Anomalous frequency-dependent conductivity near the quantum Hall transition

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The dynamical transport properties near the integer quantum Hall transition are investigated at zero temperature by means of the Dirac fermion approach. These properties have been studied experimentally at low frequency ω and low temperature near the $\nu = 1$ filling factor Hall transition, with the observation of an unusual broadening and an overall increase of the longitudinal conductivity Re$\{\sigma_{xx}\}$ as a function of ω. We find in our approach that, unlike for normal metals, the longitudinal conductivity increases as the frequency increases, while the width ΔB (or $\Delta \nu$) of the conductivity peak near the Hall transition increases. These findings are in reasonable quantitative agreement with recent experiments by Engel *et al.* [Phys. Rev. Lett. **71**, 2638 (1993)], as well as with recent numerical work by Avishai and Luck (cond-mat/9609265). [S0163-1829(99)00907-8]

I. INTRODUCTION

A two-dimensional (2D) electron gas placed in a strong perpendicular magnetic field exhibits, in the presence of a disordered one-particle potential, the exact quantization of the Hall conductivity σ_{xy} accompanied by the vanishing of the longitudinal conductivity σ_{xx}. While this integer quantum Hall effect (IQHE) is now fairly well understood,[1] the transition region between consecutive Hall plateaus has recently attracted a good deal of attention from the condensed-matter physics community. Extensive experimental,[2–6] theoretical,[7–10] and numerical[11,12] work has been carried out to characterize the transport properties in the transition region. This region arises due to the existence of delocalized electronic states responsible for the jump in the Hall conductivity between two consecutive Hall plateaus. Within this region σ_{xx} takes up a narrow peaklike form.

The detailed study of the transition region between quantum Hall plateaus is important in testing and enhancing our understanding of the IQHE and of the underlying physics of localization in the presence of a magnetic field. In this paper we contribute to the advance of the theoretical description by showing how the Dirac fermion approach,[8,13–15] a successful theoretical treatment in the description of the IQH transition, affords a detailed explanation of some puzzling recent experimental measurements by Engel *et al.*[16] and Balaban, Meirav, and Bar-Joseph[17] of the frequency-dependent ac conductivity in the IQH system at low temperatures.

Hitherto, much work has concentrated on the characterization of the static transport properties in the neighborhood of the transition region, where the approach to the localization-delocalization transition is dominated by a diverging localization length $\xi \sim |E - E_c|^{-\nu}$. More recently, some attention has been devoted to frequency-dependent properties, where the approach to the critical point E_c is also characterized by diverging time scales. The frequency-dependent ac conductivity is an interesting and probably characteristic quantity of a 2D conducting QH system where σ_{xx} is nonzero (metallic regime). In contrast to normal metals, where a Drude-like behavior is observed, the conductivity of a 2D QH system has been reported to display an *increasing* behavior with frequency.[16] Thus it is indicative of a class of systems different from normal metals (for which the conductivity *decreases* with frequency, or temperature). This can be seen from the peak shape of $\sigma_{xx}(B, \omega)$ as a function of the magnetic field B in the recent IQHE measurements of Ref. 16: the width ΔB of the conductivity peak broadens as the frequency (or temperature) is increased. Numerical simulations, based on the lowest Landau-level projection, initially gave indications only for a decreasing (yet non-Drude-like) $\sigma_{xx}(\omega)$.[18] However, a more recent numerical investigation by Avishai and Luck[19] provided strong evidence for a broadening of the longitudinal conductivity peak with frequency.

This is a convenient point where to make the case for the theoretical approach—based on Dirac fermions—that is to be used in this paper and also to put it into the right perspective. Most—although by all means not all[1,20]—of the theoretical schemes that have been so far developed to explain the IQHE on a microscopic basis, rely on the concept of localization of the single-electron states in the presence of any finite amount of disorder. Generally speaking, it is sufficient to have a finite but narrow band of extended states near the center of each Landau level, and total localization everywhere else in the density of states (DOS), to explain most features of transport in the IQHE. A detailed theory, stemming from the field-theoretic and scaling approach to Anderson localization *in the absence* of a magnetic field, has been developed by Pruisken and co-workers[21,22] and rewarded with some experimental evidence for its correctness.[23] This approach makes use of the concept of composition of the σ_{xx} and σ_{xy} conductivities, and leads to a scaling theory where these conductivities are both universal (although the actual value of σ_{xx} is yet undetermined exactly at the transition) and the degenerate extended states at each transition are concentrated

into single-energy points, say in the DOS. Outside these points, corresponding to the free-electron Landau energy levels, there is total localization, although, in the original paper by Levine, Libby, and Pruisken[21] the possibility of a *band* of extended states near the plateau-to-plateau transition could not be ruled out. This approach certainly represents a useful picture for most numerical as well as experimental studies of the IQH transition, leading to the concept of quantum critical point that may prove useful also to other areas of research in condensed-matter theory. The approach of Pruisken and co-workers[21,22] certainly remains an appealing global picture reproducing the pattern of the IQH transport experiment. However, our point of view here is that the single-energy extended-state picture cannot be the full story in a detailed microscopic understanding of the IQH transition. Infinitely narrow single-particle levels are a paradigm for bound states (point spectrum), not for extended ones (continuous spectrum). This is in line with Heisenberg's indetermination principle; for if a quantum particle can be made to sample a band of extended states with energy width W (via, e.g., an infinitesimal disturbance) it will acquire a momentum uncertainty

$$\Delta p = \sqrt{2\mu W}, \tag{1}$$

μ being some effective inertial mass (not just in the sense of periodic potential's band structure). If we take the disordered potential to be characterized by a spatial-correlation characteristic length l, then $\Delta p l \sim \hbar$ yields

$$\mu \sim \hbar^2 / (2Wl^2). \tag{2}$$

So, an infinitely narrow band would lead to an infinite effective mass or zero mobility and no conduction. This argument—though by no means a proof—works for the atomic as well as for the perfect periodic lattice limits. It is also important to understand conduction in doped semiconductors, where infinitely narrow impurity levels represent weakly bound localized states giving way to an impurity conduction band only in the limit of heavily doped samples. The outstanding example we know of a single-energy *localized* degenerate state is that of a Landau level in the impurity-free electron gas. There, the enormous degeneracy of the Landau levels is a consequence of the independence of energy on the orbital momentum L_z. In our view, as soon as impurity collisions take place there will be new states, some of them extended and ready to accept the scattered electrons. The finite bandwidth of such states ensures the mobility of the electrons being scattered, and therefore conduction. The finite bandwidth picture for the extended states would thus seem to be in agreement with basic quantum physics principles. We should stress, however, that there exist 1D and 2D models of electrons in random potentials[7,8,24–27] which do demonstrate the possibility of single energy extended states. Nevertheless, these models have a singular DOS (vanishing or divergent at this energy), a situation which is not covered by the above qualitative argument. In this paper we aim at describing physical 2D systems with a regular DOS and no extreme behavior.

The Dirac fermion model with an inhomogeneous mass, as used in this paper, is a plausible representation for electrons undergoing the IQH transition in the presence of a random potential, as discussed in Ref. 8. As also explained in

Sec. II, initial work with this model relied on perturbative calculations which led to unphysical results for the density of states.[8] However, it was shown by one of the present authors that a nonperturbative approach would cure this problem yielding all the desired features with the sole random mass ingredient. Moreover, the nonperturbative calculation leads to a narrow, but finite, bandwidth for the extended states (Refs. 10 and 28, see also Ref. 29). As we have explained, we find this feature of the model rather attractive, together with the fact that the approach lends itself to a number of predictive analytical calculations not possible otherwise with the single-energy picture. The Dirac fermion formulation was recently also strengthened by a mapping of the network model[11] for the IQH transition onto a Dirac effective Hamiltonian, with both inhomogeneous mass and scalar as well as vector potentials.[30] In our view, the inhomogeneous mass is sufficient to give the full picture, and at the same time the simplest.

The purpose of this paper is to investigate the low-frequency behavior of the conductivity peak on the basis of a theory accounting directly for the IQH delocalized states, using this effective and appealing approach based on Dirac fermions. This approach has so far been used to account for the static transport properties of the IQH transition, and the method can be extended to include thermal fluctuations.[31] Frequency-dependent behavior is, however, very similar to temperature behavior,[16] since dynamics enters in the formalism through the Matsubara frequencies, which are themselves proportional to temperature. We will therefore work only with the frequency-dependent Dirac fermion approach. A complementary approach to the frequency-dependent conductivity was worked out by Polyakov and Shklovskii using the hopping mechanism of localized states.[32] They derived power laws for the broadening of the conductivity peaks due to frequency, current, and temperature. In contrast to our approach, which works very close to the conductivity peak at low frequency, they studied the broadening in the regime of higher frequency where the electronic states become localized if the frequency goes to zero. The success of this hopping approach calls for an extension of these ideas to the regime where the states are delocalized. This is precisely what we shall do in the following using the method of Dirac fermions which seems to afford a good deal of predictive power.

The paper is organized as follows. In Sec. II we briefly recall the basic features of the Dirac fermion approach to the IQH transition. This approach is implemented in Sec. III, in order to directly evaluate the ac longitudinal conductivity $\sigma_{xx}(\omega)$ from the Dirac fermion propagator in which a weakly disordered one-particle potential is accounted for. The main features of the ac conductivity are described in the light of recent numerical as well as experimental investigations of dynamical scaling in the IQH transition region. In Sec. IV the description is specialized for the frequency dependence of the conductivity peak width, which is found to be in agreement with some recent measurements by Shahar *et al.*[33] carried out for the (related) temperature dependence of the longitudinal resistivity width and by Balaban, Meirav, and Bar-Joseph[17] for the frequency dependence. For nonvanishing frequencies the results of our calculation also agree

with the experimental data of Engel et al.,[16] and the numerical data of Avishai and Luck.[19] Section V contains our conclusions and outlook.

II. MODEL AND DERIVATION OF THE CONDUCTIVITY

The main features of the IQH transition are captured quite effectively by a tight-binding model in which, although the real system has no lattice, the electrons hop over a scale given by the magnetic length.[8] This is closely related to the Chalker-Coddington network model[11] in which electrons hop from region to region with random tunneling and random magnetic flux. Both models[8,30] lead in the large-scale approximation to an effective Hamiltonian describing the dynamics of Dirac fermions (with a random mass or coupled to a random vector potential),

$$H_D = (i\nabla_1 + A_1)\sigma_1 + (i\nabla_2 + A_2)\sigma_2 + M\sigma_3, \qquad (3)$$

where the energy is measured in units of the hopping parameter t of the original lattice model. ∇_j is the lattice differential operator in the j direction, and $\{\sigma_j\}$ are the Pauli matrices. This Hamiltonian, with a random mass term M and zero random vector potential A_j, is a reasonable starting point for the description of the IQH transition between plateaus at filling $\nu = 1$. One important issue consists in what type of randomness is realistic in the approach with Dirac fermions. Ludwig et al.[8] argued that the case of a random Dirac mass is insufficient to describe the generic IQH transition, since it has a vanishing DOS at the transition point. Yet the random Dirac mass is reasonable on an intuitive basis because it is the most relevant random contribution to the Dirac Hamiltonian in terms of symmetry breaking[34] (see the discussion in Ref. 8). However, it was shown by several other authors and approaches that the DOS becomes nonzero at the transition when treated on a nonperturbative basis.[10,28,29] Therefore, the random mass case, in contrast to that of a pure random vector potential, ought to represent a generic model for the IQH transition and this is the point of view adopted in this paper. We choose a random mass $M = m + \delta M$ with mean m and a Gaussian distribution with $\langle \delta M_r \delta M_{r'} \rangle = g\delta_{rr'}$, g being a measure of the strength of disorder.

The frequency-dependent conductivity at $T = 0$ reads[35]

$$\sigma_{xx}(\omega) = \frac{\omega}{2} \int_{-\omega}^{\omega} \hat{\sigma}_{xx}(\omega, E) dE \qquad (4)$$

with

$$\hat{\sigma}_{xx}(\omega, E) = -\frac{e^2}{h} \lim_{\eta \to 0} \sum_r r^2 \langle G(0,r; E+\omega+i\eta)G(r,0; E-\omega-i\eta) \rangle \equiv \frac{e^2}{h} \lim_{\eta \to 0} \nabla_k^2 \tilde{C}(k, \eta-i\omega, E)|_{k=0}, \qquad (5)$$

where G is the one-particle Green's function of H_D. In the following we will use the standard approximation of small ω for $\sigma_{xx}(\omega)$,[7]

$$\sigma_{xx}(\omega) \approx \omega^2 \hat{\sigma}_{xx}(\omega, 0). \qquad (6)$$

This approximation will be shown to be equivalent to the Einstein relation, as can be deduced from the expression for $\tilde{C}(k, \eta - i\omega, E)$ which will be derived below.

According to the Dirac fermion approach of Ref. 10, the two-particle Green's function describes a diffusive behavior between the Hall plateaus,

$$\tilde{C}(k, \eta - i\omega, 0) = \frac{\pi}{2} \frac{\rho}{\eta - i\omega + Dk^2}, \qquad (7)$$

where ρ is the average DOS and D the diffusion coefficient (notice that we use a notation for D different from that of Ref. 10)

$$D = 2g\eta'\alpha \left[1 + \alpha \left(\frac{(m' + i\eta')^2}{1/g - 2\alpha(m' + i\eta')^2} \right. \right.$$

$$\left. \left. + \frac{(m' - i\eta')^2}{1/g - 2\alpha(m' - i\eta')^2} \right) \right], \qquad (8)$$

with (for an infinite cutoff)

$$\alpha = \int (m'^2 + \eta'^2 + k^2)^{-2} d^2k/4\pi^2 = \frac{1}{4\pi(m'^2 + \eta'^2)}. \qquad (9)$$

The parameters m' and $\eta' \equiv \pi g\rho$ have been evaluated within a saddle point approximation.[10] They obey the following equations (taking the limit $\eta \to 0$)

$$\eta' + i\omega = \eta' gI \qquad (10)$$

and

$$m' = m/(1 + gI), \qquad (11)$$

with the integral I given by

$$I \sim \frac{1}{\pi} \int_0^\lambda (\eta'^2 + m'^2 + k^2)^{-1} k \, dk$$

$$= \frac{1}{2\pi} \ln[1 + \lambda^2/(\eta'^2 + m'^2)]. \qquad (12)$$

Here we have cut off the k integration to $|k| \leq \lambda$. This is necessary because the integral I would not otherwise exist. The cutoff corresponds to a minimal length scale in the real system, i.e., the lattice constant in our model, which is usually the mean free path of the particles. It reflects the fact that (quasi)particles cannot be considered as independent on arbitrary short scales.

III. EVALUATION OF THE FREQUENCY-DEPENDENT CONDUCTIVITY

Assuming weak disorder ($g \ll 1$), from Eq. (8) we obtain

$$D \approx 2g\eta'\alpha[1 + 2g\alpha(m'^2 - \eta'^2)]$$

$$= \frac{g\eta'}{2\pi(m'^2 + \eta'^2)}\left[1 + \frac{g}{2\pi}\frac{m'^2 - \eta'^2}{m'^2 + \eta'^2}\right], \qquad (13)$$

which for the special case $m = 0$ becomes

$$D = \frac{g}{2\pi\eta'}(1 - g/2\pi) = \frac{1}{2\pi^2\rho}(1 - g/2\pi). \qquad (14)$$

Then the conductivity reads, according to Eqs. (5)–(7)

$$\sigma_{xx}(\omega) \approx \frac{e^2}{\hbar}D\rho. \qquad (15)$$

We notice that this is the Einstein relation. Going back to the general case, for the conductivity Eqs. (13) and (15) imply the expression

$$\sigma_{xx}(\omega) \approx \frac{e^2}{h\pi}\frac{1}{1 + m'^2/\eta'^2}\left[1 + \frac{g}{2\pi}\frac{m'^2/\eta'^2 - 1}{m'^2/\eta'^2 + 1}\right]. \qquad (16)$$

This represents a simple scaling form of the type

$$\sigma_{xx}(m, \omega) = \frac{e^2}{h}G(m'/\eta'), \qquad (17)$$

since only the combination m'/η' enters the expression.

Now we have to evaluate m' and η' from Eqs. (10) and (11). For $\omega, \eta' \neq 0$, Eq. (10) can also be written as

$$e^{2\pi/g}e^{2\pi i\omega/\eta' g} = 1 + \frac{\lambda^2}{\eta'^2 + m'^2}, \qquad (18)$$

and since we are interested in the small frequency regime $\omega \approx 0$ we find it useful to work out a closed approximate analytic solution of this equation. For $\omega \approx 0$, we have $gI \approx 1$ from Eq. (10). Then Eq. (11) implies $m' \approx m/2$ and the exponential term in Eq. (18) can be expanded to give, in leading order,

$$\eta'^2 \approx \lambda^2 e^{-2\pi/g}\left(1 - \frac{2\pi}{g}i\frac{\omega}{\eta'}\right) - m^2/4$$

$$= \frac{m_c^2 - m^2}{4} - \frac{2\pi i m_c^2}{4g}\frac{\omega}{\eta'}, \qquad (19)$$

with $m_c = 2\lambda\exp(-\pi/g)$. This is a cubic equation in η' from which we take the solution

$$\eta' = \begin{cases} y^{1/3} + \dfrac{a}{3}y^{-1/3}, & a < -3(b^2/4)^{1/3} \\[2mm] -\dfrac{1}{2}\left[y^{1/3} + \dfrac{a}{3}y^{-1/3} + i\sqrt{3}\left(y^{1/3} - \dfrac{a}{3}y^{-1/3}\right)\right], & a \geq -3(b^2/4)^{1/3}, \end{cases} \qquad (20)$$

with

$$y = -i(b/2 + \sqrt{b^2/4 + a^3/27}),$$

$$a = \frac{m_c^2 - m^2}{4}, \qquad b = \frac{2\pi m_c^2}{4g}\omega. \qquad (21)$$

This solution reproduces the correct result in the limit $\omega \to 0$, namely, $\eta' \to \sqrt{a}$ for $a \geq 0$ and $\eta' \to 0$ for $a < 0$.[31] Using the approximate values of m' and η' from the above, we can study

$$\sigma_{xx}(\omega) \approx \frac{e^2}{h\pi}\frac{1}{1 + m^2/4\eta'^2}. \qquad (22)$$

For $b \propto \omega$ large compared to $(m_c^2 - m^2)/4$, we have $\eta' \propto \omega^{1/3}$ from Eq. (19). Consequently, the scaling behavior of the conductivity is given in this regime by

$$\sigma_{xx}(m, \omega) = \frac{e^2}{h}G(m\omega^{-1/3}). \qquad (23)$$

This, of course, does not hold for all values of ω, because η' is not a power law for very small ω. The general behavior of the real and imaginary parts of $\sigma_{xx}(\omega)$ is shown in Fig. 1, for the illustrative values $m_c = 0.01$ and $m = 0.009$.

IV. BROADENING OF THE σ_{xx} PEAK

Much of the experimental and numerical work on this problem has concentrated on the conjectured universal scaling behavior of the peak width of $\sigma_{xx}(B, \omega)$. It is clear from our calculation in Sec. III that the peak width of our model does not vanish with vanishing frequency. There are, at this point, two possible attitudes for this fact: either our model does not capture the physics of the real systems, or the non-vanishing width is too small to be resolved in the experiments[23] or in the computer calculations.[11,12] There are alternative models, for instance the 2D Dirac fermions with random vector potential in place of the random mass, which do have a vanishing peak width.[8] However, these models have an unphysical behavior due to a singular DOS and a peak height different from experimental observations.[6] Since within a nonperturbative calculation the peak height and the
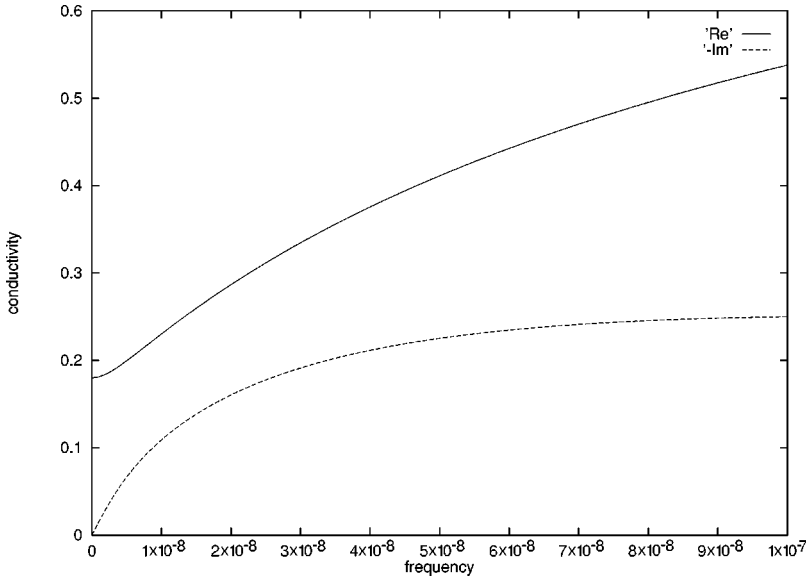
FIG. 1. Real (upper curve) and imaginary (lower curve) parts of the longitudinal conductivity $\sigma_{xx}(\omega)$ in units of e^2/h as a function of the dimensionless frequency parameter $b = 2\pi m_c^2 \omega/4g$ (see text).

smooth behavior of the DOS in the case of a random mass are in good agreement with experiments, it is more likely that the second point of view applies to our model. This we have advocated for in our presentation of the theoretical framework used in this paper, as given in Sec. I. Moreover, very recent experiments indicate that the peak width does indeed not vanish in the zero-temperature and zero-frequency limit.[33,17] The exponential dependence of the width on the disorder parameter may explain why it is difficult to measure the narrow peak width, in particular in samples with weak disorder.

The broadening of the peak width can be seen in Fig. 2, where $\mathrm{Re}(\sigma_{xx})$ is plotted as a function of the average Dirac mass m and the frequency ω for, as an illustration, a disorder strength $g=0.6$ corresponding to the value $m_c=0.01$. For $m \approx m_c$ the conductivity $\mathrm{Re}(\sigma_{xx})$ varies roughly like $\omega^{2/3}$ as one would expect from Eqs. (22) and (23). However, away from m_c the broadening does not, strictly speaking, describe a power law.

V. DISCUSSION AND CONCLUSIONS

The broadening of the conductivity peak has also been studied numerically by Avishai and Luck.[19] They studied the scaling of the real part of the conductivity $\sigma_{xx}(E,\omega)$, finding that there is indeed a scaling law of the type

$$\mathrm{Re}\,\sigma_{xx}(E,\omega) \approx \frac{e^2}{h} G(|E-E_c|\omega^{-\kappa}) \qquad (24)$$

for the real part of the conductivity. These authors found values for κ between 0.31 and 0.43. We can use our result from Eq. (16), where $E-E_c$ is replaced by the parameter m driving the IQH transition. However, in contrast to Eq. (24), we do not obtain a simple scaling form for arbitrarily small frequency (see Fig. 2). Still, for weak disorder the agreement with Ref. 19 is qualitatively reasonable, at least for small values of $m-m_c$ (the numerical values quoted are for the critical point).

In general, if we insist on a scaling law, the exponent κ is related to the dynamical exponent z and to the localization
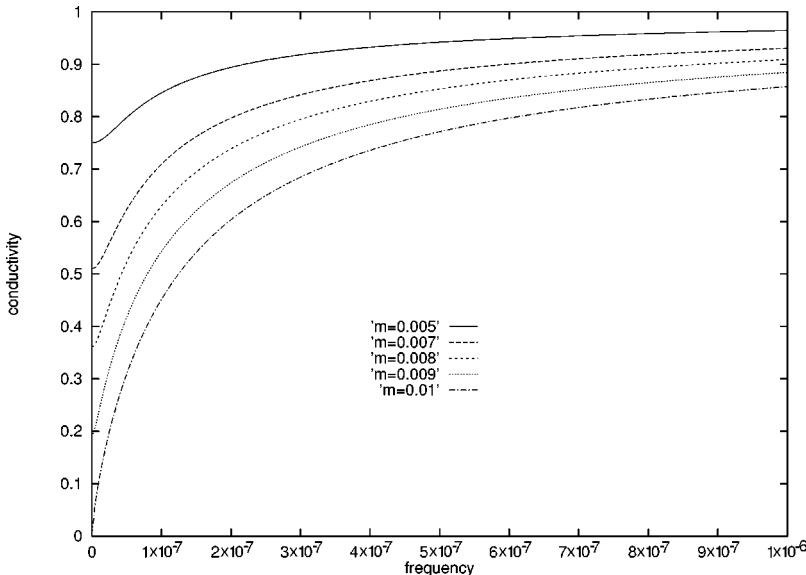

FIG. 2. Broadening of the conductivity peak with frequency. $\mathrm{Re}(\sigma_{xx})$ is plotted vs the Dirac mass m (in units of the hopping parameter t) and the dimensionless frequency parameter b.
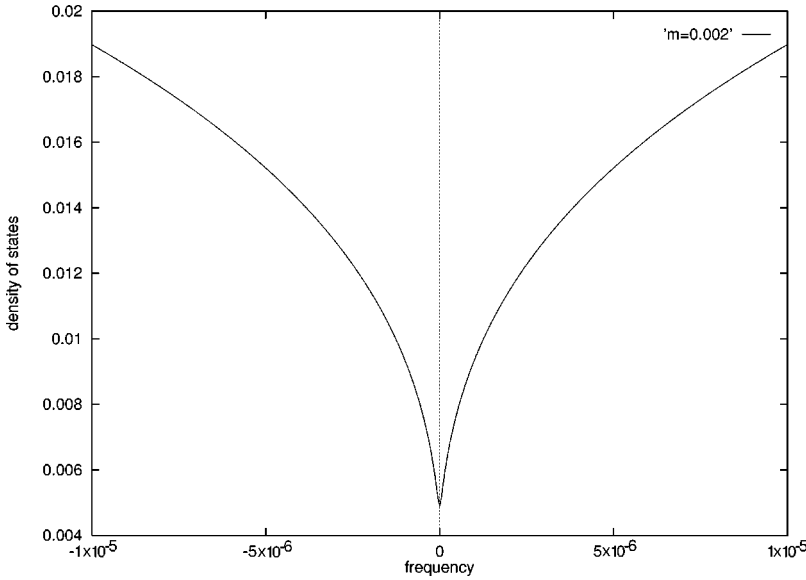
FIG. 3. Average density of states for $m_c = 0.01$.

length exponent ν by $\kappa = 1/z\nu$. From $\kappa = \frac{1}{3}$ we could evaluate ν because the usual argument for the dynamical exponent of noninteracting particles is $z = 2$.[19] The latter arises from the assumption that the diffusion coefficient and the DOS are constant near the critical point in the two-particle Green's function of Eq. (7). However, if the DOS itself behaves like a power law, $\rho \sim \omega^\alpha$, then the exponent z depends on α. For instance, the pure system of Dirac fermions has $\alpha = 1$ and there is an effective exponent $\alpha = \frac{1}{3}$ in the case of a random mass, according to the discussion below Eq. (22) (and see also Fig. 3). Then Eq. (14) implies $D \sim \omega^{-\alpha}$, and from a simple calculation for the two-particle Green's function of Eq. (7), with $k \sim L^{-1}$ and $\eta = 0$, we obtain

$$\widetilde{C}(L^{-1}, \omega) = \omega^{\alpha - 1}\overline{C}(L^{2/(1+\alpha)}\omega), \qquad (25)$$

where $\overline{C}$ is the scaling function. With $\alpha = 1$ (pure system) this implies $z = 1$, and with $\alpha = \frac{1}{3}$ it gives $z = \frac{3}{2}$. The above scaling form does not describe the asymptotic behavior for $\omega \sim 0$, but rather the effective behavior probably relevant for most of the numerical calculations.

The result in Eq. (22) is also different from the Drude theory,[36] suitably adapted to accommodate the presence of the Lorentz force on the scattered electrons,

$$\sigma_{xx}(\omega) = \sigma_0 \frac{1 - i\omega\tau}{(1 - i\omega\tau)^2 + \omega_c^2\tau^2}, \qquad (26)$$

where the zero field dc Drude conductivity σ_0 and the cyclotron frequency ω_c are defined in the usual way (τ is a collision time):

$$\sigma_0 = \frac{ne^2\tau}{m}, \qquad \omega_c = \frac{eB}{mc}. \qquad (27)$$

The frequency scale in our study is determined by the value of the hopping parameter t in the tight-binding model of Ludwig et al.[8] The physical frequency $\bar{\omega}$ is related to the dimensionless frequency ω via $\omega = \hbar\bar{\omega}/t$. Moreover, if we employ the parameter $b = 2\pi m_c^2\omega/4g$ with the illustrative value of $m_c = 0.01$, the dimensional frequency is related to b

via $\bar{\omega} \approx 4 \times 10^{16}b$ Hz for a value of $t = 1$ eV. (Note, however, that our choice $m_c = 0.01$ (i.e., $g = 0.6$) is probably only qualitatively significant, since it is difficult to estimate the value of disorder strength g in a real sample. The above value may be too large for real systems that have less disorder).

Our assumption of small frequency breaks down if $\hbar\bar{\omega}$ becomes of the order of the characteristic energy scale given by the hopping rate t. The latter is of the order of 1 eV in realistic systems; thus we expect a crossover frequency of about $\bar{\omega} \approx 10^{15}$ Hz. Typical experiments were performed for frequencies between 0.2 and 14 GHz; (Ref. 16) this is well below the crossover frequency, and our small frequency approximation should hold.

In a recent paper Shahar et al.[33] studied the behavior of the longitudinal resistivity, in the neighborhood of the transition from the QH liquid to the Hall insulator, known to be of the form

$$\rho_{xx} = e^{-\Delta\nu/\nu_0(T)}. \qquad (28)$$

These authors found a deviation from the conventional scaling form $\nu_0(T) = T^{1/z\nu}$. Instead, they suggested fitting their experimental data with the form

$$\nu_0(T) = \beta + \alpha T, \qquad (29)$$

where $\alpha, \beta > 0$ (both parameters α and β depending strongly on sample properties). A very similar result was found by Balaban, Meirav, and Bar-Joseph[17] for the frequency-dependent conductivity at zero temperature. These results are consistent with our finding of a deviation of the broadening from a power law for very low frequencies. It is a consequence of the nonzero bandwidth of delocalized states in the disorder-dependent interval $[-2\lambda \exp(-\pi/g), 2\lambda \exp(-\pi/g)]$ present in our model.

In conclusion, we have developed a theoretical treatment for the dynamical transport properties of the IQH system near the plateau-to-plateau transition. Our results indicate an increase of the longitudinal conductivity accompanied by the broadening of the conductivity peak as the frequency is in-

creased. At large frequencies, this behavior follows a power law also found in recent experiments[16] and numerical studies.[19] However, for very low frequency we found deviations from a power law, similarly to what was found in recent experiments by Shahar *et al.*[33] and Balaban, Meirav, and Bar-Joseph.[17] For larger values of the frequency, the scaling behavior of the conductivity is recovered with an exponent $\frac{1}{3}$ which is in agreement with the recent work by Avishai and Luck.[19] These results follow from a theoretical treatment in which the delocalized states responsible for the IQH transition are properly accounted for.

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