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**Electron-Electron Interactions  
and Charge Transport  
in Mesoscopic Conductors**

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Habilitationsschrift

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# Chapter 1

## Introduction

More than a hundred years ago Drude put forth a theory of metallic conduction [1, 2]. In his theory electrons are treated as a gas of particles which are assumed to move along classical trajectories until they collide with one another or with the ions. The collisions abruptly alter the velocity of the electrons as it is illustrated in Fig. 1.1. Since that time a number of details in Drude's theory have been improved. In Drude's time for example it seemed to be reasonable to assume that the electronic velocity distribution was given in equilibrium by the Maxwell-Boltzmann distribution. About a quarter of a century later Sommerfeld replaced the Maxwell-Boltzmann distribution by the Fermi-Dirac distribution. A further important step was the description of collisions beyond the relaxation time approximation, which finally lead to the Boltzmann equation for the dynamics of the distribution function. In many cases the transport theory based on the Boltzmann equation successfully describes the electrical conductivity of metals. On the other hand the Boltzmann theory still assumes that electrons move along classical trajectories, and quantum mechanical interference effects are neglected. The latter become important at a high concentration of defects or in very small systems. Therefore in the limits of strong disorder or small system size the Boltzmann equation fails and the transport becomes non-classical.

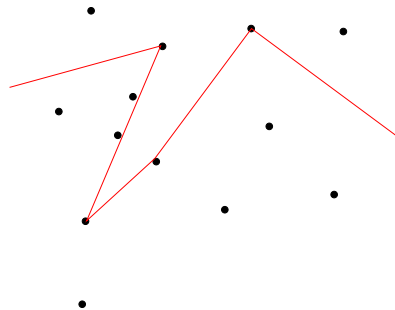


Figure 1.1: The classical picture for charge transport in metals: Electrons move through the metal along classical trajectories. From time to time they scatter and transfer momentum to the lattice.

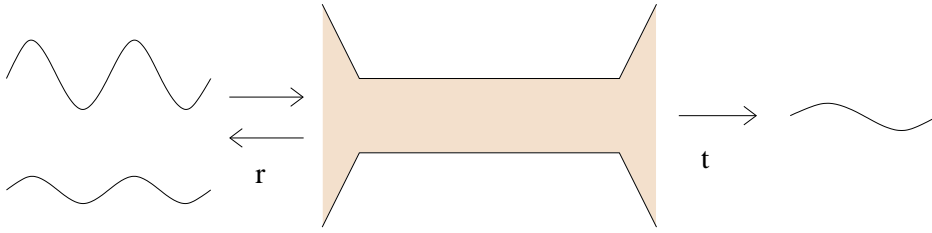


Figure 1.2: The quantum mechanical picture for electron transport: An incoming electron wave is transmitted or reflected.

Starting from the quantum mechanical description of electrons in metals, one obtains a complementary point of view for charge transport: Consider for example a constriction as shown in Fig. 1.2. An incoming electron wave is transmitted or reflected at the constriction with amplitudes  $t$  and  $r$ . The current is then proportional to the transmission probability, *i.e.* the modulus squared of the amplitude. For simplicity we restrict ourselves to zero temperature and a one-dimensional conducting channel for the moment. Assuming two different chemical potentials in the left and right reservoir,  $\mu_L = \mu_R + eV$ , the current through the constriction is given by

$$I = 2e \int_{\mu_R}^{\mu_L} d\epsilon \mathcal{N}(\epsilon) v(\epsilon) |t|^2; \quad (1.1)$$

the factor two is due to the electron spin. The product of the density of states  $\mathcal{N}(\epsilon)$  times the velocity  $v(\epsilon)$  does not depend on energy and is equal to  $\mathcal{N}v = 1/2\pi\hbar$ . As a result the conductance of the constriction is determined as

$$G = 2 \frac{e^2}{h} |t|^2, \quad (1.2)$$

with  $h/e^2 \approx 25.8 \text{ k}\Omega$ .

The connection between scattering amplitudes and the conductance was first proposed by Landauer [3]. Generalizations of the conductance formula (1.2), including for example many transport channels and more than two leads, have been discussed by several authors [4–9]. The Landauer approach to the conductance has been successfully applied to describe transport through structures, where the quantum mechanical coherence of the electron wave functions is maintained over the full system. A fascinating example is the transport through quantum point contacts, which has been studied both in semiconductors [10, 11] and through contacts consisting of single atoms [12].

We will mainly focus on the transport in metallic systems, where the classical approach to the conductivity is still a good starting point, but quantum effects give rise to corrections to the Drude conductivity. There are various types of such corrections. As the most widely known example we consider here weak localization; for reviews on this subject see [13–17]. The weak localization correction to the conductivity arises as a result of the quantum interference of electron waves in disordered systems. To get

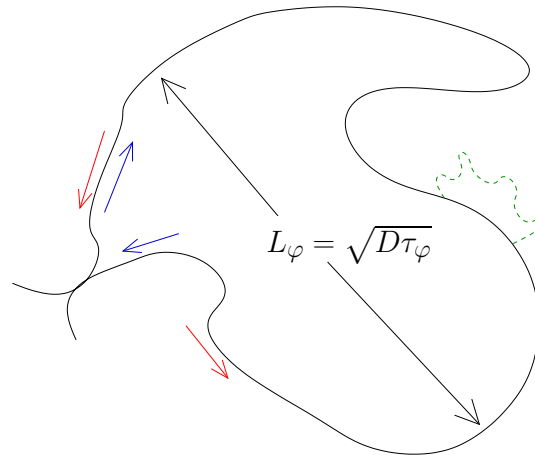


Figure 1.3: A self-intersecting path as it is relevant for the weak localization correction to the conductivity. The dashed wavy line represents an inelastic scattering event which is responsible for phase breaking;  $\tau_\varphi$  is the phase coherence time.

from point  $A$  to point  $B$  electrons move along classical paths. In order to obtain the total probability for a transfer from point  $A$  to  $B$ , in classical physics, one has to sum the probabilities for a particle to move along all possible paths. In quantum mechanics one has to sum amplitudes and take the modulus squared at the end. The relevant paths for weak localization are those with self-intersections and with the velocities of the incoming and outgoing paths in opposite directions as shown in Fig. 1.3. An electron can travel around the path clockwise or anti-clockwise. The two paths can be assigned an amplitude  $\Psi_1$  and  $\Psi_2$ . The probability is then proportional to

$$|\Psi_1 + \Psi_2|^2 = |\Psi_1|^2 + |\Psi_2|^2 + 2\text{Re}(\Psi_1^* \Psi_2). \quad (1.3)$$

The first two terms on the right hand side correspond to the classical probabilities. The third term, the interference term, only appears in quantum mechanics. For the special type of paths considered, the interference term is always positive, *i.e.* due to interference the probability of the process is enhanced. Neglecting the interference corresponds to a classical description of the electrons (Drude-Sommerfeld theory, Boltzmann equation).

This article summarizes and discusses the author's contributions to the theoretical description of transport in mesoscopic conductors. To distinguish the general references from the own publications which are attached to this article, the latter are cited as [A1], [A2], and so on. The central questions addressed are the following:

- In which way is the transport modified by electron-electron interactions?
- What happens in the presence of a large voltage, when linear response theory fails?

Both questions are relevant for the interpretation of experiments: Shortly after the discovery of weak localization, it was found that similar effects in the conductivity are caused by the electron-electron interaction [18–21]. Furthermore electron-electron interactions are also relevant for weak localization itself, since they provide a mechanism for phase breaking. However, while it is clear that inelastic scattering contributes to dephasing, the exact way this happens is by far less obvious as is evident from the recent debates in the literature [22–25]. Also, the importance of studying nonlinear transport is apparent: Many transport experiments on mesoscopic samples are carried out at low temperatures. Under these conditions inelastic scattering events which drive the electron system towards local equilibrium freeze out. Therefore non-equilibrium conditions can be realized even at rather low voltages.

## Outline

The article is arranged as follows: In chapter 2 we introduce the Green’s function technique in the non-equilibrium (Keldysh) formulation, which is the main theoretical tool throughout this article. The chapter is rather formal and may be skipped by a reader who is not interested in technical details. Nevertheless we included the material as the general background on which the results presented in the subsequent chapters have been obtained. Following the literature we will demonstrate how a Boltzmann-like theory and the Drude conductivity are found within the Green’s function formalism.

In chapter 3 transport beyond the Drude-Boltzmann theory will be considered. By extending the formalism of chapter 2 contributions to the current density due to “maximally crossed diagrams” and due to the Coulomb interaction will be calculated. The results for the Coulomb interaction have been originally published in [A1–A3].

Chapter 4 will be devoted to some applications. The general expression for the current density of chapter 3 will be evaluated explicitly for different experimental setups. We will consider the nonlinear conductivity in films and wires [A1–A3]. Furthermore we will investigate the question of whether the phase coherence time  $\tau_\varphi$ , which is a central quantity for weak localization, is also relevant for the Coulomb interaction corrections to the conductivity [A1].

In chapter 5 a rather different type of electron-electron interaction will be considered, namely electrons interacting via dynamical impurities. We will present results obtained for persistent currents in rings [A4–A6] and discuss the relevance of dynamical defects for low temperature dephasing [A5–A7].

The main focus of the article is on systems with diffusive electron motion. Interesting physics is, however, also found in other circumstances. In the last chapter of this article we will approach the vast field of transport through quantum dots. In particular we will investigate transport through a strongly interacting dot, where Kondo physics is relevant. After summarizing the established results for a quantum dot which is connected to two normal conducting leads, we will discuss the presently available

results for a quantum dot which is connected to a normal and to a superconducting lead [A8,A9].

The relevant publications, [A1-A9], are attached to the article.

# Chapter 2

## The classical theory of transport

The traditional transport theory in metals is based on the Boltzmann equation. The central object is the distribution function  $f(\mathbf{k}, \mathbf{r}, t)$ , where  $f(\mathbf{k}, \mathbf{r}, t) d\mathbf{r} d\mathbf{k} / (2\pi)^3$  is the number of electrons (per spin direction) at time  $t$  in the phase space volume  $d\mathbf{r} d\mathbf{k}$ . Charge and current density are given by

$$\rho(\mathbf{r}, t) = (-2e) \int \frac{d\mathbf{k}}{(2\pi)^3} f(\mathbf{k}, \mathbf{r}, t), \quad (2.1)$$

$$\mathbf{j}(\mathbf{r}, t) = (-2e) \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\hbar \mathbf{k}}{m} f(\mathbf{k}, \mathbf{r}, t). \quad (2.2)$$

In thermal equilibrium the distribution function reduces to the Fermi function. The Boltzmann equation determines the dynamics of the distribution function, and reads

$$\partial_t f + \mathbf{v} \cdot \nabla_{\mathbf{r}} f + \frac{1}{\hbar} \mathbf{F} \cdot \nabla_{\mathbf{k}} f = I[f]. \quad (2.3)$$

The left hand side of the equation contains information on the energy spectrum,  $\hbar \mathbf{v} = \nabla_{\mathbf{k}} \epsilon(\mathbf{k})$ , and external forces  $\mathbf{F}$ , whereas the collision term on the right hand side contains the scattering processes. For impurity scattering the collision term is given by

$$I[f] = - \int \frac{d\mathbf{k}'}{(2\pi)^3} W_{\mathbf{k}, \mathbf{k}'} [f(\mathbf{k}) - f(\mathbf{k}')]. \quad (2.4)$$

In this section we will recall how a Boltzmann-like kinetic equation is found within the Green's function formalism. To this end we introduce the non-equilibrium Green's function technique, as originally formulated by Keldysh [26]. Our notation will mainly follow [27]. We will first give general definitions, and we will then introduce the quasi-classical Green's function. In the presence of impurities a Boltzmann-like kinetic equation and the Drude conductivity are recovered within the Born approximation for the self-energy. To obtain quantum corrections to the conductivity, which will be discussed in chapters 3 and 4, more evolved approximations for the self-energy are necessary. From now on we set  $\hbar = k_B = 1$ , except in final results, where for clarity we put the constants.



$$\begin{pmatrix} G^R & G^K \\ 0 & G^A \end{pmatrix} = \begin{pmatrix} \leftarrow & \boxed{\leftarrow} \\ 0 & \leftarrow \end{pmatrix}$$

Figure 2.1: Graphical representation of the Green's function; we mark  $G^K$  with a shaded box. The shaded box can be understood as the Keldysh component of the self-energy, see Eq. (2.12).

In the Keldysh formalism the Green's functions have the matrix structure

$$\hat{G} = \begin{pmatrix} G^R & G^K \\ 0 & G^A \end{pmatrix}, \quad (2.5)$$

with

$$G^R(x_1, x_2) = -i\Theta(t_1 - t_2) (\langle \Psi(x_1)\Psi^\dagger(x_2) + \Psi^\dagger(x_2)\Psi(x_1) \rangle) \quad (2.6)$$

$$G^A(x_1, x_2) = +i\Theta(t_2 - t_1) (\langle \Psi(x_1)\Psi^\dagger(x_2) + \Psi^\dagger(x_2)\Psi(x_1) \rangle) \quad (2.7)$$

$$G^K(x_1, x_2) = -i (\langle \Psi(x_1)\Psi^\dagger(x_2) - \Psi^\dagger(x_2)\Psi(x_1) \rangle), \quad (2.8)$$

where  $\Psi$  and  $\Psi^\dagger$  are the usual fermion operators and  $x_i = (\mathbf{x}_i, t_i)$ . The brackets  $\langle \dots \rangle$  denote an average over a statistical operator,  $\text{Tr}\rho(\dots)$ . The Dyson equation reads

$$(\hat{G}_0^{-1} - \hat{\Sigma})\hat{G} = \delta(x_1 - x_2), \quad (2.9)$$

where the Green's functions  $\hat{G}_0$ ,  $\hat{G}$ , and the self-energy  $\hat{\Sigma}$  are considered as matrices in space, time, and the Keldysh index.  $\hat{G}_0$  is diagonal in the Keldysh index, the space and time dependence is given by

$$\hat{G}_0^{-1}(x_1, x_2) = \left[ i\partial_{t_1} - \frac{1}{2m} (-i\nabla_{\mathbf{x}_1} + e\mathbf{A}(x_1))^2 - e\phi(x_1) + \mu \right] \delta(x_1 - x_2). \quad (2.10)$$

Disorder and interactions are contained in the self-energy. In the Keldysh space the self-energy has the same triangular matrix structure as the Green's function,

$$\hat{\Sigma} = \begin{pmatrix} \Sigma^R & \Sigma^K \\ 0 & \Sigma^A \end{pmatrix}. \quad (2.11)$$

This relation allows expression of the Keldysh component of the Green's function as

$$G^K = G^R \Sigma^K G^A. \quad (2.12)$$

For a graphical representation see Fig. 2.1.

For the moment let us consider non-interacting electrons in thermal equilibrium. The retarded and advanced Green's functions can then be expressed in terms of the eigenfunctions  $\Psi_\lambda$  and eigenenergies  $\epsilon_\lambda$  of the single-particle Hamilton operator,

$$G_\epsilon^{R(A)}(\mathbf{x}_1, \mathbf{x}_2) = \sum_\lambda \frac{\Psi_\lambda(\mathbf{x}_1)\Psi_\lambda^*(\mathbf{x}_2)}{\epsilon - \epsilon_\lambda \pm i0}. \quad (2.13)$$

In particular these components of the Green's function do not depend on the statistical operator. On the other hand the Keldysh component of the Green's function crucially depends on the occupation of the states. In thermal equilibrium the Keldysh component is expressed in terms of the retarded and advanced components by  $G_\epsilon^K = [1 - 2f(\epsilon)](G_\epsilon^R - G_\epsilon^A)$ , where  $f(\epsilon)$  is the Fermi function. Generally, the equation of motion for  $G^K$  constitutes the quantum-kinetic equation. Approximations to this equation lead to the Boltzmann equation, Boltzmann-like equations, and generalizations.

As a first step towards the Boltzmann equation we define center-of-mass and relative variables

$$\begin{aligned} \mathbf{x} &= \frac{1}{2}(\mathbf{x}_1 + \mathbf{x}_2), & \mathbf{r} &= \mathbf{x}_1 - \mathbf{x}_2 \\ T &= \frac{1}{2}(t_1 + t_2), & t &= t_1 - t_2 \end{aligned} \quad (2.14)$$

There are various strategies how to proceed, see for example [27]. We introduce the  $\xi$ -integrated (quasi-classical) Green's function

$$\hat{g}_{t_1 t_2}(\hat{\mathbf{p}}, \mathbf{x}) = \frac{i}{\pi} \int d\xi d\mathbf{r} e^{-i\mathbf{p}\cdot\mathbf{r}} \hat{G}\left(\mathbf{x} + \frac{\mathbf{r}}{2}, t_1; \mathbf{x} - \frac{\mathbf{r}}{2}, t_2\right) \quad (2.15)$$

$$= \hat{g}(\hat{\mathbf{p}}, t; \mathbf{x}, T), \quad (2.16)$$

where  $\xi = \mathbf{p}^2/2m - \mu$ , and  $\hat{\mathbf{p}}$  is a unit vector along the momentum. In the entire article we will keep the notation of small  $\hat{g}$  for the  $\xi$ -integrated Green's functions, and capital  $\hat{G}$  for the original Green's functions. When approximating the density of states as an energy independent constant, the  $\xi$ -integration is related to an integration over the momentum  $\mathbf{p}$  according to

$$\int \frac{d\mathbf{p}}{(2\pi)^3} \rightarrow \mathcal{N}_0 \int d\xi \int \frac{d\hat{\mathbf{p}}}{4\pi}. \quad (2.17)$$

We now give some relations that are specific for impurity scattering. We treat impurity scattering within the self-consistent Born approximation. Assuming a Gaussian,  $\delta$ -correlated impurity potential one observes that the impurity self-energy is related to the  $s$ -wave part of the quasi-classical Green's function,

$$\hat{\Sigma}_{tt'}^{\text{imp}}(\mathbf{x}) = -\frac{i}{2\tau} \int \frac{d\hat{\mathbf{p}}}{4\pi} \hat{g}_{tt'}(\hat{\mathbf{p}}, \mathbf{x}). \quad (2.18)$$

Notice that here one has to determine three components of the self-energy. The retarded and advanced self-energy have a simple structure,  $\Sigma_{tt'}^{\text{imp}, R(A)} = \mp(i/2\tau)\delta(t - t')$ , so the

retarded and advanced Green's functions are

$$G^{R(A)}(\mathbf{p}, \epsilon) = \frac{1}{\epsilon - \xi \pm i/2\tau}. \quad (2.19)$$

We already mentioned that the equation of motion for the Keldysh component of the Green's function constitutes the kinetic equation. For simplicity we neglect external fields for the time being. Near the Fermi energy ( $\epsilon, \omega \ll \epsilon_F$ ) and for small momenta ( $q \ll p_F$ ) one finds from Eqs. (2.12) and (2.15)

$$g^K(\hat{\mathbf{p}}, \epsilon; \mathbf{q}, \omega) = \frac{i}{\pi} \int d\xi G^R(\epsilon + \omega/2, \mathbf{p} + \mathbf{q}/2) \times \Sigma^K(\epsilon; \mathbf{q}, \omega) G^A(\epsilon - \omega/2, \mathbf{p} - \mathbf{q}/2) \quad (2.20)$$

$$\approx \frac{i}{\tau} \frac{1}{\omega + i/\tau - v_F \hat{\mathbf{p}} \cdot \mathbf{q}} \int \frac{d\hat{\mathbf{p}}}{4\pi} g^K(\hat{\mathbf{p}}, \epsilon; \mathbf{q}, \omega). \quad (2.21)$$

In real time the kinetic equation reads

$$[\partial_T + v_F \hat{\mathbf{p}} \cdot \nabla_{\mathbf{x}}] g^K(\hat{\mathbf{p}}, \epsilon; \mathbf{x}, T) = -\frac{1}{\tau} \left[ g^K(\hat{\mathbf{p}}, \epsilon; \mathbf{x}, T) - \int \frac{d\hat{\mathbf{p}}}{4\pi} g^K(\hat{\mathbf{p}}, \epsilon; \mathbf{x}, T) \right], \quad (2.22)$$

which reminds us strongly of the Boltzmann equation. We will come back to this point at the end of the chapter. Notice that this equation is solved by any function  $g^K(\hat{\mathbf{p}}, \epsilon; \mathbf{x}, T)$  which is independent of direction  $\hat{\mathbf{p}}$ , position  $\mathbf{x}$ , and time  $T$ . This reflects the fact that any distribution function is allowed for non-interacting electrons. In the rest of this article, the diffusive limit is considered, where energies and momenta are restricted even more, namely  $\omega\tau, qv_F\tau \ll 1$ . By expanding (2.21) for small energy and momentum and taking the angular average, one finds the diffusive equation

$$(\partial_T - D\nabla_{\mathbf{x}}^2) \int \frac{d\hat{\mathbf{p}}}{4\pi} g^K(\hat{\mathbf{p}}, \epsilon; \mathbf{x}, T) = 0, \quad (2.23)$$

where the diffusion constant is  $D = v_F^2\tau/3$ .

The charge density and current density are generally related to the Keldysh component of the Green's function,

$$\rho(\mathbf{x}, t) = ieG^K(\mathbf{x}, t; \mathbf{x}, t) \quad (2.24)$$

$$\mathbf{j}(\mathbf{x}, t) = \frac{e}{2m} [\nabla_{\mathbf{x}} - \nabla_{\mathbf{x}'} + 2ie\mathbf{A}(\mathbf{x}, t)] G^K(\mathbf{x}, t; \mathbf{x}', t)|_{\mathbf{x}'=\mathbf{x}}. \quad (2.25)$$

In terms of the quasi-classical Green's functions, the charge density and current density are [27]

$$\rho(\mathbf{x}, t) = 2e\mathcal{N}_0 \left( \frac{\pi}{2} \int \frac{d\hat{\mathbf{p}}}{4\pi} g_{tt}^K(\hat{\mathbf{p}}, \mathbf{x}) - e\phi(\mathbf{x}, t) \right) \quad (2.26)$$

$$\mathbf{j}(\mathbf{x}, t) = e\pi\mathcal{N}_0 \int \frac{d\hat{\mathbf{p}}}{4\pi} v_F \hat{\mathbf{p}} g_{tt}^K(\hat{\mathbf{p}}, \mathbf{x}). \quad (2.27)$$

A more explicit expression for the current density is obtained in the diffusive limit and using the Born approximation for the self-energy. With the self-energy (2.18) the current density is

$$\mathbf{j}(\mathbf{x}, t) = -e\pi D\mathcal{N}_0\nabla_{\mathbf{x}} \int \frac{d\hat{\mathbf{p}}}{4\pi} g_{tt}^K(\hat{\mathbf{p}}, \mathbf{x}) = -D\nabla_{\mathbf{x}}\rho(\mathbf{x}, t) + 2e^2 D\mathcal{N}_0\mathbf{E}(\mathbf{x}, t). \quad (2.28)$$

Finally, we make the connection with the Boltzmann equation, as suggested in [28]. In the presence of a scalar field  $\phi$  the kinetic equation becomes

$$\begin{aligned} & [\partial_T + v_F\hat{\mathbf{p}} \cdot \nabla_{\mathbf{x}} - e(\partial_T\phi)\partial_{\epsilon}] g^K(\hat{\mathbf{p}}, \epsilon; \mathbf{x}, T) = \\ & -\frac{1}{\tau} \left[ g^K(\hat{\mathbf{p}}, \epsilon; \mathbf{x}, T) - \int \frac{d\hat{\mathbf{p}}}{4\pi} g^K(\hat{\mathbf{p}}, \epsilon, \mathbf{x}, T) \right]. \end{aligned} \quad (2.29)$$

Defining the distribution function as

$$f(\mathbf{p}, \mathbf{x}, t) = \frac{1}{2} [1 - g^K(\hat{\mathbf{p}}, \epsilon; \mathbf{x}, T)|_{\epsilon=\xi-e\phi}], \quad (2.30)$$

and ignoring the explicit dependence of  $g^K$  on  $\hat{\mathbf{p}}$  so that the derivatives of  $f$  are

$$\nabla_{\mathbf{p}}f = -\frac{1}{2}\nabla_{\mathbf{p}}\xi\partial_{\epsilon}g^K \quad (2.31)$$

$$\nabla_{\mathbf{x}}f = -\frac{1}{2}[\nabla_{\mathbf{x}}g^K - e(\nabla_{\mathbf{x}}\phi)\partial_{\epsilon}g^K] \quad (2.32)$$

$$\partial_Tf = -\frac{1}{2}[\partial_Tg^K - e(\partial_T\phi)\partial_{\epsilon}g^K], \quad (2.33)$$

one recovers from the kinetic equation (2.29) the Boltzmann equation (2.3) with the external force given by  $\mathbf{F} = e\nabla_{\mathbf{x}}\phi$ .

# Chapter 3

## The Coulomb interaction in diffusive conductors – general formalism

Quantum effects give rise to deviations from the classical expression of the electrical conductivity of a metal: The conductivity depends on the sample specific realization of the impurity potential, and even after averaging the conductivity over all possible realizations of the impurity potential corrections to the conductivity remain. The quantum corrections to the average conductivity in a metal with diffusive electron motion are weak localization, and the interaction contributions to the conductivity. The latter are often classified as the particle-particle (Cooper) channel, which is related to exchange of superconducting fluctuations, and the particle-hole channel, which is related to the exchange of charge fluctuations (spin singlet channel) or spin fluctuations (spin triplet channel). In this article we concentrate on the average conductivity, and in particular on the Coulomb interaction in the particle-hole channel. We will neglect the Cooper channel. This is justified in non-superconducting metals, since in this situation the relevant interaction parameter scales downwards under the renormalization group. For completeness we will also discuss briefly weak localization. In this chapter we will give the general expressions for the contribution to the current density due to weak localization and due to the electron-electron interaction. The following chapter 4 will contain specific applications.

### 3.1 Weak localization

In a weakly disordered metal, quantum interferences lead at low temperature to deviations from the Drude-Boltzmann theory of transport. Gorkov *et al.* [29] and Abrahams *et al.* [30] showed that the summation of maximally crossed diagrams gives rise to divergences in the conductivity for arbitrarily weak disorder in dimensions less than two. This so-called weak localization correction to the conductivity is due to electrons

diffusing along closed paths, where quantum interference causes an enhanced backscattering probability, as discussed in the introduction of this article. The weak localization contribution to the current density is given by

$$\delta \mathbf{j}_{\text{WL}}(t) = -e^2 D \tau \frac{4}{\pi} \int_{\tau}^{\infty} d\eta C_{\eta, -\eta}^{t-\eta/2}(\mathbf{x}, \mathbf{x}) \mathbf{E}(t - \eta), \quad (3.1)$$

where  $D$  is the diffusion constant,  $\tau$  the elastic scattering time and  $C_{\eta, -\eta}^t(\mathbf{x}, \mathbf{x})$  is the cooperon at two coinciding points in space. In the presence of a vector potential, the cooperon is given by the solution of the differential equation

$$\left\{ 2 \frac{\partial}{\partial \eta} - D(\nabla_{\mathbf{x}} + ie\mathbf{A}_C)^2 \right\} C_{\eta, \eta'}^t(\mathbf{x}, \mathbf{x}') = \frac{1}{\tau} \delta(\mathbf{x} - \mathbf{x}') \delta(\eta - \eta'). \quad (3.2)$$

with  $\mathbf{A}_C = \mathbf{A}(\mathbf{x}, t + \eta/2) + \mathbf{A}(\mathbf{x}, t - \eta/2)$ . We recall now the results for the conductivity. In the absence of external fields the cooperon at two coinciding points in space is given by

$$C_{\eta, -\eta}^t(\mathbf{x}, \mathbf{x}) = \frac{1}{2\tau} \left( \frac{1}{4\pi D \eta} \right)^{d/2} e^{-\eta/\tau_{\varphi}}, \quad (3.3)$$

where  $d$  is the spatial dimension and a phenomenological dephasing time  $\tau_{\varphi}$  has been introduced; microscopically dephasing arises from inelastic scattering. Inserting the above into Eq. (3.1), one arrives at

$$\delta \sigma_{\text{WL}} = \begin{cases} -\frac{e^2}{\pi \hbar} \sqrt{D \tau_{\varphi}} & (d = 1) \\ -\frac{e^2}{2\pi^2 \hbar} \ln(\tau_{\varphi}/\tau) & (d = 2) \\ \frac{e^2}{2\pi^2 \hbar} \sqrt{1/D \tau_{\varphi}} + \text{const} & (d = 3). \end{cases} \quad (3.4)$$

The connection to the Green's function formalism is the following: In chapter 2 we demonstrated that by approximating the impurity self-energy by  $\hat{\Sigma}_{\text{Born}}$  the Boltzmann equation for the distribution function and the Drude conductivity are recovered. The weak localization correction is found when considering also the maximally crossed diagrams,

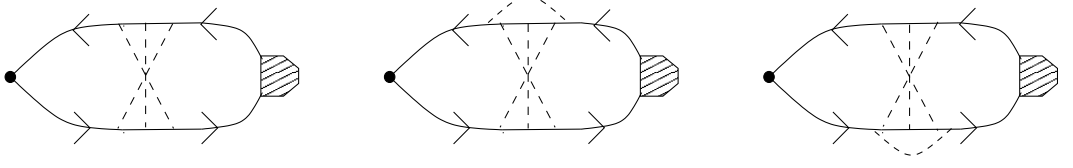
$$\hat{\Sigma} = \hat{\Sigma}_{\text{Born}} + \hat{\Sigma}_{\text{mc}}, \quad (3.5)$$

with

$$\hat{\Sigma}_{\text{Born}} = \text{---} \overbrace{\text{---}}^{\text{---}} \text{---} \quad (3.6)$$

$$\hat{\Sigma}_{\text{mc}} = \text{---} \overbrace{\text{---}}^{\text{---}} \text{---} + \text{---} \overbrace{\text{---}}^{\text{---}} \text{---} + \text{---} \overbrace{\text{---}}^{\text{---}} \text{---} + \dots \quad (3.7)$$

The self-energy has to be determined self-consistently. The starting point for the evaluation of the current density is Eq. (2.25), where we replace  $G^K$  by the product

Figure 3.1: Relevant graphs for the current density  $\mathbf{j}_{\text{mc}}$ .

$G^R \Sigma^K G^A$ . The contributions of the crossed diagrams to the retarded and the advanced self-energy are small and are therefore neglected [31]. The contribution to  $\Sigma_{\text{mc}}^K$  from the diagram with  $m$  impurity lines contains the sum of the product over  $(2m - 1)$  Green's functions,  $\sum_{n=0}^{2m-2} (G^R)^n G^K (G^A)^{2m-2-n}$ , *i.e.* integrals involving all combinations of the Green's function from  $G^K G^A \dots G^A$  to  $G^R \dots G^R G^K$ . The momentum integrals reduce to integrals over pairs of Green's functions of the form

$$\eta^{RR} = \frac{1}{2\pi N_0 \tau} \int \frac{d\mathbf{k}}{(2\pi)^3} G^R(\mathbf{k}) G^R(-\mathbf{k} + \mathbf{q}), \quad (3.8)$$

and the analogously defined integrals  $\eta^{AA}$  and  $\eta^{RA}$ . For small  $\mathbf{q}$  the integrals  $\eta^{RR}$  and  $\eta^{AA}$  are of the order  $1/(\epsilon_F \tau)$ , whereas the integrals  $\eta^{RA}$  are of order one. Terms involving  $\eta^{RR}$  or  $\eta^{AA}$  will thus be neglected. Following this rule the graphs contributing to  $\mathbf{j}_{\text{mc}}$  are shown in Fig. 3.1. After some algebra the current density is found as

$$\mathbf{j}_{\text{mc}}(\mathbf{x}, t) = 2eD\tau \int_{\tau}^{\infty} d\eta C_{\eta, -\eta}^{t-\eta/2}(\mathbf{x}, \mathbf{x}) \nabla_{\mathbf{x}} \int \frac{d\hat{\mathbf{p}}}{4\pi} g_{t-\eta, t-\eta}^K(\hat{\mathbf{p}}, \mathbf{x}). \quad (3.9)$$

The cooperon arises from a summation over integrals of the type  $\eta^{RA}$ . The total weak localization correction to the current density is

$$\delta \mathbf{j}_{\text{WL}} = \delta \mathbf{j}_{\text{Born}} + \mathbf{j}_{\text{mc}}, \quad (3.10)$$

where  $\delta \mathbf{j}_{\text{Born}}$  takes care of weak localization corrections to the distribution function [32],  $g^K \rightarrow g^K + \delta g^K$ , so that

$$\delta \mathbf{j}_{\text{Born}} = -e\pi D N_0 \nabla_{\mathbf{x}} \int \frac{d\hat{\mathbf{p}}}{4\pi} \delta g_{tt}^K(\hat{\mathbf{p}}, \mathbf{x}). \quad (3.11)$$

In equation (3.10) the sum of both terms is needed in order to ensure charge conservation. Recall also the general relation between the charge density and  $g^K$ , Eq. (2.26). In the special case where the electric field and the charge density are homogeneous in space, the weak localization correction to the current density, as given in Eq. (3.1), is recovered.

### 3.2 Interaction correction to diffusive transport

Shortly after the discovery of weak localization it was found [19–21] that similar effects in the conductivity can also be caused by the electron-electron interaction in weakly disordered electron systems. The interaction correction to the conductivity in the particle-hole singlet channel, for example, is given by [A1]

$$\delta\sigma_{\text{EEI}} = -\frac{4e^2D}{\pi d} \int_{\tau}^{\infty} d\eta \left( \frac{\pi T\eta}{\sinh(\pi T\eta)} \right)^2 \left( \frac{1}{4\pi D\eta} \right)^{d/2} \quad (3.12)$$

which leads to

$$\delta\sigma_{\text{EEI}} \approx \begin{cases} -4.91 \frac{e^2}{\pi^2\hbar} \sqrt{\hbar D/k_B T} & (d=1) \\ -\frac{e^2}{2\pi^2\hbar} \ln(\hbar/k_B T\tau) & (d=2) \\ 1.83 \frac{e^2}{6\pi^2\hbar} \sqrt{k_B T/\hbar D} + \text{const.} & (d=3) \end{cases} \quad (3.13)$$

Inclusion of the triplet channels does not change the functional form of the temperature dependence of the correction to the conductivity, but modifies the prefactor, which then depends on the strength of the electron-electron interaction in the spin triplet channel [33, 34].

Whereas a simple and convincing physical interpretation of weak localization exists, we are not aware of as simple an interpretation of the interaction effect. However, attempts have been made [35, 36], and we present the main ideas. First, one observes that impurities perturb the charge distribution in the metal,  $n \rightarrow n + \delta n$ ; for a single impurity, for example, there is a density oscillation around the impurity, the Friedel oscillations, even at large distance,  $\delta n(r) \sim \sin(2k_F r)/r^3$ . In the case of many impurities an inhomogeneous electron density forms, which depends on the distribution of the impurity positions. In the presence of interactions the charge inhomogeneity acts as an additional scattering potential, which in Hartree approximation is given by

$$V_H(\mathbf{r}) = \int V(\mathbf{r} - \mathbf{r}') \delta n(\mathbf{r}') d\mathbf{r}'. \quad (3.14)$$

It is clear that this additional scattering potential may affect the elastic mean free path. The way this happens, however, is by far less obvious. In the following we give an argument [35] which shows that the interaction contribution to the conductivity is due to quantum interference. Consider two classical paths an electron can travel to get from point A to point B. Let path one and two be identical up to an extra closed loop in path two, so that there is a phase difference between the two amplitudes,  $\Psi_2 = \exp(i\varphi_{\text{loop}})\Psi_1$ . The sign of the interference term  $\text{Re}(\Psi_1^*\Psi_2)$  is positive or negative, depending on the phase  $\varphi_{\text{loop}}$ , and therefore processes of this type give a negligible small contribution to the total probability for traveling from A to B. This becomes different in the presence of electron-electron interactions. To first order in the Hartree potential  $V_H$  we may write the interference term as  $\text{Re}(\Psi_1^*\Psi_2 V_H)$ . Now notice that the charge



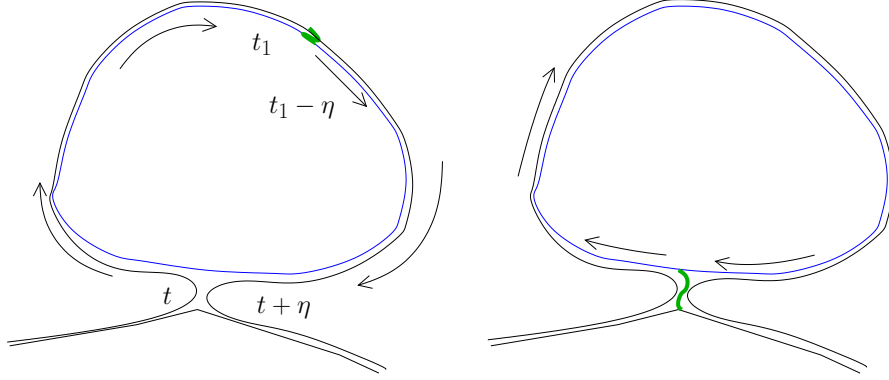


Figure 3.2: Graphical representation of processes leading to the interaction correction to the conductivity. A virtual particle is created at  $(\mathbf{x}_1, t_1 - \eta)$ , propagates and is absorbed at time  $t_1$  at the same point in space. A conduction electron propagates in the same direction along the same closed path.

inhomogeneity  $\delta n(\mathbf{r})$  and therefore  $V_H(\mathbf{r})$  is related to (virtual) electrons or holes which propagate along closed paths, since

$$\delta n(\mathbf{r}) = \frac{i}{\pi} \int d\epsilon f(\epsilon) [G_\epsilon^R(\mathbf{r}, \mathbf{r}) - G_\epsilon^A(\mathbf{r}, \mathbf{r})]. \quad (3.15)$$

In the case where a virtual hole goes around the same closed loop as the path number two the phase factor  $\exp(i\varphi_{\text{loop}})$  cancels, and  $\Psi_1$  and  $\Psi_2$  interfere coherently. A graphical representation of relevant processes is shown in Fig. 3.2, which is obtained by translating Feynman graphs including the electron-electron interaction [34].

## The current density

The electron-electron interaction enters the kinetic equation and modifies the distribution function. We will not consider the kinetic equation in presence of disorder and interaction, and only refer to the relevant literature [18, 27, 37, 38].

In [A3] we have derived a general expression for the current density in the presence of disorder and interaction. The result obtained there generalizes (3.12) allowing us to describe non-equilibrium effects and spatially inhomogeneous situations. The starting point of the derivation is the self-energy [18, 27]

$$\hat{\Sigma} = \hat{\Sigma}_{\text{Born}} + \hat{\Sigma}_V, \quad (3.16)$$

where  $\hat{\Sigma}_{\text{Born}}$  is the previously defined impurity self-energy and  $\hat{\Sigma}_V$  arises from the interaction,

$$\begin{aligned} \Sigma_{V,ij}(x, x') &= i \sum_{i'j'kk'} \int dx_2 dx_3 dx_4 dx_5 \Gamma_{ii'}^k(x_5; x, x_3) \\ &\times V^{kk'}(x_5, x_4) G_{i'j'}(x_3, x_2) \tilde{\Gamma}_{jj'}^{k'}(x_4; x_2, x'). \end{aligned} \quad (3.17)$$

Figure 3.3: The self-energy containing both disorder and interaction; the interaction vertex is dressed with impurity lines.

Here  $i, j, \dots$  denote the components in Keldysh space of the self-energy  $\Sigma$ , Green's function  $G$ , interaction  $V$ , and vertices  $\Gamma$  and  $\tilde{\Gamma}$ . Notice that in the Keldysh triangular representation the “absorption” and “emission” vertices have to be distinguished [27]. The “absorption” vertex  $\Gamma$  is given by

$$\begin{aligned} \Gamma_{ij}^k(x; x_1, x_2) &= \gamma_{ij}^k + \frac{1}{2\pi\mathcal{N}_0\tau} \sum_{i'j'} \int dx'_1 dx'_2 G_{ii'}(x_1, x'_1) \\ &\times \Gamma_{i'j'}^k(x; x'_1, x'_2) G_{j'j}(x'_2, x_2). \end{aligned} \quad (3.18)$$

An analogous equation holds for the “emission” vertex  $\tilde{\Gamma}$ , which is of the same structure, except that the bare vertex  $\gamma$  is replaced by  $\tilde{\gamma}$ . The bare vertices  $\gamma, \tilde{\gamma}$  are local in space and time. The structure in Keldysh space is  $\gamma_{ij}^1 = \tilde{\gamma}_{ij}^2 = \delta_{ij}/\sqrt{2}$  and  $\gamma_{ij}^2 = \tilde{\gamma}_{ij}^1 = \sigma_{ij}^x/\sqrt{2}$ . A diagrammatic representation of both the self-energy and vertex equations is shown in Fig. 3.3. The self-energy  $\Sigma_V$  as considered here only consists of an exchange term. The inclusion of the Hartree term, as represented in Fig. 3.2, is straightforward, and corresponds to the inclusion of the spin-triplet channels. Although the structure of the self-energy in Fig. 3.3 seems to be harmless at first glance, the diagrams can become rather complex in explicit calculations, see for example the calculation of the density response function in [34].

After some algebra the contribution of the Coulomb interaction to the current density  $\delta\mathbf{j}_{\text{EEI}} = \delta\mathbf{j}_{\text{Born}} + \delta\mathbf{j}_V$  is found as

$$\begin{aligned} \delta\mathbf{j}_{\text{Born}}(\mathbf{x}, t) &= -e\pi D\mathcal{N}_0 \nabla_{\mathbf{x}} \int \frac{d\hat{\mathbf{p}}}{4\pi} \delta g_{tt}^K(\hat{\mathbf{p}}, \mathbf{x}) \\ \delta\mathbf{j}_V(\mathbf{x}, t) &= e4\pi D\mathcal{N}_0\tau^2 \int d\eta dx_1 dx_2 \\ &\times \text{Re} [F_{t-\eta, t}(\mathbf{x}) D_{t-\eta/2, t_1-\eta/2}^\eta(\mathbf{x}, \mathbf{x}_1) F_{t_1, t_1-\eta}(\mathbf{x}_1) \\ &\times V_{t_1, t_2}^R(\mathbf{x}_1, \mathbf{x}_2) (-i\nabla_{\mathbf{x}}) D_{t_2, t-\eta}^0(\mathbf{x}_2, \mathbf{x})]. \end{aligned} \quad (3.20)$$

In this equation  $F_{tt'}(\mathbf{x})$  is the distribution function, which is related to the  $s$ -wave part of the quasi-classical Green's function according to

$$g_{s;tt'}^K(\mathbf{x}) = \int \frac{d\hat{\mathbf{p}}}{4\pi} g_{tt'}^K(\hat{\mathbf{p}}, \mathbf{x}) = \int dt_1 [g_{s;tt_1}^R(\mathbf{x}) F_{t_1 t'}(\mathbf{x}) - F_{tt_1}(\mathbf{x}) g_{s;t_1 t'}^A(\mathbf{x})], \quad (3.21)$$

and  $D_{tt'}^\eta(\mathbf{x}, \mathbf{x}')$  is the diffuson, given by the solution of the differential equation

$$\left\{ \frac{\partial}{\partial t} - D(\nabla_{\mathbf{x}} + ie\mathbf{A}_D)^2 - ie\phi_D \right\} D_{tt'}^\eta(\mathbf{x}, \mathbf{x}') = \frac{1}{\tau} \delta(\mathbf{x} - \mathbf{x}') \delta(t - t'), \quad (3.22)$$

with  $\mathbf{A}_D = \mathbf{A}(\mathbf{x}, t + \eta/2) - \mathbf{A}(\mathbf{x}, t - \eta/2)$ , and  $\phi_D = \phi(\mathbf{x}, t + \eta/2) - \phi(\mathbf{x}, t - \eta/2)$ . The expression for the current density (3.20) has been derived in [A3]. It generalizes earlier results, which are valid in the absence of the external vector potential [39] or for the electron system near local equilibrium [A1,A2]. In [A1] the main focus was on the dephasing problem. In [A2] the spin-triplet channels and Fermi liquid renormalizations were included, which make the theory applicable even in strongly interacting Fermi liquids. In [A3] the formalism was made gauge invariant and has been extended to situations far from equilibrium. The diagrammatic calculations have been confirmed by another technique [A3], namely the Keldysh version [40–42] of the nonlinear sigma model [33]. Notice that the expression of the current density is valid for an arbitrary form of the distribution function and the diffuson. This allows the examination of the current in different experimental and geometrical setups.

# Chapter 4

## The Coulomb interaction in diffusive conductors – applications

The equation for the current density is still rather complex. In [A1-A3] the formalism has been applied to several questions, and we will now briefly summarize some of the results. Starting with a remark on gauge invariance, we continue by outlining how to recover the well known interaction correction to the linear conductivity within the present formalism. New results on the nonlinear conductivity and on phase breaking will be discussed in the following sections. This chapter closes with an outlook on possible future applications.

### 4.1 Gauge invariance

We begin by demonstrating explicitly the gauge invariance of the expression for the current density. At first one notices that  $\delta\mathbf{j}_{\text{Born}} = -D\nabla\delta\rho$  is gauge invariant. For  $\delta\mathbf{j}_V$  an explicit check is necessary. Given the gauge transformation

$$\mathbf{A} \rightarrow \mathbf{A} + \nabla\chi \quad (4.1)$$

$$\phi \rightarrow \phi - \partial_t\chi \quad (4.2)$$

the distribution function and the diffuson transform according to

$$F_{tt'}(\mathbf{x}) \rightarrow F_{tt'}(\mathbf{x}) \exp\{-ie[\chi(\mathbf{x}, t) - \chi(\mathbf{x}, t')]\} \quad (4.3)$$

$$\begin{aligned} D_{tt'}^\eta(\mathbf{x}, \mathbf{x}') &\rightarrow D_{tt'}^\eta(\mathbf{x}, \mathbf{x}') \\ &\times \exp\{-ie[\chi(\mathbf{x}, t + \frac{\eta}{2}) - \chi(\mathbf{x}, t - \frac{\eta}{2})]\} \\ &\times \exp\{ie[\chi(\mathbf{x}', t' + \frac{\eta}{2}) - \chi(\mathbf{x}', t' - \frac{\eta}{2})]\}. \end{aligned} \quad (4.4)$$

By applying the above transformation to  $\delta\mathbf{j}_V$  as given in Eq. (3.20), one can easily verify that the function  $\chi(\mathbf{x}, t)$  drops, so that the expression is manifestly gauge invariant.

## 4.2 Linear response

Since the expression for  $\delta\mathbf{j}_{\text{EEI}}$  is rather complex it is worthwhile checking the linear conductivity. We work in a vector gauge,  $\mathbf{A} = -t\mathbf{E}$ ,  $\phi = 0$ , and assume that the electron distribution function has the equilibrium form,  $F(\epsilon, \mathbf{x}) = \tanh(\epsilon/2T)$ . Equivalently, in the time domain this means that  $F_{t't'}(\mathbf{x}) = -iT/\sinh[\pi T(t-t')]$ . For a system which is homogeneous in space (after averaging over the disorder), also the charge density is homogeneous, and therefore  $\delta\mathbf{j}_{\text{Born}}$  is zero.

In order to calculate  $\delta\mathbf{j}_V$  we have to know the screened Coulomb interaction and the diffuson. The dynamically screened Coulomb interaction as a function of frequency and momentum is

$$V^R(\mathbf{q}, \omega) = \frac{4\pi e^2}{q^2 + 8\pi e^2 \mathcal{N}_0 \frac{Dq^2}{-i\omega + Dq^2}} \approx \frac{1}{2\mathcal{N}_0} \frac{-i\omega + Dq^2}{Dq^2}. \quad (4.5)$$

The expression in the middle of this equation is valid in three dimensions. On the right hand side we assumed good screening, *i.e.* the screening vector  $\kappa$ , with  $\kappa^2 = 8\pi e^2 \mathcal{N}_0$ , is set to infinity. The perfectly screened Coulomb interaction in one and two dimensions is identical to the one in three dimensions as given on the right hand side in Eq. (4.5). Now we calculate the two diffusons  $D_{t't'}^\eta(\mathbf{q})$  entering the current density. It is important to note that they appear with different time arguments  $\eta$ . In the second of the two diffusons in  $\delta\mathbf{j}_V$  the time  $\eta$  is zero with the consequence that the diffuson does not depend on the vector potential  $\mathbf{A}$ , and is thus given by

$$D^0(\mathbf{q}, \omega) = \frac{1}{\tau} \frac{1}{-i\omega + Dq^2}. \quad (4.6)$$

The convolution of the interaction with the second of the two diffusons appearing in the formula for the current gives then

$$\int dt_2 V_{t_1, t_2}^R(\mathbf{q}) D_{t_2, t-\eta}^0(\mathbf{q}) = \frac{1}{2\mathcal{N}_0\tau} \frac{1}{Dq^2} \delta(t_1 - t + \eta) \quad (4.7)$$

and the expression for the current density becomes

$$\delta\mathbf{j}_V(t) = -\frac{2e\tau}{\pi} \sum_{\mathbf{q}} \int_{\tau}^{\infty} d\eta \frac{\mathbf{q}}{q^2} \left( \frac{\pi T}{\sinh(\pi T\eta)} \right)^2 D_{t-\eta/2, t-3\eta/2}^\eta(\mathbf{q}). \quad (4.8)$$

The electric field enters via the remaining diffuson, which is given by

$$D_{t-\eta/2, t-3\eta/2}^\eta(\mathbf{q}) = \frac{1}{\tau} \exp[-D(\mathbf{q} - e\mathbf{E}\eta)^2\eta] \quad (4.9)$$

$$= \frac{1}{\tau} e^{-Dq^2\eta} (1 + 2De\mathbf{q} \cdot \mathbf{E}\eta^2 + \dots). \quad (4.10)$$

After performing the momentum integration one arrives at equation (3.12) for the correction to the conductivity.

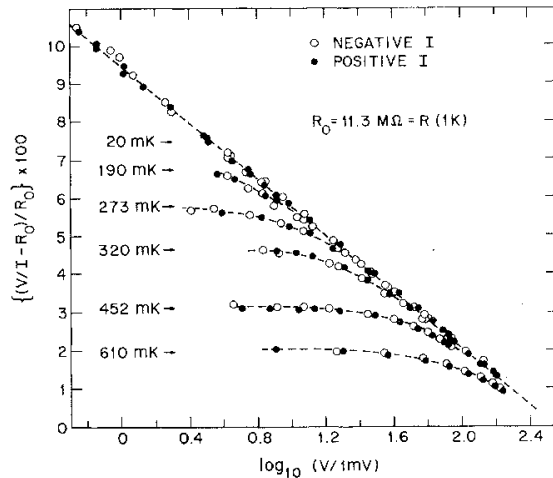


Figure 4.1: Resistivity as a function of voltage for a thin AuPd film, taken from [43]. The resistance  $R_0 = 11.3 \text{ M}\Omega$  corresponds to  $R_{\square} \approx 4500 \text{ }\Omega$ .

### 4.3 Nonlinear conductivity in films

In 1979 Dolan and Osheroff [43] observed a logarithmic variation of the resistivity of thin metallic films as a function of the applied voltage; experimental data is shown in Fig. 4.1. In order to explain the experiment Anderson *et al.* [44] argued that the logarithm as a function of voltage is directly related to the logarithm as a function of temperature (from weak localization in two dimensions) since the dissipated power heats the electron gas: In the case of a strong electric field, the electron temperature is of the order of the voltage drop on the relevant inelastic scattering length, the electron-phonon length, *i.e.*  $T \sim eEL_{e\text{-ph}}$ , with  $L_{e\text{-ph}} = \sqrt{D\tau_{e\text{-ph}}}$ . When  $L_{e\text{-ph}}$  is proportional to a power of the temperature,  $L_{e\text{-ph}} \sim T^{-p}$ , electric field and temperature are related as  $T^{1+p} \sim E$  so that a logarithmic temperature dependence of the linear resistivity causes the logarithmic voltage dependence.

Shortly after the first experiments, it was discussed whether heating is the only origin of the nonlinear conductivity or if an electric field – in analogy to a magnetic field – can directly destroy weak localization via dephasing [45,46]. The correct answer to the second question is “no” [47,48], as one can easily verify by calculating the phase shifts of a pair of time reversed paths. In the presence of a vector potential an electron which propagates along a path  $\mathbf{x}_1(t)$  or  $\mathbf{x}_2(t)$  accumulates an extra phase

$$\varphi_{1,2} = \frac{e}{\hbar} \int_0^\eta dt' \dot{\mathbf{x}}_{1,2} \cdot \mathbf{A}. \quad (4.11)$$

For a pair of closed time reversed paths,  $\mathbf{x}_2(t) = \mathbf{x}_1(\eta - t)$ , and a static electric field,  $\mathbf{A}(t) = -\mathbf{E}t$ , the difference of the two phases vanishes, *i.e.* there is no “dephasing” from a static electric field.

However, in a later examination of the current-voltage characteristics of gold films, Bergmann *et al.* [49] noted that the experimental data is not completely compatible with a pure heating model. As a possible explanation of the experimental findings they suggested that the Coulomb interaction contribution to the resistivity shows non-ohmic behavior with

$$\delta R_{\text{EEI}}/R^2 = -\frac{e^2}{4\pi^2\hbar} \ln [(k_B T)^2 + \alpha\hbar D(eE)^2/k_B T], \quad (4.12)$$

where  $\alpha$  is a factor of the order one.

Indeed in formulating the phase shift argument for the interaction contribution, a sensitivity to a static electric field cannot be excluded [A1]: The interaction correction to the conductivity is related to the propagation of a particle and a hole along closed paths. Hence one may think of this as a particle starting for example at  $t = 0$  and arriving at  $t = \eta$ . This particle is interacting with a hole which is traversing the same closed path. Since the point of interaction  $\mathbf{x}(t_1)$  can be anywhere along the path, the particle and hole traverse the loop at different times, compare Fig. 3.2. In the absence of a vector potential the phases of particle and hole cancel, whereas in the presence of a vector potential the accumulated phase difference is

$$\delta\varphi = \frac{e}{\hbar} \int_{t_1-\eta}^{t_1} dt' \dot{\mathbf{x}}_1 \cdot \mathbf{A} - \frac{e}{\hbar} \int_0^\eta dt' \dot{\mathbf{x}}_2 \cdot \mathbf{A}. \quad (4.13)$$

The relevant paths obey the relations  $\mathbf{x}_1(t) = \mathbf{x}_2(t)$  for  $0 < t < t_1$  and  $\mathbf{x}_1(t-\eta) = \mathbf{x}_2(t)$  for  $t_1 < t < \eta$ , which allows us to write the phase as

$$\delta\varphi = \frac{e}{\hbar} \int_{t_1-\eta}^0 dt' \dot{\mathbf{x}}_1 \cdot [\mathbf{A}(t') - \mathbf{A}(t' + \eta)]. \quad (4.14)$$

For the particular case of a static electric field described by  $\mathbf{A} = -\mathbf{E}t$ , the above given phase shift becomes  $\delta\varphi = \frac{e}{\hbar}\eta(\mathbf{x}_2 - \mathbf{x}_1) \cdot \mathbf{E}$ . This suggests that the interaction correction should be sensitive to a static electric field, leading to a nonlinear conductivity. The quantitative calculation in fact gives [A1]

$$\delta R_{\text{EEI}}/R^2 \approx -\frac{e^2}{\pi^2\hbar} \left( \ln k_B T + 1.62 \frac{\hbar D(eE)^2}{(\pi k_B T)^3} \right), \quad (4.15)$$

verifying the non-ohmic behavior of the resistivity with the characteristic electric field scale as it has been suggested in [49]. In order to obtain Eq. (4.15), a thermal distribution function with electron temperature  $T$  has been assumed. Equation (4.15) is then obtained from the current formula (3.20) under the condition  $\hbar D(eE)^2 \ll (\pi k_B T)^3$ . Note that the functional form (4.12) of the non-ohmic resistivity cannot be confirmed theoretically. In [A1,A2] various situations have been examined, including strong electric fields, time dependent fields, one to three dimensions, and magnetic field effects.

## 4.4 Nonlinear conductivity in wires

For a thin wire of length  $L$  the analog of Eq. (4.15) is [A2]

$$\delta R_{\text{EEI}}/R^2 \approx \frac{e^2}{\pi^2 \hbar} \frac{L_T}{L} \left( 4.91 - 0.21 \frac{\hbar D (eV/L)}{(k_B T)^3} \right), \quad (4.16)$$

where  $L_T = \sqrt{\hbar D/k_B T}$  is the thermal diffusion length, and  $V = EL$  is the applied voltage. Again, the result has been obtained under the assumption of a thermal distribution function with a constant temperature  $T$ . This is reasonable for macroscopic samples, it fails, however, in samples which are shorter than the electron-phonon scattering length. In short samples it is therefore necessary to study the Coulomb interaction correction to the conductivity far from equilibrium [39, 50][A3].

For an evaluation of the current-voltage characteristics the diffuson and the distribution function in the wire are required. The diffuson is found by solving the differential equation (3.22) with the conditions that the derivative normal to an insulating boundary vanishes

$$(\mathbf{n} \cdot \nabla) D(\mathbf{x}, \mathbf{x}') \Big|_{\mathbf{x} \in \text{i.b.}} = 0 \quad (4.17)$$

and the diffuson itself vanishes at a metallic boundary,

$$D(\mathbf{x}, \mathbf{x}') \Big|_{\mathbf{x} \in \text{m.b.}} = 0. \quad (4.18)$$

The latter condition corresponds to the assumption that an electron arriving at the metallic boundary escapes into the leads with zero probability to come back into the wire. Furthermore it is assumed that the left and right leads of the wire are in thermal equilibrium,

$$F(\epsilon, \mathbf{x}) \Big|_{\mathbf{x} \in \text{l.l., r.l.}} = \tanh \left( \frac{\epsilon \pm eV/2}{2T} \right), \quad (4.19)$$

and the voltage difference is  $V$ . The distribution function inside the wire is found by solving the kinetic equation. It is found that the distribution function depends on the various relaxation mechanisms governing the collision integral [23, 51–54]:

- a) When  $L \gg L_{e\text{-ph}}$ , the distribution function acquires the equilibrium form with a local chemical potential and temperature,

$$F(\epsilon, x) = \tanh \left( \frac{\epsilon + eV(L - 2x)/2L}{2T(x, V)} \right), \quad (4.20)$$

where  $x = 0 \dots L$  is the distance from the left lead. The local temperature  $T(x)$  may be determined from an energy balance argument, assuming that the dissipated power equals the gradient of the heat flow. In the limit considered here the heat flow is dominated by the phonons. If the “hot” phonons escape



ballistically into the substrate, the temperature in the bulk of the wire does not depend on the position  $x$  but is voltage dependent,  $T = T(V)$ . Neglecting the region near the leads, where the temperature rises from  $T_{\text{lead}}$  to  $T(V)$ , Eq. (4.16) is recovered for the voltage dependent resistivity.

- b) When  $L_{\text{e-ph}} \gg L \gg L_{\text{in}}$  one still expects a distribution function near local equilibrium, due to inelastic electron-electron scattering. The local temperature is determined from the relation  $\sigma E^2 = -\nabla[\kappa \nabla T(x)]$ , where  $\kappa$  is the thermal conductivity. Using the Wiedemann-Franz law,  $\kappa = \frac{\pi^2}{3} T \sigma (k_B/e)^2$ , the temperature profile in the wire is determined as

$$T^2(x) = T_{\text{lead}}^2 + \frac{3}{\pi^2} \left( \frac{eV}{L} \right)^2 x(L-x). \quad (4.21)$$

- c) When  $L_{\text{in}} \gg L$  the distribution function is a linear superposition of the distribution function of the leads,

$$F(\epsilon, x) = [(L-x)F(\epsilon, 0) + xF(\epsilon, L)] / L. \quad (4.22)$$

It is found that in both limits b) and c) the current can be written as [A3]

$$\delta I_{\text{EEI}}(V, T) = \frac{e^2 L_T}{\hbar L} f(eV/k_B T) V, \quad (4.23)$$

where the function  $f$  depends on the distribution function and on the length of the wire;  $T$  is the temperature in the leads. Numerical results are shown in Fig. 4.2. Notice that  $f(eV/k_B T)$  is proportional to  $\delta I_{\text{EEI}}/V$ , so that  $f(eV/k_B T)$  also represents the voltage dependent conductance in units of  $(e^2/\hbar)(L_T/L)$ . For low voltage and large system size  $L \gg L_T$  the standard result  $\delta I/V \approx -0.5(e^2/\hbar)(L_T/L)$  is approached. For shorter systems the correction to the current remains smaller, since electrons escape quickly from the wire into the leads. The full lines show the voltage dependent conductance for case c); the long dashed line corresponds to case b). The short dashed line ( $L/L_T = 5$ ) is obtained within a simple approximation: Instead of evaluating the full expression for  $\delta \mathbf{j}_{\text{EEI}}$  we take the linear conductivity as a function of temperature, and average over the temperature profile,

$$\delta \sigma_{\text{heating}} = \frac{1}{L} \int_0^L dx \delta \sigma(T(x)). \quad (4.24)$$

Important results are the following: In both cases b) and c) the conductance scales with voltage over temperature. In case b) (hot electrons) the main effect is simple heating, *i.e.* the non-ohmic effects are hard to be observed. Far from equilibrium, on the other hand, the current-voltage characteristics is quantitatively different from the hot electron regime. The temperature dependence of the Coulomb interaction contribution

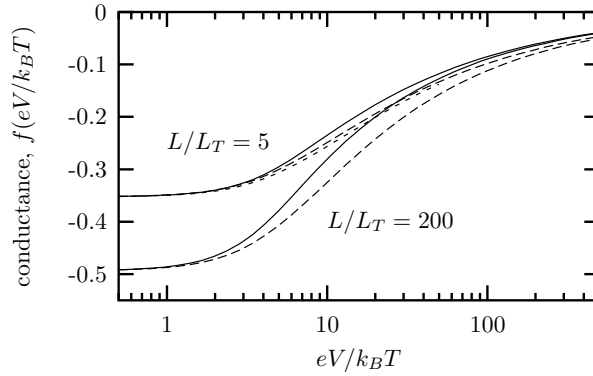


Figure 4.2: Interaction correction to the conductance  $I/V$  for a mesoscopic wire as a function of voltage [A3].  $I/V$  is plotted in units of  $(e^2/\hbar)L_T/L$ . The full line corresponds to the non-equilibrium distribution function (4.22). The line with long dashes corresponds to the local equilibrium distribution function (4.20) with the  $x$ -dependent temperature. The line with short dashes ( $L/L_T = 5$ ) is the nonlinear conductivity due to the heating contribution only, Eq. (4.24).

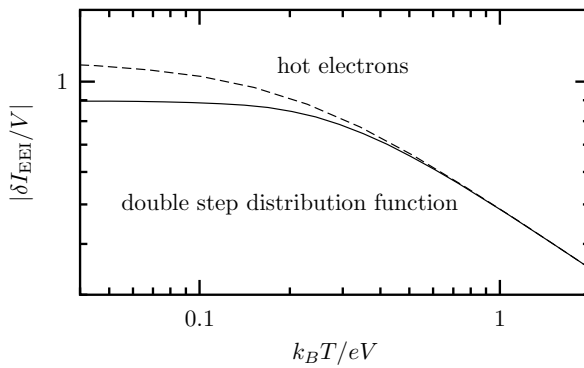


Figure 4.3: Temperature dependence of the Coulomb interaction correction to the conductance of a mesoscopic wire in arbitrary units. The full line corresponds to the double-step distribution function, the dashed line corresponds to hot electrons with temperature profile, as explained in the text.  $T$  is the electron temperature in the leads.

to the conductance is shown in Fig. 4.3 in a double logarithmic plot. The curves shown are obtained from the same numerical data for  $f(eV/k_B T)$  as shown in Fig. 4.2 ( $L/L_T = 200$ ). At high temperature the Coulomb correction to the conductance follows  $1/\sqrt{T}$ , which is seen as a linear behavior in the double logarithmic plot. When the temperature in the leads becomes less than the voltage, the conductance saturates. In the absence of inelastic scattering, c), this low temperature saturation appears at a higher temperature than in b).

What about the experimental situation? The distribution function in mesoscopic wires was measured by Pothier *et al.* [55]. For short wires and low temperature ( $L \approx 1.5 \mu\text{m}$ ,  $T \approx 25 \text{ mK}$ ), the double step like distribution function was observed. Unfortunately, as far as we know, there is no detailed investigation of the temperature and voltage dependence of the conductivity in this experiment. Weber *et al.* [56] measured the current-voltage characteristics of nanobridges which were attached to large reservoirs. They observed a scaling of the voltage with the temperature, which agrees qualitatively with our prediction. However, for a quantitative explanation of the experiment it might be important to take into account scattering of the electrons at the interface between the leads and the bridge [56], or charging effects [57].

## 4.5 Electron dephasing

The dephasing time  $\tau_\varphi$  is a central object in weak localization. A comprehensive review on recent experimental studies is given in [58]. The dephasing time sets the scale over which an electron propagates without losing phase coherence, and determines the amplitude of the weak localization correction to the conductivity. The amplitude of the interaction correction to the conductivity, on the other hand, is set by the thermal time  $\tau_T = \hbar/k_B T$ , see Eq. (3.13). In most cases the dephasing time is much longer than the thermal time. In some experiments, however, an extraordinarily strong phase breaking has been reported: Recently a low temperature saturation of  $\tau_\varphi$  in gold wires [59, 60] has attracted much attention [22–24, 61–65]. Furthermore in a number of cuprates, the dephasing rate decreases only slowly with decreasing temperature [66–68]. For example in  $\text{Bi}_2\text{Sr}_2\text{CuO}_6$ , [66], the dephasing rate varies as  $1/\tau_\varphi \sim T^{1/3}$ , with  $\tau_\varphi$  much shorter than  $\tau_T$ . In this case phase breaking may become relevant also in the interaction contribution to the conductivity.

Unfortunately, in the experiments cited above, the microscopic mechanism which is responsible for the strong phase breaking is unknown. Nevertheless we believe it is important to answer the following questions: (i) Is phase breaking also relevant in the interaction contribution to the conductivity? (ii) If yes, is the phase breaking rate which is relevant in the particle-hole channel (Coulomb interaction) the same as in the particle-particle channel (weak localization)? While Castellani *et al.* [69] came to the conclusion that the answer is “yes” for both questions, Raimondi *et al.* [A1] reexamined the problem, studying dephasing due to internal electric field fluctuations. They

confirmed the suggestion that dephasing is possible also in the particle-hole channel, but found different dephasing times for the particle-hole and particle-particle channels. Raimondi *et al.* [A1] started from the expression for the interaction contribution to the current density,  $\delta \mathbf{j}_V$ , and then averaged over internal electric field fluctuations using a path integral formalism for the diffuson. In the following we will give a semi-quantitative summary of their analysis [A1].

In thermal equilibrium the internal electric field fluctuations are given by

$$e^2 \langle E^i E^j \rangle_{\mathbf{q}, \omega} = q^i q^j \frac{2T}{\omega} \text{Im} V^R(\mathbf{q}, \omega), \quad (4.25)$$

where  $V^R(\mathbf{q}, \omega)$  is the retarded Coulomb interaction and low frequencies ( $\omega \ll T$ ) have been assumed. The phase shift of an electron which is propagating along a path  $\mathbf{x}_{1,2}(t)$  is  $\varphi_{1,2} = e \int dt \dot{\mathbf{x}}_{1,2}(t) \cdot \mathbf{A}(t)$ . Both for weak localization and the interaction effect the phase difference  $\delta\varphi$  for a pair of classical paths is relevant. Averaging the phase factor  $\exp(i\delta\varphi)$  over the electric field fluctuations, using the relation

$$\langle \exp(i\delta\varphi) \rangle = \exp\left(-\frac{1}{2} \langle (\delta\varphi)^2 \rangle\right) = \exp(-S), \quad (4.26)$$

one finds

$$\begin{aligned} S_{\text{p.p.}} &= \frac{1}{2} \int_{-\eta}^{\eta} dt_1 \int_{-\eta}^{\eta} dt_2 \int_{-T}^T \frac{d\omega}{2\pi} \sum_{\mathbf{q}} \frac{2T}{\omega} [-\text{Im} V^R(\mathbf{q}, \omega)] \\ &\times \exp\{i\mathbf{q} \cdot [\mathbf{x}_1(t_1) - \mathbf{x}_1(t_2)]\} \left[ \cos\left(\omega \frac{t_1 - t_2}{2}\right) - \cos\left(\omega \frac{t_1 + t_2}{2}\right) \right] \end{aligned} \quad (4.27)$$

in the case of weak localization, and

$$\begin{aligned} S_{\text{p.h.}} &= \int_0^{\eta} dt_1 \int_0^{\eta} dt_2 \int_{-T}^T \frac{d\omega}{2\pi} \sum_{\mathbf{q}} \frac{2T}{\omega} [-\text{Im} V^R(\mathbf{q}, \omega)] \\ &\times \exp\{i\mathbf{q} \cdot [\mathbf{x}_1(t_1) - \mathbf{x}_1(t_2)]\} e^{-i\omega(t_1 - t_2)} [1 - \cos(\omega\eta)] \end{aligned} \quad (4.28)$$

in the case of the interaction effect; both in  $S_{\text{p.p.}}$  and in  $S_{\text{p.h.}}$  the velocities  $\dot{\mathbf{x}}(t_{1,2})$  were removed by a partial integration, and the relation between the two relevant paths  $\mathbf{x}_1(t)$  and  $\mathbf{x}_2(t)$  were exploited. The expressions for  $S_{\text{p.p.}}$  and  $S_{\text{p.h.}}$  appear similar, but clearly the time dependent factors in the second line differ, which can lead to quite different dephasing times in the particle-particle and particle-hole channels. In two dimensions, for example, the final results are

$$S_{\text{p.p.}} \approx \frac{T\eta}{4\pi D\mathcal{N}_0} \ln(T\eta); \quad \frac{1}{\tau_{\varphi}} \approx \frac{T}{4\pi D\mathcal{N}_0} \ln(4\pi D\mathcal{N}_0) \quad (4.29)$$

and

$$S_{\text{p.h.}} \approx \frac{T\eta}{4\pi D\mathcal{N}_0} \ln(D\kappa^2 T\eta^2); \quad \frac{1}{\tau_{\varphi}} \approx \frac{T}{4\pi D\mathcal{N}_0} \ln[D\kappa^2 (4\pi D\mathcal{N}_0)^2 / T], \quad (4.30)$$

where  $\kappa$  is the inverse screening length. The dephasing time is here determined from the condition  $S(\eta = \tau_\varphi) = 1$ . In the particle-particle channel the standard result of Altshuler, Aronov and Khmelnitskii [70–72] is found. The dephasing time in the particle-hole channel is different from the one in the particle-particle channel, and is identical to the inelastic scattering rate in the two-particle propagators (diffuson or cooperon), as first calculated by Fukuyama and Abrahams [73, 74].

The low frequency electric field fluctuations which we consider here, cannot explain the strong phase breaking observed in [59, 66]. However we expect similar results for other phase breaking mechanisms: We expect that a mechanism leading to strong dephasing in the particle-particle channel will also cause strong dephasing in the particle-hole channel. In particular when  $\tau_\varphi$  becomes comparable to or shorter than the thermal time  $\tau_T$  the relevant time scale which sets the amplitude of the Coulomb interaction contribution to the conductivity will be  $\tau_\varphi$  instead of  $\tau_T$ . This is consistent with the experiments: In the gold wires of [59] the dephasing rate saturated below  $T \sim 1$  K to values of the order  $\hbar/\tau_\varphi \sim 1$ -10 mK. In the samples with the strongest phase breaking a saturation of the interaction correction to the conductivity has been observed below  $T \sim 100$  mK [60]. In  $\text{Bi}_2\text{Sr}_2\text{CuO}_6$ , a compound with a single  $\text{CuO}_2$  plane per unit cell, the in-plane zero field resistivity increases as  $\ln T$  below  $\sim 18$  K, consistent with quantum interference effects in two dimensions. The shape of the orbital magnetoresistance is well fitted by the weak localization expression [66], but with an unconventionally large dephasing rate which varies as  $T^{1/3}$ . The spin component of the magnetoresistance varies at low fields as  $[R(B) - R(0)]/R(0) \sim (B/B_s)^2$ . From the standard theory for the Coulomb interaction [15, 75] one would expect a magnetic field scale which is linear in the temperature,  $B_s \sim 1/\tau_T$ . Experimentally the magnetic field scale varies as  $T^{0.4}$ , which is close to the temperature variation of the dephasing rate. This suggests that the origin of the spin component of the magnetoresistance might indeed be the Coulomb interaction contribution to the conductance, but in the presence of an up to now not identified phase breaking mechanism.

## 4.6 Summary and outlook

In this chapter we discussed the contribution of the Coulomb interaction to the current density in a disordered conductor, with emphasis on applications of the formalism developed in chapter 3. A gauge invariant expression for the Coulomb correction to the current density has been given. In the linear response limit the previously known results are reproduced. The real-time formalism allows a well controlled calculation of the dephasing time. Results beyond linear response have been given.

Clearly, however, there is still room for further investigations. For example it would be interesting to study time dependent phenomena. Response to time dependent fields with frequencies in the microwave regime has been studied in some detail for weak localization, both theoretically [13, 76, 77] and experimentally [78, 79]. In these exper-

iments the influence of a microwave field on the DC-conductivity of thin films was investigated. In the experimental studies not only weak localization but also the interaction effect is present, however there are only few theoretical studies of the interaction correction in the presence of high frequency fields. Altshuler *et al.* [13] calculated the high-frequency (linear) conductivity. A first attempt to calculate the DC-conductivity in the presence of a microwave was made in [A1]. There the microwave field was included in the theory via the diffusion propagators. The distribution function, on the other hand, was considered stationary. Going beyond this analysis, the distribution function could be determined by explicitly solving the kinetic equation in the presence of the microwave field. The distribution function obtained in this way should then be used in the expression for the current density, Eq. (3.20), in order to calculate the interaction contribution to the conductivity in the presence of the microwave field.

Even for a stationary distribution function further investigations are important, for example, in the case of transport through ultrasmall systems. Here, in contrast, we concentrated on systems with diffusive electron motion for systems which are larger than the thermal diffusion length  $L_T = \sqrt{\hbar D/k_B T}$ . The reason for this latter restriction is that a simple interface between the mesoscopic conductor and the leads was assumed: The boundary condition (4.18) describes an ideal interface, *i.e.* the conductance of the leads is infinitely large so that an electron which arrives at the interface escapes into the leads and never comes back. For a realistic description of small conductors a more realistic treatment of the interface is necessary. Assume for example that there is a finite conductance of the interface which can be described by a tunneling Hamiltonian. It is well known that the Coulomb interaction plays an important role in the transport through tunnel junctions in a disordered system, since the density of states is reduced near the Fermi energy [19, 20, 33, 34, 40, 80–82]. This leads to a suppression of the tunnel conductance and to a nonlinear current voltage characteristics at low bias. On the other hand Coulomb blocking of tunneling may also be due to charging effects [83–87]. These two classes of phenomena were described on the same footing for the first time by Nazarov [88, 89] and later by various authors [81, 90–95]. In structures where the total conductance is neither fully determined by the interfaces, nor by the diffusive motion of the electrons, it is necessary to treat the Coulomb blocking of tunneling and Coulomb corrections to diffusive transport on the same footing. This might be relevant for the experiment of Weber *et al.* [56], where transport through a metallic nanobridge connected to two large reservoirs has been studied.

Last but not least in very clean or very small samples the electron motion is ballistic, not diffusive. A theory for the Coulomb correction to transport in ballistic systems has recently been put forward by Zala *et al.* [38] in order to describe transport at intermediate temperatures, where the thermal time  $\tau_T$  is comparable to the (elastic) scattering time  $\tau$ . At present, however, the theory has not yet been applied to finite size systems.

# Chapter 5

## Dynamical defects in metals

Impurities in a metal are often considered as static defects. In this case the impurities contribute only to elastic but not to inelastic scattering. This is different in the presence of dynamical defects. Possible realizations of dynamical defects are magnetic impurities or defects which can tunnel between two or several positions. The interaction of conduction electrons with dynamical defects has been suggested to be relevant for certain experiments in mesoscopic physics, such as the zero-bias anomalies observed in nanoconstrictions [96–99], persistent currents in rings [A4–A6], energy relaxation rate in wires [55, 100–102], and low temperature dephasing [62, 104–107][A7]. In this chapter we summarize results on the subjects where the present author was involved.

### 5.1 Persistent currents in rings

The magnetic response of a small metallic ring varies periodically as a function of the enclosed magnetic flux  $\phi$ . Lévy *et al.* [108] measured the nonlinear magnetic response of an ensemble of  $10^7$  mesoscopic copper rings. The measured signal varied periodically with the period of half a flux quantum  $\phi_0/2 = h/2e$ , corresponding to a current  $I \approx I_{h/2e} \sin(4\pi\phi/\phi_0)$  circulating in each ring. For temperatures in the mK regime the amplitude was  $|I_{h/2e}^{\text{exp}}| \approx 0.3$  nA per ring, which is of the order of one elementary charge divided by the time electrons need to diffuse around a ring,  $|I_{h/2e}^{\text{exp}}| \approx 0.6 e/\tau_d$ . For reviews on the subject see [109].

Calculations which neglect electron-electron interactions underestimate the experimentally observed current by about two orders of magnitude. The Coulomb interaction enhances the average current considerably above the value for non-interacting electrons [110], with the result  $I_{h/2e}^{\text{Coul}} \sim \mu^* e/\tau_d$ , where  $\mu^*$  is a dimensionless parameter which characterizes the strength of the interaction in the Cooper channel; for a metal which does not become superconducting even at the lowest temperatures,  $\mu^*$  is expected to be positive, and much smaller than one. The theory of Ambegaokar and Eckern [110] describes well certain aspects of the experiments: the periodicity of  $I(\phi)$  is predicted correctly, the amplitude  $I_{h/2e}$  is of the order  $e/\tau_d$ , and also the temperature

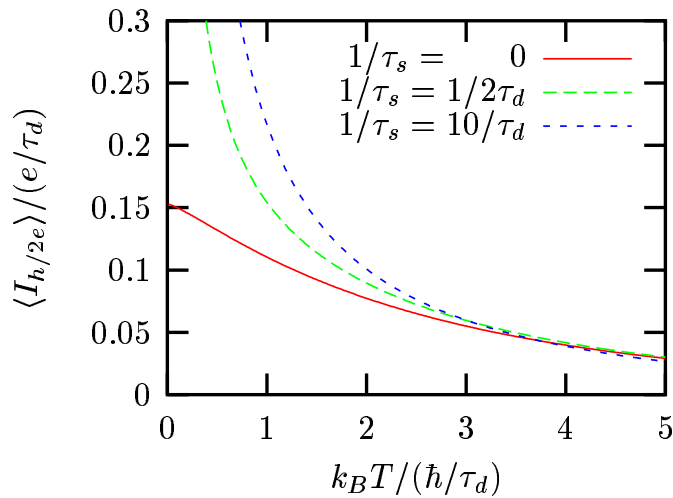


Figure 5.1: Theoretical prediction for the persistent current in the presence of magnetic impurities and Coulomb interaction [A6]. Here the Coulomb interaction parameter has been estimated as  $\mu^* = 0.06$ .

dependence is reasonable. Nevertheless the agreement is not satisfactory: Although the current is of the right order of magnitude, the theoretical prediction is still numerically smaller than the observed value.

In [A4-A6] it has been pointed out that in the presence of dynamical impurities, the persistent current may have a sizeable contribution through what can be called an “effective impurity mediated interaction”. For example, in the case of magnetic impurities which are weakly coupled to the conduction electrons (coupling constant  $J$ ), this effective interaction is proportional to the impurity susceptibility and the density of impurities,  $V^{\text{eff}} \sim nJ^2\chi$ . Figure 5.1 shows the theoretically predicted temperature dependence of the persistent current in the presence of both magnetic impurities and Coulomb interaction. When the spin-flip scattering time  $\tau_s$  is comparable to the diffusion time  $\tau_d$  the low temperature persistent current is enhanced.

Similar results have been obtained in [A4] for magnetic impurities which are strongly interacting with the conduction electrons, *i.e.* below the Kondo temperature, and in [A5] for conduction electrons interacting via two-level systems. Unfortunately, a systematic experimental study of the persistent current in the presence of magnetic impurities does not yet exist.

Recently it has been suggested by Mohanty [111] and Kravtsov and Altshuler [112] that the “large” experimentally observed persistent current might be related to another problem in mesoscopic physics, the unexpectedly large electron dephasing rate. To substantiate this point of view they calculated the dephasing time and the persistent current in the presence of non-equilibrium electric field fluctuations. They found that a random electric field with short temporal correlations induces a DC current in



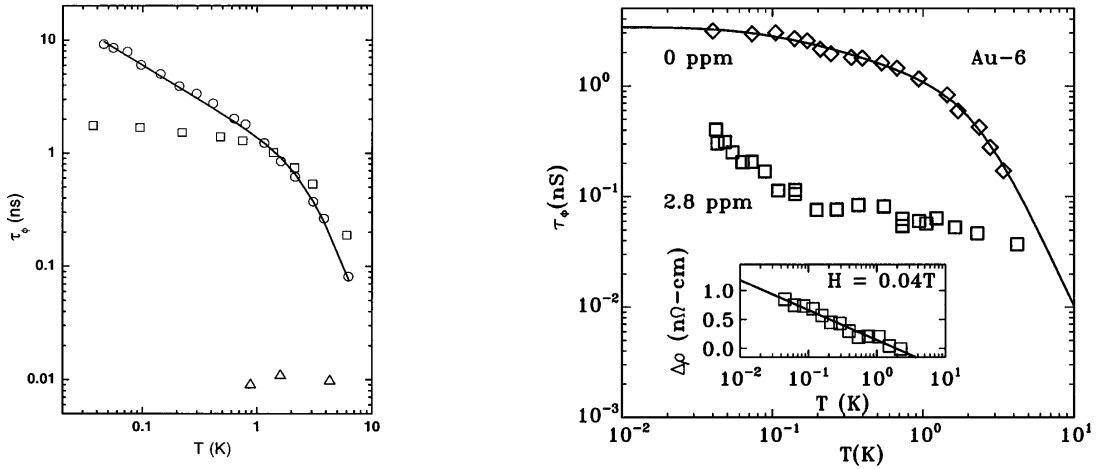


Figure 5.2: On the left hand side, the phase coherence time for the silver (circles), copper (squares), and gold (triangles) samples, taken from [64]. The right figure shows the phase coherence time in a gold wire before and after iron implantation, [59]. In the inset the log  $T$  behavior of the resistivity due to magnetic impurities is seen.

mesoscopic rings of order of

$$I_{h/2e} \sim \frac{e}{\tau_\phi} \exp(-L/L_\phi), \quad (5.1)$$

where  $\tau_\phi$  and  $L_\phi$  are the phase coherence time and length due to the same random electric field. So far, in all experiments on persistent currents the circumference  $L$  of the rings was of the order  $L_\phi$ . In this case  $I_{h/2e}$  is of the order  $e/\tau_d$  as the experimentally observed current. Qualitatively the same observation has been made for other mechanisms [A4,A5]. If phase breaking is dominated by magnetic impurities or by two-level systems, and  $\tau_\phi \sim \tau_d$ , then the persistent current due to the impurity mediated electron-electron interaction is of the order  $e/\tau_d$ , as shown in Fig. 5.1 for the case of magnetic impurities.

A brief review of the existing experimental and theoretical results on this subject can be found in [A6].

## 5.2 Electron dephasing

At low temperature all inelastic scattering processes are expected to freeze out and accordingly the inelastic scattering time and the dephasing time are expected to diverge when the temperature decreases to zero. Contrary to this expectation, in many experiments  $\tau_\phi$  saturates in the low temperature limit. Traditionally the low temperature saturation has been attributed to magnetic impurities or heating which both are difficult to exclude experimentally. Some data from Gougam *et al.* [64] and Mohanty *et al.* [59] are shown in Fig. 5.2. Gougam *et al.* found a low temperature saturation in gold and copper samples, but not in silver. At first they considered it unlikely that magnetic

impurities could be the reason for the saturation of the dephasing time. Meanwhile, however, this explanation is considered to be very likely, since in the samples with strong phase breaking, a logarithmic temperature dependence of the resistance due to the Kondo effect has been observed. Also the measured energy relaxation rate can consistently be explained with the assumption of magnetic impurities [55, 100–102]. In a later experiment on very pure gold samples, Pierre *et al.* [103] found a temperature dependent  $\tau_\varphi$  down to the lowest measured temperatures. Experimental data from Mohanty *et al.* [59] for one particular gold wire before and after iron implantation are shown on the right hand side in Fig. 5.2. Here the situation is different from the one discussed above. Before iron implantation the dephasing rate saturated at low temperature. In some samples also the resistance saturated [60], only after iron implantation a  $\log T$  behavior due to the Kondo effect was seen. Due to the absence of the  $\log T$  behavior before iron implantation we believe that magnetic impurities cannot explain the low temperature saturation of  $\tau_\varphi$  in these samples. Non-equilibrium effects are difficult to be excluded as the reason for the low temperature saturation of  $\tau_\varphi$ . For example, when increasing the voltage a saturation of the dephasing rate was seen by Ovadyahu [107] even though the resistance continued to increase with decreasing temperature. Mohanty *et al.* [59] stressed that they observed a saturation of  $\tau_\varphi$  even after excluding these possibilities. The experiment triggered off a large amount of theoretical and experimental activity, as documented in a recent review [58].

The possibility of dephasing by dynamical defects with a rate proportional to the temperature was mentioned in [113]. In [A7] and [62] it has been pointed out that a temperature independent rate is also possible, which could then explain the observed low temperature saturation of  $\tau_\varphi$ . For a model of a dynamical defect consider, for example, a two-level system with asymmetry  $\Delta$  and tunneling matrix element  $\Delta_0$ . The inelastic two-level system electron cross section is then proportional to

$$\sigma_{\text{in}} \sim \frac{\Delta_0^2}{\Delta^2 + \Delta_0^2} \quad (5.2)$$

as long as the temperature is larger than  $\sqrt{\Delta^2 + \Delta_0^2}$ . In the presence of many two-level systems the total inelastic scattering rate and its temperature dependence are a function of the distribution of the parameters  $\Delta$  and  $\Delta_0$ ,  $P(\Delta, \Delta_0)$ . With the standard assumption [114] of a flat distribution for both  $\Delta$  and the logarithm of  $\Delta_0$ , *i.e.*  $P(\Delta, \Delta_0) \sim 1/\Delta_0$ , the inelastic scattering rate remains temperature independent as long as  $k_B T > \Delta_0^{\text{max}}$ , and goes to zero for lower temperature.

The question is then whether the number of dynamical defects in the materials is large enough to explain the dephasing. The answer depends on the specific material. Aleiner *et al.* [115] concluded that the saturation of  $\tau_\varphi$  in the gold wires of Mohanty *et al.* [59] would require a two-level system density of states which exceeds its typical value for metallic glasses by one to two orders of magnitude. Similar results were obtained by Ahn and Mohanty [104] who estimated the two-level systems required to

explain  $\tau_\varphi$  in open quantum dots [116,117]. Aleshin *et al.* [105] observed a temperature independent  $\tau_\varphi$  in heavily doped polyacetylene. They emphasized that for the polymer materials the possibility for formation of multistable defects is favorable due to presence of “free volume”. In particular the measured value of  $\tau_\varphi$  is consistent with a reasonable defect concentration. Also in  $\text{In}_2\text{O}_{3-x}$  films [107] and in heavily doped Si samples [106] dephasing is believed to be dominated by dynamical defects, although in these latter examples  $\tau_\varphi$  is temperature dependent with  $1/\tau_\varphi \sim T$ .

Since a large number of dynamical defects is required in order to be relevant for phase breaking, the same defects also modify other physical quantities. According to Aleiner *et al.* [115] substantial effects are expected in the specific heat and ultrasound attenuation. Implications on persistent currents in rings were analyzed in [A5].

Imry *et al.* [A7] related phase breaking to conductance noise. This theory applies to materials where the noise is dominated by a mechanism which is closely related to the so-called universal conductance fluctuations, and which was first described by Feng, Lee and Stone [118]. In particular, it was predicted in [A7] that when dephasing is dominated by defect motion, then the conductance noise from a phase coherent volume is near the maximum possible value, *i.e.*  $\langle(\delta G)^2\rangle \sim (e^2/h)^2$ , and the noise is “saturated”. This prediction is consistent with the observations made in [106].

Zawadowski *et al.* [62] suggested a somewhat different mechanism for dephasing by two-level systems. For two-level systems with sufficiently small intervalley barriers “Kondo physics” is important. The low temperature physics is that of a two-channel Kondo model and exhibits non Fermi liquid behavior. In the non Fermi liquid regime, the single-to-many particle scattering rate remains finite as the temperature decreases to zero, see [62] for details. Zawadowski *et al.* argued that the single-to-many particle scattering causes dephasing, and therefore they expect a finite dephasing rate as the temperature drops to zero. So far this argument has not been checked in an explicit calculation of  $\tau_\varphi$ , but the clarification of the question whether zero temperature dephasing is possible in non Fermi liquids certainly is an urgent issue.

# Chapter 6

## Transport through a quantum dot

A quantum dot is a small device where the electrons are confined in all three spatial dimensions. Typical dimensions range from nanometers to a few microns. Due to the confinement the energy levels in the dot are quantized. A voltage applied to one of the gate electrodes controls the number of electrons that are confined on the dot. In recent years transport through quantum dots, and in particular the Coulomb blockade and the Kondo effect, have attracted much attention [119–127]: Adding a charge  $Q$  to the dot costs a charging energy  $Q^2/2C$ , where  $C$  is the capacitance. The Coulomb blockade occurs when the Fermi energy of the leads falls in the gap between the ground state energies of the dot with  $N$  and  $N+1$  electrons. If an odd number of electrons is trapped in the dot, the total spin is necessarily non-zero. The localized spin which is coupled to the electrons in the leads mimics a magnetic impurity in the metal, see Fig. 6.1. As a consequence a Kondo resonance forms at low temperature, and transport will occur due to resonance tunneling. The quantum dot which is coupled to two normal conducting leads has been studied in much detail.

In the following we summarize results on the linear conductance. Then we switch to the quantum dot which is coupled to a normal and a superconducting lead. In order

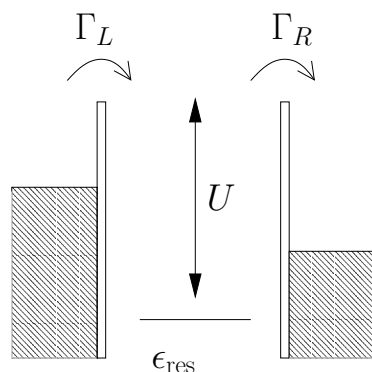


Figure 6.1: Schematic energy diagram of a quantum dot which is coupled to two leads with different chemical potential. The position of the level in the dot can be controlled by a gate voltage. The charging energy is  $U = e^2/C$ .

to obtain transport through such a device, pairs of electrons have to traverse the dot which then form a Cooper pair in the superconducting lead. Thus we may ask, does the Kondo resonance lead to resonant tunneling also in this case? Or does strong Coulomb repulsion always win and block transport? We will summarize the available theoretical results [128–132][A8,A9].

The conductance of the dot which is coupled to two normal conducting leads shows a universal scaling, *i.e.* the temperature dependence can be fitted to a function with only the Kondo temperature as a free parameter. Denoting the tunnel rates through the two barriers by  $\Gamma_L$  and  $\Gamma_R$  the total conductance of the quantum dot is [127]

$$G_{\text{NN}} = \frac{2e^2}{h} \frac{4\Gamma_L\Gamma_R}{(\Gamma_L + \Gamma_R)^2} f(T/T_K). \quad (6.1)$$

For example, at low temperature the universal function is  $f(T/T_K) = 1 - \pi^2 T^2/T_K^2$ , hence the conductance of a symmetric quantum dot ( $\Gamma_L = \Gamma_R$ ) always reaches the value  $G = 2e^2/h$  in the low temperature limit. The function  $f$  has been determined in the full temperature range with the help of the numerical renormalization group in [133] and the Bethe-Ansatz in [134] and agrees well with what is found in experiments [135,136].

What will happen if the quantum dot is coupled to a normal and a superconducting lead? Beenakker [128] addressed this problem neglecting the Coulomb interaction, and found the zero temperature conductance to be given by

$$G_{\text{NS}} = \frac{4e^2}{h} \left( \frac{2\Gamma_N\Gamma_S}{4\epsilon_{\text{res}}^2 + \Gamma_N^2 + \Gamma_S^2} \right)^2, \quad (6.2)$$

where  $\epsilon_{\text{res}}$  is the energy of the resonant level, and  $\Gamma_N, \Gamma_S$  are the tunnel rates through the two tunnel barriers. The conductance on resonance,  $\epsilon_{\text{res}} = 0$ , is maximal when  $\Gamma_N = \Gamma_S$ , and is then equal to  $4e^2/h$ , twice the conductance for resonant tunneling with two normal leads.

In the presence of a strong Coulomb repulsion the situation is not as clear. So far, neither the temperature dependence of the conductance nor the zero temperature conductance as a function of the parameters  $\Gamma_N, \Gamma_S, \epsilon_{\text{res}}$  are known exactly. In [129] it was shown that the current through the dot can be expressed in terms of the retarded and advanced Green's functions of the dot. For  $T \ll \Delta$  the conductance reads

$$G_{\text{NS}} = \frac{e^2}{h} i \int d\epsilon \left( -\frac{\partial f}{\partial \epsilon} \right) \Gamma_N \text{Tr} \left\{ \hat{\tau}_z \hat{G}^R[\hat{\Sigma}^R, \hat{\tau}_z] \hat{G}^A \right\}, \quad (6.3)$$

where  $f(\epsilon)$  is the Fermi function. The Green's functions are two by two Nambu matrices. The quasi-particle current is neglected in (6.3); this is manifest in the commutator  $[\hat{\Sigma}^R, \hat{\tau}_z]$ , which selects the anomalous components of the self-energy.

The Green's functions have been calculated within different approximations. Neglecting the Coulomb interaction on the quantum dot the self-energy is given by

$$\hat{\Sigma}^R \approx -\frac{1}{2} \begin{pmatrix} i\Gamma_N & \Gamma_S \\ \Gamma_S & i\Gamma_N \end{pmatrix}. \quad (6.4)$$

Considering the zero temperature limit, Beenakker's result, Eq. (6.2), is recovered. In the presence of Coulomb interaction, double occupancy of the dot becomes unfavorable and therefore the anomalous Green's function is suppressed. For strong Coulomb interactions and within an equation-of-motion approach [129][A9] the anomalous self-energy is fully suppressed at the Fermi energy, and grows linearly with the distance from the Fermi energy,  $\Sigma_{12}^R \approx \epsilon\Gamma_S/\pi\Delta$ . As a consequence the conductance of the dot is small and vanishes when the temperature decreases to zero.

However, it is known that the equation-of-motion approach is reliable only above the Kondo temperature. The slave-boson mean field approximation, applied to the problem in [A8], is believed to capture qualitatively the correct physical behavior in the opposite limit, when  $T_K \gg \Delta \gg T$ . The Green's function in this regime is given by

$$\hat{G}^R = b_0^2 \left[ \epsilon - \tilde{\epsilon}_{\text{res}}\tau_z + i\frac{1}{2}\tilde{\Gamma}_N + \frac{1}{2}\tilde{\Gamma}_S\tau_x \right]^{-1}, \quad (6.5)$$

where  $b_0$  is a mean field parameter which is controlled by the occupancy  $n_{\text{res}}$  of the resonant level,  $b_0^2 + n_{\text{res}} = 1$ . The zero temperature conductance is of the form (6.2), with effective parameters  $\tilde{\Gamma}_{N,S} = b_0^2\Gamma_{N,S}$ , and  $\tilde{\epsilon}_{\text{res}} \approx 0$ . From this analysis one concludes that the Kondo effect enhances the conductance, and  $G_{\text{NS}} = 4e^2/h$  for the symmetric quantum dot.

Clerk *et al.* [131] studied the situation  $\Delta > T_K$  within a modified version of the non-crossing-approximation. They did not find an enhancement of the zero bias conductance. A possible reason for this is that in [131] only the symmetric situation  $\Gamma_N = \Gamma_S$  has been studied. It has been suggested in [A8] and confirmed by Cuevas *et al.* [132] that the maximum conductance  $G_{\text{NS}} = 4e^2/h$  can also be reached when  $\Delta > T_K$ , however not in the symmetric case, but in a asymmetrically coupled dot with fine-tuned tunnel rates  $\Gamma_N, \Gamma_S$ .

In summary, despite considerable efforts it is at present not completely clear how strong Coulomb interaction modifies transport through a quantum dot which is attached to a normal and a superconducting lead. The reason is that quite different qualitative results – enhanced or suppressed conductance – have been obtained in the regions where the respective approximations are valid. Results obtained with a numerically exact method which works over a large parameter range, such as the numerical renormalization group or a Monte Carlo simulation, could help to settle the issue. Also, an experimental realization of such a device would be most helpful.

# Chapter 7

## Concluding remarks

In this article we discussed transport properties of mesoscopic samples with emphasis on electron-electron interaction effects. Since the article summarizes the authors contributions to the field and is not intended as a comprehensive review, several aspects have been omitted above. We take this opportunity to mention some of these aspects and to speculate about future prospects of the field.

A major part of this article has been devoted the regime of diffusive transport, where we worked out a theory for electron-electron interaction effects out of equilibrium. There is a long list of possible extensions: We concentrated on the ensemble average of the current density, but did not touch the problem of current noise [50, 137] or sample specific conductance fluctuations [138, 139]. We discussed transport properties of films and wires, but so far the formalism has not been applied to more complicated devices. In that case one has to include interfaces in the theoretical description which can be interfaces between normal metals, superconductors, magnets, etc. We restricted ourselves to the limit of diffusive electron motion; in small and clean samples, however, the mean free path can be longer than the systems size, which requires a theory valid for ballistic electron motion.

In relation to the problem of electron dephasing, we have shown that (i) dephasing affects also the interaction correction to the conductivity, and (ii) a temperature independent dephasing rate is possible in the presence of a sufficiently large number of dynamical defects. However there are still many open questions: What is the microscopic nature of the dynamical defects? Do dynamical defects which exhibit non Fermi liquid physics at low temperature, destroy phase coherence even in the zero temperature limit? What is the microscopic origin of the dephasing rate which varies as  $\sqrt{T}$  in gold films [60], and  $T^{1/3}$  in  $\text{Bi}_2\text{Sr}_2\text{CuO}_6$  [66]? Why is the dephasing rate, as determined from the weak localization magnetoresistance, different from the rate determined from conductance fluctuations measurements [140, 141]?

For quantum dots we restricted our discussion to one question, namely the influence of a Kondo resonance on the linear conductivity. This is only one aspect in a large field of phenomena. Quantum effects in the Coulomb blockade have recently been reviewed

by Aleiner *et al.* [127]. Moreover the promising attempts [142, 143] to use quantum dots as devices for quantum computing should be mentioned, too.

To summarize, transport in mesoscopic structures is an active field of research, with several open questions ranging from fundamental problems of quantum theory to possible technological application in the future.



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# Appendix A

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