A power-weighted variant of the EU27 Cambridge Compromise

G.R. Grimmett[a], K.-F. Oelbermann[b], F. Pukelsheim[b,*]

[a] *Statistical Laboratory, Centre for Mathematical Sciences, Cambridge University, Wilberforce Road, Cambridge CB3 0WB, UK*

[b] *Institut für Mathematik, Universität Augsburg, D-86135 Augsburg, Germany*

1. Introduction

The Cambridge Compromise "5 + Upw" is a mathematical formula to apportion the 751 seats of the European Parliament (EP) among the 27 Member States (Grimmett, 2012; Grimmett et al., 2011). It operates in two steps. In the first step, five *base seats* are allocated to the citizenry of each Member State. In the second step, the 616 (=751 − 27 · 5) *remaining seats* are apportioned in proportion to the States' population figures[1] $p_1, \ldots, p_{27}$. The divisor method with rounding upwards secures the minimum threshold of six seats per Member State. Seat contingents are capped at 96 if need be, staying in line with Article 14 II of the Treaty of Lisbon that decrees that no Member State shall be allocated less than 6 or more than 96 seats (TEU-Lisbon, 2010).

This note develops a *power-weighted variant* "5 + Pwr + Upw" of the Cambridge Compromise, in the spirit of Arndt (2008) and Theil and Schrage (1977). The variant refers the apportionment of the 616 remaining seats to power-weighted population indices p_i^E, rather than to the original population figures p_i. When necessary, an exponent $E < 1$ is calculated in order that the contingent of the largest Member State be reduced to exactly 96 seats.

It is shown in Section 2 that, for a given apportionment vector $x = (x_1, \ldots, x_{27})$, the exponent E ranges across an interval $[E_{\min}(x), E_{\max}(x)]$ without alteration to x. We describe in Section 3 how to determine an exponent E that secures 96 seats for the largest State. Section 4 contains a discussion of the merits of the variant from the viewpoint of general electoral principles, and of the requirements of the Union's primary law.

The power-weighted variant stays closer to the *status quo* composition of the EP than do (at least) many of the other methods reviewed by Grimmett et al. (2011). It affords a possible transition from the present composition to the eventual Cambridge Compromise "5 + Upw" across an enlarged Union, and it satisfies the amended definition of degressive proportionality of Grimmett et al. (2011). On the other hand, the use of power-weighted population indices entails a lack of transparency, and constitutes a breach of the principle of equal voting power.

2. Exponent-ranges

Suppose the *seat vector* x originates from population indices that are weighted with some given exponent $E > 0$. Since the exponent is a continuous variable while the seat contingents are integer-valued, there exists a range of values of E across which the seat vector x is constant. In this section, we investigate the induced *exponent-range* $[E_{\min}(x), E_{\max}(x)]$. The allocation of base seats plays no role in this investigation, and will therefore be neglected in this section. Similarly, we shall take no account of the 96-seat cap in this section.

The seats to be apportioned are the 616 remaining seats. Let $y = y(E)$ be a seat apportionment vector generated with a given exponent $E > 0$. The components y_i are obtained from the population indices p_i^E via scaling with a common divisor $D > 0$ and rounding the resulting quotients upwards, where the divisor is chosen so that the sum of the components exhausts the seats available:

$$y_i = \left\lceil \frac{p_i^E}{D} \right\rceil \quad \text{for } i = 1, \ldots, 27, \qquad \sum_{i=1}^{27} y_i = 616.$$

* Corresponding author.
E-mail addresses: g.r.grimmett@statslab.cam.ac.uk (G.R. Grimmett), Oelbermann@math.uni-augsburg.de (K.-F. Oelbermann), Pukelsheim@math.uni-augsburg.de (F. Pukelsheim).
URLs: http://www.statslab.cam.ac.uk/~grg/ (G.R. Grimmett), http://www.math.uni-augsburg.de/stochastik/oelbermann/ (K.-F. Oelbermann), http://www.math.uni-augsburg.de/stochastik/pukelsheim/ (F. Pukelsheim).

[1] We find it useful to sort populations from largest to smallest. Thus p_1 refers to the largest Member State (at present, Germany), and p_{27} to the smallest (at present, Malta).

Table 1
Power-weighted variant, below and beyond the 96-seat restriction. When $E = 0.01$, seats are apportioned almost equally. When $E = 0.9$ and $E = 0.91$, Germany is allocated 96 seats, stars * indicating intervening seat transfers. With $E = 1$, population figures are unweighted. When $E = 27.5$, seats are apportioned as unequally as possible.

Member State	Population	Exponent E for power-weighted variant							
(EU27)	(1.1.2011)	0.01	0.89	0.899	0.9	0.91	0.912	1	27.5
Germany	81 802 257	28	95	95	*96	96	97	104	595
France	64 714 074	28	78	78	78	*79	79	83	6
United Kingdom	62 008 048	28	75	*76	76	76	76	80	6
Italy	60 340 328	28	74	74	74	74	74	78	6
Spain	45 989 016	28	59	59	59	59	59	61	6
Poland	38 167 329	28	51	51	51	51	51	51	6
Romania	21 462 186	28	33	33	*32	32	32	31	6
Netherlands	16 574 989	28	27	27	27	27	27	25	6
Greece	11 305 118	28	21	21	21	21	20	19	6
Belgium	10 839 905	28	20	20	20	20	20	19	6
Portugal	10 637 713	28	20	20	20	20	20	18	6
Czech Republic	10 506 813	28	20	20	20	20	20	18	6
Hungary	10 014 324	28	19	19	19	19	19	18	6
Sweden	9 340 682	28	18	18	18	18	18	17	6
Austria	8 375 290	28	17	17	17	17	17	16	6
Bulgaria	7 563 710	28	16	16	16	16	16	15	6
Denmark	5 534 738	28	14	*13	13	13	13	12	6
Slovakia	5 424 925	28	13	13	13	13	13	12	6
Finland	5 351 427	28	13	13	13	13	13	12	6
Ireland	4 467 854	28	12	12	12	12	12	11	6
Lithuania	3 329 039	28	11	11	11	*10	10	10	6
Latvia	2 248 374	28	9	9	9	9	9	8	6
Slovenia	2 046 976	27	9	9	9	9	9	8	6
Estonia	1 340 127	27	8	8	8	8	8	7	6
Cyprus	803 147	27	7	7	7	7	7	6	6
Luxembourg	502 066	27	6	6	6	6	6	6	6
Malta	412 970	27	6	6	6	6	6	6	6
Sum	501 103 425	751	751	751	751	751	751	751	751

For a given y, the exponent-range $[E_{\min}(y), E_{\max}(y)]$ is calculated as follows.

We need consider only seat vectors y with $y_i \geq 1$ for all i. Consider two States indexed i and j. There exists a unique *critical exponent* $E(i, j)$, depending on y, such that, when $E = E(i, j)$, the allocations (y_i, y_j) and $(y_i + 1, y_j - 1)$ are equally justified. Only situations with $y_j \geq 2$ are relevant in order to make sure that $y_j - 1 \geq 1$ plus five base seats achieve the six seat minimum required by the Treaty of Lisbon.

The tie entails the identities

$$\frac{p_i^{E(i,j)}}{D} = y_i \quad \text{and} \quad \frac{p_j^{E(i,j)}}{D} = y_j - 1.$$

In view of the rule of rounding upwards, this indicates that State i may be allocated y_i or $y_i + 1$ seats, and that State j is eligible for $y_j - 1$ or y_j seats. The identities allow us to eliminate the divisor and to solve thus for the critical exponent,

$$E(i, j) = \frac{\log(y_i/(y_j - 1))}{\log(p_i/p_j)}.$$

As E increases, the seat vector y is unchanged until E attains the value

$$E_{\max}(y) = \min\{E(i, j) : p_i > p_j,\ y_j \geq 2\}.$$

By a similar argument as E decreases, the lower boundary point of the exponent-range is

$$E_{\min}(y) = \max\{E(i, j) : p_i < p_j,\ y_j \geq 2\}.$$

The exponent-range for y is found to be $[E_{\min}(y), E_{\max}(y)]$. This range necessarily contains the given exponent E used to generate the seat vector y.

For example, consider the case of unweighted population figures, $E = 1$. The seat numbers $x_i(1) = 5 + y_i(1)$ are exhibited in the penultimate column of Table 1. It transpires that $y(E) = y(1)$ is unchanged as E ranges from $E_{\min}(y(1)) = 0.9956$ to $E_{\max}(y(1)) = 1.0010$. The "nicest" value in this exponent-range is evidently $E = 1$.

For a general compact interval $[E_{\min}, E_{\max}]$, we may pick a "nice" representative exponent E by rounding the midpoint of the interval to as few significant digits as the interval permits. The smallest and greatest intervals are half-open, however. In these two intervals, we may choose a representative exponent near the closed boundary, in order to avoid numerical difficulties.

The transfer argument proves also that, if $E < E'$, the seat vector $y(E)$ is majorized[2] by the seat vector $y(E')$. In particular, the largest State $i = 1$ has seat numbers that are nondecreasing in E, that is $E < E' \Rightarrow y_1(E) \leq y_1(E')$. We have that $y_1(0.01) = 28 - 5 = 23$ and $y_1(27.5) = 595 - 5 = 590$, see Table 1. Hence, the equation $y_1(E) = 91$ is solvable for E. We consider the determination of a solution in the next section.

3. Choice of initial exponent

We may determine an exponent E with $y_1(E) = 91$ in the following algorithmic manner. As starting point, we may expect the ideal share of the largest State to be close to its target contingent of 91 seats,

$$\frac{p_1^E}{\sum_{i=1}^{27} p_i^E} \cdot 616 = 91.$$

This equation may be solved numerically, and yields (with the 1.1.2011 Eurostat population figures) an initial exponent $E_{\text{init}} = 0.8888$. The largest State $i = 1$ misses its target by a single seat,

[2] The property of majorization amounts to the fact that, for any k, the aggregate number of seats of the k largest States is nondecreasing. As a consequence, the aggregate number of seats of the $27 - k$ smallest States is nonincreasing. See Marshall et al. (2002) for more details.

Table 2
Comparison of five EP compositions. Column A is the Cambridge Compromise "5 + Upw" of Grimmett et al. (2011). Column B is the parabolic allotment of Ramírez-González (2010). Columns C and D are power-weighted variants, with exponents 0.91 and 0.9 respectively, and associated population indices as shown. Column E is the *status quo*.

EU27	Population	$Popn^{0.91}$	$Popn^{0.9}$	A CC	B Par.	C $x(0.91)$	D $x(0.9)$	E Now
DE	81 802 257	15 871 442.9	13 227 834.7	96	96	96	96	99
FR	64 714 074	12 823 567.3	10 712 698.1	85	80	79	78	74
UK	62 008 048	12 334 675.7	10 308 684.6	81	78	76	76	73
IT	60 340 328	12 032 419.8	10 058 816.8	79	76	74	74	73
ES	45 989 016	9 397 563.0	7 877 505.3	62	61	59	59	54
PL	38 167 329	7 931 211.1	6 660 741.7	52	52	51	51	51
RO	21 462 186	4 697 029.6	3 967 405.2	32	33	32	32	33
NL	16 574 989	3 712 807.7	3 144 183.8	26	27	27	27	26
EL	11 305 118	2 621 080.4	2 228 166.2	19	20	21	21	22
BE	10 839 905	2 522 744.0	2 145 472.4	19	20	20	20	22
PT	10 637 713	2 479 887.2	2 109 421.9	18	19	20	20	22
CZ	10 506 813	2 452 102.5	2 086 046.1	18	19	20	20	22
HU	10 014 324	2 347 284.3	1 997 834.3	18	19	19	19	22
SE	9 340 682	2 203 152.3	1 876 466.1	17	18	18	18	20
AT	8 375 290	1 994 940.1	1 700 982.6	16	16	17	17	19
BG	7 563 710	1 818 229.6	1 551 891.6	15	15	16	16	18
DK	5 534 738	1 368 416.6	1 171 621.6	12	13	13	13	13
SK	5 424 925	1 343 687.6	1 150 679.5	12	13	13	13	13
FI	5 351 427	1 327 111.3	1 136 639.2	12	13	13	13	13
IE	4 467 854	1 126 134.0	966 249.0	11	11	12	12	12
LT	3 329 039	861 608.9	741 458.8	10	10	10	11	12
LV	2 248 374	602 837.7	520 812.9	8	8	9	9	9
SI	2 046 976	553 493.6	478 631.7	8	8	9	9	8
EE	1 340 127	376 446.1	326 912.4	7	7	8	8	6
CY	803 147	236 245.5	206 212.8	6	7	7	7	6
LU	502 066	154 060.9	135 109.1	6	6	6	6	6
MT	412 970	128 969.2	113 325.2	6	6	6	6	6
Sum	501 103 425			751	751	751	751	754

$y_1(0.8888) = 90$. The induced exponent-range turns out to be $[0.8884, 0.8977]$, with "nice" exponent 0.89. The machinery of Section 2 yields the next seat vector $y(0.899)$, with exponent-range $[0.8978, 0.8998]$, which still fails to allocate 91 seats to the largest State. One further iteration yields an exponent $0.9 \in [0.8999, 0.9035]$ with which the discrepancy vanishes. See Table 1.

An improved initialization procedure is available. The divisor method with rounding upwards is known to be biased, in that it has a tendency on average to favour smaller States at the expense of larger States. The seat-bias of the largest State may be approximated by the formula

$$-\frac{1}{2}\left\{\left(\sum_{n=1}^{27}\frac{1}{n}\right)-1\right\} = -1.4457,$$

see Heinrich et al. (2005). With this term included, the above initialization equation becomes

$$\frac{p_1^E}{\sum_{i=1}^{27} p_i^E}\cdot 616 = 91 + 1.4457,$$

with numerical solution $E_{\text{init}} = 0.9055$, and hence the "nice" exponent 0.91. The ensuing seat vector $y(0.91)$ achieves the target for the largest State, $y_1(0.91) = 91$, with exponent-range $[0.9036, 0.9109]$. Thus $y(0.91)$ differs from $y(0.9)$.

We return now to the original setting in which each State receives in addition five base seats, and we write $x_i = 5 + y_i$. If an exponent lies below 0.8999 or above 0.9109, the corresponding allocation fails to allocate to the largest State its target contingent of 96 seats. The exponent-range $[0.8999, 0.9035]$ gives rise to the feasible seat vector $x(0.9)$, and the subsequent exponent-range $[0.9036, 0.9109]$ yields the apportionment $x(0.91)$. Thus we obtain two apportionment vectors, $x(0.9)$ and $x(0.91)$, each of which allocates 96 seats to the largest Member State. See[3] Table 1.

As pointed out to the authors by a referee and also by Svante Janson, there are alternative ways to calculate sensible initial exponents E_{init}, including the asymptotic bias formula of Janson (2011). However, when implementing the procedure into the open source Augsburg software BAZI[4] we found that the naïve initialization $E = 1$ works very well, for all practical purposes. Practical problems are of such low complexity that special initialization routines would appear redundant.

We close this section with some comments based on the current and foreseeable population profile of the European Union.

1. There can exist several values of E for which the seat vector $x(E)$ satisfies $x_1(E) = 96$. Of these, the smallest such exponent leads to the composition closest to the *status quo* composition and, presumably, risks the greatest acclaim of the incumbent Parliament. Similarly, the composition with the largest exponent comes closest to the principled approach of the Cambridge Compromise. See Table 2.

2. A composition based on an exponent satisfying $E \leq 1$ automatically satisfies the revised definition of degressive proportionality of Grimmett et al. (2011). Conversely, when $E > 1$, the representation might possibly be called "progressive". A progressively proportional system is employed for the translation of votes into seats for the Estonian *Riigikogu* (parliament), using the divisor method with rounding downwards (Jefferson/D'Hondt/Hagenbach-Bischoff) with exponent $E = 10/9$.

[3] Exponent-ranges for E and applicable divisors D are:

$E_{\min}$	0	0.8884	0.8978	0.8999	0.9036	0.9110	0.9956	27.2202
Exponent	0.01	0.89	0.899	0.9	0.91	0.912	1	28
$E_{\max}$	0.0123	0.8977	0.8998	0.9035	0.9109	0.9125	1.0010	∞
Divisor	0.0526	121 400	144 400	146 960	174 600	180 800	830 000	$6.12 \cdot 10^{218}$.

[4] Available from www.uni-augsburg.de/bazi.

3. The power-weighted variant is a smoother allocation in situations where the Cambridge Compromise "5+Upw" hits the upper cap. Its implementation is thus in two steps. First, calculate the CamCom apportionment. If this caps the largest State at 96, then calculate the power-weighted variant.

4. As new States accede, the exponents E that achieve $x_1(E) = 96$ approach unity. When unity is reached, $E = 1$, the downweighting of population figures becomes neutral, and the ensuing apportionment is that of the Cambridge Compromise.

5. The identity $5+\text{Upw} = 5.5+\text{Std} = 6+\text{Dwn}$ (Zachariasen, 2011) is invalid for *non-linear* variants of the Cambridge Compromise including that considered here.

6. Serious issues of transparency and interpretation arise in the use of power-weighted population indices. For example, in the composition with exponent $E = 0.9$, the number of (non-base) seats allocated to Italy is in proportion to a population index of 10 058 816.8 power-weighted "apportionment units", rather than to the Italian population 60 340 328. How should this be interpreted, or explained to an Italian citizen? This problem does not arise in the linear case when $E = 1$. In this case there is a divisor-value $D = 830\,000$ with the clear interpretation that every group of 830 000 Union citizens accounts for one seat in Parliament (subject to rounding).

4. Discussion

The legal principles for the composition of and election to the European Parliament (EP) may be found in Article 14 of the Treaty of Lisbon (TEU-Lisbon, 2010):

(II1) The EP shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats.

(III) The members of the EP shall be elected for a term of five years by direct universal suffrage in a free and secret ballot.

The concept of "equality" appears in several other articles of the Treaty (including Articles 2, 4, 9, 10), but is notably absent from Article 14 III.

Moreover, under the Treaty of Lisbon the task of deciding on the EP's composition has become a matter for secondary (parliamentary) law, and therefore subject to challenge in the Court of Justice of the European Union.

The European Commission for Democracy through Law [Venice Commission] (2002) lists five principles of Europe's electoral heritage.

(I) The five principles underlying Europe's electoral heritage are *universal, equal, free, secret and direct suffrage*.

The 27 Member States of the Union count among the 47 members of the Council of Europe, and as such endorse the Venice Commission's Code of Good Practice in Electoral Matters. Furthermore, the Union has resolved to accede to the European Convention for the Protection of Human Rights and Fundamental Freedom, and will then become accountable to the European Court of Human Rights.

The concept of electoral equality is not self-explanatory, but requires interpretation. Constitutional courts take pains to distinguish between large electorates (such as a national parliament), medium-size electorates (such as a provincial legislature), and small electorates (as in local communities). The European Union is on a scale in excess of these, and the notions of electoral equality entertained within the 27 Member States cannot be extended automatically to the entirety of the Union. The EP has thus substantial freedom in specifying an interpretation of the concept of electoral equality.

The two-stage process of the Cambridge Compromise may be interpreted as a type of "dual electoral equality". This dual concept is a merger of the *one state, one vote* rule of international law, and of the *one person, one vote* principle of equal representation of citizens. There is a potential ambiguity in the term "state" over whether it refers to *government* or to *people*. Our belief that many Members of the EP consider themselves representatives of their *citizenry* is supported by the statement of Article 14 II that says just this. We shall thus write here of "citizenries" rather than of "states".

The Cambridge Compromise merges the two aspects of electoral equality, and may be interpreted as follows. The base component is directed towards equality of citizenries, and the proportional allocation of the remaining seats is aimed at equality among Union citizens. In contrast, it is considerably harder to justify the power-weighted variant of the Cambridge Compromise, since it violates the principle of equal suffrage as set out by the Venice Commission:

(2.2) *Equal voting power:* seats must be evenly distributed between the constituencies.

There seems little doubt in the context of the EP that the word "constituencies" should be interpreted as "Member States". Yet the power-weighted apportionment is decidedly unequal[5]. For example, whereas the Italian population-index accounts for just a sixth of its population, the Maltese index accounts for one quarter.

A further principle of electoral affairs, the *continuity principle*, shields the legislator from abstract rules in situations where more sensitive action is needed. During a period of significant institutional development, the EP may adopt the power-weighted variant $5 + \text{Pwr} + \text{Upw}$ as a step along a continuous transition from the negotiated *status quo* composition to the constitutionally principled Cambridge Compromise. This variant honours the equality principle for citizenries, and converges for an enlarged Union towards the equality principle for individual citizens. The transitional period with $E < 1$ is justifiable by an appeal to the principle of continuity.

The above discussion may be summarized as follows. While the power-weighted variant of the Cambridge Compromise is technically sound and feasible, it is in conflict with the principle of equal voting power. Since the power-weighting will diminish as the Union grows, this variant may be justified by the continuity principle.

We close with a word of caution. The lack of transparency of the power-weighted variant may be more harmful than helpful to public reception. The current rapporteur Andrew Duff MEP has proposed an intermediate step of negotiation as a bridge between the *status quo* and the Cambridge Compromise. This pragmatic proposal may be superior in communication and implementation.

Acknowledgments

We would like to thank Svante Janson, Daniel Lübbert and a referee for valuable comments on an earlier version of the paper.

References

Arndt, F., 2008. Ausrechnen statt aushandeln: Rationalitätsgewinne durch ein formalisiertes Modell für die Bestimmung der Zusammensetzung des Europäischen Parlaments (with English summary). Zeitschrift für ausländisches öffentliches Recht und Völkerrecht – Heidelberg Journal of International Law 68, 247–279.

[5] In the terminology of the German Federal Constitutional Court, equal voting power is termed *Zählwertgleichheit*.

European Commission for Democracy through Law [Venice Commission] 2002. Code of good practice in electoral matters: Guidelines and explanatory report. Opinion no. 190/2002, CDL-AD 23 rev.

Grimmett, G., 2012. European apportionment via the Cambridge Compromise. Mathematical Social Sciences 63 (2), 68–73.

Grimmett, G., Laslier, J.-F., Pukelsheim, F., Ramírez-González, V., Rose, R., Słomczyński, W., Zachariasen, M., Życzkowski, K., 2011. The allocation between the EU member states of the seats in the European parliament. European Parliament, Directorate-General for Internal Policies, Policy Department C: Citizen's Rights and Constitutional Affairs, Note 23.03.2011 (PE 432.760).

Heinrich, L., Pukelsheim, F., Schwingenschlögl, U., 2005. On stationary multiplier methods for the rounding of probabilities and the limiting law of the Sainte-Laguë divergence. Statistics & Decisions 23, 117–129.

Janson, S., 2011. Asymptotic bias of some election methods. Working note.

Marshall, A.W., Olkin, I., Pukelsheim, F., 2002. A majorization comparison of apportionment methods in proportional representation. Social Choice and Welfare 19, 885–900.

Ramírez-González, V., 2010. Degressive proportionality: composition of the European Parliament. The parabolic method. In: Cichocki, M., Zyczkowski, K. (Eds.), Institutional Design and Voting Power in the European Union. Ashgate, London, pp. 215–234.

TEU-Lisbon,, 2010. Treaty on European Union, articles 1–19. Offical Journal of the European Union C 83 (30.2.2010), 13–27.

Theil, H., Schrage, L., 1977. The apportionment problem and the European Parliament. European Economic Review 9, 247–263.

Zachariasen, M., 2011. On the relationship between base and rounding method. Working note.