Tails of the density of states of two-dimensional Dirac fermions

Simon Villain-Guillot, Giancarlo Jug, Klaus G. Ziegler

Self-consistent theory of dynamic melting of a vortex lattice

Staffan Grundberg and Jørgen Rammer

Department of Theoretical Physics, Umeå University, S-90187 Umeå, Sweden[1]

Abstract. The dynamic melting of vortex lattices in type II superconductors is considered. A field-theoretic formulation of the pinning problem allows the average over the quenched disorder to be performed exactly. A self-consistent theory is constructed using a functional method for the effective action, allowing a determination of the pinning force and the vortex fluctuations. The phase diagram for the dynamic melting transition is determined numerically. In contrast to perturbation theory, the self-consistent theory is in quantitative agreement with the prediction of a recent phenomenological theory and simulations and experimental data.

1 Introduction

The advent of high temperature superconductors has stimulated a renewed interest in the dynamics of vortices in type II superconductors. In this letter we consider the influence of quenched disorder on the dynamic melting of a vortex lattice. This non-equilibrium phase transition has been studied experimentally [1] as well as through numerical simulation and a phenomenological theory and perturbation theory [2], [3], [4]. The notion of dynamic melting refers to the melting of a moving vortex lattice where in addition to the thermal fluctuations, fluctuations in vortex positions are induced by the disorder. A temperature-dependent critical velocity distinguishes a transition between a phase where the vortices form a moving lattice, the solid phase, and a vortex liquid phase.

2 Equation of motion and effective action

We consider a two-dimensional system (normal to $\hat{\mathbf{n}}$) since we have a thin superconducting film in mind, or a 3D layered superconductor with uncorrelated disorder between the layers. The description of the vortex dynamics will be based on the Langevin equation [2], [5]

$$\eta \dot{\mathbf{u}}_{\mathbf{R}t} + \sum_{\mathbf{R}'} \Phi_{\mathbf{R}\mathbf{R}'} \mathbf{u}_{\mathbf{R}'t} = \mathbf{F} - \nabla V(\mathbf{R} + \mathbf{u}_{\mathbf{R}t}) + \xi_{\mathbf{R}t} , \qquad (1)$$

[1] E-mail: grundberg@tp.umu.se, rammer@tp.umu.se

where $\mathbf{u}_{\mathbf{R}t}$ is the displacement at time t of the vortex which initially has equilibrium position $\mathbf{R}$, and η is the friction coefficient (per unit length of the vortex). The dynamic matrix, $\Phi_{\mathbf{R}\mathbf{R}'}$, of the triangular Abrikosov vortex lattice describes the harmonic interaction between the vortices, and is specified within the continuum theory of elastic media by the compression modulus, c_{11}, and the shear modulus, c_{66} [6],

$$\Phi_{\mathbf{q}} = \frac{\varphi_0}{B} \begin{pmatrix} c_{11}q_x^2 + c_{66}q_y^2 & (c_{11} - c_{66})\, q_x q_y \\ (c_{11} - c_{66})\, q_x q_y & c_{66}q_x^2 + c_{11}q_y^2 \end{pmatrix}, \tag{2}$$

where $\varphi_0 = h/2e$ is the flux quantum, and B the magnitude of the external magnetic field, $\mathbf{B} = B\hat{\mathbf{n}}$, and φ_0/B is therefore equal to the area, a^2, of the unit cell of the vortex lattice. The force (per unit length) on the right hand side of Eq. (1) consists of the Lorentz force, $\mathbf{F} = \varphi_0 \mathbf{j} \times \hat{\mathbf{n}}$, due to the (assumed constant) transport current density $\mathbf{j}$, and the thermal white noise stochastic force, $\xi_{\mathbf{R}t}$, is specified according to the fluctuation-dissipation theorem $\langle \xi_{\mathbf{R}t}^\alpha \xi_{\mathbf{R}'t'}^{\alpha'} \rangle = 2\eta T\delta(t - t')\, \delta_{\alpha\alpha'}\delta_{\mathbf{R}\mathbf{R}'}$, and V is the pinning potential due to quenched disorder. The pinning is described by a Gaussian distributed stochastic potential with zero mean, and thus characterized by its correlation function $\langle V(\mathbf{x})\, V(\mathbf{x}') \rangle = \nu(\mathbf{x} - \mathbf{x}') = \nu_0/(2\pi r_p^2) \exp\{-(\mathbf{x} - \mathbf{x}')^2/(2r_p^2)\}$, taken to be a Gaussian function with range r_p and strength ν_0 in our numerical calculations.

The average vortex motion is conveniently described by reformulating the stochastic problem in terms of the dynamic field theory [7]. The probability functional for a realization $\{\mathbf{u}_{\mathbf{R}t}\}_{\mathbf{R}}$ of the motion of the vortex lattice is expressed as a functional integral over a set of auxiliary variables $\{\tilde{\mathbf{u}}_{\mathbf{R}t}\}_{\mathbf{R}}$, and we are led to consider the generating functional

$$\mathcal{Z}[\mathbf{F},\, \mathbf{J}] = \int \prod_{\mathbf{R}} \mathcal{D}\mathbf{u}_{\mathbf{R}t} \int \prod_{\mathbf{R}'} \mathcal{D}\tilde{\mathbf{u}}_{\mathbf{R}'t'}\, e^{i\mathcal{S}[\mathbf{u},\, \tilde{\mathbf{u}}]}, \tag{3}$$

where in the action, $\mathcal{S}[\mathbf{u},\, \tilde{\mathbf{u}}] = \tilde{\mathbf{u}}(D_R^{-1}\mathbf{u} + \mathbf{F} - \nabla V + \xi) + \mathbf{J}\mathbf{u}$, matrix notation is used in order to suppress the integrations over time and summations over vortex positions and Cartesian indices. The retarded Green's operator is given by $-D_R^{-1}\mathbf{u} = \eta\dot{\mathbf{u}}_{\mathbf{R}t} + \sum_{\mathbf{R}'} \Phi_{\mathbf{R}\mathbf{R}'}\mathbf{u}_{\mathbf{R}'t}$, and its Fourier transform is the matrix in Cartesian space $D_R^{-1}(\mathbf{q},\, \omega) = i\eta\omega\mathbf{1} - \Phi_{\mathbf{q}}$. The average with respect to thermal noise and quenched disorder is immediately performed and we obtain the averaged generating functional [8]

$$Z[f,\, K] = \langle\!\langle \mathcal{Z} \rangle\!\rangle = \int \mathcal{D}\varphi\, e^{iS[\varphi] + if\varphi + \frac{i}{2}\varphi K\varphi}, \tag{4}$$

where in order to obtain a self-consistent equation for the two-point Green's function, we have added a two-particle source term K. We have introduced the notation $\varphi = (\tilde{\mathbf{u}},\, \mathbf{u})$ and $f = (\mathbf{F},\, \mathbf{J})$, and the source term, $\mathbf{J}$, coupling to the vortex positions, $\mathbf{u}$, allow us to generate the vortex correlation functions, say,

$$\langle\!\langle \mathbf{u}_{\mathbf{R}t}\mathbf{u}_{\mathbf{R}'t'} \rangle\!\rangle = -\frac{\delta^2 Z}{\delta \mathbf{J}_{\mathbf{R}t}\, \delta \mathbf{J}_{\mathbf{R}'t'}}\bigg|_{K=0,\, \mathbf{J}=\mathbf{0}}. \tag{5}$$

The action obtained upon averaging, $S = S_0 + S_V$, consists of a quadratic term, $S_0[\varphi] = \varphi D^{-1}\varphi/2$, specified in terms of the free inverse symmetric matrix Green's

function (where the matrix D^{-1} in the dynamical, or Keldysh, indices in addition is a matrix in Cartesian indices, and time and vortex positions)

$$D^{-1} = \begin{pmatrix} 2i\eta T\delta_{\alpha\beta}\delta_{\mathbf{RR}'}\delta(t-t') & D_R^{-1} \\ D_A^{-1} & 0 \end{pmatrix}, \tag{6}$$

and a term originating from the disorder

$$S_V[\varphi] = \frac{-i}{2} \sum_{\mathbf{RR}'} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \, \tilde{u}_{\mathbf{R}t}^{\alpha} \nabla_{\alpha}\nabla_{\beta}\nu(\mathbf{u}_{\mathbf{R}t} - \mathbf{u}_{\mathbf{R}'t'}) \, \tilde{u}_{\mathbf{R}'t'}^{\beta}. \tag{7}$$

In order to obtain an equation for the pinning force, we consider the effective action $\Gamma[\bar{\varphi}, G] = W[f, K] - f\bar{\varphi} - \bar{\varphi}K\bar{\varphi}/2 - i\,\text{Tr}\,(GK)/2$, the generator of two-particle irreducible vertex functions, i.e., the Legendre transform of the generator of connected Green's functions, $W[f, K] = -i\ln Z[f, K]$, which satisfies the equations

$$\frac{\delta\Gamma}{\delta\bar{\varphi}} = -f - K\bar{\varphi}, \qquad \frac{\delta\Gamma}{\delta G} = -\frac{i}{2}\,K, \tag{8}$$

where $\bar{\varphi}_{\mathbf{R}t}^{\alpha}$ is the average field, and G is the full connected two-point matrix Green's function of the theory, and Tr denotes the trace over all variables and indices.

In the physical problem of interest the sources K and $\mathbf{J}$ vanish, and the full matrix Green's function has, due to the normalization of the generating functional, $Z[\mathbf{F}, \mathbf{J}=\mathbf{0}, K=0] = 1$, the structure in Keldysh space

$$G_{ij} = \begin{pmatrix} 0 & G^A \\ G^R & G^K \end{pmatrix} = -i\begin{pmatrix} 0 & \langle\langle\tilde{u}^{\alpha}\,\tilde{u}^{\beta}\rangle\rangle \\ \langle\langle u^{\alpha}\,\tilde{u}^{\beta}\rangle\rangle & \langle\langle\delta u^{\alpha}\,\delta u^{\beta}\rangle\rangle \end{pmatrix}, \tag{9}$$

where $\delta\mathbf{u}_{\mathbf{R}t} = \mathbf{u}_{\mathbf{R}t} - \langle\langle\mathbf{u}_{\mathbf{R}t}\rangle\rangle$. The retarded Green's function $G_{\alpha\beta}^R$ gives the linear response to the force F_{β}, and $G_{\alpha\beta}^K$ is the correlation function (both matrices in Cartesian indices as indicated).

The effective action can be written on the form $\Gamma[\bar{\varphi}, G] = S[\bar{\varphi}] + i\,\text{Tr}\,(D_S^{-1}G - \ln(D^{-1}G) - 1)/2 - i\ln\langle e^{iS_{\text{int}}[\bar{\varphi}, \psi]}\rangle_G^{2\text{PI}}$, where $D_S^{-1} = \delta^2 S[\bar{\varphi}]/\delta\bar{\varphi}\,\delta\bar{\varphi}$, and $S_{\text{int}}[\bar{\varphi}, \psi]$ is the part of $S[\bar{\varphi} + \psi]$ which is higher than second order in ψ in the expansion around the average field $\bar{\varphi}$ [9]. The superscript "2PI" on the last term indicates that only two-particle irreducible vacuum diagrams should be kept in the interaction part of the effective action, and the subscript that propagator lines represent G, i.e., the brackets with subscript G denote the average $\langle e^{iS_{\text{int}}[\bar{\varphi}, \psi]}\rangle_G = (\det iG)^{-1/2} \int \mathcal{D}\psi \, e^{i\psi G^{-1}\psi/2} \, e^{iS_{\text{int}}[\bar{\varphi}, \psi]}$.

3 Hartree approximation

In order to get a closed expression for the self-energy in terms of the two-point Green's function we expand the exponential and keep only the first term, $-i\ln\langle e^{iS_{\text{int}}[\bar{\varphi}, \psi]}\rangle_G^{2\text{PI}} \simeq \langle S_{\text{int}}[\bar{\varphi}, \psi]\rangle_G^{2\text{PI}}$, i.e., the Hartree approximation where diagrams with propagators connecting different impurity correlators are neglected (which can also be expressed as a Gaussian fluctuation corrected saddle-point approximation [10]).

For the problem of interest, the two-particle source, K, vanishes, and the second equation in (8) yields the Dyson equation, $G^{-1} = D^{-1} - \Sigma[\bar{\varphi}, G]$, with the Keldysh matrix self-energy given by

$$\Sigma_{ij} = \begin{pmatrix} \Sigma^K & \Sigma^R \\ \Sigma^A & 0 \end{pmatrix} = 2i \left. \frac{\delta \langle S_V[\bar{\varphi} + \psi] \rangle_G^{\text{2PI}}}{\delta G_{ij}} \right|_{\substack{K=0 \\ \mathbf{J}=\mathbf{0}}}. \tag{10}$$

The Dyson equation and Eq. (10) constitute a set of self-consistent equations for the Green's functions and the self-energies. Since the expectation value of the auxiliary field vanishes due to the normalization of the generating functional, the average field entering Eq. (10) is $\bar{\varphi} = (\langle\!\langle \tilde{\mathbf{u}}_{\mathbf{R}t} \rangle\!\rangle, \langle\!\langle \mathbf{u}_{\mathbf{R}t} \rangle\!\rangle) = (\mathbf{0}, \mathbf{v}t)$, where $\mathbf{v}$ is the average lattice velocity. The matrix self-energy has two independent components since $\Sigma_{\beta\alpha}^A(\mathbf{R}t, \mathbf{R}'t') = \Sigma_{\alpha\beta}^R(\mathbf{R}'t', \mathbf{R}t)$. From Eq. (10) we obtain for the case of N vortices

$$\Sigma_{\alpha\beta}^K(\mathbf{R}t, \mathbf{R}'t') = -i/(Na^2) \sum_{\mathbf{k}} \nu(\mathbf{k}) \, k_\alpha k_\beta \, e^{-\tilde{\varphi}_{\mathbf{k}}}, \tag{11}$$

and $\Sigma^R(\mathbf{q}, \omega) = \sigma^R(\mathbf{q}, \omega) - \sigma^R(\mathbf{q} = \mathbf{0}, \omega = 0)$, where the Fourier transform has the Cartesian components

$$\sigma_{\alpha\beta}^R(\mathbf{R}t, \mathbf{R}'t') = 1/(Na^2) \sum_{\mathbf{k}} \nu(\mathbf{k}) \, k_\alpha k_\beta (\mathbf{k} G^R(\mathbf{R}t, \mathbf{R}'t') \, \mathbf{k}) \, e^{-\tilde{\varphi}_{\mathbf{k}}}. \tag{12}$$

The influence of thermal and disorder induced fluctuations is described by the fluctuation or damping exponent $\varphi_{\mathbf{k}}(\mathbf{R}, t; \mathbf{R}', t') = i\mathbf{k}(G^K(\mathbf{R}, t; \mathbf{R}, t) - G^K(\mathbf{R}, t; \mathbf{R}', t'))\mathbf{k}$ contained in $\tilde{\varphi}_{\mathbf{k}}(\mathbf{R}, t; \mathbf{R}', t'; \mathbf{v}) = -i\mathbf{k} \cdot (\mathbf{R} - \mathbf{R}' + \mathbf{v}(t - t')) + \varphi_{\mathbf{k}}(\mathbf{R}, t; \mathbf{R}', t')$. Using the Langevin equation and the first equation in Eq. (8) we obtain for the pinning force, $\mathbf{F}_p = -\langle\!\langle \nabla V \rangle\!\rangle$,

$$\mathbf{F}_p = i \sum_{\mathbf{R}'} \int_{-\infty}^{\infty} \mathrm{d}t \int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^2} \, \mathbf{k}\nu(\mathbf{k}) \, (\mathbf{k} G^R(\mathbf{R}t, \mathbf{R}'t') \, \mathbf{k}) \, e^{-\tilde{\varphi}_{\mathbf{k}}}. \tag{13}$$

4 Results

Let us recall the argument for determining the phase diagram for dynamic melting of a vortex lattice [2]. There the disorder induced fluctuations were estimated considering the correlation function, $\varkappa_{\beta\alpha}(\mathbf{x}, t) = \langle\!\langle f_\beta(\mathbf{x}, t) f_\alpha(\mathbf{0}, 0) \rangle\!\rangle$, of the pinning force density of the vortices, $\mathbf{f}(\mathbf{x}, t) = -\sum_\nu \delta(\mathbf{x} - \mathbf{R}_\nu(t)) \nabla V(\mathbf{x} - \mathbf{v}t)$. Neglecting the interdependence of the fluctuations of the vortex positions and the fluctuations in the disorder potential, the pinning force correlation function factorizes, and since in the fluidlike phase the motion of different vortices are "incoherent", one gets $\varkappa_{\beta\alpha}(\mathbf{x}, t) \simeq -n_V \delta(\mathbf{x}) \nabla_\beta \nabla_\alpha \nu(\mathbf{v}t)$, n_V being the density of vortices. In analogy with the noise correlator, the effect of disorder induced fluctuations is represented by a "shaking temperature"

$$T_{\text{sh}} = \frac{1}{4\eta n_V} \sum_\alpha \int \mathrm{d}\mathbf{x} \int \mathrm{d}t \, \kappa_{\alpha\alpha}(\mathbf{x}, t) = \frac{\nu_0}{4\sqrt{2\pi} \, F r_p^3}, \tag{14}$$

where in the last equality it is assumed that the pinning force is small compared to the friction force, i.e., $\eta v \simeq F$. An effective temperature is then obtained by adding

the "shaking temperature" due to disorder to the temperature, and according to Eq. (14) the effective temperature decreases with increasing external force (i.e., with increasing average velocity of the vortices). As the external force is increased the fluid thus freezes into a lattice. The value of the external force for which the moving lattice melts, the transition force F_t, is in reference 2 taken to be the value for which the effective temperature equals the melting temperature in the absence of disorder, $T_{\mathrm{eff}}(F = F_t) = T_m$, and has, according to Eq. (14), the temperature dependence $F_t = \nu_0/(4\sqrt{2\pi}\, r_p^3 (T_m - T))$ for temperatures below the melting temperature of the ideal lattice (we note that the transition force for strong enough disorder exceeds the critical force for which the lattice is pinned $F_t > F_c \sim \nu_0^{1/2}/r_p^2$).

We now describe the calculation within the self-consistent theory of the physical quantities of interest for dynamic melting. The conventional way of determining the melting transition is to use the Lindemann criterion, which states that a lattice melts when the displacement fluctuations reach a critical value $\langle\!\langle \mathbf{u}^2 \rangle\!\rangle = c_L^2 a^2$, where c_L is the Lindemann parameter which is typically ranging in the interval from 0.1 to 0.2. In two dimensions the position fluctuations of a vortex diverge even for a clean system, and the Lindemann criterion implies that a two-dimensional vortex lattice is always unstable against thermal fluctuations. However a quasi long-range translational order persists up to a certain melting temperature [3]. As a criterion for the loss of long-range translational order a modified Lindemann criterion involving the relative vortex fluctuations, $\langle\!\langle (\mathbf{u}(\mathbf{R} + \mathbf{a}_0, t) - \mathbf{u}(\mathbf{R}, t))^2 \rangle\!\rangle = 2c_L^2 a^2$, where $\mathbf{a}_0$ is a primitive lattice vector, has successfully been employed [3], and its validity verified within a variational treatment [11]. The relative displacement fluctuations are specified in terms of the correlation function, $\langle\!\langle (\mathbf{u}(\mathbf{R} + \mathbf{a}_0, t) - \mathbf{u}(\mathbf{R}, t))^2 \rangle\!\rangle = 2i\,\mathrm{tr}\,(G^K(\mathbf{0}, 0) - G^K(\mathbf{a}_0, 0))$, where tr denotes the trace with respect to the Cartesian indices. The correlation function is determined by the Dyson equation $G_{\mathbf{q}\omega}^K = G_{\mathbf{q}\omega}^R (\Sigma_{\mathbf{q}\omega}^K - 2i\eta T)\, G_{\mathbf{q}\omega}^A$, where the influence of the quenched disorder appears explicitly through Σ^K and implicitly through Σ^R and Σ^A in the advanced and retarded response functions. Furthermore, the self-energies depend self-consistently on the response and correlation functions. We have calculated numerically the Green's functions and self-energies and thereby the vortex fluctuations for a vortex lattice of size 8×8, and evaluated the pinning force from Eq. (13).

We determine the phase diagram for dynamic melting of the vortex lattice by calculating the relative displacement fluctuations for a set of velocities, and interpolate to find the transition velocity, v_t, i.e., the value of the velocity at which the fluctuations fulfill the modified Lindemann criterion (the determination of the Lindemann parameter is discussed shortly). The transition force is given by the averaged equation of motion, $F_t = \eta v_t + F_p(v_t)$, and is then determined by using the numerically calculated pinning force. Repeating this procedure for various temperatures determines the melting curve, i.e., the temperature dependence of the transition force, $F_t(T)$, separating two phases in the F-T-plane: a high velocity phase where the vortices form a moving solid when the external force exceeds the transition force $F > F_t(T)$, and a liquid phase for forces below the transition force.

In order to be able to compare the results of the self-consistent theory to the simulation results, we choose the same parameters [2]. There the melting temperature in the absence of disorder is taken to be $T_m = 0.007\,(2(\varphi_0/4\pi\lambda)^2$ is chosen as the unit of energy per unit length, where λ is the London penetration depth), the

value obtained by simulations of clean systems, and assumed equal to the Koster-litz-Thouless temperature, $T_{\mathrm{KT}} = c_{66}a^2/4\pi$, determining thereby the shear modulus to have the value $c_{66} = 0.088$ (a is chosen as the unit of length) [12]. The the range of the vortex interaction, λ, was approximately equal to the lattice spacing, a_0, giving for the compression modulus $c_{11} = 16\pi\lambda^2 c_{66}/a_0^2 \simeq 50c_{66} \simeq 4.4$ [5]. The range and strength of the disorder correlator in the simulations are $r_p = 0.2$ and $\nu_0 = 1.42 \cdot 10^{-5}$ in the chosen units.

Once the Lindemann parameter is determined, our numerical results for the relative displacement fluctuations can be used to obtain the dynamic phase diagram. In order to do so we calculate "melting curves", as described above, using the self-consistent theory for a set of different values of the Lindemann parameter. We find that these curves have the same shape, close to the melting temperature, as the melting curve obtained from the shaking theory, $T = C_1 - C_2/F_t$. The curve which intersects closest to the melting temperature $T_m = 0.007$, the one depicted in the inset in the figure, then determines the Lindemann parameter to be given by the value $c_L = 0.124$. The corresponding phase diagram obtained from the self-consistent theory is shown in the figure, as well as the simulation data [2], and the
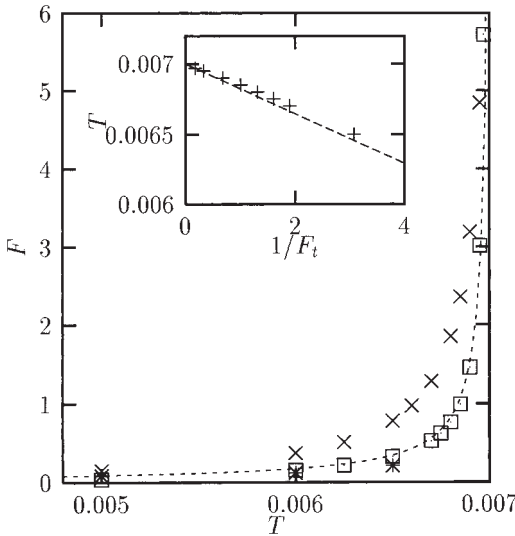
Fig. 1 The dynamic melting phase diagram. The points on the melting curve separates the two phases — for values of the external force larger than the transition force the moving vortices form a solid, and for smaller values a liquid. The dots in the boxes represent points on the melting curve obtained from the self-consistent theory for an 8×8 lattice, while the three stars represent the simulation results [2]. The crosses represent the results of lowest order perturbation theory. The dashed line is the curve $F_t(T) = 1.77 \cdot 10^{-4}/(0.007 - T)$, the melting curve obtained from the shaking theory. Inset: Temperature as a function of the inverse transition force obtained from the self-consistent theory (plus signs), close to the melting temperature, for the particular value of the Lindemann parameter $c_L = 0.124$, for which the curve intersects the T-axis at $T_m = 0.00701$. The points lie on a straight line just as the prediction of the shaking theory, i.e., the dashed line $F_t \sim 1/(T_m - T)$, yielding the value one for the critical exponent.

melting curve obtained from the shaking theory. The melting of the vortex lattice was in the simulations indicated by an abrupt increase in the structural disorder, a different melting criterion, and the agreement of the self-consistent theory with the simulation data are therefore independently validating the use of the modified Lindemann criterion. The shaking theory is seen to be in remarkably good agreement with the self-consistent theory even at lower velocities where the shaking argument is not *a priori* valid, a feature which, however, is less pronounced for stronger disorder. Whereas the deviation between the self-consistent and shaking theory for the previous parameter values typically is five percent, in the case of a five-fold stronger disorder, $\nu_0 = 7.1 \cdot 10^{-5}$, it is more than fifteen percent. It is also of interest to recall, that while the melting curve obtained from the shaking theory was based on arguments only valid in the liquid phase, i.e., freezing of the vortex liquid was considered, the melting curve we obtained from the self-consistent theory is calculated assuming a lattice, i.e., we consider melting of the moving lattice. As apparent from the inset in the figure, the critical exponent obtained from the self-consistent theory, 1.0, is in excellent agreement with the prediction of the shaking theory, where the critical exponent equals one. Furthermore, we find that the self-consistent theory yields the value $1.65 \cdot 10^{-4}$ for the constant C_2, a value close to the one predicted by the shaking theory, $\nu_0/(4\sqrt{2\pi}\, r_p^3) = 1.77 \cdot 10^{-4}$.

The melting curve predicted by lowest order perturbation theory is also shown in the figure. The correlation function is in this case given by $G_{\mathbf{q}\omega}^{K(1)} = D_{\mathbf{q}\omega}^R [\Sigma_{\mathbf{q}\omega}^{K(1)} - 2i\eta T - 2i\eta T(\Sigma_{\mathbf{q}\omega}^{R(1)} D_{\mathbf{q}\omega}^R + D_{\mathbf{q}\omega}^A \Sigma_{\mathbf{q}\omega}^{A(1)})] D_{\mathbf{q}\omega}^A$, where the self-energies are calculated to first order in ν_0. As to be expected, the perturbation theory results are at high velocities in good agreement with the self-consistent theory. However, we observe that the melting curve obtained from lowest order perturbation theory deviates markedly at intermediate velocities from the prediction of the self-consistent theory, and thereby the shaking theory, which is known to account well for the measured melting curve, Hellerqvist et al. [1].

In conclusion, we have developed a self-consistent theory of the dynamic melting transition of a vortex lattice, enabling us to determine numerically the melting curve directly from the dynamics of the system. The self-consistent theory corroborates the phase diagram obtained by the phenomenological shaking theory, whereas lowest order perturbation theory does not. The melting curve obtained from the self-consistent theory is found to be in good quantitative agreement with the phenomenological theory as well as with simulations and experimental data.

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