

The Energy-Energy Correlation Function of the Random Bond Ising Model in Two Dimensions

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Abstract

The energy-energy correlation function of the two-dimensional Ising model with weakly fluctuating random bonds is evaluated in the large scale limit. Two correlation lengths exist in contrast to one correlation length in the pure 2D Ising model: one is finite whereas the other is divergent at the critical points. The corresponding exponent of the divergent correlation length is $\nu_e = 1/2$ in contrast to the pure system where $\nu_e = 1$. The calculation is based on a previously developed effective field theory for the energy density fluctuations.

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The two-dimensional Ising model [1] with random bonds is an interesting example for the competition of thermal fluctuations and quenched disorder. According to the Harris criterion [2] neither of these two effects dominates over the other near the critical point. This picture is supported by a number of investigations for weak disorder, treated in perturbation theory with respect to the variance of the bond distribution, which came to the conclusion that the thermodynamic properties are not dramatically changed by quenched disorder [3–6]. This is a consequence of the fact that the fixed point of the pure Ising model is stable against disorder (at least in one-loop order). The effect of disorder on the power law of the spin-spin correlation $\langle S_R S_0 \rangle \sim R^{-1/4}$ of the pure system is controversial. While some authors claim there is no effect on the exponent $1/4$ [4–6] others find a change for the correlation function to $\exp\{-const.[\log(\log R)]^2\}$ [3]. On a naive basis one expects a reduction of the correlation near the critical point due to the additional fluctuations of the random bonds. Numerical simulations [7,8] do not indicate a significant change of the correlation function.

Apart from the perturbative approaches it was found more recently in a non-perturbative treatment that disorder modifies the phase diagram of the two-dimensional Ising model [9,10]. In particular, disorder creates a new phase between the ferro- and the paramagnetic phase. However, the width of this phase is small and vanishes for any finite order in perturbation theory. For the same reason it is rather unlikely that this phase can be observed in a numerical simulation. Nevertheless, there is a rigorous proof for its existence [11]. This phase is characterized by a new order parameter. It was not possible in previous studies to understand its role in terms of the thermodynamic functions. However, it was related to the inverse correlation length of the average spin-spin correlation function by Braak [12]. In the following it will be shown that it also plays the role of an inverse correlation length of the average energy-energy correlation function.

The Ising model describes a system of discrete spins $S_r = \pm 1$ on a lattice. In this article a square lattice will be considered. The spins on nearest neighbor sites of the lattice are coupled with the energy

$$\mathcal{H} = - \sum_{r,r'} J_{r,r'} S_r S_{r'} \quad (1)$$

where $J_{r,r'}$ is a random coupling variable. A statistical system of spins at the inverse temperature β is defined by the partition function

$$Z = \sum_{\{S_r\}} \exp(-\beta\mathcal{H}). \quad (2)$$

Using this quantity the energy-energy correlation function (EECF) reads

$$\langle \mathcal{E}_r \mathcal{E}_{r'} \rangle = \frac{1}{\beta^2} \sum_{r'',r'''} J_{r,r''} J_{r',r'''} \frac{\partial}{\partial J_{r,r''}} \frac{\partial}{\partial J_{r',r'''}} \log Z. \quad (3)$$

In order to calculate Z a number of mappings and approximations is useful. Firstly, the statistics of Ising spins can be mapped onto the statistics of closed polygons which excludes multiple occupation of links between lattice sites. This leads to $Z = Z_0 Z_P$ where Z_0 is a spin-independent part and Z_P the partition function of the closed polygons. Secondly, Z_P is equivalent with the partition function of non-interacting fermions with four colors. As a consequence, Z_P^2 is as fermion determinant. An approximation for weak disorder and large scales reads as a fermion determinant for complex fermions with two colors $Z_P^2 \approx \det(H_0 + \delta y \sigma_3)$. The Fourier components of the lattice matrix H_0 are

$$\tilde{H}_0 = m \sigma_3 + \bar{y}_c (\sigma_2 \sin k_1 + \sigma_1 \sin k_2). \quad (4)$$

δy_r represents the random fluctuations of the couplings with variance g and $m = y - y_c$, $y = \tanh(\beta J)$, $y_c = \sqrt{2} - 1$. σ_j are the Pauli matrices. Introducing the Green's function $G(\varepsilon) = (H_0 + \delta y \sigma_3 + i\varepsilon \sigma_0)^{-1}$ the EECF reads in this approximation

$$\langle \mathcal{E}_r \mathcal{E}_{r'} \rangle \approx \lim_{\varepsilon \rightarrow 0} \langle (\bar{y} + \delta y_r) (\bar{y} + \delta y_{r'}) \text{Tr}[G_{r,r'} G_{r',r}] \rangle \quad (5)$$

with a positive constant \bar{y} . Naively, one would neglect in a weak disorder approximation the δy contributions in front of the G 's. However, we will see in the following that these terms are important. This is the first step for the derivation of the EECF in the disordered Ising system.

As given in (3) the fluctuations of the random coupling $J_{r,r'}$ are conjugate to the energy density \mathcal{E}_r . Therefore, it is convenient to transform the distribution of the fluctuations δy into a distribution of the energy fluctuations. The latter was performed in Ref. [9] by mapping the random bond fluctuations to a random matrix

$$\delta y_r \sigma_3 \rightarrow Q_r. \quad (6)$$

Q_r is a Hermitean 2×2 matrix with random matrix elements. It describes the energy density fluctuations. For instance, the average energy density is proportional to the average trace of Q . Without discussing the structure of this random matrix model (the details can be found in [9]) we turn directly to the saddle point approximation of the model. It is sufficient to consider the upper left 2×2 block matrix $Q_r = Q_0 + \delta Q_r$ where Q_0 is given by a saddle point approximation [9] with

$$Q_0 = -\frac{i}{2}\eta\sigma_3 - \frac{1}{2}m_s\sigma_0. \quad (7)$$

η and m_s are solutions of the saddle point equations

$$\eta = \eta g I \quad (8)$$

$$\bar{m} = m + m_s = \frac{m}{1 + gI} \quad (9)$$

with the integral $I = 2 \int [\bar{m}^2 + \eta^2 + |\kappa|^2]^{-1} d^2k / (2\pi)^2$. The solution, which describes the new intermediate phase, is given by $\eta \neq 0$ as

$$\eta^2 = e^{-2\pi\bar{y}_c^2/g} - m^2/4 \quad (10)$$

and $\bar{m} = m/2$. η vanishes at the critical points $m_c = \pm 2e^{-\pi\bar{y}_c^2/g}$ and is zero for $m^2 > m_c^2$.

The EECF can be expressed by the Gaussian fluctuations around the saddle point solution Q_0 as

$$\langle \mathcal{E}_r \mathcal{E}_{r'} \rangle \approx \frac{1}{\mathcal{N}} \int [\bar{y} Tr(Q_r) + Tr(Q_r^2)] [\bar{y} Tr(Q_{r'}) + Tr(Q_{r'}^2)] \exp[-(\delta Q, \hat{I} \delta Q)] \prod d\delta Q \quad (11)$$

with normalization \mathcal{N} . The first order terms in Q are the contribution from $\bar{y}G$ whereas the quadratic terms $Tr(Q^2)$ are a result of δyG in (5). Higher order terms in the exponential function are neglected in this approximation. Taking them into account (e.g. third order terms in Q) could lead to a renormalization of \hat{I} . On the other hand, the saddle point approximation corresponds with the $N \rightarrow \infty$ limit of a more general model [10]. Fluctuations appear as corrections with $1/N^{-1/2}$ for each δQ . Therefore, expression (11) should be a good approximation in terms of the $1/N$ -expansion.

The eigenvalues $\lambda_1, \dots, \lambda_3$ of the 4×4 matrix $\hat{I}(k)$ are positive whereas λ_4 vanishes like η^2 if one approach the critical points m_c :

$$\lambda_4 \sim const. |m + m_c| + bk^2 + o(k^3). \quad (12)$$

The mode of λ_4 is $\delta Q_{11} - \delta Q_{22}$. Therefore, the terms with $Tr(Q) = m_s + \delta Q_{11} + \delta Q_{22}$ are *not* critical in contrast to terms with $Tr(Q^2)$. This fact explains why the δy contributions are important in the expression of the EECF in (5). The mode $\delta Q_{11} + \delta Q_{22}$, which corresponds to λ_3 , has a characteristic finite correlation length $\xi_3 \sim const. \lambda_3^{-1/2}$. Furthermore, there is a divergent correlation length from the correlation of $Tr(Q_r^2)$ which is proportional to $|\eta|^{-1}$, the order parameter of the intermediate phase. It reads in terms of m $\xi_4 \sim \xi_0 |m + m_c|^{-1/2}$.

Thus, the EECF is characterized by two different length scales. One is the *finite* correlation length ξ_3 , the other is the *divergent* correlation length ξ_4 which is related to the order parameter $|\eta|^{-1}$ of the new phase. In contrast to this, the pure system has only one correlation length which diverges at the critical point $m = 0$ as $\xi_{pure} \sim |m|^{-1}$.

In conclusion, quenched disorder reduces significantly the correlation length of the energy fluctuations in the two-dimensional Ising model (see table 1). This result indicates that disorder is not only marginal in this model. A similar reduction is expected for the spin-spin correlation function in opposition to the controversial results obtained previously in perturbation theory [3–6]. However, Braak [12] found in saddle point approximation that the correlation length of the average spin-spin correlation function in the intermediate phase is proportional to η^{-2} ; i.e., the critical exponent for this correlation length is $\nu = 1$ as in the

pure Ising system.

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TABLES

TABLE I. Table of critical points and exponents for correlation lengths of the energy-energy correlation function (with the constant $c = \pi\bar{y}_c^2 \approx 0.54$).

	pure	disordered
m_c	0	$\pm 2e^{-c/g}$
ν_e	1	1/2