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New Electromagnetic Mode in Graphene

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A new, weakly damped, *transverse* electromagnetic mode is predicted in graphene. The mode frequency ω lies in the window $1.667 < \hbar\omega/\mu < 2$, where μ is the chemical potential, and can be tuned from radio waves to the infrared by changing the density of charge carriers through a gate voltage.

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In the past two years, a great deal of attention has been attracted by the discovery of graphene, a truly two-dimensional (2D) electronic system [1,2] (for recent reviews see Refs. [3,4]). Graphene is a monolayer of carbon atoms, and the band structure of electrons in graphene consists of six Dirac cones at the corners of the hexagon-shaped Brillouin zone [5,6], with the massless, linear electron-hole dispersion with the effective "velocity of light" $V \approx 10^8$ cm/s. The special spectrum of the charge carriers leads to a number of interesting transport properties, which have been intensively studied in the literature, see, e.g., Refs. [7–21] and review articles [3,4].

Electrodynamic properties of graphene have been studied in Refs. [11–14,22–30]. Frequency-dependent conductivity [11,12,22–24] and collective excitations of the graphene layer—plasmons [13,14,25,26], thermo-plasma polaritons [27]—have been theoretically investigated. It has been shown that the specific band structure of graphene leads to a certain modification of the plasmon and plasmon-polariton spectra, as compared to conventional 2D electron systems with the parabolic dispersion of electrons.

In this Letter we show that the Dirac spectrum of electrons leads to a radically new feature of the electrodynamic response of the electron-hole plasma in graphene, as compared to conventional electron systems. We predict the existence of a *transverse* (TE) electromagnetic mode in graphene, the mode which cannot exist in systems with the parabolic electron dispersion. The new mode propagates along the graphene layer with the velocity close to the velocity of light, has a weak damping, and its frequency is tunable across a broad frequency range from radio waves to the infrared. These properties may have a strong potential for future electronic and optoelectronic applications of graphene.

To explain the essence of our finding, we briefly discuss the nature of electromagnetic modes in conventional electronic systems. Isotropic and uniform three-dimensional plasmas can support (in zero magnetic field) both longitudinal (plasmons) and transverse electromagnetic modes. The electric field vector in the longitudinal (transverse) wave is parallel (perpendicular) to the wave vector. In the 2D electron gas, however, only the longitudinal (more exactly, transverse magnetic, TM) modes—2D plasmons and plasmon-polaritons—may exist under standard experimental conditions [31]. The spectrum of electromagnetic modes, propagating along and localized near the 2D electron gas layer, has the form

$$1 + \frac{2\pi i\sigma(\omega)\sqrt{q^2 - \omega^2/c^2}}{\omega} = 0 \qquad (1)$$

for the TM waves [32], and

$$1 - \frac{2\pi i\omega\sigma(\omega)}{c^2\sqrt{q^2 - \omega^2/c^2}} = 0 \qquad (2)$$

for the TE waves [33]; here $\sigma(\omega)$ is the local dynamic conductivity of the 2D gas and c is the velocity of light. As seen from (1) and (2), the TM (TE) modes may exist if and only if the imaginary part of $\sigma(\omega)$ is positive (negative). In conventional 2D electron systems, realized, for instance, in GaAs/AlGaAs quantum-well structures, the conductivity can be described by the Drude model $\sigma(\omega) = in_s e^2/m(\omega + i\gamma)$, where n_s, e, m, and γ are the density, the electric charge, the effective mass, and the scattering rate of 2D electrons, respectively. As $\sigma''(\omega) > 0$, only the TM waves (plasmons, plasmon-polaritons) can propagate in such structures.

In graphene with the massless Dirac form of the electron/hole dispersion the situations is different. To demonstrate this, we begin with the Hamiltonian $\hat{H} = V\sigma_\alpha \hat{p}_\alpha$, which determines the energy spectrum of charge carriers in the electron ($l = 2$) and hole ($l = 1$) bands,

$$E_{\mathbf{k}l} = (-1)^l \hbar V k, \qquad l = 1, 2, \qquad (3)$$

and the corresponding wave functions $|\mathbf{k}l\rangle$. Here $\alpha = (x, y)$, σ_α are Pauli matrixes, $\hat{p}_\alpha$ is the momentum operator, and $\mathbf{k} = (k_x, k_y)$, $k = \sqrt{k_x^2 + k_y^2}$ is the wave vector. Using the self-consistent-field approach [34] (alternatively, the Kubo formalism [8], or the random-phase approximation [35]) we calculate the high-frequency conductivity of the system $\sigma_{\alpha\beta}(\omega) = \sigma(\omega)\delta_{\alpha\beta}$. Scattering is ignored (i.e., $\gamma = 0$) here. The conductivity consists of the intraband and the interband contributions [12,22],

$$\sigma^{\rm intra}_{\alpha\beta}(\omega) = \frac{-ie^2}{\hbar^2(\omega + i0)S}\sum_{\mathbf{k}l}\frac{\partial E_{\mathbf{k}l}}{\partial k_\alpha}\frac{\partial f(E_{\mathbf{k}l})}{\partial E_{\mathbf{k}l}}\frac{\partial E_{\mathbf{k}l}}{\partial k_\beta}, \quad (4)$$

$$\sigma^{\rm inter}_{\alpha\beta}(\omega) = \frac{ie^2\hbar}{S}\sum_{\mathbf{k},l\neq l'}\frac{f(E_{\mathbf{k}l'}) - f(E_{\mathbf{k}l})}{E_{\mathbf{k}l'} - E_{\mathbf{k}l} - \hbar(\omega + i0)} \times \frac{1}{E_{\mathbf{k}l'} - E_{\mathbf{k}l}}\langle \mathbf{k}l|\hat{v}_\alpha|\mathbf{k}l'\rangle\langle \mathbf{k}l'|\hat{v}_\beta|\mathbf{k}l\rangle, \quad (5)$$

where $f(E_{\mathbf{k}l})$ is the Fermi distribution function, $\hat{v}_\alpha = V\sigma_\alpha$ is the velocity operator, and S is the sample area. In the gate controlled graphene systems, where the density of charge carriers can be tuned by the gate voltage, we obtain that the intraband conductivity (4) at $T/\mu \to 0$ assumes the Drude-like form,

$$\sigma_{\rm intra}(\omega) = \frac{in_s e^2 V^2}{(\omega + i0)\mu} = \frac{e^2 g_s g_v}{16\hbar}\frac{4i}{\pi\Omega}, \quad (6)$$

while the interband contribution (5) gives

$$\sigma_{\rm inter}(\omega) = \frac{e^2 g_s g_v}{16\hbar}\left(\theta(|\Omega| - 2) - \frac{i}{\pi}\ln\left|\frac{2+\Omega}{2-\Omega}\right|\right); \quad (7)$$

see Fig. 1. Here $\Omega = \hbar\omega/\mu$, μ is the chemical potential, $n_s = g_s g_v \mu^2/4\pi\hbar^2 V^2$ is the density of electrons at $T = 0$, and $g_s = 2$ and $g_v = 2$ are the spin and valley degeneracies in graphene (six Dirac cones give the valley degeneracy $g_v = 2$ because only one third of each cone belongs to the first Brillouin zone [6]).

As seen from Eqs. (6) and (7), as well as from Fig. 1, the imaginary part of $\sigma_{\rm intra}(\omega)$ is positive, like in the standard 2D electron systems, so that the intraband contribution alone could not provide the conditions of the TE-mode existence. The imaginary part of the interband contribution, however, is negative and diverges logarithmically at $\Omega \to 2$. This divergency is associated with the steplike behavior of the real part $\sigma'_{\rm inter}(\omega)$, which describes the interband absorption of radiation at $\hbar\omega > 2\mu$. Using (6) and (7), the TE-mode dispersion (2) can now be written as

$$\sqrt{Q^2 - \Omega^2} = \frac{g_s g_v}{4}\frac{e^2}{\hbar c}\left(\frac{\Omega}{2}\ln\frac{2+\Omega}{2-\Omega} - 2\right), \qquad \Omega < 2, \quad (8)$$

where $Q = \hbar cq/\mu$. Notice that apart from the dimensionless wave vector Q and the frequency Ω, Eq. (8) depends only on the fine structure constant $\alpha = e^2/\hbar c$. The TE mode (8) does not decay at $T = 0$ and exists in the window $1.667 < \Omega < 2$, where the term in large parentheses in (8) is positive. To give a numerical example, at $n_s \simeq 6\times 10^{12}$ cm^{-2} the TE mode should exist in the window 115 THz $\lesssim f \lesssim$ 140 THz, at $n_s \simeq 10^{11}$ cm^{-2}—in the window 15 THz $\lesssim f \lesssim$ 18 THz, and so on. Choosing an appropriate value of the gate voltage, one can tune the TE mode to a desired frequency range. This may be useful in designing devices for infrared, terahertz, and microwave electronics. The TM mode (2D plasmon-polariton) does not exist at $1.667 < \Omega < 2$; its dispersion relation (1) has real solutions only at $\Omega < 1.667$.

Figure 2 illustrates the dependence of $\sqrt{Q^2 - \Omega^2}$ on the frequency Ω, Eq. (8). Because of the small factor $\alpha = e^2/\hbar c$ in the right-hand side of (8), the deviation of the wave vector Q from the frequency Ω in the wave is small as compared to the values of Q and Ω themselves. This means that the TE mode propagates along the 2D graphene layer (x direction) with the velocity close to the velocity of light, $\omega \lesssim cq$, and that the localization length of the wave

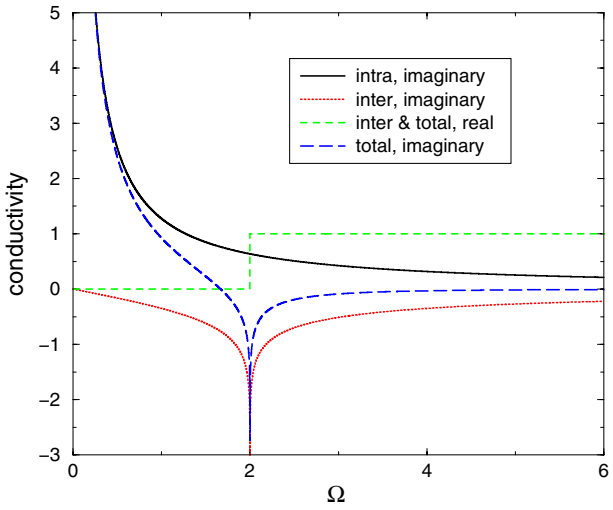

FIG. 1 (color online). The dynamic conductivity of the graphene layer, in units $e^2 g_s g_v/16\hbar$, as a function of the frequency $\Omega = \hbar\omega/\mu$ in the collisionless limit at zero temperature $T/\mu = 0$. Imaginary and real parts of the intraband and interband contributions, as well as their sum (the total conductivity) are shown.

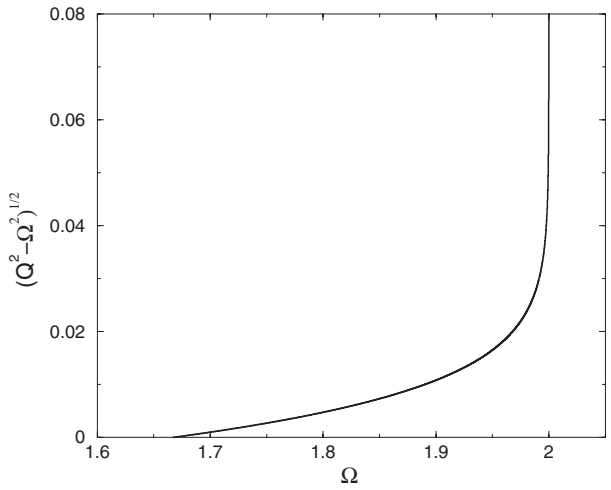

FIG. 2. The value of $\sqrt{Q^2 - \Omega^2}$ as a function of the dimensionless frequency Ω for the TE mode.

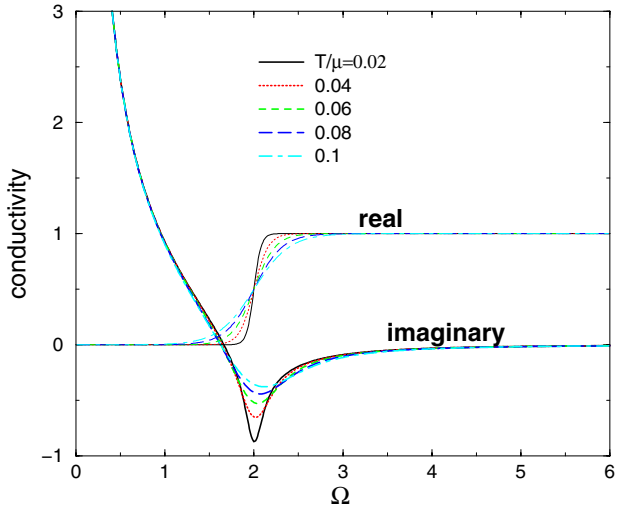

FIG. 3 (color online). The real and imaginary parts of the dynamic conductivity of graphene, in units $e^2 g_s g_v/16\hbar$, as a function of frequency $\Omega = \hbar\omega/\mu$ at finite temperatures.

in the perpendicular (z) direction is much larger than its wavelength in the x direction.

The frequency dependence of the conductivity at finite temperature is shown in Fig. 3. At $T > 0$, the real part $\sigma'(\omega)$ becomes finite at $\Omega < 2$, and the TE mode acquires a finite damping, $Q = Q' + iQ''$. This damping, however, is very small. Using that $\alpha \ll 1$ we get from Eq. (8)

$$\frac{Q''}{Q'} = \frac{q''}{q'} = \left(\frac{\pi\alpha}{2}\right)^2 \tilde{\sigma}'(\omega)[-\tilde{\sigma}''(\omega)], \qquad (9)$$

where $\tilde{\sigma}$ is the normalized conductivity of the layer, $\sigma = (e^2 g_s g_v/16\hbar)\tilde{\sigma}$. Figure 4 exhibits the factor $\tilde{\sigma}'(\omega) \times [-\tilde{\sigma}''(\omega)]$ as a function of Ω at different temperatures. One sees that $\tilde{\sigma}'(\omega)[-\tilde{\sigma}''(\omega)]$ is smaller than 1 and hence that q''/q' does not exceed 10^{-4} even at $T \simeq 0.1\mu$. At the

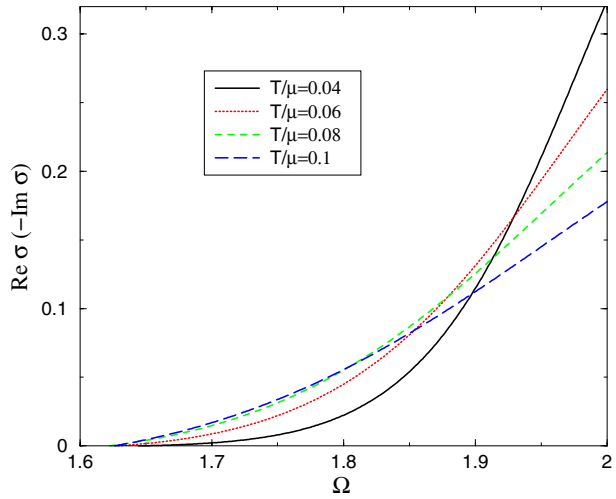

FIG. 4 (color online). The factor $\tilde{\sigma}'(\omega)[-\tilde{\sigma}''(\omega)]$ from Eq. (9) as a function of $\Omega = \hbar\omega/\mu$ at finite temperatures.

density $n_s \simeq 6 \times 10^{12}$ cm^{-2} the Fermi energy μ corresponds to $T \simeq 3000$ K in graphene, so the TE mode should be easily observable at room temperature.

To conclude, we have predicted a new transverse electric mode in graphene. The existence of the mode is directly related to the linear "relativistic" spectrum of charge carriers in graphene; in a conventional system of 2D electrons with the parabolic dispersion such a mode cannot exist. The mode frequency is widely tunable across the range from radio waves to infrared frequencies, depending on the density of electrons or holes in the system. The damping of the mode is very weak even at room temperatures.

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