The upper one-sided modified EWMA chart triggers a signal at time t, suggesting that an increase in the process mean has occurred, if

$$Z_t - E_{0,1}(Z_t) > c\sqrt{\operatorname*{Var}_{0,1}(Z_t)}\,,$$

for some fixed $c > 0$. This signal is called a valid alarm, in case the process is out-of-control, and a false alarm, otherwise. We denote the RL of this chart conditioned on the Model (2.1) by

$$N_{\delta,\Delta} = \inf\left\{ t \in \mathbb{N} : Z_t - E_{0,1}(Z_t) > c\sqrt{\operatorname*{Var}_{0,1}(Z_t)} \right\}, \tag{2.5}$$

the number of observations taken after $t = 0$ until a signal is triggered by the control chart.

It is worth mentioning here that the in-control asymptotic variance is frequently preferred in practice to $\mathrm{Var}_{0,1}(Z_t)$, since this substitution avoids updating the control limits at each time point. Assuming that $\{\gamma_v\}$ is absolutely summable we get the asymptotic variance

$$\begin{aligned}\overline{\sigma}^2_{\delta,\Delta} &= \Delta^2 \lim_{t\to\infty} \underset{0,1}{\mathrm{Var}}(Z_t) \\ &= \Delta^2\, \overline{\sigma}^2_{0,1} \\ &= \Delta^2\, \gamma_0\, \frac{\lambda}{2-\lambda}\left[1 + 2\sum_{v=1}^{\infty} \frac{\gamma_v}{\gamma_0}(1-\lambda)^v\right]. \end{aligned} \tag{2.6}$$

Table 2.1 In-control asymptotic variances for white noise and stationary AR(1) and ARMA(1,1) models.

Model	γ_0	$\gamma_v/\gamma_0,\ v = 1, 2, \ldots$	$\overline{\sigma}^2_{0,1}$
WN	σ^2	0	$\gamma_0\ \frac{\lambda}{2-\lambda}$
AR(1)	$\frac{\sigma^2}{1-\alpha^2}$	α^v	$\gamma_0\ \frac{\lambda}{2-\lambda}\ \frac{1+\alpha(1-\lambda)}{1-\alpha(1-\lambda)}$
ARMA(1,1)	$\frac{\sigma^2\ (1+2\alpha\beta+\beta^2)}{1-\alpha^2}$	$\frac{(1+\alpha\beta)(\alpha+\beta)}{1+2\alpha\beta+\beta^2}\alpha^{v-1}$	$\gamma_0\ \frac{\lambda}{2-\lambda}\left[1+2\ \frac{(1+\alpha\beta)(\alpha+\beta)}{1+2\alpha\beta+\beta^2}\ \frac{1-\lambda}{1-\alpha(1-\lambda)}\right]$

For future reference we list in Table 2.1 the in-control asymptotic variances of the EWMA summary statistic ($\overline{\sigma}^2_{0,1}$) when the target process is governed by a white noise (WN) process, and by stationary AR(1) and ARMA(1,1) models (i.e. $Y_t = \varepsilon_t$, $Y_t = \alpha Y_{t-1} + \varepsilon_t$ and $Y_t = \alpha Y_{t-1} + \varepsilon_t + \beta\varepsilon_{t-1}$, respectively, with $\{\varepsilon_t\} \sim \mathrm{WN}(0, \sigma^2)$ and $\alpha \in (-1, 1)$). For more details about these three models see, for example, [4] or [3].

3 Main results

Before we proceed with the main results of the paper, we need two preparatory definitions.

The random variable X is said to be stochastically smaller than Y in the usual sense – $X \le_{st} Y$ – if and only if their survival functions satisfy $P(X > x) \le P(Y > x)$, $-\infty < x < \infty$ (see [22, p. 4]).

Let X_θ be a random variable whose distribution depends on parameter $\theta \in \Theta$. Then we denote the fact that X_θ stochastically increases with $\theta \in \Theta$ in the usual sense by – $X_\theta \uparrow_{st}$ with θ – if and only if $X_\theta \le_{st} X_{\theta'}$, $\theta \le \theta'$ $(\theta, \theta' \in \Theta)$.

Theorem 3.1 *Let*

$$\delta_L = \inf_{t=1,2,\ldots} \frac{c}{\sqrt{\gamma_0}}\, \frac{\sqrt{\mathrm{Var}_{0,1}(Z_t)}}{1-(1-\lambda)^t} \tag{3.1}$$

$$\delta_U = \sup_{t=1,2,\ldots} \frac{c}{\sqrt{\gamma_0}}\, \frac{\sqrt{\mathrm{Var}_{0,1}(Z_t)}}{1-(1-\lambda)^t}. \tag{3.2}$$

Then the following stochastic monotone behaviours hold for the run length defined in (2.5) for the upper one-sided modified EWMA chart:

$$N_{\delta,\Delta} \downarrow_{st} \text{ with } \Delta, \text{ for fixed } \delta \leq \delta_L \tag{3.3}$$

$$N_{\delta,\Delta} \uparrow_{st} \text{ with } \Delta, \text{ for fixed } \delta \geq \delta_U. \tag{3.4}$$

Proof: Properties (3.3) and (3.4) follow by virtue of the monotone behaviour (in terms of Δ) of the survival function of $N_{\delta,\Delta}$

$$P_{\delta,\Delta}(N_{\delta,\Delta} > k) = P_{\delta,\Delta}\left[\frac{Z_t - E_{\delta,\Delta}(Z_t)}{\sqrt{\mathrm{Var}_{\delta,\Delta}(Z_t)}} \leq \frac{A_t(\delta)}{\Delta}, t = 1, \ldots, k\right], \tag{3.5}$$

for $k = 1, 2, \ldots$, where

$$\begin{aligned} A_t(\delta) \;&=\; \Delta\, \frac{c\sqrt{\mathrm{Var}_{0,1}(Z_t)} + E_{0,1}(Z_t) - E_{\delta,\Delta}(Z_t)}{\sqrt{\mathrm{Var}_{\delta,\Delta}(Z_t)}} \\ &= c - \frac{\delta\,\sqrt{\gamma_0}\,\left[1-(1-\lambda)^t\right]}{\sqrt{\mathrm{Var}_{0,1}(Z_t)}}. \end{aligned}$$

In fact, the survival function of $N_{\delta,\Delta}$ decreases (increases) with respect to Δ if $A_t(\delta) \geq (\leq)\, 0$ for $t = 1, 2, \ldots$, that is, if

$$\delta \leq (\geq)\, \frac{c}{\sqrt{\gamma_0}}\, \frac{\sqrt{\mathrm{Var}_{0,1}(Z_t)}}{1-(1-\lambda)^t}, \text{ for all } t = 1, 2, \ldots$$

The bounds in Equations (3.1) and (3.2) immediately follow. □

Remark 3.2 The results of Theorem 3.1 certainly deserve a few comments.

1.) Property (3.3) enables us to conclude that, for fixed $\delta \leq \delta_L$, the larger the shift in scale (i.e., as Δ increases), the sooner a signal is produced, yielding to a more sensitive chart in stochastic terms.

2.) Result (3.4) essentially means that the upper one-sided modified EWMA chart regretfully decreases its ability to trigger a signal as shifts in scale become more severe, provided that δ is fixed and $\delta \geq \delta_U$. This disadvantage is surely due to the fact that the upper one-sided modified EWMA chart is essentially designed to detect shifts in the location of the model, under the assumption that the scale remains constant and equal to its target value.

3.) We are not able to establish the stochastic behaviour of $N_{\delta,\Delta}$ with respect to Δ, for any values of δ in the interval (δ_L, δ_U).

We close this section by stating the analogue of Theorem 3.1 for

$$\overline{N}_{\delta,\Delta} = \inf\{t \in \mathbb{N} : Z_t - E_{0,1}(Z_t) > c\overline{\sigma}_{0,1}\}, \tag{3.6}$$

the RL of the modified upper one-sided EWMA scheme with control limits based on the asymptotic variance.

Corollary 3.3 *If asymptotic control limits are at use then Theorem 3.1 remains valid if the bounds δ_L and δ_U are replaced by*

$$\overline{\delta}_L = \frac{c}{\sqrt{\gamma_0}}\,\overline{\sigma}_{0,1} \tag{3.7}$$

$$\overline{\delta}_U = \frac{\overline{\delta}_L}{\lambda}. \tag{3.8}$$

Proof: This lemma can be proved in a similar way to Theorem 3.1 if we take into account that in this case

$$\begin{aligned} A_t(\delta) &= \Delta \frac{c\,\overline{\sigma}_{0,1} + E_{0,1}(Z_t) - E_{\delta,\Delta}(Z_t)}{\sqrt{\mathrm{Var}_{\delta,\Delta}(Z_t)}} \\ &= \frac{1}{\sqrt{\mathrm{Var}_{0,1}(Z_t)}}\ \left\{c\,\overline{\sigma}_{0,1} - \delta\,\sqrt{\gamma_0}\,\left[1-(1-\lambda)^t\right]\right\}. \end{aligned}$$ □

Remark 3.4 1.) Considering that there is a shift at the very first point of surveillance is a very common assumption in the SPC literature. However, the assumption of a shift at an arbitrary time point $q > 1$ can be easily dealt with. In fact, additional calculations led to the conclusion that this assumption has no impact on the results of Theorem 3.1 if the variance of the EWMA statistic is an increasing function of Δ (this holds if, for instance, the autocovariance function $\{\gamma_\nu\}$ is nonnegative).

2.) A comment ought to be made about situations in which the mean and the variance do not change simultaneously. These situations, though interesting, are, as far as we know, not considered in SPC literature. Nevertheless, it can be also dealt with considering, without any loss of generality, a shift in the mean at time $t = 1$ as in the paper and a shift in σ^2 at time $t = q$. In fact, we can also prove that the bounds in Equations (3.1) and (3.2) still hold if the variance of the EWMA statistic is an increasing function of Δ.

3.) Moreover, when the target values of the mean μ and the autocovariance function $\{\gamma_\nu\}$ are unknown, we are dealing with a slightly different setting. However, the assessment of the impact of estimation on the distribution of the run length and its stochastic behaviour is beyond the scope of this paper.

4 Further results

The bounds δ_L and δ_U, defined by (3.1) and (3.2), are often difficult to obtain. However, under mild conditions, the bounds δ_L and δ_U may be replaced by simpler expressions, namely, related to the asymptotic in-control variance, as stated in the next corollary.

Corollary 4.1 *Let the target process $\{Y_t\}$ be stationary with mean μ_0 and absolutely summable autocovariance function $\{\gamma_v\}$. If*

$$\frac{\sqrt{\text{Var}_{0,1}(Z_t)}}{1-(1-\lambda)^t} \text{ decreases with } t \tag{4.1}$$

then the bounds δ_L and δ_U are equal to

$$\delta'_L = \frac{c}{\sqrt{\gamma_0}}\,\overline{\sigma}_{0,1} \tag{4.2}$$

$$\delta'_U = c. \tag{4.3}$$

Proof: Under the assumptions of the lemma the bounds δ_L and δ_U in Equations (3.1) and (3.2) are given by

$$\delta_L = \inf_{t=1,2,\dots} \frac{c}{\sqrt{\gamma_0}} \frac{\sqrt{\text{Var}_{0,1}(Z_t)}}{1-(1-\lambda)^t} = \frac{c}{\sqrt{\gamma_0}}\,\overline{\sigma}_{0,1} = \delta'_L$$

$$\delta_U = \sup_{t=1,2,\dots} \frac{c}{\sqrt{\gamma_0}} \frac{\sqrt{\text{Var}_{0,1}(Z_t)}}{1-(1-\lambda)^t} = \frac{c}{\sqrt{\gamma_0}} \frac{\sqrt{\text{Var}_{0,1}(Z_1)}}{1-(1-\lambda)} = c = \delta'_U. \qquad \square$$

Now, it is natural to inquire whether there are target processes verifying (4.1).

Lemma 4.2 *Let $\{Y_t\}$ be a stationary process with mean μ_0 and absolutely summable autocovariance function $\{\gamma_v\}$. If $\{\gamma_v\}$ is a nonnegative decreasing sequence then the ratio $\sqrt{\text{Var}_{0,1}(Z_t)}/[1-(1-\lambda)^t]$ decreases with t.*

The proof of this decreasing behaviour is excruciatingly long and was compacted in the Appendix.

Note that Lemma 4.2 holds for: any WN process; any stationary AR(1) model, with parameter $\alpha \in [0, 1)$; and any stationary ARMA(1,1) model, with parameters such that $\alpha \in [0, 1)$ and $\frac{(1+\alpha\beta)(\alpha+\beta)}{1+2\alpha\beta+\beta^2} > 0$.

Remark 4.3 In Table 4.1, we can find the corresponding expressions of the ratio

$$\eta_t = \left[\frac{\lambda}{2-\lambda}\,\gamma_0\right]^{-1} \frac{\text{Var}_{0,1}(Z_t)}{[1-(1-\lambda)^t]^2} \tag{4.4}$$

and of the lower bound $\delta_L = \delta'_L$, for the WN process and the stationary AR(1) and ARMA(1,1) models.

Table 4.1 Expressions of $\delta_L = \delta'_L$, for the WN process and the stationary AR(1) and ARMA(1,1) models (with $\xi_t = \sum_{v=1}^{t-1} \alpha^v (1-\lambda)^v [1-(1-\lambda)^{2(t-v)}]$).

Model	η_t	$\delta_L = \delta'_L$
WN	$\frac{1+(1-\lambda)^t}{1-(1-\lambda)^t}$	$c\sqrt{\frac{\lambda}{2-\lambda}}$
AR(1)	$\frac{1+(1-\lambda)^t}{1-(1-\lambda)^t} + \frac{2\xi_t}{[1-(1-\lambda)^t]^2}$	$c\sqrt{\frac{\lambda}{2-\lambda}}\sqrt{\frac{1+\alpha(1-\lambda)}{1-\alpha(1-\lambda)}}$
ARMA(1,1)	$\frac{1+(1-\lambda)^t}{1-(1-\lambda)^t} + \frac{(1+\alpha\beta)(\alpha+\beta)}{\alpha(1+\alpha\beta+\beta^2)}\frac{2\xi_t}{[1-(1-\lambda)^t]^2}$	$c\sqrt{\frac{\lambda}{2-\lambda}}\sqrt{1+2\,\frac{(1+\alpha\beta)(\alpha+\beta)}{1+2\alpha\beta+\beta^2}\,\frac{1-\lambda}{1-\alpha(1-\lambda)}}$

η_t is clearly a decreasing function of t for WN output since $\lambda \in (0, 1]$.

Now, notice that the sum ξ_t and η_t can be written in a recursive way for $t = 1, 2, \ldots$:

$$\begin{aligned} \xi_{t+1} &= \sum_{v=1}^{t} \alpha^v (1-\lambda)^v \left[1-(1-\lambda)^{2(t+1-v)}\right] \\ &= \alpha(1-\lambda)\left\{\xi_t + \left[1-(1-\lambda)^{2t}\right]\right\}; \end{aligned} \tag{4.5}$$

$$\begin{aligned} \eta_{t+1} &= \frac{[1+(2k-\alpha)(1-\lambda)] - (1-\lambda)^{2(t+1)}[1+(2k-\alpha)/(1-\lambda)]}{[1-(1-\lambda)^{t+1}]^2} \\ &\quad + \alpha(1-\lambda)\left[\frac{1-(1-\lambda)^t}{1-(1-\lambda)^{t+1}}\right]^2 \eta_t, \end{aligned} \tag{4.6}$$

where $k = (1+\alpha\beta)(\alpha+\beta)/(1+2\alpha\beta+\beta^2)$, for the ARMA(1,1) model; and $k = \alpha$, for the AR(1) model. However, from (4.5) we conclude that ξ_t is not a monotone function of t and (4.6) fails to help us prove that η_t is a decreasing function of t. The complete proof of this monotone behaviour can be found in the Appendix, as mentioned before.

Finally, note that our numerical studies using *Mathematica* ([25]) led us to conjecture that η_t (and therefore the ratio $\sqrt{\text{Var}_{0,1}(Z_t)}/[1-(1-\lambda)^t]$) either decreases with t, or has two decreasing subsequences (for odd and even values of t) converging to the same limit, for all constellations of set parameters considered for the stationary AR(1) and ARMA(1,1) processes, regardless of their autocorrelation functions. If this conjecture holds then the bounds in Corollary 4.1 are valid for all stationary AR(1) and ARMA(1,1) processes.

Remark 4.4 It should be added that Lemma 4.2 implies

$$\frac{\text{Var}_{0,1}(Z_{t+1})}{\text{Var}_{0,1}(Z_t)} \leq \left[\frac{1-(1-\lambda)^{t+1}}{1-(1-\lambda)^t}\right]^2, \tag{4.7}$$

while [21] has shown that, for an arbitrary stationary Gaussian process with nonnegative autocovariance function, we have

$$\begin{aligned}\frac{\mathrm{Var}_{0,1}(Z_{t+1})}{\mathrm{Var}_{0,1}(Z_t)} &\geq \frac{1-(1-\lambda)^{2(t+1)}}{1-(1-\lambda)^{2t}} \\ &= \left[\frac{1-(1-\lambda)^{t+1}}{1-(1-\lambda)^{t}}\right]^2 \\ &\quad \times \left\{1 - \frac{2\lambda(1-\lambda)^t}{\left[1+(1-\lambda)^t\right]\left[1-(1-\lambda)^{t+1}\right]}\right\}. \end{aligned} \tag{4.8}$$

In case (4.1) does not hold, replacing the exact bounds δ_L and δ_U for easy-to-compute ones is also of practical use. For instance, if $\{Y_t\}$ is stationary with mean μ_0 and absolutely summable autocovariance function $\{\gamma_v\}$, we get

$$\underset{0,1}{\mathrm{Var}}(Z_t) \leq \gamma_0 \, \frac{\lambda}{2-\lambda} \left[1 + 2\sum_{v=1}^{\infty}(1-\lambda)^v\right] = \gamma_0.$$

Thus,

$$\delta_U = \sup_{t=1,2,\ldots} \frac{c}{\sqrt{\gamma_0}} \, \frac{\sqrt{\mathrm{Var}_{0,1}(Z_t)}}{1-(1-\lambda)^t} \leq \frac{c}{\lambda}.$$

In the light of this result we infer that an inaccurate upper bound is c/λ. However, we can further improve such an inexact upper bound and also provide an alternative to the exact lower bound δ_L.

Corollary 4.5 *The stochastic order relations (3.3) and (3.4) are still valid if δ_L and δ_U are replaced by*

$$\begin{aligned}\delta_L^* &= c\lambda \\ \delta_U^* &= \frac{c}{\lambda\sqrt{\gamma_0}}\,\overline{\sigma}_{0,1},\end{aligned}$$

provided that the autocovariance function $\{\gamma_v\}$ is nonnegative and absolutely summable.

Proof: Bounds δ_L^* and δ_U^* are obtained as follows:

$$\delta_L \geq \inf_{t=1,2,\ldots} \frac{c}{\sqrt{\gamma_0}} \sqrt{\underset{0,1}{\mathrm{Var}}(Z_t)} = \frac{c}{\sqrt{\gamma_0}} \sqrt{\underset{0,1}{\mathrm{Var}}(Z_1)} = c\lambda = \delta_L^* \;;$$

$$\delta_U \leq \sup_{t=1,2,\ldots} \frac{c}{\lambda\sqrt{\gamma_0}} \sqrt{\underset{0,1}{\mathrm{Var}}(Z_t)} = \frac{c}{\lambda\sqrt{\gamma_0}}\,\overline{\sigma}_{0,1} = \delta_U^* \,. \qquad \square$$

Remark 4.6 A few comments about the exact and alternative lower and upper bounds.

1.) For i.i.d. output, the bounds $\delta_L = \delta'_L$ and $\delta_U = \delta'_U$ obtained here are distinct from the ones in [16] for the upper one-sided EWMA chart they considered, since this chart has a summary statistic different from (2.2) and uses control limits that are functions of the asymptotic variance. In fact, the lower bound corresponding to $\delta_L = \delta'_L$ in [16] is equal to $-c(1-\lambda)/\sqrt{\lambda(1-\lambda)}$, thus, negative. As for the upper bound, these authors obtained $c/\sqrt{\lambda(1-\lambda)}$, which is larger than the corresponding $\delta_U = \delta'_U = c$.

2.) Note that, for a stationary target process with mean μ_0, an absolutely summable autocovariance function $\{\gamma_v\}$ and a decreasing ratio $\frac{\sqrt{\text{Var}_{0,1}(Z_t)}}{[1-(1-\lambda)^t]}$, we get $\delta_L = \delta'_L = \overline{\delta}_L$ and $\delta_U = \delta'_U \leq \delta^*_U = \overline{\delta}_U$. Thus, the range of the interval where we cannot predict the monotone behaviour of $\overline{N}_{\delta,\Delta}$, $(\overline{\delta}_L, \overline{\delta}_U)$, is larger than the interval associated to $N_{\delta,\Delta}$, (δ_L, δ_U). Using exact limits does indeed pay off.

5 Simulation results and concluding remarks

To give the reader further insight into how the ARL behaves as a function of Δ, for fixed values of δ, we present a simulation study, since there is no other possibility of obtaining an ARL defined as $E_{\delta,\Delta}(N_{\delta,\Delta}) = 1 + \sum_{k=1}^{+\infty} P_{\delta,\Delta}(N_{\delta,\Delta} > k)$, where the term of this sum is defined by Equation (3.5) and therefore related to the distribution function of a k-dimensional multivariate normal r.v.

The simulation results refer to an upper one-sided modified EWMA chart for the mean of a few stationary AR(1) and ARMA(1,1) processes and were obtained by considering 10^6 replications, $\lambda = 0.1$, $\mu_0 = 0$ and $\sigma_0 = 1$.

The constant c which defines the upper control limit was chosen, via simulation, in such way that the scheme requires an in-control ARL close to 500 samples. The remaining parameters, namely the autoregressive and moving average parameters, the in-control asymptotic variances of the EWMA summary statistic and the bounds δ'_L and δ'_U (as defined in Corollary 4.1) can be also found in Table 5.1.

Table 5.1 Parameters, in-control ARLs, and bounds δ'_L and δ'_U, for upper one-sided modified EWMA chart for the mean of stationary AR(1) and ARMA(1,1) processes.

α	β	In-control ARL	γ_0	$\overline{\sigma}^2_{0,1}$	$\delta'_L = \frac{c}{\sqrt{\gamma_0}}\,\overline{\sigma}_{0,1}$	$\delta'_U = c$
0.5	0	500.004	1.333333	0.185008	0.888915	2.386350
0.8	0	499.994	2.777778	0.898079	1.239752	2.180351
0.6	0.3	499.995	2.265625	0.460991	1.043992	2.314434
0.6	–0.3	500.000	1.140625	0.139195	0.836669	2.395045

Figure 6.1 presents several ARL curves for values of Δ ranging from an 80 % reduction of the process standard deviation ($\Delta = 0.2$) to a 300 % increase in this parameter ($\Delta = 3$).

On one hand, some of these plots illustrate the fact that $E_{\delta,\Delta}(N_{\delta,\Delta})$ is a decreasing (an increasing) function of Δ for $\delta = \delta'_L$ ($\delta = \delta'_U$), when we are dealing with upper one-sided modified EWMA for the mean of stationary AR(1) and ARMA(1,1) processes under the conditions of Corollary 4.1 and Lemma 4.2.

On the other hand, the three intermediate curves on the left hand side of Figure 6.1, set separately and in a different scale on the right hand side of the figure, portray a completely different scenario for $\delta = 0.7\delta'_L + 0.3\delta'_U, 0.6\delta'_L + 0.4\delta'_U, 0.5\delta'_L + 0.5\delta'_U$: $E_{\delta,\Delta}(N_{\delta,\Delta})$ can have a nonmonotonous behaviour in terms of Δ. A plausible explanation for this nonmonotonous behaviour as the level of the standard deviation increases, for a fixed δ belonging to the interval (δ'_L, δ'_U), is the fact that $N_{\delta,\Delta}$ starts by having a stochastic decreasing behaviour in Δ when $\delta \leq \delta'_L$, and becomes stochastically increasing in Δ when $\delta \geq \delta'_U$.

Moreover, the percentage points of $N_{\delta,\Delta}$ in Table 6.2 either have no monotone behaviour or change their monotone behaviour in terms of Δ, for $\delta = 0.7\delta'_L + 0.3\delta'_U$, $\delta = 0.6\delta'_L + 0.4\delta'_U$ and $\delta = 0.5\delta'_L + 0.5\delta'_U$. Take for instance the ARMA (1,1) process with parameters $\alpha = 0.6$, $\beta = 0.3$, in case $\delta = 0.6\delta'_L + 0.4\delta'_U$: the 5, 10, 25% percentage points, the median and the mean of $N_{\delta,\Delta}$ all decrease with Δ suggesting an increasing ability to trigger an out-of-control signal; however, the 75, 90 and 95 % percentage points increase with Δ suggesting just the opposite.

As for the ARL results in Figure 6.1 and Table 6.2, we can add that not only the charts do not give any protection to decreases or increases in the standard deviation but also that an increase in σ can lead to an increase in the out-of-control ARL for large values of δ. Both situations should not be tolerated by the practitioners.

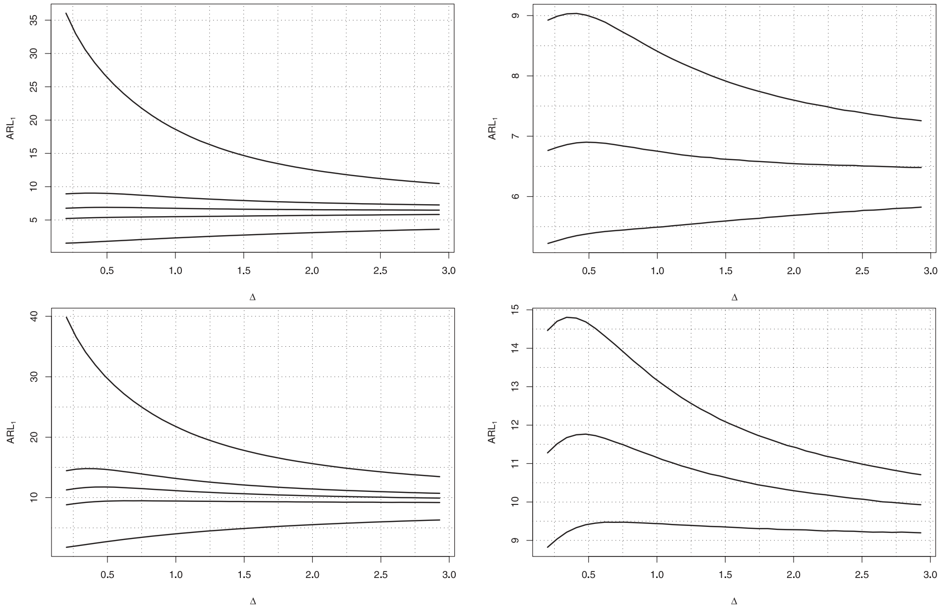
Control schemes are often used to monitor a process for the sole purpose of detecting assignable causes that result in changes in parameters which in turn may result in lower-quality output.

Unlike some authors who argue that consideration of the theoretical properties of the control schemes (the "probabilistic" approach) reduces the usefulness of the techniques (as observed by [26]), we strongly believe that the knowledge of the RL distribution and its stochastic monotone behaviour in terms of the model parameters provides decisive insights into how schemes work in practice and helps practitioners better understand the ability of the control scheme to monitor process quality and the way its performance changes or can be improved. In particular, we proved that under certain conditions the discriminating effect of the upper one-sided EWMA chart for the mean of correlated output decreases with the magnitude of the shift in the process variance. The financial impact of such behaviour can only be avoided by using a joint scheme for μ and σ^2.

6 Appendix

The reader should be reminded that this appendix has the sole purpose of presenting a short version of the proof that the sufficient condition (4.1) of Corollary 4.1 holds for any stationary process with mean μ_0 and nonnegative decreasing and absolutely summable autocovariance function $\{\gamma_v\}$, as stated in Lemma 4.2.

(a) AR(1) processes: $\alpha = 0.5$ (top), 0.8 (bottom)



(b) ARMA(1,1) processes: $\alpha = 0.6$, $\beta = 0.3$ (top), $\alpha = 0.6$, $\beta = -0.3$ (bottom)

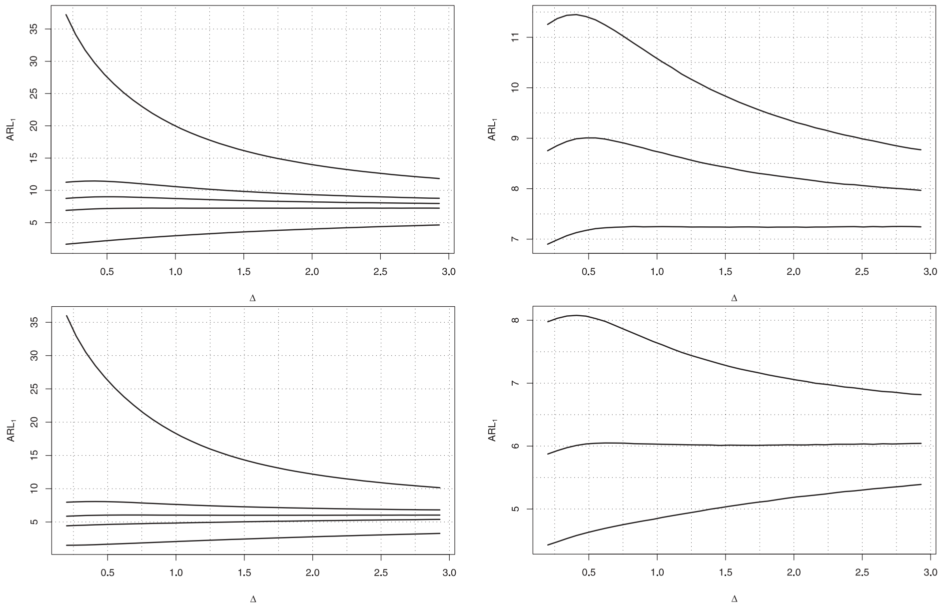


Figure 6.1 Plots of $E_{\delta,\Delta}(N_{\delta,\Delta})$, for upper one-sided modified EWMA charts and $\Delta \in [0.2, 3]$. On the left: $\delta = \delta_L'$, $0.7\delta_L' + 0.3\delta_U'$, $0.6\delta_L' + 0.4\delta_U'$, $0.5\delta_L' + 0.5\delta_U'$, δ_U from top to bottom. On the right: $\delta = 0.7\delta_L' + 0.3\delta_U'$, $0.6\delta_L' + 0.4\delta_U'$, $0.5\delta_L' + 0.5\delta_U'$.

Table 6.2 AR(1) and ARMA(1,1) processes – $N_{\delta,\Delta}$ percentage points (p.p.) and $E_{\delta,\Delta}(N_{\delta,\Delta})$ values, for upper one-sided modified EWMA charts.

	$\alpha = 0.5$								
	$\delta = 0.7\delta'_L + 0.3\delta'_U$			$\delta = 0.6\delta'_L + 0.4\delta'_U$			$\delta = 0.5\delta'_L + 0.5\delta'_U$		
$N_{\delta,\Delta}$ p.p.	Δ			Δ			Δ		
	0.5	1.0	2.0	0.5	1.0	2.0	0.5	1.0	2.0
5 %	3	1	1	2	1	1	2	1	1
10 %	4	1	1	3	1	1	2	1	1
25 %	6	3	1	4	2	1	3	2	1
50 %	8	7	4	6	5	3	5	4	3
75 %	12	12	11	9	9	9	7	8	8
90 %	15	18	19	12	14	16	9	12	14
95 %	17	22	26	13	18	22	10	15	19
$E_{\delta,\Delta}(N_{\delta,\Delta})$	9.03	8.39	7.60	6.88	6.72	6.51	5.38	5.50	5.68

	$\alpha = 0.8$								
	$\delta = 0.7\delta'_L + 0.3\delta'_U$			$\delta = 0.6\delta'_L + 0.4\delta'_U$			$\delta = 0.5\delta'_L + 0.5\delta'_U$		
$N_{\delta,\Delta}$ p.p.	Δ			Δ			Δ		
	0.5	1.0	2.0	0.5	1.0	2.0	0.5	1.0	2.0
5 %	3	1	1	2	1	1	1	1	1
10 %	4	1	1	3	1	1	2	1	1
25 %	7	3	1	5	2	1	4	2	1
50 %	12	8	5	10	7	4	8	5	3
75 %	20	19	16	16	16	14	13	13	12
90 %	29	32	32	23	27	29	19	24	26
95 %	35	41	44	29	36	40	24	31	36
$E_{\delta,\Delta}(N_{\delta,\Delta})$	14.64	13.21	11.42	11.71	11.09	10.36	9.46	9.45	9.26

	$\alpha = 0.6, \beta = 0.3$								
	$\delta = 0.7\delta'_L + 0.3\delta'_U$			$\delta = 0.6\delta'_L + 0.4\delta'_U$			$\delta = 0.5\delta'_L + 0.5\delta'_U$		
$N_{\delta,\Delta}$ p.p.	Δ			Δ			Δ		
	0.5	1.0	2.0	0.5	1.0	2.0	0.5	1.0	2.0
5 %	3	1	1	3	1	1	2	1	1
10 %	4	2	1	3	1	1	3	1	1
25 %	7	4	1	5	3	1	4	2	1
50 %	10	8	5	8	7	4	6	5	4
75 %	15	15	13	12	12	11	9	10	10
90 %	20	23	24	16	19	21	13	16	19
95 %	23	29	32	18	24	28	15	20	25
$E_{\delta,\Delta}(N_{\delta,\Delta})$	11.41	10.55	9.34	8.99	8.76	8.19	7.16	7.23	7.23

	$\alpha = 0.6, \beta = -0.3$								
	$\delta = 0.7\delta'_L + 0.3\delta'_U$			$\delta = 0.6\delta'_L + 0.4\delta'_U$			$\delta = 0.5\delta'_L + 0.5\delta'_U$		
$N_{\delta,\Delta}$ p.p.	Δ			Δ			Δ		
	0.5	1.0	2.0	0.5	1.0	2.0	0.5	1.0	2.0
5 %	3	1	1	2	1	1	2	1	1
10 %	3	1	1	2	1	1	3	1	1
25 %	5	3	1	4	2	1	3	2	1
50 %	7	6	4	6	5	3	4	4	3
75 %	10	11	10	8	8	8	6	7	7
90 %	14	16	18	10	13	15	8	10	13
95 %	16	20	24	12	16	21	9	13	17
$E_{\delta,\Delta}(N_{\delta,\Delta})$	8.07	7.61	7.03	6.03	6.05	6.03	4.62	4.83	5.14

Let

$$
\begin{aligned}
B(t) &= \frac{\mathrm{Var}_{0,1}(Z_{t+1})}{\gamma_0(1-x)^2} \\
&= \frac{1-x^{2(t+1)}}{1-x^2} + \frac{2}{1-x^2}\sum_{\nu=1}^{t}\rho_\nu x^\nu[1-x^{2(t+1-\nu)}], \qquad (6.1)
\end{aligned}
$$

where $x = 1-\lambda \in [0,1)$ and $\{\rho_\nu\}$ represents the autocorrelation function of a stationary process. Then from (2.4) we conclude that (4.1) is equivalent to

$$
B(t-1)(1-x^{t+1})^2 \geq B(t)(1-x^t)^2, \forall t. \qquad (6.2)
$$

Taking into account the symmetry of the autocorrelation function and that

$$
B(t) - B(t-1) = 2\sum_{\nu=0}^{t-1}\rho_{t-\nu}x^{t+\nu} + x^{2t} \qquad (6.3)
$$

$$
B(t) - x^2 B(t-1) = 1 + 2\sum_{\nu=1}^{t}\rho_\nu x^\nu \qquad (6.4)
$$

$$
x^t + \sum_{\nu=0}^{t-1}\rho_{t-\nu}x^\nu + x^t\sum_{\nu=1}^{t}\rho_\nu x^\nu = \sum_{\nu=0}^{2t}\rho_{t-\nu}x^\nu, \qquad (6.5)
$$

(6.2) can be successively rewritten as

$$
B(t) - x\,B(t-1) - \sum_{\nu=0}^{2t}\rho_{t-\nu}x^\nu \geq 0 \qquad (6.6)
$$

$$
(1-x)B(t-1) + 2\sum_{\nu=0}^{t-1}\rho_{t-\nu}x^{t+\nu} + x^{2t} - \sum_{\nu=0}^{2t}\rho_{t-\nu}x^\nu \geq 0. \qquad (6.7)
$$

Now, noting that

$$
\begin{aligned}
B(t-1) &= \frac{1-x^{2t}}{1-x^2} + \frac{2}{1-x^2}\sum_{\nu=1}^{t-1}\rho_\nu x^\nu[1-x^{2(t-\nu)}] \\
\sum_{\nu=0}^{t-1}\rho_{t-\nu}x^{t+\nu} &= \sum_{\nu=1}^{t-1}\rho_\nu x^{2t-\nu} + \rho_t x^t \qquad (6.8) \\
\sum_{\nu=0}^{2t}\rho_{t-\nu}x^\nu &= \rho_t(1+x^{2t}) + x^t + \sum_{\nu=1}^{t-1}\rho_t x^{t-\nu} + x^t\sum_{\nu=1}^{t-1}\rho_\nu x^\nu, \qquad (6.9)
\end{aligned}
$$

we obtain

$$
\begin{aligned}
&\sum_{\nu=1}^{t-1}\rho_\nu\left(2\,\frac{x^\nu + x^{2t-\nu+1}}{1+x} - x^{t-\nu} - x^{t+\nu}\right) - \rho_t(1-x^t)^2 \\
&\qquad + \frac{(1-x^t)(1-x^{t+1})}{1+x} \geq 0. \qquad (6.10)
\end{aligned}
$$

which is still equivalent to (6.7), and to

$$\sum_{\nu=0}^{t} \rho_\nu \left(2\,\frac{x^\nu + x^{2t-\nu+1}}{1+x} - x^{t-\nu} - x^{t+\nu} \right) \geq \frac{(1-x^t)(1-x^{t+1})}{1+x}. \tag{6.11}$$

After dividing both members of (6.11) by $\frac{(1-x^t)(1-x^{t+1})}{1+x}$, we get

$$\frac{1}{1-x^t}\sum_{\nu=0}^{t} \rho_\nu (x^\nu - x^{t-\nu}) + \frac{1}{1-x^{t+1}}\sum_{\nu=0}^{t} \rho_\nu (x^\nu - x^{t-\nu+1}) \geq 1. \tag{6.12}$$

This inequality can be rewritten as

$$\frac{1}{1-x^t}\sum_{\nu=0}^{t} \rho_\nu (x^\nu - x^{t-\nu}) + \frac{1}{1-x^{t+1}}\sum_{\nu=0}^{t+1} \rho_\nu (x^\nu - x^{t+1-\nu}) \geq 1 - \rho_{t+1}. \tag{6.13}$$

Since $x \in [0,1)$ and we assumed that $\{\rho_\nu\}$ is a nonnegative and decreasing sequence, it can be shown that $\sum_{\nu=1}^{t-1} \rho_\nu (x^\nu - x^{t-\nu}) \geq 0$. In fact, for even values of t, $t = 2l$, we have

$$\sum_{\nu=1}^{2l-1} \rho_\nu (x^\nu - x^{2l-\nu}) = \sum_{\nu=1}^{l-1} (\rho_\nu - \rho_{2l-\nu})(x^\nu - x^{2l-\nu}) \geq 0 \tag{6.14}$$

and, for odd values of t, $t = 2l+1$, we get

$$\sum_{\nu=1}^{2l} \rho_\nu (x^\nu - x^{2l+1-\nu}) = \sum_{\nu=1}^{l} (\rho_\nu - \rho_{2l+1-\nu})(x^\nu - x^{2l+1-\nu}) \geq 0. \tag{6.15}$$

Consequently, for $w = t, t+1$, it follows

$$\begin{aligned}\sum_{\nu=0}^{w} \rho_\nu (x^\nu - x^{w-\nu}) &= (1-x^w)(1-\rho_w) + \sum_{\nu=1}^{w-1} \rho_\nu (x^\nu - x^{w-\nu}) \\ &\geq (1-x^w)(1-\rho_w).\end{aligned} \tag{6.16}$$

Thus, the left member of (6.13) is greater or equal to $(1-\rho_t) + (1-\rho_{t+1})$. Hence, the inequality (6.13) holds. This completes the proof of Lemma 4.2.

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