List Apparentements in Local Elections: A Lottery

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1 List Apparentements

List apparentements form a peculiarity of certain proportional representation systems. In some countries they are employed at the national level, as in Switzerland and Israel. In Germany they are restricted to the local level. Here we elucidate their role in a case study, the 2008 local elections in the German State of Bavaria. Bochsler (2009) presents a more general overview of the subject.

Political parties, or groups of citizens who submit a list of candidates, may register a *list apparentement*[1] with the electoral bureau prior to the election. On Election Day, the conversion of votes into seats then takes place in two stages. Firstly, in the *super-apportionment*, the votes cast for the partners of the apparentement are totaled, and this total enters as a single count into the calculation to apportion all available seats.

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[1] The French term "apparentement" is also used in English, see Gallagher and Mitchell (2005), p. 631.

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Table 1 List apparentements in the local electionsof the State of Bavaria, 2 March 2008

Number of lists	3	4	5	6	7	8	9	10	11	12	13	Sum
One apparentement	72	141	116	64	40	17	5	1				456
Two apparentements		13	45	54	30	27	13	6	2	1		191
Three apparentements				3	4	6	3	4			1	21
Number of communities	72	154	161	121	74	50	21	11	2	1	1	668

Secondly, in the *sub-apportionment*, every list apparentement undergoes a follow-up calculation. Here the seats that the apparentement earned as a whole are apportioned among its partners, proportionally to the vote count for each partner list.

Apparentements do not commit the partner lists during the upcoming legislative period, neither to strive for common goals, nor to enter into a formal coalition. Any party may team up with any other party. There is an affinity of conservative parties to go along with other conservative parties, of course, and liberal groups with other liberal groups. Yet, in our Bavarian case study, we could not identify a definite pattern of who joins which apparentement. Everything is possible, and almost everything is realized.

In the 2008 local elections in Bavaria, just one list apparentement was registered in 456 communities,[2] with the number of campaigning lists running from 3 through 10. Two apparentements emerged in 191 communities, three in 21. Altogether the election featured 901 list apparentements,[3] in 668 out of 2,127 communities. See Table 1.

List apparentements must not be taken as an oath of disclosure towards voters, as is apparent on the ballot paper. Partner lists are not marked in a way that every voter instantly recognizes the affiliation of a party to an apparentement. But seek, and ye shall find. On Bavarian ballot sheets it is the small print, down in the bottom line.

The partners of a list apparentement join companionship only for the day of reckoning. As soon as the electoral results are publicized, the composition of the apparentement disappears from the statistical tables, as if documenting them would constitute an embarrassment to those concerned. What, then, makes list apparentements attractive?

List apparentements are beset with the mystic aura that they even out detrimental disparities of the electoral system. We shall show that such speculations are sometimes right, and sometimes wrong. Moreover, the 2008 Bavarian elections featured thirty-six instances where list apparentements grotesquely reversed the popular vote, in that of two lists the weaker list won more seats.

[2] We use the term *community* as a generic synonym for political entities where voters elect a local council, such as cities, counties, townships, villages, and the like, as in Pukelsheim et al. (2009).

[3] We get $456 \times 1 + 191 \times 2 + 21 \times 3 = 901$.

2 Seat Biases

The element of the electoral system that is notorious for its built-in disparity is the formula for the conversion of votes into seats that comes under the names of D′Hondt, Hagenbach-Bischoff, or Jefferson. We prefer to call it the *divisor method with rounding down (D′Hondt)*, in order to indicate how it works. Any vote count is divided by a common divisor, the *electoral key*, and the resulting quotient is rounded down to its integer part to obtain the seat allocation. The value of the electoral key ascertains that all available seats are handed out (Pukelsheim 2002).

The divisor method with rounding down (D′Hondt) is notorious for its seat biases in favor of larger parties and at the expense of smaller parties. On average, larger parties are allocated more seats than strict proportionality would grant them, and these seats are taken away from smaller parties. There are unbiased alternatives which are increasingly taking over, especially in Germany (Pukelsheim 2003). Among them are the quota method with residual fit by largest remainders (Hamilton/Hare) and, as of recently, the divisor method with standard rounding (Webster/Sainte-Laguë).

Historically, the coupling of D′Hondt with list apparentements is the rule and, in German States, prevails in Bavaria and the Saarland. List apparentements are removed from the law as soon as an unbiased electoral formula is implemented provided the law-makers understand their electoral system, as in the Swiss Cantons of Zürich, Schaffhausen, and Aargau (Pukelsheim and Schuhmacher 2004). Otherwise they remain in the law as a relict of times passed (Rhineland-Palatia). And occasionally an electoral law with old ballast is recycled to give democracy a new start (Thuringia).

The notion of *seat bias* designates the mean deviation of the seats practically apportioned, from the ideal share of seats granted by theoretical proportionality. The mean is evaluated uniformly across all conceivable vote outcomes. Surplus and deficit materialize per each election, and stay practically constant over all council sizes. Seat bias formulas for the divisor method with rounding down (D′Hondt) are listed in Table 2.[4]

Without list apparentements, seat biases exhibit a clear trend. The decrease from profits to losses follows the final vote count ranking. The upper third of stronger lists (in terms of votes received) is granted a surplus of seats. But one man's meat is another man's poison. The lower two thirds of weaker lists have to endure a seat deficit.

With list apparentements the seat biases do remain calculable. However, the clear order from top to bottom is lost, and a bewildering diversity of results comes to light. The bewilderment is caused by the double application of the divisor method with rounding down (D′Hondt), thus reinforcing its built-in seat biases. Whether a party wins or loses seats turns into a lottery.

[4] Without list apparentements the formulas are derived in Schuster et al. (2003). With list apparentements the formulas are new and due to Leutgäb (2008).

Table 2 Formulas for the D′Hondt seat biases

Without any list apparentement, the D′Hondt seat bias of the *j*-strongest (in terms of votes received) list is

$\mathrm{D'H(j)} = \frac{1}{2}(\ell\, s(j) - 1)$, where $s(j) = \frac{1}{\ell}\left(\frac{1}{j} + \cdots + \frac{1}{\ell}\right)$.

Here $s(j)$ is the expected vote share of the j -strongest of ℓ lists.

With ℓ lists partitioned into the apparentements $L_1, \ldots, L_k$, the D′Hondt seat bias of the *j*-strongest list becomes

$\mathrm{D'H}\,(j \mid L_1, \ldots, L_k) = \frac{1}{2}\left(k\, s(j) + (p-1)\,\frac{s(j)}{s(V)} - 1\right)$, where $s(V) = \sum_{i \in V} s(i)$.

Here V is the apparentement in which the *j*-strongest list figures as one of p partners, and $s(V)$ is its expected seat share.

Example: City of Friedberg, 2008. Of six lists, the second-, third- and fifth-strongest lists joined in an apparentement (case B).

	A: 1,2,3,4,5,6	B: 2 + 3+5,1,4,6
List 1	0.725	0.317
List 2	0.225	0.507
List 3	−0.025	0.160
List 4	−0.192	−0.295
List 5	−0.317	−0.245
List 6	−0.416	−0.444

Without list apparentements (case A), the strongest List 1 may expect an advantage of about 3 seats in 4 elections (3/4 $\approx$ 0.725). However, Lists 2, 3 and 5 formed an apparentement while Lists 1, 4 and 6 stood for themselves (case B). In this constellation, the largest bonus (0.507) goes to List 2. The total bias increases from 0.950 (= 0.725 + 0.225) in case A, to 0.984 (= 0.317 + 0.507 + 0.160) in case B.

The City of Friedberg (AGS[5] 09771130) provides an instructive example. Six lists campaigned which we retrospectively number from 1 to 6 according to their popular support. That is, List 1 finished strongest and won a larger popular vote than List 2. List 2 entered into an apparentement with Lists 3 and 5, while the others stood alone. The apparentement ranked top in the super-apportionment, where it won a rank-1-bonus. In the sub-apportionment the bonus was passed on to List 2 which was strongest among the partners of the apparentement. The arrangement thus secured a top rank for List 2 twice, in the super-apportionment and in the sub-apportionment. In the end, the weaker List 2 won more seats than the stronger List 1. The Bavarian electoral law circumnavigates the popular vote, by way of list apparentements.

[5] AGS = Amtlicher Gemeindeschlüssel = official community key. The key defines a standard order for German communities. It may also be used to retrieve some basic statistical information about the community via www.destatis.de/gv/.

Table 3 D′Hondt seat biases for three-party systems, empirical and theoretical values, Bavarian local elections 2008

List partitions	A: 1, 2, 3		B: 1, 2 + 3		C: 1 + 2, 3		D: 1 + 3, 2	
	Empir.	Theor.	Empir.	Theor.	Empir.	Theor.	Empir.	Theor.
Strongest list	0.218	0.416	0.000	0.111	0.333	0.455	0.266	0.534
Median list	−0.019	−0.083	0.137	0.135	0.000	−0.066	−0.133	−0.222
Weakest list	−0.019	−0.333	−0.137	−0.246	−0.333	−0.389	−0.133	−0.312
Total bias	0.218	0.416	0.137	0.246	0.333	0.455	0.266	0.534

3 Three-Party Systems

The analysis remains somewhat more transparent in three-party systems, the simplest constellation where list apparentements come into play. With only a single list the election turns into a simple majority vote. When there are two lists (2008 in Bavaria in about four hundred communities), they are lacking a third against whom it would pay to join into an apparentement.

Although three-party systems represent the simplest case, it is sufficient to indicate potential complications since there exist four ways of partitioning the lists. In case A (1, 2, 3) all lists stand alone. In the cases B, C und D a two-partner list apparentement is formed. In partition B (1, 2 + 3) the two weaker lists join in an apparentement, in C (1 + 2, 3) the two stronger lists. This leaves case D (1 + 3, 2), where the strongest and the weakest list unite against the median list.

In the 2008 Bavarian local elections there were 585 communities where just three lists campaigned. Of these, 513 fell into the apparentement-free category A, while fifty-one communities featured partition B (1, 2 + 3), six C (1 + 2, 3), and fifteen D (1 + 3, 2).

Table 3 shows the seat biases incurred by the partitions A–D when the divisor method with rounding down (D′Hondt) is used. The empirical values are the averages, among the communities where in 2008 the partition occurred, of the D′Hondt apportionment from the (unbiased) allocation of the divisor method with standard rounding (Webster/Sainte-Laguë). Most often the latter yields the same seat allocation as does the quota method with residual fit by largest remainders (Hamilton/Hare).

The theoretical values are the means calculated using the formulas in Table 2. Empirical and theoretical values conform quite satisfactorily. The total bias (=sum of all positive seat biases) is dampened in case B, as compared to the apparentement-free case A, enlarged in case C, and maximized in case D.

Practicalities defy theoretical predictions. In the 2008 Bavarian local elections it happened not once, but several times that the strongest list secured a double bonus by teaming up with weaker parties.

4 Large Parties Uniting with Small Parties

Table 4 presents an example of a double bonus, in Unterallgäu County. The divisor method with standard rounding (Webster/Sainte-Laguë) allocates about one seat per each 51,900 votes. The strongest list, with quotient 1,377,975/51,900 = 26.55, is allocated 27 seats (Column A). Even with no apparentement, the divisor method with rounding down (D′Hondt) gives an advantage by awarding it 28 seats since, with electoral key 48,000, the quotient 1,377,975/48,000 = 28.7 is rounded down (Column B).

However, two list apparentements had been registered. The strongest List 1 united with the fifth-strongest List 5, and the forth- and sixth-strongest lists joint together. Table 4 shows what happened. Without list apparentements, List 1 and 5 would have gained 28 + 3 = 31 D′Hondt seats. With list apparentements, they won 32 seats (Column C1). The sub-apportionment assigns the second bonus seat to the stronger of the two partners, List 1 (Column C2). In the end List 1 is apportioned 29 seats, rather than its unbiased share of 27 seats (Column D).

5 Lottery Effects

Formation of list apparentements turns into a lottery for the reason that there is a plethora of ways as to how a set of lists may be partitioned into different apparentements. The six lists in Table 2 admit 201 apparentements; for the seven lists of Table 4 the count[6] grows to 875. The information for voters that "some lists form an apparentement" is much too vague to be of any value. The abundance of possible list apparentements makes it impossible to intuitively assess their consequences.

A first rule applies to list apparentements just as it applies to any other game: *Nothing ventured, nothing gained.* Lists who prefer to maintain their independence and do not join an apparentement must, on average, endure a seat deficit so that their competitors may be served with a seat surplus.[7]

The second rule is a counterpart of the first: *If there is just one list apparentement, its partners are guaranteed to be on the winner's side*. On the average the partners of a sole apparentement receive a seat surplus as compared to the apparentement-free D′Hondt apportionment.[8] In 2008 two thirds of the Bavarian

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[6] Our counts neglect the borderline cases (1) "everyone stands alone" (1, 2,…, $\ell-1$, ℓ) and there is no sub-apportionment, and (2) "all join together" (1 + 2+… + "$\ell-1$" + ℓ) and there is no super-apportionment.

[7] The formulas from Table 2 yield $\mathrm{D'H}(j\,|\,L_1,\ldots,L_k) - \mathrm{D''H}(j) = -(\ell-k)\;s(j)\,/\,2<0$, assuming that List j remains alone while other lists enter into an apparentement of two or more partners ($k<\ell$).

[8] The formulas give $\mathrm{D'H}(j\,|\,V;\ \{i\},\ i\notin V) - \mathrm{D''H}(j) = \ (1-s(V))\,(p-1)\,s(j)\,/\,(2\,s(V)) > 0$, assuming List j is one of p partners of the (sole) list apparentement V, the other $\ell-p$ lists running by themselves.

Table 4 Double bonus for the strongest list, Unterallgäu County 2008

List (apparentement)	Votes	(A) S-L = H/H w/o a.	(B) D′H w/o a.	(C1) D′H with a.	(C2) D′H 1 + 5	(C3) D′H 4 + 6	(D) Final seats
List 1	1,377,975	27	28		29		29
List 2	730,846	14	15	14			14
List 3	337,937	7	7	6			6
List 4	189,648	4	3			4	4
List 5	181,235	3	3		3		3
List 6	163,465	3	3			3	3
List 7	85,511	2	1	1			1
Apparentement 1 + 5	(1,559,210)			32			
Apparentement 4 + 6	(353,113)			7			
Sum	3,066,617	60	60	60	32	7	60
Divisor		51,900	48,000	48,724	47,000	44,000	

communities (456 of 668, see Table 1) featured just one list apparentement. Its partners could look forward to a bonus simply because their competitors were napping.

In 212 communities, however, two or more list apparentements were registered. These are the instances when the elections turn into a lottery. Surpluses and deficits constitute a zero-sum game. It is plainly impossible that each and every protagonist finishes up with a bonus. But who is advantaged, and who is disadvantaged, is predictable only after extensive calculations, and in practice turns into mere luck.

It is not even recognizable what happens to the total bias of the system. It is a wide-spread belief that list apparentements *always* dampen the total bias. This belief is erroneous, as has already been seen in Table 3. Moreover, of the 201 apparentements into which six lists may be partitioned, 73 were realized in the 2008 Bavarian local elections. Of these, barely 44 diminished the total bias. With the other 29 partitions—that is, in more than a third of all cases—the total bias became larger, not smaller.

Here is a seemingly balanced example worth mulling over, from the previous Bavarian local elections in 2002. In Bad Füssing (AGS 09275116) nine lists campaigned, and formed three apparentements of three partners each, namely $1 + 3 + 5$, $2 + 4 + 7$ und $6 + 8 + 9$. Again lists are numbered according to their ranking by votes received. Who paid the bill? Who made the best cut? In case the gentle reader would like to ponder the example, we masquerade the answers as reference Xyz (2002).

6 Discordant Seat Assignments

We consider it a system defect when the popular vote is turned upside down, and fewer votes finish up with more seats. We call a setting in which of two lists that one with fewer votes gets more seats, a *discordant seat assignment*, or simply a discordance.

Table 5 further elaborates on the Friedberg example of Table 2, illustrating how discordances evolve. The second-strongest list ranks by more than five-thousand votes behind the winning list. Yet List 2 wins 13 seats, while List 1 acquires only 12. The theoretical formulas in Table 2 already foreshadowed this mishap.

Table 6 assembles all thirty-six discordances which emerged during the 2008 Bavarian local elections.[9] The Friedberg example is not a singular exception. In seven instances the second-strongest list leapt to the top as far as seats are concerned; while the strongest list dropped down to rank two. In Eurasburg (AGS

[9] Vote counts reflect council sizes, as every voter has as many votes as there are council seats to fill.

Table 5 Discordance victory of the second-strongest list, City of Friedberg 2008

List (apparentement)	Votes	(A) S-L w/o a.	(B) D′H w/o a.	(C) H/H w/o a.	(D1) D′H with a.	(D2) D′H 2 + 3+5	(E) Final seats
List 1	150,615	12	12	12	12		12
List 2	145,292	12	12	11		13	13
List 3	30,558	2	2	2		2	2
List 4	28,428	2	2	2	2		2
List 5	18,291	1	1	2		1	1
List 6	12,010	1	1	1	0		0
Apparentement 2 + 3+5	(194,141)				16		
Sum	385,194	30	30	30	30	16	30
QuotalDivisor		1,2000	1,2400	12,839.8	1,2100	1,1000	

Table 6 Discordant seat assignments, Bavaria 2008

AGS	Community|Council	Size	D'Hondt discordances	List partitionings
09173123	Eurasburg	16	(1)6206 − 3:(2)6172 − 4	2 + 6 + 7, 3 + 4, 1, 5, 8
09175122	Grafing	24	(3)20229 − 3:(4)19282 − 4	1, 2 + 4, 3, 5
09179121	Fürstenfeldbruck	40	(4)43625 − 3:(5)42345 − 4	2 + 3 + 5 + 6, 1, 4
09179145	Puchheim	24	(4)19148 − 2:(5)19146 − 3	1, 2, 3 + 5, 4
09180125	Oberammergau	20	(7)4498 − 1:(8)4321 − 2	1 + 3 + 7, 4 + 5 + 6 + 8 + 9, 2
09180129	Saulgrub	12	(1)5267 − 5:(2)5137 − 6	2 + 3, 1
09186	Pfaffenhofen County	60	(5)169961 − 3:(6)169250 − 4	1, 2, 3, 6 + 7, 4, 5
09186122	Geisenfeld	20	(5)4382 − 0:(6)4032 − 1	1 + 4, 2 + 6, 3, 5
09272118	Freyung	20	(3)6291 − 1:(5)5950 − 2 (4)5953 − 1:(5)5950 − 2	1 + 7, 2, 5 + 6, 3, 4
09273	Kelheim County	60	(5)162316 − 3:(6)152563 − 4	1 + 4 + 5, 2, 3, 6 + 7 + 8
09277111	Arnstorf	20	(2)6300 − 2:(3)5692 − 3	3 + 4 + 5 + 6 + 8, 1 + 7 + 9, 2
09279128	Moosthenning	16	(2)5574 − 2:(3)5201 − 3	3 + 5 + 7, 1, 2, 4, 8 + 9, 6
09376163	Schwarzenfeld	20	(2)8023 − 2:(3)7678−3	1 + 5, 3 + 4 + 6 + 7, 2
09376169	Stulln	12	(2)4642 − 2:(3)3962 − 3	1, 3 + 4, 2
09472143	Goldkronach	16	(3)4866 − 2:(4)4577 − 3	4 + 5 + 6, 1 + 8, 2, 3, 7
09472167	Mistelgau	16	(1)6922 − 3:(2)6016 − 4	2 + 3 + 4 + 5 + 6 + 7 + 8, 1
09472197	Waischenfeld	16	(4)2723 − 1:(5)2394 − 2	1, 5 + 6, 3 + 8, 2, 4, 7, 9
09474121	Ebermannstadt	20	(4)9525 − 2:(5)8022 − 3	1 + 7, 3 + 5 + 6, 2, 4
09474123	Eggolsheim	20	(2)8468 − 2:(3)8289 − 3	1 + 6, 3 + 4, 5 + 7 + 9 + 10, 2, 8
09474129	Gößweinstein	16	(2)7611 − 3:(3)7529 − 4	1 + 5, 3 + 4, 2
09572111	Adelsdorf	20	(2)21082 − 5:(3)20852 − 6	1 + 4, 3 + 5, 2
09673172	Sulzdorf	12	(4)2263 − 1:(5)1984 − 2	2 + 3 + 5, 1, 4, 6 + 7, 8
09673173	Sulzfeld	12	(4)3322 − 1:(5)3314 − 2	1, 3 + 5, 2, 4
09678170	Röthlein	16	(3)5885 − 2:(4)5704 − 3	1 + 6, 4 + 5, 2, 3, 7
09678193	Werneck	24	(3)13129 − 2:(4)11690 − 3	4 + 7 + 8 + 10 + 11, 2 + 6 + 9, 3 + 12 + 13, 1, 5

(continued)

Table 6 (continued)

AGS	Community|Council	Size	D′Hondt discordances	List partitionings
09771130	Friedberg	30	(1)150615 − 12:(2)145292 − 13	2 + 3 + 5, 1, 4, 6
09772147	Gersthofen	30	(2)42811 − 5:(3)41234 − 6	1, 3 + 4 + 5 + 6, 2
09772177	Meitingen	24	(2)18634 − 3:(3)17483 − 4	1 + 6, 3 + 4 + 5 + 7, 2
09772178	Mickhausen	12	(1)2416 − 4:(2)2288 − 5	2 + 3, 1
09773117	Bissingen	16	(4)3577 − 1:(5)3346 − 2	1 + 3 + 5 + 7 + 10, 2 + 6 + 8 + 9, 4
09774135	Günzburg	24	(1)47078 − 7:(2)44800 − 8	1 + 4 + 5, 2 + 3 + 6
09774171	Offingen	16	(1)10916 − 5:(2)10540 − 6	2 + 5, 1, 3, 4
09777129	Füssen	24	(2)24004 − 4:(3)22502 − 5	3 + 4 + 6 + 7 + 9 + 10, 1 + 5, 2 + 8
09779147	Fremdingen	14	(3)6748 − 2:(4)6597 − 3	2 + 4 + 5, 1, 3
09780117	Buchenberg	16	(3)3041 − 1:(4)2642 − 2	1, 2, 4 + 5, 3

09173123) the strongest list (1) with 6,206 votes got 3 seats, while the second-strongest list (2) fell back in votes (6,172), but jumped ahead in seats (4).

The partitioning of the apparentements are exhibited in the right-most column of Table 6, demonstrating the abundance of possibilities of who may go together with whomever else. Eurasburg featured two apparentements. The second-, sixth-, and seventh-strongest lists united (2 + 6 + 7), and finished first in terms of votes. The third- and fourth-strongest lists (3 + 4) came in second. The others stood alone (1, 5, 8).

Since list apparentements entail repeated apportionment calculations with multiple steps of rounding, *every* electoral formula is prone to discordant seat assignments. In particular, neither the divisor method with standard rounding (Webster/Sainte-Laguë) nor the quota method with residual fit by largest remainders (Hamilton/Hare) are immune to discordances. However, due to its notorious seat biases the D′Hondt method gives rise to discordances about twice as often as compared to its unbiased competitors. While D′Hondt systematically favors the stronger partners within an apparentement, the unbiased methods behave unpredictably and, when producing discordances, may favor lists within the apparentements, or lists that stand alone.

7 Constitutional Principles

May local elections turn into a lottery? Article 28 of the *Grundgesetz,* the German constitution, defines the standard. Elections in Germany must be universal and direct, as well as free, equal, and secret. The principle of electoral equality acquires a double meaning, *Chancengleichheit der Parteien* (equal chances for parties) aiming at parties and candidates, and *Erfolgswertgleichheit der Stimmen* (equal success values of votes) honoring the role of voters.

The lottery character of list apparentements certainly honors the equality principle as far as equal chances for parties are concerned. Officials of all parties have an equal opportunity to place their stakes in the game. If some players miss their turn, as in Friedberg Lists 1, 4 and 6, such negligence does not render the law unconstitutional.

We believe that constitutionality of list apparentements is much more problematic when considered from the voters' point of view. It is questionable whether the election can rightly claim to be direct. After all, two apportionment calculations are called for, and this detour hardly qualifies as a direct route from votes to seats.

Furthermore we find it more than unclear whether votes can be considered free. From the voters' viewpoint it is unknown third parties who interfere and decide whether the votes first undergo a preliminary evaluation via list apparentements, or not.

And what about electoral equality? If the constitution requires all votes to achieve an equal success value, how does it happen that fewer votes can lead to

more seats? In order to justify such a contradiction, a sophistic vindication is called for that we are unable to offer with our modest talents as statisticians.[10]

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Xyz (2002). List 1 paid a major portion of the bill, its bonus shrank by a third of a seat ($0.548-0.914=-0.366$). List 6 benefitted most, gaining close to half a seat ($0.202-(-0.227)=0.429$). In the 2002 election, List 6 won two seats, and thus got ahead of List 5 who had to resign themselves to just one seat.

[10] The German Federal Constitutional Court shares a critical stand on list apparentements: *Every list apparentement* [*leads*] *to a violation of the principle of electoral equality, since votes are assigned unequal weights without justifying the deviation from equality by a forceful, substantive argument,* see BVerfGE 82 (1991) 322–352 [345]. The decision concerned the by-passing of the five-percent hurdle in the first all-German elections in 1990, not the role of list apparentements in local elections.