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Interacting bosons in an optical lattice

A strongly interacting Bose gas in an optical lattice is studied using a hard-core interaction. Two different approaches are introduced, one is based on a spin$1 / 2$ Fermi gas with attractive interaction, the other one on a functional integral with an additional constraint (slave-boson approach). The relation between fermions and hard-core bosons is briefly discussed for the case of a one-dimensional Bose gas. For a threedimensional gas we identify the order
 parameter of the Bose-Einstein condensate through a Hubbard-Stratonovich transformation and treat the corresponding theories within a mean-field approximation and with Gaussian fluctuations. The role of quantum and thermal fluctuations are studied in detail for both approaches, where we find good agreement with the Gross-Pitaevskii equation and with the Bogoliubov approach in the dilute regime. In the dense regime, which is characterized by the phase transition between the Bose-Einstein condensate and the Mott insulator, we discuss a renormalized Gross-Pitaevskii equation. Finally, we compare the results of the attractive spin-1/2 Fermi gas and those of the slave-boson approach and find good agreement for all physical quantities.

