Dissipationless directed transport in rocked single-band quantum dynamics

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Using matter waves that are trapped in a deep optical lattice, dissipationless directed transport is demonstrated to occur if the single-band quantum dynamics is periodically tilted on one half of the lattice by a monochromatic field. Most importantly, the directed transport can exist for almost all system parameters, even after averaged over a broad range of single-band initial states. The directed transport is theoretically explained within ac-scattering theory. Total reflection phenomena associated with the matter waves traveling from a tilting-free region to a tilted region are emphasized. The results are of relevance to ultracold physics and solid-state physics, and may lead to powerful means of selective, coherent, and directed transport of cold particles in optical lattices.

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I. INTRODUCTION

Optical lattices [1] have offered new opportunities for fundamental research in condensed-matter physics [2–6]. One important example is Bloch oscillations (BO) [7–9] associated with a periodic potential. Due to BO, a static bias becomes useless in generating a net current in the single-band dynamics of a periodic potential. Hence examining how dissipation helps generate directed current of cold atoms and/or molecules across an optical lattice would shed light on how electron current gradually emerges from the interplay of a bias and collision events [10].

Given this circumstance under which no directed transport can be coherently generated by a static bias, an intriguing question arises: how can we, if possible, achieve robust directed transport in an ideal periodic potential with an oscillating force, in the absence of any collision effects? More specifically, are there simple designs to realize generic directed transport involving only one energy band (e.g., the lowest band) of a periodic potential, for a broad range of initial states? Two motivating approaches attacked this fundamental question, but neither of them was able to reach a very positive and definite answer. In particular, the first approach directly copes with BO, with a driving force in resonance with the BO frequency [11–13]. Unfortunately, the direction of the net transport thus obtained depends sensitively on the initial state and on the phase of the driving force. Hence it is not expected that the directed transport survives if the dynamics is averaged over many initial conditions. The second approach relies solely on a driving field that mixes different harmonics of a fundamental frequency [14,15]. However, in addition to the requirement of initial state coherence (consistent with similar findings in "coherent control" [16]), the relative phase between different harmonics should not fluctuate [15]. If the relative phase does fluctuate, then the directed transport was simply transient in the absence of a bath [15], thereby confronted again with the usage of dissipation to generate current in periodic structures.

Dissipationless directed transport in driven single-band quantum dynamics, if exists, can be regarded as a type of "Hamiltonian ratchet effect" [17–21], a timely topic that attracts great interests recently. Many studies of Hamiltonian ratchet effects have focused on model systems with kicking periodic potentials [22–27]. In these model studies the system is a free particle between neighboring kicks, hence it is not trapped inside the periodic potential. As such, if a static bias is allowed to apply to the system, dissipationless directed transport can easily be generated in these systems. Conceptually different is the consideration of Hamiltonian ratchet effect in single-band quantum dynamics, where a static bias simply does not work. Evidently then, dissipationless directed transport in driven single-band quantum dynamics, if established, would constitute a unique class of the Hamiltonian ratchet effect [17–21].

Using matter waves in a deep optical lattice as a possible realization of a tight-binding model Hamiltonian, we propose in this paper a straightforward and powerful approach to *dissipationless, single-band*, and *robust* directed transport in one-dimensional periodic potentials in the presence of a monochromatic driving field. The directed transport results from fully coherent quantum dynamics associated with a zero-mean driving field, and is hence unrelated to any sort of system-bath interaction. Furthermore, the current, irrespective of the details of system parameters, exists even after averaged over a broad range of initial states. The results expose a new face of the interplay of a driving force, energy-band properties, and symmetry breaking in inducing directed transport. Experimental and theoretical implications of our finding are vast.

Computational as well as theoretical results also suggest that an optical lattice with its one half periodically tilted carries important applications for ultracold physics itself. In particular, total reflection of matter waves traveling from a tilting-free region to a tilted region is emphasized in this paper. Such an intriguing aspect of matter waves in an optical lattice can be very useful for blocking or filtering out one particular component in a cold gas mixture, an important topic that is attracting considerable attention [28,29]. How particle-particle interactions might affect the total reflection of the matter waves in a half-tilted optical lattice will be addressed elsewhere [30].

This paper is organized as follows. We first propose in Sec. II our model system describing matter waves moving in a deep optical lattice, half of which are subject to a driving field. This is followed by computational results that demonstrate the dramatic consequences due to the driving field. In Sec. III we develop a simple scattering theory to explain and understand the results. Finally, in Sec. IV we discuss a subtle symmetry-breaking issue, compare this work with other related studies of directed transport of cold atoms, and then draw conclusions.

II. MATTER WAVES IN A HALF-TILTED DEEP OPTICAL LATTICE

A deep optical lattice can be formed by two interfering and counter-propagating strong laser beams. The basic element here is to periodically tilt one half of an optical lattice. Although this is experimentally more demanding than periodically tilting the entire lattice via lattice acceleration, we assume it can be realized and discuss three possible scenarios. One possibility is to apply a driving electric field to one half of the lattice, with the strength of the electric field linearly changing with the lattice site. If cold atoms are in the lattice, then they will experience the static Stark shifts as a linear function of the lattice site. If cold dipolar molecules are in the lattice, then the interaction between the electric dipole and the driving electric field can give an even stronger tilting potential. The second possibility is to take advantage of the magnetic dipole moment of the trapped particles: applying a linearly increasing magnetic field to one half of the lattice will create a half-tilted optical lattice as well. The third scenario is motivated by the so-called phase imprinting technique in manipulating Bose Einstein condensates [31,32]. That is, an additional far off-resonance laser beam covering only one half of the lattice is applied, with the laser intensity linearly varying in space and periodically modulated. Such a laser beam interacts with the cold particles through their induced dipole moment, due to the same mechanism as the optical lattice itself.

With these considerations, the quantum dynamics of the cold particle matter wave can be described by a tight-binding Hamiltonian as follows:

$$H = -J\sum_{n} (|n\rangle\langle n+1| + |n+1\rangle\langle n|) + \cos(\omega t)\sum_{n} nF_n|n\rangle\langle n|, \tag{1}$$

with

$$F_{n\geqslant 0} = F, \quad F_{n<0} = 0. \tag{2}$$

Here, J is the tunneling constant (positive) between neighboring lattice sites, ω is the tilting frequency of an external force, and F is the tilting strength of the force. As clearly indicated by Eq. (2), only the right half of this lattice is tilted periodically. Spatial symmetry is thus broken, but the mean force is zero. Note also that between the tilted region and the tilting-free region, there is no sudden change in the field strength because the tilting field linearly increases its strength from zero. Below we assume $\hbar=1$, and that all systems parameters are scaled dimensionless variables (e.g., the quasimomentum of the system will be given in units of $1/d$, where d is the lattice constant). While focusing on the optical lattice realization, one should recognize that the above tight-binding Hamiltonian may be realized in other contexts, e.g., electrons moving in a semiconductor superlattice with a driving electric field applied to the right half of the superlattice.

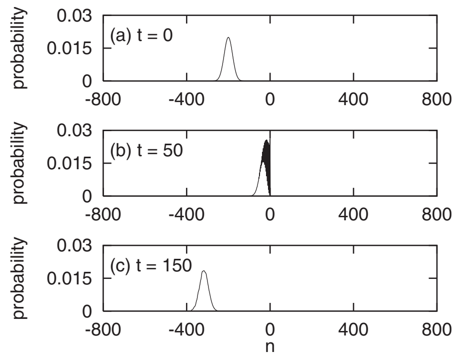

FIG. 1. Complete reflection of a wave packet traveling from left (not tilted) to right (tilted). The system parameters are $\omega=10$, $F=20$, $J=2.0$, and the initial Gaussian wave packet [see Eq. (3)] has a central quasimomentum $k_1=\pi/3$, and a position variance $\Delta_1=20$. Panel (a) shows the initial wave packet, panel (b) shows the wave packet when it is hitting the $n=0$ boundary of tilting, and panel (c) indicates that the wave packet is bounced back to the tilting-free region.

The significant impact of this tilting-half-lattice scenario on the quantum transport of cold particles trapped in the lattice can be first appreciated by directly examining some wave-packet dynamics calculations. As one illuminating example, consider the case of $\omega=10$, $F=20$, and $J=2.0$. The reason why we choose a relatively high driving frequency ω is related to a simple scattering theory developed in the next section (nonetheless, computationally speaking, using a driving field with relatively low frequencies, e.g., $\omega=1.0$, can generate similar, but less generic results). The initial wave packet, denoted $C_1(n)$, is given by

$$C_1(n) = A\exp(ik_1 n)\exp\left[-\frac{(n-n_0)^2}{4\Delta_1^2}\right]. \tag{3}$$

Here $\Delta_1=20$, A is just a normalization constant, $k_1=+\pi/3$ ($k_1=-\pi/3$), and $n_0=-200$ ($n_0=200$) for a wave packet launched from the left (right) side of the lattice.

Figure 1 depicts the fate of such a wave packet initially traveling from left to right. At about $t=50$, this wave packet hits the $n=0$ boundary of the tilting field. Interestingly enough, as manifested by its location at a later time, e.g., at $t=150$, no wave-packet amplitudes are seen to make their journey all the way to the right half of the lattice that is being tilted. Instead, the entire wave packet is seen to bounce back to the tilting-free region. The reflection probability numerically calculated is larger than 99.9%, indicating that this scattering is essentially an event of total reflection.

In clear contrast, Fig. 2 depicts the result if an analogous wave packet is launched from right to left. The first difference is that the wave packet travels at a group velocity much

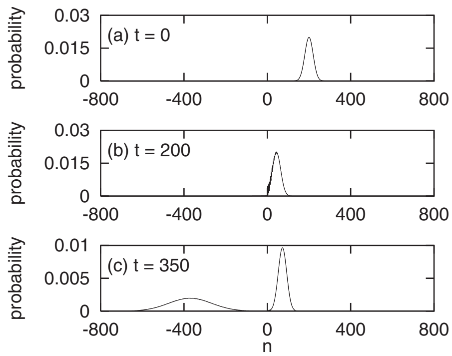

FIG. 2. Significant transmission of a wave packet traveling from the right tilted region to the left tilting-free region. Panels (a), (b), and (c) show the wave-packet shape at three different times. Systems parameters are the same as in Fig. 1, and the initial Gaussion wave packet is a mirror image of that shown in Fig. 1(a). At $t=350$, the probability of finding the atom being at the left half of the lattice is about 50%. This should be compared with the total reflection case seen in Fig. 1.

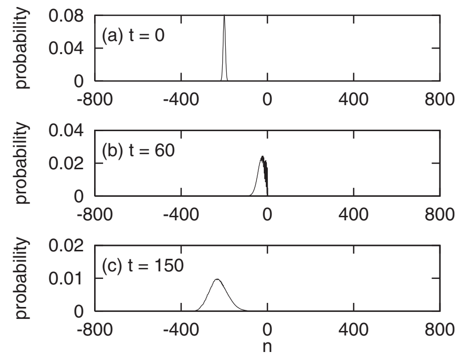

FIG. 3. Complete reflection of a wave packet traveling from left (not tilted) to right (tilted). The system parameters are $\omega=12$, $F=36$, $J=2.0$, and the initial Gaussian wave packet [see Eq. (4)] has a central quasimomentum $k_2=0.8$, and a position variance $\Delta_2=5$. Panel (a) shows the initial wave packet, panel (b) shows the wave packet when it is hitting the $n=0$ boundary, and panel (c) indicates that the wave packet is bounced back to the tilting-free region.

slower than in Fig. 1. Indeed, only until about $t=200$, does the wave packet start to collide with the $n=0$ boundary. But at a later time, about half of this wave packet is able to travel across the $n=0$ boundary, and then continue its travel in the tilting-free region. The other amplitudes of this wave packet are bounced back to the right.

Consider a second sampling case in our wave-packet dynamics calculations. Here $\omega=12$, $F=36$, and $J=2.0$. The initial Gaussian wave packet, denoted by $C_2(n)$, is now given by

$$C_2(n) = A \exp(ik_2 n)\exp\left[-\frac{(n-n_0)^2}{4\Delta_2^2}\right]. \qquad (4)$$

Here n_0 is the same as before, but we choose $\Delta_2=5.0$ to consider much narrower wave packets as initial conditions. As for the central quasimomentum, we choose $k_2=0.8$ for a wave packet launched from the left side of the lattice. For a reason to be explained below, which is related to an expression for the group velocity of wave packets in the tilted region, we find that we should still choose $k_2=0.8$ (instead of $k_2=-0.8$) to launch an analogous wave packet traveling from the right half to the left.

As we deduce from Fig. 3, total reflection of the matter wave also occurs when the wave packet travels from left to right. Because the wave packet in Fig. 3(a) has much larger quasimomentum variance than that in Fig. 1(a), its ensuing spreading is also faster. So when this wave packet hits the boundary [Fig. 3(b)] it is possible to see a similar position variance as in Fig. 1(b). We then place this initial wave packet much closer to the boundary. Total reflection is observed again, and in this case the position variance at the time of boundary hitting is much smaller. By contrast, when an analogous wave packet is launched from right to left (see Fig. 4), significant probability ($>50\%$) can eventually be found in the tilting-free region. These results further confirm that our previous observations made from Fig. 1 and Fig. 2 are general.

The computational results depicted and elucidated above provide a clear-cut case of symmetry breaking: i.e., more particles are transported from right to left than from left to right. However, because the details of the wave-packet dynamics depend on the coherence properties of the initial wave packets and hence differ from shot to shot (especially in experiments), the important question is then if directed transport of cold particles from right to left can survive when we average the quantum dynamics over a distribution of initial conditions, and if yes, can we develop a simple theory to identify the conditions and hence guide the experiments. This is exactly what we will elaborate in the next section.

III. SIMPLE SCATTERING THEORY

To rationalize the computational results we first consider a well-understood approximation in treating a globally tilted lattice by an oscillating linear force $fn\cos(\omega t)$ [4,5]. It can

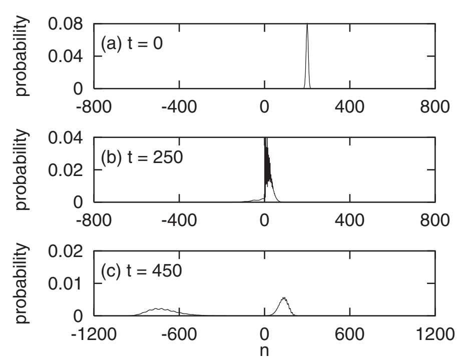

FIG. 4. Significant transmission of an initially narrow wave packet traveling from the right tilted region to the left tilting-free region. Panels (a), (b), and (c) show the wave-packet shape at three different times. System parameters are the same as in Fig. 3, and the initial Gaussion wave packet has a central quasimomentum $k_2=0.8$, and a position variance $\Delta_2=5.0$. The transmission probability is larger than 50%. Results here should be compared with those in Fig. 3, where no transmission is observed.

be easily shown, even at a level of classical Hamiltonian dynamics, that the primary effect of a high-frequency tilting can be accounted for by rescaling the tunneling constant J down to $J\mathcal{J}_0(f/\omega)$, where $\mathcal{J}_0$ is the ordinary Bessel function of order zero. Formally speaking, this approximation arises from a "$1/\omega$" expansion of an exact Floquet theory of the driven quantum dynamics [19]. In the "$1/\omega$" expansion of the Floquet theory, a static Hamiltonian $\tilde{H}$ as the zero-order approximation to the Floquet spectrum is given by

$$\tilde{H} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt \exp[i(f/\omega)n\sin(\omega t)]H_0 \times\exp[-i(f/\omega)n\sin(\omega t)], \tag{5}$$

where H_0 denotes the undriven Hamiltonian. In representation of quasimomentum k, the tight-binding Hamiltonian of a deep optical lattice can be written as $H_0=-2J\cos(k)$. Then, using

$$\exp[i(f/\omega)n\sin(\omega t)]\cos(k)\exp[-i(f/\omega)n\sin(\omega t)] = \cos[k+(f/\omega)\sin(\omega t)], \tag{6}$$

and

$$\exp[iz\sin(\omega t)] = \sum_{l=-\infty}^{+\infty}\mathcal{J}_l(z)\exp(il\omega t), \tag{7}$$

one immediately obtains that Eq. (5) does yield a scaling of J by the factor $\mathcal{J}_0(f/\omega)$. Clearly, this approximation is valid if the tilting frequency is high enough. That is, for a large tilting frequency ω, the probability of finding the system absorbing (releasing) a net photon (energy of $\hbar\omega$) from (to) the driving field in the end is negligible due to a too large energy exchange. Then an effective static Hamiltonian $\tilde{H}$ suffices to describe the driven quantum dynamics. Certainly, within this approach the system is still allowed to absorb and release an equal number of virtual photons.

We now adapt this effective Hamiltonian approach to the case of a half-tilted deep optical lattice. That is, for the right half of the lattice, the primary effect of the tilting can be accounted for by rescaling the tunneling constant J down to J_R, i.e.,

$$J_R = J\mathcal{J}_0(F/\omega). \tag{8}$$

Note that J_R can be negative. Because the left half of the lattice is not tilted, the associated tunneling constant, now denoted J_L, is still given by

$$J_L = J. \tag{9}$$

Given these considerations, we can describe our system by effective Hamiltonians $\tilde{H}_L$ and $\tilde{H}_R$, for the left and right halves of the lattice. That is,

$$\tilde{H}_L = -J_L\sum_{n\leqslant 0}(|n-1\rangle\langle n| + |n\rangle\langle n-1|); \tag{10}$$

$$\tilde{H}_R = -J_R\sum_{n\geqslant 0}(|n\rangle\langle n+1| + |n+1\rangle\langle n|). \tag{11}$$

In representation of the associated quasimomentum k_L or k_R for particles moving on the left or on the right, we have

$$\tilde{H}_L = -2J_L\cos(k_L), \tag{12}$$

$$\tilde{H}_R = -2J_R\cos(k_R). \tag{13}$$

Such dispersion relations yield the following group velocities

$$v_L = 2J_L\sin(k_L), \tag{14}$$

$$v_R = 2J_R\sin(k_R). \tag{15}$$

In particular, the above expression of v_R indicates that when J_R is negative, then one needs to have a positive $\sin(k_R)$ to have a group velocity in the negative direction. This explains why in the case of Fig. 4 we use $k_2=0.8$, instead of $k_2=-0.8$, to launch a wave packet from right to left.

The essence of the quantum dynamics for our system is now reduced to a quantum scattering problem as a particle travels across two regions with different dispersion relations. Great caution, however, is required because these dispersion relations are distinctively different from those for free particles. A trial wave function for a left-to-right scattering event can be written as

$$\psi_L(n\leqslant 0) = \exp(ik_L n) + r_{LR}\exp(-ik_L n), \tag{16}$$

$$\psi_R(n\geqslant 0) = t_{LR}\exp(ik_R n), \tag{17}$$

where

$$1 + r_{LR} = t_{LR}, \tag{18}$$

$$J_L\cos(k_L) = J_R\cos(k_R). \tag{19}$$

Considering the sign of the group velocity v_L, we require $k_L\in[0,\pi]$. Otherwise the group velocity v_L of the incoming wave would be negative, contradicting our assumption. Analogously, $k_R\in[0,\pi]$ if $J_R\geqslant 0$ and $k_R\in[-\pi,0]$ if $J_R\leqslant 0$. Substituting $\psi_R(n)$ and $\psi_L(n)$ into the discrete Schrödinger equation associated with $\tilde{H}_L$ and $\tilde{H}_R$, and then evaluating the coefficient at site $n=0$, we obtain

$$J_L(r_{LR} - r_{LR}^*) = J_R(t_{LR} - t_{LR}^*)\exp[-i(k_L + k_R)]. \tag{20}$$

Equation (20), together with the condition (19), suffice to guarantee that r and s are real variables. Moreover, requiring that the probability at site $n=0$ is constant, we obtain

$$2J_L\sin(k_L) = 2J_L\sin(k_L)r_{LR}^2 + 2J_R\sin(k_R)t_{LR}^2. \tag{21}$$

Indeed, recalling the group velocities v_L and v_R, the left-hand side of Eq. (21) is seen to represent the total incoming flux, which equals the reflected flux $2J_L\sin(k_L)r_{LR}^2$ plus the transmitted flux $2J_R\sin(k_R)t_{LR}^2$.

With Eqs. (18), (19), and (21), one finds

$$t_{LR} = \frac{2J_L \sin(k_L)}{J_R \sin(k_R) + J_L \sin(k_L)}, \tag{22}$$

with

$$k_R = \arccos\left[\frac{\cos(k_L)}{\mathcal{J}_0(F/\omega)}\right] \tag{23}$$

for $\mathcal{J}_0(F/\omega) \geqslant 0$, and

$$k_R = -\pi + \arccos\left[\frac{\cos(k_L)}{\mathcal{J}_0(F/\omega)}\right] \tag{24}$$

for $\mathcal{J}_0(F/\omega) < 0$. The same procedure can be applied to right-to-left scattering. In particular, the analogous transmission amplitude t_{RL} for right-to-left scattering is found to be

$$t_{RL} = \frac{2J_R \sin(k_R)}{J_L \sin(k_L) + J_R \sin(k_R)}, \tag{25}$$

with k_L given by

$$k_L = \arccos[\mathcal{J}_0(F/\omega)\cos(k_R)]. \tag{26}$$

With regard to the derivations of the reflection and transmission amplitudes, additional remarks are necessary. It is very tempting to apply familiar free-space scattering treatments to the situation here. For example, one may naively require the derivative $\partial\psi_L(n)/\partial n$ to be continuously connected with the derivative $\partial\psi_R(n)/\partial n$ at $n=0$. This would be an incorrect procedure because the connection between the flux operator and the momentum operator is much different from that in free space. However, a less rigorous, but enlightening approach in deriving Eq. (22) does exist by making a more sensible analog to the familiar scattering theory in free space. Specifically, in virtue of the fact that the quantum flux operator here is directly related to $J_L \sin(i\partial/\partial n)$ and $J_R \sin(i\partial/\partial n)$, we have

$$J_L \sin\left(i\frac{\partial}{\partial n}\right)\psi_L(0) = J_R \sin\left(i\frac{\partial}{\partial n}\right)\psi_R(0). \tag{27}$$

With this requirement and Eq. (18) one can obtain the same scattering results as above.

Intriguing physics can be deduced upon inspecting Eqs. (23) and (24). That is, if the right half of the lattice is tilted such that

$$\left|\frac{\cos(k_L)}{\mathcal{J}_0(F/\omega)}\right| > 1, \tag{28}$$

then for such k_L there is no solution for k_R, implicitly assumed to be real in the trial wave function. One might argue that when a real solution of k_R does not exist, then an imaginary k_R could offer a solution describing a state exponentially decaying in the right half of the lattice. Interestingly, this is not the case, because the trial wave function $\psi_R(n \geqslant 0)$ [see Eq. (17)] with an imaginary k_R can never satisfy the effective, stationary Schrödinger equation of the discrete system here. Clearly, when the inequality (28) holds, then k_R does not exist and hence t_{LR} must be zero. That is, no transmission is allowed for the left-to-right scattering, thereby theoretically confirming our previous observations made from Fig. 1 and Fig. 3. By contrast, in the case of right-to-left scattering, for arbitrary k_R a solution for k_L is guaranteed [see Eq. (26)]. This is evident because $\arccos[\mathcal{J}_0(F/\omega)\cos(k_R)]$ is always well defined [note that $|\mathcal{J}_0(F/\omega)| \leqslant 1$]. This identifies a strongly broken symmetry, suggesting the possibility of more particles transported from right to left than transported from left to right.

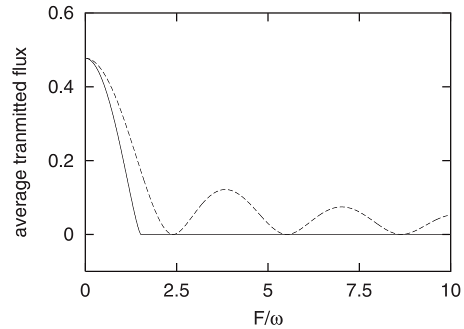

FIG. 5. The transmitted flux as a function of F/ω, averaged over a broad range of single-band initial states (see the text for details). Solid line is for left-to-right scattering $[\bar{\Phi}_{LR}(\Delta_k)]$ and dashed line is for right-to-left scattering $[\bar{\Phi}_{RL}(\Delta_k)]$, with $\Delta_k=\pi/3$. $\bar{\Phi}_{LR}(\Delta_k)$ is seen to be generically smaller than $\bar{\Phi}_{RL}(\Delta_k)$. Note that when F/ω exceeds a threshold value, even the averaged left-to-right flux is always zero. The difference between $\bar{\Phi}_{LR}(\Delta_k)$ and $\bar{\Phi}_{RL}(\Delta_k)$ gives rise to a net right-to-left flux of cold particles.

To emphasize that the above observation is a rather general feature for a broad range of initial states, we now consider the average transmitted flux $\bar{\Phi}_{LR}$ for left-to-right scattering. The averaging is over a range $[0,\Delta_k]$, with the convenient assumption that each quasimomentum state within this range has equal probability. Then

$$\bar{\Phi}_{LR}(\Delta_k) = \frac{1}{\Delta_k}\int_0^{\Delta_k} t_{LR}^2 J_R \sin(k_R) dk_L. \tag{29}$$

In the same fashion, the average transmitted flux $\bar{\Phi}_{RL}$ for right-to-left scattering can be defined, i.e.,

$$\bar{\Phi}_{RL}(\Delta_k) = \frac{1}{\Delta_k}\int_0^{\Delta_k} t_{RL}^2 J_L \sin(k_L) dk_R \tag{30}$$

for $J_R \geqslant 0$, and

$$\bar{\Phi}_{RL}(\Delta_k) = \frac{1}{\Delta_k}\int_{-\pi}^{-\pi+\Delta_k} t_{RL}^2 J_L \sin(k_L) dk_R \tag{31}$$

for $J_R < 0$.

Figure 5 compares $\bar{\Phi}_{LR}(\Delta_k)$ with $\bar{\Phi}_{RL}(\Delta_k)$ as a function of F/ω, for $\Delta_k=\pi/3$, a case representing severe averaging over a broad range (but still less than half of the entire range) of initial quasimomentum states. It is seen that except for zero-measure cases (also discussed below), $\bar{\Phi}_{LR}(\Delta_k)$ is always less than $\bar{\Phi}_{RL}(\Delta_k)$. Their difference indicates that, for arbitrary tilting frequency ω and arbitrary tilting strength F, there generically exists a net transport of particles from right to left. Even more significant, when F/ω exceeds a threshold value (i.e., when $\mathcal{J}_0(F/\omega)<0.5$ in the case of $\Delta_k=\pi/3$), then total reflection occurs for any k_L within $[0,\Delta_k]$, hence $\bar{\Phi}_{LR}(\Delta_k)=0$ whereas $\bar{\Phi}_{RL}(\Delta_k)$ can be significant. As seen from Fig. 5, this leads to a truly dramatic effect with a broad range of initial states averaged over: only particles launched from the right can travel to the left, and no particle is allowed to travel from the left half to the right half. For these cases, the results in Fig. 5 are also indicative of how F/ω must be tuned in order to generate an optimal transmission flux from right to left.

It should be noted, however, that the zero flux from left to right, as shown in Fig. 5, is a theoretical result based on a treatment with the static Hamiltonians $\tilde{H}_L$ and $\tilde{H}_R$. The actual flux from left to right might not be mathematically zero, but should be extremely small. Indeed, in the complete reflection cases considered in Fig. 1 and Fig. 3 where initial Gaussian wave packets are used, the reflection probability never assumes exactly 100%, but is extremely close to 100%.

Notably, as also elucidated with Fig. 5, when $\mathcal{J}_0(F/\omega)=0$ for $F/\omega=2.4\ldots,\ 5.5\ldots,\ \ldots,$ then both $\bar{\Phi}_{LR}(\Delta_k)$ and $\bar{\Phi}_{RL}(\Delta_k)$ are zero and no directed transport can be generated. Indeed, in these cases the "communication" between the left and right is cutoff, as a direct consequence of tilting-induced localization [4,5,19,33–36]: particles on the right half cannot even tunnel between neighboring sites. A similar situation happens if we apply a static force only to the right half of the lattice. Then, particles on the right cannot travel due to Bloch oscillations, and particles in the left half can travel and will be bounced back from the $n=0$ boundary. Because a nonzero right-to-left transmission is necessary to achieve directed transport from the right end to the left end of the lattice, it becomes clear that the directed transported induced by a half-tilted lattice with $\mathcal{J}_0(F/\omega)\neq 0$ lies not only in the total reflection in left-to-right scattering, but also in the significant transmission in right-to-left scattering.

Can we still have directed transport if we average the dynamics over all possible single-band initial states? Interestingly, it can be easily proved that if $\Delta_k=\pi/2$ (averaging over a half-filled band) or $\Delta_k=\pi$ (averaging over a completely filled band), then under the strong assumption that each quasimomentum state still has equal probability one obtains

$$\bar{\Phi}_{LR}(\pi/2)=\bar{\Phi}_{RL}(\pi/2) \tag{32}$$

and

$$\bar{\Phi}_{LR}(\pi)=\bar{\Phi}_{RL}(\pi), \tag{33}$$

both of which result in a *vanishing* net flux. Together with the results shown in Fig. 5, this theoretical result has implications for experiments. That is, to observe a net transport of particles from right to left, one must have a certain degree of control over how particles are injected into the lattice. For example, if particles are injected such that more particles occupy the states at the bottom of the single band than other states, then the result $\bar{\Phi}_{LR}(\pi/2)=\bar{\Phi}_{RL}(\pi/2)$ or $\bar{\Phi}_{LR}(\pi)=\bar{\Phi}_{RL}(\pi)$ becomes irrelevant. Indeed, for these cases the averaging should be over a range $\Delta_k<\pi/2$. The associated results are then expected to be analogous to that seen in Fig. 5 and directed transport of cold particles can be safely predicted.

The required control of how cold particles should be injected into the optical lattice suggests that certain degree of spatial coherence of the initial states is needed in order to observe the directed transport. As already implied by the results in Fig. 3 and Fig. 4 where narrow Gaussian wave packets are considered as initial conditions, this requirement of initial state coherence properties can be easily met. Indeed, using an uncertainty relation, one obtains that as long as the initial wave packet spans over several lattice sites, then the variance in the quasimomentum will be sufficiently small (e.g., $<\pi/3$) to ensure the directed transport. Fortunately this requirement does not present any difficulty in today's experiments with cold particles. Indeed, loading cold atoms into an optical lattice with a particular quasimomentum in a particular energy band was achieved experimentally in Refs. [37,38].

IV. DISCUSSION AND CONCLUSION

The simple scattering theory in Sec. III explains well our computational findings. The theory is based upon an effective, static Hamiltonian arising from the zero-order approximation of a high frequency "$1/\omega$" expansion of the exact Floquet theory. Because the static effective Hamiltonian is always time-reversal symmetric, one might wonder how it is possible to have directed transport of cold particles that seemingly contradicts with the time-reversal symmetry. To clarify this issue, we point out that our results do not contradict with well-established symmetry requirements for directed transport. In particular, for a static Hamiltonian system, one always has [19]

$$\langle n_L|U(t)|n_R\rangle=\langle n_R|U(t)|n_L\rangle, \tag{34}$$

where $|n_R\rangle$ and $|n_L\rangle$ are quantum states describing an atom being localized exclusively at lattice sites n_R and n_L, and $U(t)$ is the propagator associated with the static effective Hamiltonian. Equation (34) hence indicates that, due to the time-reversal symmetry, the probability of transporting a particle exclusively localized at site n_L to site n_R is identical with the probability of transporting a particle exclusively localized at site n_R to site n_L. This is exactly one consequence of Eq. (33). Specifically, for these initial states without any spatial coherence, the initial quasimomenta fill the entire single band with equal probability, therefore a zero net flux is also predicted from our scattering theory. This makes it clear that Eqs. (32) and (33) originate ultimately from the time-reversal symmetry of the system. This leads to the rather formal conclusion that one prerequisite for directed transport to occur in our time-reversal symmetric system is a certain degree of spatial coherence in the initial states.

We now stress again the important advantages afforded by this work as compared with those in Refs. [11–15]. First, only a single-frequency driving field is used here, with no special condition imposed on the driving frequency. Indeed, given the robustness of our approach, one might conjecture that directed transport may survive fluctuations in the driving frequency. Second, because the results depicted in Fig. 5 have been averaged over a broad range of single-band initial conditions, it becomes clear that even highly mixed quantum states can generate dissipationless directed transport. In other words, only very limited "quantum purity" in the initial states is needed to ensure dissipationless directed transport. These advantages make the tilting-half-lattice scenario a generic and robust approach for directed transport in rocked single-band quantum dynamics.

It is also interesting to compare this work with other related studies of directed transport of cold atoms in optical lattices [39–41]. Experiments in Ref. [39,40] used dissipative optical lattices arising from near-resonant laser beams. To verify dissipationless current here a far detuned, and hence conservative, optical lattice is required. The interesting recent work of Ref. [41] exploits a harmonic-mixing field, a chaotic layer, and peculiar features in the Floquet states as system parameters are suitably tuned, possessing a complex dynamics. By contrast, in our system the directed transport, which occurs in wide parameter regimes, is generated by a regular single-band dynamics.

The ability to induce fully coherent and directed transport of cold particles in its lowest energy band might lead to building blocks in constructing atom circuits with unusual characteristics. For example, the net transport rate here $[\sim(\bar{\Phi}_{RL}-\bar{\Phi}_{LR})]$ is an oscillating function of F/ω, instead of being proportional to a "voltage" $\sim F$. The revealed simple mechanism of a quantum "Maxwell demon" without dissipation also suggests that cold particles in a mixture may be selectively transported in a fully coherent fashion. Likewise, applying the results to single-band quantum transport of electrons, new electronic devices with abnormal current-voltage characteristics, and even new designs of coherent electron pumps become possible.

In conclusion, we show that dissipationless and generic directed transport can emerge from single-band quantum dynamics driven by a monochromatic field, even after averaged over a broad range of initial states. The underlying mechanism of the directed transport is related to total reflection vs significant transmission as the matter wave in a half-tilted optical lattice moves in opposite directions. The results are of fundamental interest to solid-state physics and ultracold physics. Experiments using cold atoms and/or molecules in a deep and half-tilted optical lattice should be able to verify the results of this study.

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