

## Reply to the comment by A. A. Nersesyan and A. M. Tsvelik on "Non-zero fermi level density of states for a disordered d-wave superconductor in two dimensions"

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**Ziegler *et al.* Reply:** In their Comment on our recent Letter [1], Nersesyan and Tselik (NT) [2] have raised questions about the applicability of our calculation of the density of states (DOS) of a 2D  $d$ -wave superconductor (SC), though they do not actually dispute the result of a nonzero Fermi level DOS for a  $d$ -wave SC. They question the relevance of our exact calculation of the DOS using a Lorentzian disorder distribution, claiming “that models with Lorentzian and Gaussian disorder belong to different universality classes,” on the grounds of a comparison with a straightforward perturbation expansion in the disorder potential. We use a different method [1,3] to obtain lower bounds for the DOS for other (e.g., Gaussian) distributions supporting our contention of a nonzero DOS at the Fermi level. NT challenge this method by alleging that “similar discrepancies exist between the results obtained by this method for another model of disorder—(2 + 1)-dimensional fermions with random mass”; i.e., they contend that the method used in [4] gives incorrect results.

We have prepared a paper [5] which explicitly proves the existence of a nonzero lower bound for the Fermi level DOS for 2D SC's with nodes for a large class of disorder distributions, including the Gaussian. The proof applies the work of Ref. [3] to the case of interest. It also shows that the contribution of the “tails” of the distribution are unimportant for the Fermi level DOS. Therefore, our method of dealing with disorder in our Letter [1] (using a Lorentzian distribution) produces generic results for the physical system of a 2D SC with nodes, with disorder in the chemical potential. NT have not identified an error in our proof. Their argument against it is based on inconclusive numerical evidence, as discussed below.

The approach of NT [6] approximates a lattice system with disorder in the chemical potential by a continuum model of Dirac fermions with random gauge fields. The more general model of Dirac fermions with all kinds of disorder allowed (e.g., random mass) has been analyzed by Mudry *et al.* [6] who found that the critical line corresponding to pure gauge field disorder is *unstable* with respect to the other kinds of disorder. They conclude that “unless there exists a symmetry of the underlying lattice regularization” forbidding other disorder than random gauge fields the non-Gaussian cumulants are relevant perturbations. In short, the model considered by NT in Ref. [6] is most likely not generic, since the physical system does not provide a symmetry that allows the disorder to be described solely by random gauge fields. In this regard we emphasize that our work [1] is not “aimed to debunk the entire field theoretical approach to disordered systems,” but is actually in agreement with the conclusions of the no less field theoretical work by Mudry *et al.* [7].

(1) *Comparison with perturbation theory.*—The Lorentzian disorder distribution for which we obtained our exact results is quite special in terms of perturbation theory, but we do not believe that for the Fermi level DOS it will give qualitatively different results than the

impurity model with Gaussian disorder considered by NT. The fact that the Lorentzian disorder leaves the DOS of an  $s$ -wave SC unaffected shows that its tails do not necessarily lead to unphysical results. In any case, it is disingenuous to argue that because we cannot reproduce a perturbative calculation with a Lorentzian distribution the result of our Letter must be “in a different universality class.” Since every moment of the distribution is infinite, perturbative arguments do not apply to this case.

(2) *Dirac fermions with random mass.*—In contrast to what NT imply, there is analytical [3,4] and numerical [8] evidence that Dirac fermions with random mass have a nonzero DOS at the Fermi level. For a Gaussian disorder distribution with width  $\gamma$  the DOS at the Fermi level was estimated in [4] to be  $\propto \exp(-\pi/\gamma)$ , a nonanalytic behavior with respect to the disorder that cannot be obtained within perturbation theory. The related 2D random bond Ising model is a highly controversial field of research [9]. Different groups have obtained different results, e.g., Refs. [11,12] of the Comment by NT disagree with their Ref. [8] on almost everything but the specific heat. The numerical work quoted by NT as Ref. [10] agrees partially with their Refs. [8,11,12] as well as with the result of Ziegler [4]. Because of the extreme weakness of the  $\ln \ln(T - T_c)$  divergence the numerical data are not able to resolve the question of divergence or finiteness of the specific heat; the data can be fitted to both theories [10]. Therefore, there is no evidence that the methods of Ref. [4] produce incorrect results.

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