# Mathematical modeling of electoral systems: analysis, evaluation, optimization. In memory of Bruno Simeone (1945-2010) 

Isabella Lari • Friedrich Pukelsheim • Federica Ricca

## 1 Foreword

This volume of the Annals of Operations Research is dedicated to the memory of our dear friend and colleague Bruno Simeone (1945-2010). It collects a selection of papers on mathematical modeling of electoral systems.

In his long and fruitful academic and scientific career, Bruno Simeone's interest in the study of electoral systems was driven by the absence of a sound appraisal of the mechanics of electoral formulae in the intense debate over electoral reform in Italy and the awareness of the contradictions and flaws of the (past and current) Italian Electoral Law. Since 1993 he studied different electoral problems with a mathematical approach-from gerrymandering to proportional electoral formulae, passing through governability and representation issues, always highlighting the constructive solutions that this can lead to and the practical policy guidance that could be derived from this kind of analysis.

The first results on this topic gave rise to the volume Evaluation and Optimization of Electoral Systems (SIAM, 1999), a book that soon became a basic reference in the literature as an introductory view of the adoption of a mathematical approach in the design of methods to translate votes into seats. From the publication of the volume onwards, Bruno made substantial contributions to the field of electoral systems, especially with respect to issues that can be formulated in terms of discrete optimization problems. The main topics of his work are political districting and biproportional apportionment, but Bruno also studied a variety of other related problems arising in the particular Italian context. In fact, in spite of the lively debate of the last 20 years about electoral reform, the Italian law is

[^0]still unsatisfactory, from both the technical and the political viewpoint. In this context the work of Bruno represented a step forward for the design of a sound and democratic electoral system in Italy. The way in which Bruno was able to properly formalize every electoral problem he studied, and to find a rigorous and elegant solution for each, was something that characterized him as a unique figure, and is the confirmation of his cleverness and expertise.

With his work Bruno Simeone has enhanced the development of mathematical methods for electoral systems, publishing many technical and non-technical papers with the double aim of promoting this field of application in operations research and spreading the use of rational methods in the political realm. He always tried to push OR toward real-world applications, including those having a social and ethical interest, always believing that the main task of an operations researcher is to find solutions for real-life concerns.

In 2005 he organized in Erice (Sicily) the workshop Mathematics and DemocracyRecent Advances in Voting Systems and Collective Choice with the "aim of collecting different viewpoints... in the emerging multidisciplinary science of voting systems and social choice... and emphasizing the role of mathematics towards a deeper understanding, a rational assessment, and a sound design of voting procedures."

The above quote is taken from the preface of the workshop proceedings published by Springer in the volume Mathematics and Democracy—Recent Advances in Voting Systems and Collective Choice. The event was considered by the international community to be a step forward in promoting and spreading the use of OR in this field. The Erice attendees recognized Bruno Simeone as one of the leading experts in the field of electoral mathematics, and his views were instrumental in drawing up "The Erice Decalogue," a collection of basic principles for the design of a fair electoral system. The Decalogue aimed at promoting a sound basis for the multidisciplinary science of electoral systems.

The call for papers of this special volume of the Annals of Operations Research invited authors to submit new contributions in the area of the mathematics of electoral systems. In this particular area, there is no unique approach to tackle problems, but, on the contrary, different mathematical tools suit the analysis and are used to bring solutions. Indeed, we received a large number of high quality submissions from the areas of mathematical programming, combinatorial optimization, game theory, and graph theory. Among these, we selected 14 original contributions to appear in this special volume.

We want to recall here that Bruno Simeone was originally invited to be the editor of this volume, but, unfortunately, on October 10, 2010, he passed away unexpectedly and the project could not start. We know how much Bruno loved the idea of this volume and we have no doubt that he would have been satisfied with this collection of papers. We are proud to have realized the volume and dedicate it to the memory of Bruno Simeone.

We want to thank Endre Boros, a very close friend and colleague of Bruno, who proposed and encouraged the realization of this volume. We also thank all the authors of this special volume for their contributions, and all our colleagues-experts in the field-who served as volunteer reviewers for their help in selecting and improving the submitted papers. Our special thanks are given to Katie D'Agosta, the Managing Editor of the journal, for her continuous support and help during the long period required to complete this volume.

## 2 Bruno Simeone (1945-2010)

At 65 years of age Bruno Simeone had published 8 volumes, and more than 100 papers, nine of which appeared after his death-proof of his lively research activity and highly
qualified work. Besides being a prominent researcher, he was a master of culture and a very talented teacher, and he left a wide research inheritance to his collaborators. One of the qualities that made Bruno extraordinary was his ability to "make difficult notions look simple." This was one of his characteristic traits both in teaching students of every level and in working and leading research teams.

Bruno Simeone was deeply convinced that research is best conducted in a group and in a multidisciplinary setting. He was rarely the unique author of his publications and never protective of his results. His precious and innovative ideas combined with his rigorous methods, made Bruno Simeone a brilliant researcher, a special teacher, and a hard worker. He provided many important results and was a constant source of inspiration for his collaborators, but in spite of his superior knowledge and scientific depth, he always appreciated learning of the ideas and results found by his (often less experienced) coauthors.

In 1989 Bruno Simeone was one of the founders of the Ph.D. program in Operations Research in Rome which he directed from 1989 to 1994. He was also the Director of the Graduate School of "Operations Research and Decision Strategies" for several years (from 1994 to 2004), and, later, when the School was converted into the new Master of Science in "Data Intelligence and Decision Strategies," he was an active member of the Scientific and Teaching Committees, guiding the constant improvement and the upgrading of the quality of the courses provided by the Master. His drive for quality also touched undergraduate education and, since 1981, when he entered the Faculty of Statistical Sciences in Rome La Sapienza as full professor (at the age of 35 -young for Italian standards), he had been involved in curriculum development, in planning and organizing classes of studies, and in operational training.

Besides ordinary courses for undergraduate students of his university, he taught many advanced courses in other Italian or international universities, for example in Canada and the USA. He taught in international summer schools, but also gave courses outside academia, such as several industrial courses in operations research, and advanced courses in management of health care processes addressed to professionals in medicine.

He gave seminars all over the world: at MIT, Columbia University, New York University, Rutgers University, University of Waterloo, Université de Montréal, Ecole Polytechnique de Montréal, Université Scientifique et Médicale de Grenoble, INRIA, Ecole Polytechnique Féderale de Lausanne, Universität Bonn, Universität Köln, Universität Osnabrück, Universidad de La Laguna (Tenerife), Universidad de Sevilla, University of Cape Town, Curtin University of Technology (Perth), and the like, and in several Italian universities and research institutions. In many of them he also played an active part in tutoring Ph.D. students and in contributing to scientific research.

Bruno Simeone had a dynamic editorial role, being an associate editor of important scientific journals, such as Discrete Optimization and the Annals of Operations Research; he was a referee for the main international journals in the area of operations research, discrete mathematics, combinatorial theory, graph theory, management science, mathematical programming, and so on.

He was involved in many research proposals at a national and international level and played an active role in promotional activities, covering responsible positions, and organizing conferences, workshops, and meetings, always with the aim of improving and developing both research and education in Italy.

Besides his internationally recognized scientific activity, Bruno Simeone was an active member of the Italian operations research community, contributing to its development. He played a fundamental role in the development, within Italian academia, of combinatorial
optimization, of which he later became one of the most prominent scholars at the international level.

As one of his best friends and colleagues said, Bruno was among the rare scholars to look beyond the borders of his own field of research, with a view not only of encouraging cross-fertilization between different cultural or disciplinary areas, but, more importantly, to bridge the gap between scientific interests and ethical and civil beliefs. Bruno has shown how mathematics may provide insights and tools to make our society better and how relevant societal problems can give rise to good mathematics.

In the following, we give an account of the main scientific research interests of Bruno Simeone. He contributed to the development of the operations research discipline, from both a theoretical and an applications viewpoint. In a number of cases, his work generated completely new lines of research, frequently in the difficult field of interdisciplinary collaboration between researchers of different areas, a kind of research that Bruno Simeone was really fond of and for which he was an obstinate supporter.

In his Ph.D. thesis ("Quadratic 0-1 Programming, Boolean Functions and Graphs," University of Waterloo, 1979) and in numerous subsequent papers, he unveiled fundamental results on the interplay among quadratic $0-1$ programming, graph theory, and the theory of Boolean functions. In particular, in the seminal paper "Roof-duality, complementation, and persistency in quadratic $0-1$ optimization" (Mathematical Programming, 1984), he introduced the roof-duality theory which set up a systematic framework for the computation of upper-bounds of quadratic $0-1$ optimization problems. This was one of many works done jointly with Peter L. Hammer, who was his Ph.D. advisor, but, most of all, a special colleague and a very close friend. The results in this area gave rise to the development of many theoretical and computational results on quadratic $0-1$ programming, including the joint work of Bruno with Giorgio Gallo on quadratic knapsack problems (Mathematical Programming, 1988), as well as, very recent and fruitful applications to image processing and segmentation problems.

In other seminal works (Discrete Applied Mathematics, 1985; Annals of Discrete Mathematics, 1994), Uri Peled and Bruno Simeone solved a long-standing open question in electrical engineering by showing that threshold Boolean functions can be recognized in polynomial time. This breakthrough was obtained as a byproduct of an efficient algorithm for obtaining all minimal solutions of so-called "regular" set covering problems. It opened the way for much subsequent research on related dualization problems.

On the topic of connected graph partitioning, Bruno helped to significantly modify the state of complexity results and to apply a wide variety of solution strategies. He found many results on path, tree, and grid partitioning problems and contributed greatly to the treatment of contiguity constrained clustering problems as optimization problems. Bruno started the research in this field with Enzo L. Aparo, who, besides being one of his masters in Italy, was one of his best friends and colleagues. Later, some important works were developed in collaboration with Mario Lucertini and Stefano Pallottino, two friends and colleagues who experienced with Bruno an intensive period of research in the years between the end of the 1970s and the beginning of the 1980s that were characterized by a growing interest in combinatorial and discrete optimization in Italy. Bruno also considered the application of graph partition problems to political districting and he was the first to provide theoretical results on the problem of gerrymandering. His fruitful activity in this research area is proven by the numerous papers he published on this topic.

In 1999 Bruno Simeone published the volume Evaluation and Optimization of Electoral Systems, SIAM Monographs on Discrete Mathematics and Applications, thus starting productive research on electoral systems and political districting, which remained
throughout the years one of his fond topics. Bruno Simeone's scientific oeuvre reveals his inquiries into the mathematical analysis of electoral systems through an active involvement in organizing conferences and workshops on electoral subjects in order to bring together researchers from all over the world and feed the development of a network of specialists in the field. The organization in 2005 of the international workshop on "Mathematics and Democracy: Voting Systems and Collective Choice" (http://www.ccsem.infn.it/ ccsem2005/Simeone.pdf), with the publication of the proceedings volume of the workshop (Springer, 2006), is an example of such activity that obviously he developed in the academic context, but that he supported in his everyday life-always pursuing his firm belief that operations research must serve real-life problems.

In this area of research Bruno Simeone can be considered one of the first promoters of operation research, optimization, and combinatorics for the analysis and the study of electoral problems. His interest in this topic started in the period after the last Italian electoral reform (2005), but it characterized his scientific production also in the following years, with many papers published both in Italian and international journals.

An expert in operations research and network theory, Bruno Simeone was naturally driven to employ graph theory in the drawing of electoral districts, and mathematical programming for the design of biproportional apportionment methods. The very Italian "give-up problem," arising when elected candidates who ran in several constituencies have to select one mandate and give up the others, was solved by Simeone using network arguments (Mathematical Social Sciences, 2011).

In the last years of his academic career Bruno Simeone contributed to the birth of another important field of research, always motivated by the idea that the powerful tools of discrete mathematics and combinatorics should be disseminated and applied to assist the decision process. In 2004 (Discrete Applied Mathematics, 2004) he introduced a new methodology called Box-clustering (BC) which is a branch of a more general methodology, called Logical Analysis of Data (LAD), pioneered in 1986 by Peter L. Hammer and his research group at Rutgers University, NJ, USA. Both LAD and BC can be applied to different data analysis problems and, in particular, they have proved to be very useful when adopted in medical diagnosis where a group of patients is followed in order to detect whether or not they suffer from a given syndrome. Still a quite young methodology, in its first experimental applications BC showed a high classification power. One of the aims in Simeone's research was to show that BC—which was employed by him and his group for the first time in an actual medical application in Italy-is a particularly valuable tool for medical diagnosis, since, besides its classification capability, it is also able to produce information about typical profiles characterizing patients affected by the syndrome. This provides additional support for medical doctors who can easily read and interpret the outcome of a BC application and use this information to support their decisions.

In the very style of Bruno Simeone's research, BC also became a source of inspiration for obtaining further important theoretical results. Based on the relations between the observations of a BC dataset, he introduced a new class of graphs, called "Incompatibility Graphs," which, besides their relevance for the applications in data mining, also have an intrinsic interest from a theoretical viewpoint, since they generalize some important classes of graphs, namely, chordal and weakly chordal graphs. The special structure of the Incompatibility Graphs can be exploited to efficiently solve some key problems related to BC, such as the "Maximum Box" and the "Minimum Covering by Boxes." Actually these problems can be formulated in an Incompatibility Graph as vertex packing and vertex coloring ones, respectively, and they can be solved in polynomial time for some important subclasses of instances.

During the last years of his life, Bruno Simeone applied discrete mathematics also to music, another of his passions. His already addressed capacity to give rise to good mathematics in different and sometimes unexpected fields, and his special capability to recognize the combinatorial flavor of every problem, made him able to find very elegant results also in this context. He formalized musical harmony concepts and introduced innovative ideas for extending the musical meaning of tonal signatures using Boolean operators (Discrete Applied Mathematics, 2013).

## 3 List of publications of Bruno Simeone

### 3.1 Books and edited volumes

1 FORTRAN codes for network optimization, Annals of Operations Research 13, Baltzer, Basel, 1988 (Editor with P. Toth, G. Gallo, F. Maffioli, S. Pallottino)
2 Combinatorial Optimization, Lecture Notes in Mathematics, 1403, Springer, Heidelberg, 1989 (Editor)
3 Special issue dedicated to Enzo Aparo for his 70th birthday, Ricerca Operativa, 57 (1991) (Editor with G. Gallo)

4 Partitioning and Decomposition in Combinatorial Optimization, Discrete Applied Mathematics, 62 (1995) (Editor with M. Lucertini, G. Rinaldi, A. Sassano)
5 Evaluation and Optimization of Electoral Systems. SIAM Monographs on Discrete Mathematics and Applications, SIAM, Philadelphia, 1999 (co-auth.: P. Grilli di Cortona, C. Manzi, A. Pennisi, F. Ricca)
6 Mathematics and Democracy. Recent Advances in Voting Systems and Collective Choice, Springer, Berlin, 2006 (Editor with F. Pukelsheim)
7 "Chapter 6: Quadratic Boolean Functions", in Boolean Functions, Theory, Algorithms, and Applications, Cambridge University Press, 2011 (Y. Crama and P.L. Hammer eds.)
8 Special issue on The Mathematics of Peter L. Hammer (1936-2006): Graphs, Optimization, and Boolean Models, Annals of Operations Research, 188, Springer, 2011 (Editor with E. Boros, Y. Crama, D. de Werra, P. Hansen, and F. Maffray)
3.2 Published articles and research reports

9 "Programmazione su un gruppo: un algoritmo branch \& bound", Atti Giornate di Lavoro AIRO, Torino 1971, 85-97 (co-auth.: G. Gallo)
10 "Sull'integrazione delle equazioni della meteorologia: III, l'integrazione nel tempo", Rivista di Meteorologia Aeronautica, 4 (1972) 317-325 (co-auth.: L. La Valle)
11 "Group optimization techniques and some numerical results via a branch \& bound approach", Atti Simposio AICA su "Tecniche di simulazione e algoritmi" (Tamburini, Milano, 1972) (co-auth.: G. Gallo, E. Martino)
12 "An Automatic River Planning Operating System (ARPOS)" in: R. Conti, A. Ruberti (eds.) Proc. 5th IFIP Symposium on Optimization Techniques, Roma 1973, Lecture Notes in Computer Science (Springer, Berlin, 1973) (co-auth.: E. Martino, T. Toffoli)
13 "Un algoritmo di equipartizione e il suo impiego in un problema di contrasto ottico", Ricerca Operativa 3 (1973), 31-42 (co-auth.: E. L. Aparo)
14 "Un modello di flusso in una rete per la gestione di risorse idriche", Atti Giornate di Lavoro AIRO, Palermo 1974, 91-102

15 "Modelli di programmazione lineare per la gestione di risorse idriche", in: I. Galligani (ed.): Applicazioni del Calcolo (CNR, Roma, 1975) 37-48
16 "Equipartizione su un grafo: un'applicazione all'informatica medica", ibidem 77-86 (co-auth.: E.L. Aparo)
17 "New cuts for integer programming via group optimization", Pubbl. IAC, Ser. III, no. 105, 1975 (co-auth.: G. Gallo)
18 "Degree sequences of threshold graphs", Proc. 9th Southeastern Conf. on Combinatorics, Graphs, and Computing, Boca Ratòn 1978, 329-355 (co-auth.: P.L. Hammer, T. Ibaraki)
19 "Optimal graph partitioning", Atti Giornate di Lavoro AIRO, Urbino 1978, 57-73
20 "Metodi di programmazione matematica nella pianificazione e nella gestione delle risorse idriche: modelli, algoritmi e software", Pubbl. IAC, Ser. III, no. 145, 1978 (coauth.: G. Silveri)
21 "Structural aspects of discrete problems", Session Report, Advanced Research Institute on Discrete Optimization and Systems Applications, Banff 1977, in: P.L. Hammer, E.L. Johnson, B. Korte (eds.): Discrete Optimization, Annals of Discrete Mathematics 4 (1979) 177-181 (co-auth.: P. Hansen)
22 "Cutting planes", ibidem, Annals of Discrete Mathematics 5 (1979) 193-194 (coauth.: M. Gondran)
23 "Quadratic 0-1 Programming, Boolean Functions, and Graphs", Doctoral Dissertation, Dept. of Combinatorics and Optimization, University of Waterloo, Ontario, 1979
24 "A conic algorithm for the group minimization problem", Cahiers du Centre d'Etudes de Récherche Opérationnelle 22, Bruxelles, Belgium (1980)
25 "Quadratic knapsack problems", Mathematical Programming Studies 12 (1980) 132-149 (co-auth.: G. Gallo, P.L. Hammer)
26 "Quasimonotone Boolean functions and bistellar graphs", Annals of Discrete Mathematics 9 (1980) 107-119 (co-auth.: P.L. Hammer)
27 "Graphes de conflit des fonctions pseudo-booléennes quadratiques", in: P. Hansen, D. de Werra (eds.): Régards sur la théorie des graphes, (Presses Polytechniques Romandes, Lausanne, 1980) 165-170 (co-auth.: C. Benzaken, P.L. Hammer)
28 "Some remarks on conflict graphs of quadratic functions", Proc. Symposium "Konstruktive Methoden der finiten nichtlinearen Optimierung", Oberwolfach 1980, Intern. Ser. Numer. Math. (Birkhauser, Basel 1980) 11-30 (co-auth.: C. Benzaken, P.L. Hammer)
29 "A switching algorithm for the solution of quadratic Boolean equations", Information Processing Letters 11 (1980) 193-198 (co-auth.: R. Petreschi)
30 "Threshold sequences", SIAM Journal on Algebraic and Discrete Methods 2 (1981) 39-49 (co-auth.: P.L. Hammer, T. Ibaraki)
31 "The splittance of a graph", Combinatorica 1 (1981) 275-284 (co-auth.: P.L. Hammer)
32 "Algoritmi di equipartizione e di clustering su grafi, e loro uso nella distrettizzazione territoriale", Notiziario UMI 8 (1981) 44-51
33 "A polynomial algorithm for recognizing threshold functions" (extended abstract), Methods of Operations Research 41 (1981) (co-auth.: U.N. Peled)
34 "Upper planes of quadratic 0-1 functions and stability in graphs", in: O. Mangasarian, R.R. Meyer, S.M. Robinson (eds.): Nonlinear Programming 4 (Academic Press, New York, 1981) 395-414 (co-auth.: P.L. Hammer, P. Hansen)
35 "Vertices belonging to all or to no maximum stable set of a graph", SIAM Journal on Algebraic and Discrete Methods 3 (1982) 511-522 (co-auth.: P.L. Hammer, P. Hansen)

36 "Isotone interval relations in linearly ordered sets", Ars Combinatoria 15 (1983) 275-278
37 "Adjonts of pure bidirected graphs", Congressus Numerantium 39 (1983) 123-144 (co-auth.: C. Benzaken, S. Boyd, P.L. Hammer)
38 "Weighted König-Egerváry graphs and quadratic $0-1$ optimization", (extended abstract) Methods of Operations Research 45 (1983) (co-auth.: J.-M. Bourjolly, P.L. Hammer)
39 "Roof-duality, complementation, and persistency in quadratic 0-1 optimization", Mathematical Programming 28 (1984) 121-155 (co-auth.: P.L. Hammer, P. Hansen)
40 "Box-threshold graphs", Journal of Graph Theory 8 (1984) 331-345 (co-auth.: U.N. Peled)
41 "Degree sequences of matrogenic graphs", Discrete Mathematics 51 (1984) 47-61 (co-auth.: P. Marchioro, A. Morgana, R. Petreschi)
42 "Node-weighted graphs having the König-Egerváry property", Mathematical Programming Studies 22 (1984) 44-63 (co-auth.: J.-M. Bourjolly, P.L. Hammer)
43 "Consistency of quadratic Boolean equations and the König-Egerváry Property for graphs", in: G. Ausiello, M. Lucertini (eds.): Analysis and Design of Algorithms for Combinatorial Problems, Annals of Discrete Mathematics 25 (1985) 281-290
44 "Polynomial-time algorithms for regular set covering and threshold synthesis", Discrete Applied Mathematics 12 (1985) 57-70 (co-auth.: U.N. Peled)
45 "Lower planes of submodular functions", Dipartimento di Informatica, Univ. di Pisa, 1985 (co-auth.: G. Gallo)
46 "Unimodular functions", Discrete Applied Mathematics 14 (1986) 269-281 (coauth.: P. Hansen)
47 "An asymptotically exact polynomial algorithm for equipartition problems", ibidem, 283-293
48 "How hard is it to partition a path", in: M. Loui, J.V. Medanic (eds.): Proc. XXIV Annual Allerton Conference on Communications, Control, and Computing 1987, 39-94 (co-auth.: M. Lucertini, Y. Perl)
49 "Order relations of variables in 0-1 programming", in: S. Martello, G. Laporte, M. Minoux, C.C. Ribeiro (eds.) Surveys in Combinatorial Optimization, Annals of Discrete Mathematics 31 (1987) 83-112 (co-auth.: P.L. Hammer)
50 "Product form parametric representation of the solutions to a quadratic Boolean equation", RAIRO Récherche Opérationnelle 21 (1987) 287-306 (co-auth.: Y. Crama, P.L. Hammer, B. Jaumard)
51 "On the complexity of the maximum satisfiability problem for Horn formulas", Information Processing Letters 26 (1987) 1-4 (co-auth.: B. Jaumard)
52 "Adamant digraphs", Discrete Mathematics 69 (1988) 253-261 (co-auth.: P. Marchioro, A. Morgana, R. Petreschi)
53 "From linear separability to unimodality: a hierarchy of pseudo-Boolean functions", SIAM Journal on Discrete Mathematics 1 (1988) 174-184 (co-auth.: P.L. Hammer, T. Liebling, D. de Werra)
54 "A selected Artificial Intelligence bibliography for Operations Researchers", in: R. G. Jeroslow (ed.) Approaches to intelligent decision support, Annals of Operations Research 12 (1988) 1-50 (co-auth.: B. Jaumard, P.S. Ow)
55 "An $\mathrm{O}\left(\mathrm{n}^{* *} 3\right)$ on line algorithm for 2-satisfiability", Atti Giornate di Lavoro AIRO, Pisa 1988, 391-399 (co-auth.: B. Jaumard, P. Marchioro, A. Morgana, R. Petreschi)
56 "On the supermodular knapsack problem", Mathematical Programming 45 (1988) 295-309 (co-auth.: G. Gallo)

57 "A feasible directions method for computing Bayes E-optimal block designs", Computational Statistics and Data Analysis 7 (1989) 23-38 (co-auth.: I. Verdinelli)
58 "Quadratic functions of binary variables", in: B. Simeone (ed.): Combinatorial Optimization, Lecture Notes in Mathematics, 1403 (Springer, Heidelberg, 1989) (coauth.: P.L. Hammer)
59 "Image enhancement by path partitioning", in: V. Cantoni, S. Levialdi (eds.): Recent Issues in Image Analysis, Lecture Notes in Computer Science (Springer, Heidelberg, 1989) (co-auth.: M. Lucertini, Y. Perl)

60 "The threshold measure of a graph", in: M.F. Capobianco, M. Guan, F. Hsu, F. Tian (eds.): Graph Theory and its Applications: East and West, Annals of the New York Academy of Sciences, vol. 576, 1989, 441-447 (co-auth.: U.N. Peled)
61 "Combinatorial methods for bounding quadratic pseudo-boolean functions", RUTCOR, New Brunswick, N.J., RRR 27-89, 1989 (co-auth.: J.-M. Bourjolly, P.L. Hammer, W.R. Pulleyblank)
62 "Fair dissections of spiders, worms, and caterpillars", Networks 20 (1990) 323-344 (co-auth.: C. De Simone, M. Lucertini, S. Pallottino)
63 "On the equivalence of paved duality and standard linearization in nonlinear 0-1 optimization", Discrete Applied Mathematics 29 (1990) 187-193 (co-auth.: P. Hansen, S.H. Lu)
64 "On line 2-satisfiability", Annals of Mathematics and Artificial Intelligence 1 (1990) 155-165 (co-auth.: B. Jaumard, P. Marchioro, A. Morgana, R. Petreschi)
65 "Boolean approaches to combinatorial optimization" (in chinese) Journal of Mathematical Research and Exposition 10 (1990) 300-312, 455-468, 619-628 (coauth.: P.L. Hammer, Y. Liu)
66 "Optimal hypergraph partitioning in VLSI circuit layout", Atti Giornate di Lavoro AIRO, Sorrento 1990, 985-1000 (co-auth.: G. Antenucci, S. Nicoloso)
67 "Recognition of a class of unimodular functions", ibidem, 243-250 (co-auth.: M. Cochand, D. de Werra). Erratum: Discrete Applied Mathematics 32 (1991)
68 "General theoretical results on rectilinear embeddability of graphs", (extended abstract) Acta Mathematicae Applicatae Sinica, 7 (1991) 187-192 (co-auth.: Y. Liu, A. Morgana)

69 "On theoretical results of at most 1-embeddability of graphs", (extended abstract) Chinese Science Bulletin 36 (1991) 1054-1055 (co-auth.: Y. Liu, P. Marchioro, R. Petreschi)
70 "Experimental comparison of 2-satisfiability algorithms", RAIRO Recherche Opérationnelle 25 (1991) 241-264 (co-auth.: R. Petreschi)
71 "Optimal grouping of researchers into departments", Ricerca Operativa 57 (1991) 45-69 (co-auth.: G. Gallo)
72 "Boolean and graph-theoretic formulations of the simple plant location problem", Transportation Science 26 (1992) 138-148 (co-auth.: P.M. Dearing, P.L. Hammer)
73 "Boolean-combinatorial bounding of maximum 2 -satisfiability", in: O. Balci, R. Sharda, S.A. Zenios (eds.): Computer Science and Operations Research: New Developments in their Interfaces (Pergamon Press, London, 1992) 23-42 (co-auth.: J.M. Bourjolly, P.L. Hammer, W.R. Pulleyblank)

74 "Theoretical results on at most 1-bend embeddability of graphs", Acta Mathematicae Applicatae Sinica 8 (1992) 188-192 (co-auth.: Y. Liu, P. Marchioro, R. Petreschi)
75 "Clustering su un albero con massima separazione", Ratio Mathematica 5 (1992) 65-73 (co-auth.: M. Maravalle)

76 "A graph-theoretic model of integrated ownership in business groups", Dipartimento di Economia, Università "La Sapienza", Discussion Paper n. 16, 1992 (co-auth.: M.T. Salvemini, R. Succi)
77 "Most uniform path partitioning and its use in image processing", Discrete Applied Mathematics 42 (1993) 227-256 (co-auth.: M. Lucertini, Y. Perl)
78 "Sharp bounds for the maximum of the Chi square index in a contingency class with given marginals", Computational Statistics and Data Analysis 16 (1993) 19-34 (coauth.: B. Kalantari, I. Lari, A. Rizzi)
79 "Clustering heuristics for set covering", Annals of Operations Research 43 (1993) 95-308 (co-auth.: R.K. Kwatera)
80 "An O(mn)-time algorithm for computing the dual of a regular Boolean function", Annals of Discrete Mathematics 49 (1994) 309-323 (co-auth.: U.N. Peled)
81 "Un metodo baconiano per l'agglomerazione delle USL nel comune di Roma", (extended abstract) in: M. Paolucci, M. Schenone (eds.): Atti Giornate di Lavoro AILOG-AIRO, Savona, 1994, 67-70 (co-auth.: U.L. Aparo, F.Grasso, E. Sesti)
82 "Un metodo baconiano per l'agglomerazione delle USL nel comune di Roma", (expanded version of ref. 81) Ricerca Operativa, 72 (1994) 57-79 (co-auth.: U. L. Aparo, F. Grasso, E. Sesti)

83 "A spanning tree heuristic for regional clustering", Communications in StatisticsTheory and Methods, 24 (1995) 623-639 (co-auth.: M. Maravalle)
84 "Agglomerazione di USL: generazione automatica di scelte alternative", Annali di Igiene 7 (1995) 121-131(co-auth.: U. L. Aparo, F. Grasso, E. Sesti)
85 "Analisi del possesso integrato nei gruppi di imprese mediante grafi" L' Industria, A. XVI, (1995) (co-auth.: M. T. Salvemini, R. Succi) 641-662
86 "Path composition and multiple choice in bimodal transportation networks", in: D. Hensher, J. King, T. Oum (eds.) Modelling Transport Systems, Proceedings of the 7th World Conference on Transportation Research, Sydney 1995 (co-auth.: M. G. Battista, M. Lucertini)

87 "Espaliers: a generalization of dendrograms", Journal of Classification 13 (1996) (coauth.: P. Hansen, B. Jaumard)
88 "A graph partition problem", Acta Mathematicae Applicatae Sinica 12 (1996) 393-400 (co-auth.: Y. Liu, A. Morgana)
89 "Political districting: traps, criteria, algorithms, and trade-offs", Atti Giornate di Lavoro AIRO, Perugia 1996, R. De Leone, F. Malucelli, S. Pallottino (eds.), 37-58 (co-auth.: F. Ricca)
90 "Equipartizione di cammini nella norma $\mathrm{L}_{\infty}$ " (extended abstract), Atti Giornate di Lavoro AIRO, Perugia 1996, R. De Leone, F. Malucelli, S. Pallottino (eds.), 235-237 (co-auth.: M. Liverani, A. Morgana, G. Storchi)
91 "Clustering on trees", Computational Statistics and Data Analysis 24 (1997) 217-234 (co-auth.: M. Maravalle, R. Naldini)
92 "Filling the missing names of towns in a map: a graph theoretic approach", The Journal of Combinatorial Mathematics and Combinatorial Computing, 25 (1997) 121-127 (co-auth.: F. Harary, A. Morgana)
93 "Political districting: traps, criteria, algorithms, and trade-offs", (expanded version of ref. 89) Ricerca Operativa XXVII (1997) 81-119 (co-auth.: F. Ricca)
94 "A linear algorithm for 2-bend embeddings of planar graphs in the 2-dimensional grid", Discrete Applied Mathematics 81 (1998) 69-91 (co-auth.: Y. Liu, A. Morgana)
"Consensus of hierarchical classifications", in: C. Hayashi, N.Ohsumi, K. Yajima, Y.Tanaka, H.-H. Bock, Y. Baba (eds.) Data Science, Classification, and Related Methods, Proc. IFCS 96, Kobe, (Springer, Tokyo, 1998) (co-auth.: M. Vichi)
"Max-min partitioning of grids into connected components", Networks 32 (1998) 115-225 (co-auth.: R. I. Becker, I. Lari, M. Lucertini)
97 "A linear programming heuristic for a hard clustering problem on trees", in: H.-H. Bock, A. Rizzi, M. Vichi (eds.) Data Science, Classification, and Related Methods, Proc. IFCS 98, Rome, (Springer, Heidelberg, 1998), (co-auth.: I. Lari, M. Maravalle) "How to make a polynomial movie from pseudo-polynomially many frames: continuous shifting algorithms for tree partitioning", in: E. Lodi, L. Pagli, N. Santoro eds.), Fun with Algorithms: Proceedings of the International Conference, Portoferraio, 1998, Carleton Scientific, Ottawa, 1999, 20-31 (co-auth.: R. I. Becker, Y. B. Chiang)
99 "On a 2-dimensional equipartition problem", European Journal of Operational Research, 113 (1999) 215-231 (co-auth.: F. Conti, F. Malucelli, S. Nicoloso)
100 "Sunday Opening Calendar for Shops in Rome through Random Utility, Matching, and Travelling Salesman Models", Ricerca Operativa XXVIII (1999) 3-25 (coauth.: A. Fiocco, D. Tersigni)
101 "Mathematics and Democracy: Evaluating and Optimising Electoral Systems", in: R: Ragaini (ed.) Proceedings International Seminar on Nuclear War and Planetary Emergencies, 24th Session, Erice, 1999 (World Scientific, Singapore, 2000), 14-25 (co-auth.: A. Pennisi, F. Ricca)
102 "Path equipartition in the Chebyshev norm", European Journal of Operational Research 123 (2000) 428-435 (co-auth.: M. Liverani, A. Morgana, G. Storchi)
103 "Combinatorial problems related to origin-destination matrices", Discrete Applied Mathematics 115 (2001) 15-36 (co-auth.: E. Boros, P. L. Hammer, F. Ricca)
104 "A polynomial time algorithm for max-min partitioning of ladders", Theory of Computing Systems 34 (2001) 353-374 (co-auth.: R. I. Becker, I. Lari, M. Lucertini)
105 "A satisfiability formulation of problems in level graphs", Electronic Notes in Discrete Mathematics, (Henry Kautz and Bart Selman, eds.) Elsevier, 9 (2001) (coauth.: E. Boros, O. Cepek, P.L. Hammer, A. Kogan, K. Makino, B. Randerath, E. Speckenmeyer)
106 "Continuous max-min tree partitioning", Theoretical Computer Science 282 (2002) 353-380 (co-auth.: R. I. Becker, Y. B. Chiang)
107 "Spazi ternari di Hamming e compressione algebrica efficiente dei dati", in: M. Cerasoli (ed.), Proc. Gian-Carlo Rota Memorial Conference, Barisciano, 2002, Dipartimento Sistemi e Istituzioni per l'Economia, Università de L'Aquila, 107-122 (co-auth.: E. Boros, P. L. Hammer, F. Ricca)
108 "Nuggets in Matching Theory", ibidem, 123-151. Reproduced in: AIRONews, XI, n. 2 (2006), 1-11

109 "On the maximum box problem", Computational Optimization and its Applications, 23 (2002) 285-298 (co-auth.: J. Eckstein, P.L. Hammer, Y. Liu, M. Nedjak)
110 "Polynomial algorithms for nested univariate clustering problems", Discrete Mathematics 245 (2002) 93-105 (co-auth.: P. Hansen, B. Jaumard)
111 "Cardinality constrained path covering problems in trees", Dipartimento di Statistica, Probabilità e Statistiche Applicate, Università "La Sapienza", Roma, Ser. A, n. 18-2002 (co-auth. N. Apollonio, L. Caccetta)
112 "Maximum weight path covering problems in graphs: formulation, relaxations, and algorithmic approaches", Technical Report, Curtin University of Technology, Perth 2002, (co-auth.: L. Caccetta)

113 "Special classes of path covering problems in graphs", Proc. Australian Optimization Day, Perth 2002 (co-auth. N. Apollonio, L. Caccetta).
114 "Preferenze, decisioni multicriterio e scelte collettive: tre risultati chiave", in: A. Agnetis, G. Di Pillo (eds.), Modelli e Algoritmi per l'Ottimizzazione di Sistemi Complessi, Atti della Scuola CIRO, Siena, 2002, Pitagora, Bologna, 2003, 473-511
115 "Maximum split clustering under connectivity constraints", Journal of Classification, 20 (2003) 143-180 (co-auth.: P. Hansen, B. Jaumard, C. Meyer, V. Doring)
116 "Pareto-optimal patterns in logical analysis of data", Discrete Applied Mathematics 144 (2004) 79-102 (co-auth.: P.L. Hammer, A. Kogan, S. Szedmák)
117 "Saturated systems of homogeneous boxes and the logical analysis of numerical data", presented at the 24th Annual Conference of the German Classification Society, Passau 2000, Discrete Applied Mathematics 144 (2004) 103-109 (co-auth.: P.L. Hammer, Y. Liu, S. Szedmák)

118 "Consensus algorithms for the generation of all maximal bicliques", Discrete Applied Mathematics 145 (2004) 11-21 (co-auth.: G. Alexe, S. Alexe, Y. Crama, S. Foldes, P.L. Hammer),
119 "Cardinality constrained path covering problems in grid graphs". Networks 44 (2004) 120-131 (co-auth.: N. Apollonio, L. Caccetta)

120 "Block linear majorants in quadratic 0-1 optimization", Discrete Applied Mathematics, 145 (2004) 52-71 (co-auth.: E. Boros, I. Lari)
121 "A Voronoi diagram approach to political districting" (Extended abstract), Workshop on "Analysis and Design of Electoral Systems", Oberwolfach, 2004 (co-auth.: I. Lari, F. Ricca)
122 "Legge elettorale con paradosso" and "E' proprio un paradosso", digital journal http://lavoce.info, November 11 and December 5, 2005 (co-auth.: A. Pennisi, F. Ricca)
123 "Matematica e Scienze Sociali per lo studio dei sistemi elettorali: un crocevia a senso unico?", digital journal http://matematica.uni-bocconi.it, December 1st, 2005 (co-auth.: A. Pennisi, F. Ricca)
124 "Malfunzionamenti dell'allocazione biproporzionale di seggi nella riforma elettorale italiana", Dipartimento di Statistica, Probabilità e Statistiche Applicate, Università "La Sapienza", Roma, Ser. A, n. 21-2005 (co-auth.: A. Pennisi, F. Ricca)
125 "Bachi e buchi della legge elettorale italiana nell'allocazione biproporzionale di seggi", Sociologia e Ricerca Sociale, 79 (2006) 55-76 (co-auth.: A.Pennisi, F. Ricca)
126 "Drawing Political Districts by Weighted Voronoi Regions and Local Search", (extended abstract), E. Yashin (ed.) Proc. VII International Conference on "Modernization of Economy and the State", Moscow, April 2006, 337-350 (coauth.: F. Ricca, A. Scozzari, I. Lari)
127 "The Sunfish against the Octopus: opposing compactness to gerrymandering", in: B. Simeone, F. Pukelsheim (eds.) Mathematics and Democracy. Recent Advances in Voting Systems and Collective Choice, Springer, Berlin, 2006, 19-41 (co-auth.: N. Apollonio, R.I. Becker, I. Lari, F. Ricca)
128 "Amending and enhancing electoral laws through mixed integer programming: the case of Italy", in: E. Yashin (ed.): Proc.VIII International Academic Conference on "Economic Modernization and Social Development", Moscow, April 2007, Higher School of Economics (co-auth. A. Pennisi, F. Ricca, P. Serafini)
129 "Weighted Voronoi Region Algorithms for Political Districting", in: D. Chaum, M. Kutylovski, R.L. Rivest, P. Ryan (eds.): Frontiers of Electronic Voting, Dagstuhl

Seminar Proceedings 07311, Dagstuhl, 2007 http://drops.dagstuhl.de/opus/frontdoor. php?source_opus=1302
130 "On the complexity of general matrix scaling and entropy minimization via the RAS algorithm", (co-auth. B. Kalantari, I. Lari, F. Ricca), Mathematical Programming, ser. A, 112 (2008) 371-401
131 "A pseudo-Boolean consensus approach to nonlinear 0-1 optimization", Discrete Applied Mathematics, 156 (2008), 2449-2458 (revised version of: CORR 79-38, Dept. of Combinatorics and Optimization, Univ. of Waterloo, Ontario, 1979)
132 "Polynomial algorithms for partitioning a tree into single-center subtrees to minimize flat service costs" Networks, 51 (2008), 78-89 (co-auth.: N. Apollonio, I. Lari, J. Puerto, F. Ricca)
133 "Local search algorithms for political districting", Operational Research, 189 (2008) 1409-1426 (co-auth.: F. Ricca)
134 "Weighted Voronoi Region Algorithms for Political Districting", Mathematical and Computer Modelling, 48 (2008) 1468-1477 (co-auth: F. Ricca, A. Scozzari)
135 "Ottimizzazione Combinatoria per la Progettazione di Sistemi Elettorali", in "Scienza delle decisioni in Italia: applicazioni della Ricerca Operativa ai problemi aziendali", G. Felici \& A. Sciomachen (eds.), ECIG, Genova, 2008, 75-87 (co-auth: A. Pennisi, F. Ricca, P. Serafini)

136 "Computing sharp bounds for hard clustering problems on trees", Discrete Applied Mathematics, 157 (2009) 991-1008, (co-auth.: I. Lari, M. Maravalle)
137 "Bicolored graph partitioning, or: gerrymandering at its worst, Discrete Applied Mathematics, 157 (2009) 3601-3614 (co-auth.: N. Apollonio, R. Becker, I. Lari, F. Ricca)
138 "Una legge elettorale sistematicamente erronea", Polena n. 2 (2009) 65-72 (coauth.: A. Pennisi, F. Ricca).
139 "On the Iterative Proportional Fitting Procedure: Structure of Accumulation Points and L1-Error Analysis", Technical report, University of Augsburg, n. 5-2009 (coauth.: F. Pukelsheim)
140 "Classification Techniques and Error Control in Logic Mining", in: DATA MINING: Annals of Information Systems, vol. 8 (2010) 99-119 (co-auth.: G. Felici, V. Spinelli)
141 "Boolean harmonies, and a tractable class of set covering problems", Dipartimento di Statistica Probabilità e Statistiche Applicate, Università degli Studi di Roma "La Sapienza", Serie A Ricerche n. 14-2010 (co-auth.: I. Lari)
142 "Political Districting: from classical models to recent approaches" (invited paper), 4OR A Quarterly Journal of Operations Research, 9 (2011) 223-254 (co-auth.: A. Scozzari, F. Ricca)
143 "The Give-up problem for blocked regional lists with multi-winners", Mathematical Social Sciences, 62 (2011) 14-24 (co-auth.: A. Scozzari, F. Ricca)
144 "Network flow methods for electoral systems", Networks, 59 (2012) 73-88 (coauth.: F. Pukelsheim, F. Ricca, A. Scozzari, P. Serafini)
145 "Error minimization methods in biproportional apportionment", TOP, The Official Journal of the Spanish Society of Statistics and Operations Research, 20 (2012) 547-577 (co-auth.: F. Ricca, A. Scozzari, P. Serafini)
146 "Parametric maximum flow methods for minimax approximation of target quotas in biproportional apportionment", Networks, 59 (2012) 191-208 (co-auth.: P. Serafini)
147 "Certificates of optimality: the third way to biproportional apportionment", Social Choice and Welfare, 38 (2012) 247-268 (co-auth.: P. Serafini)

148 "The Maximum Vertex Coverage Problem on Bipartite Graphs", Discrete Applied Mathematics special Issue CTW 2011, http://dx.doi.org/10.1016/j.dam.2013.05.015 (co-auth.: N. Apollonio)
149 "A Boolean theory of signatures for tonal scales", Discrete Applied Mathematics special Issue CTW 2011, http://dx.doi.org/10.1016/j.dam.2013.10.024 (co-auth.: G. Nouno, M. Mezzadri, I. Lari)

### 3.3 Submitted papers

Incompatibility Graphs in Data Mining, submitted to Journal of Graph Theory (co-auth: E. Boros, F. Ricca, V. Spinelli)

Carpal Tunnel Syndrome Automatic Classification: Electromyography vs. Ultrasound imaging, submitted to TOP, The Official Journal of the Spanish Society of Statistics and Operations Research (co-auth: M. Maravalle, F. Ricca, V. Spinelli)
Improved Approximation of Maximum Vertex Coverage Problem on Bipartite Graphs, submitted to SIAM Journal on Discrete Mathematics (co-auth.: N. Apollonio)


[^0]:    I. Lari ( $\triangle$ ) • F. Ricca

    Dipartimento di Scienze Statistiche, Sapienza, Università di Roma, Rome, Italy
    e-mail: isabella.lari@uniroma1.it
    F. Ricca
    e-mail: federica.ricca@uniroma1.it
    F. Pukelsheim

    Institut für Mathematik, Universität Augsburg, Augsburg, Germany
    e-mail: pukelsheim@math.uni-augsburg.de

