Nonadiabatic electron heat pump

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We investigate a mechanism for extracting heat from metallic conductors based on the energy-selective transmission of electrons through a spatially asymmetric resonant structure subject to ac driving. This quantum refrigerator can operate at zero net electronic current as it replaces hot with cold electrons through two energetically symmetric inelastic channels. We present numerical results for a specific heterostructure and discuss general trends. We also explore the conditions under which the cooling rate may approach the ultimate limit given by the quantum of cooling power.

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I. INTRODUCTION

The increasing miniaturization of electronic devices requires a deep understanding of the generation and flow of heat accompanying electron motion.[1,2] The quantum of thermal conductance, which is independent of the carrier statistics,[3] has been recently measured for phonons[4] and photons.[5] A practical and fundamental issue is the identification of possible cooling mechanisms for electron systems, a subject less developed than its atom counterpart.[6] Adiabatic electron[7,8] and molecular[9] pumps may provide reversible heat engines which would cool with minimum work expenditure. It has also been proposed that normal-superconductor interfaces can efficiently cool the normal metal under appropriate conditions of electron flow.[10,11]

In this paper, we explore an alternative electron cooling mechanism that can operate at zero electric current because it relies on the idea of replacing hot electrons with cold electrons. The cooling concept is schematically depicted in Fig. 1. An asymmetric resonant-tunneling structure is formed by two wells, each of which hosts two quasibound states. The four levels are symmetrically disposed so that the energy difference is smaller in the right (R) than in the left (L) well. On the other hand, the difference between the two upper levels is taken to be the same as that between the two lower ones, both being equal to the driving frequency: $E_{2\mathrm{L}}-E_{2\mathrm{R}}=E_{1\mathrm{R}}-E_{1\mathrm{L}}=\hbar\Omega>0$. In those conditions, electron transport is dominated by two processes: (i) electrons in the R electrode with energy $E_{2\mathrm{R}}$ are inelastically transmitted to the L electrode, where they enter with energy $E_{2\mathrm{L}}=E_{2\mathrm{R}}+\hbar\Omega$, and (ii) electrons in the left with energy $E_{1\mathrm{L}}$ are transmitted to the right while also absorbing a photon. Unlike in thermionic refrigeration,[12,13] we may assume a common chemical potential $\mu=\mu_{\mathrm{L}}=\mu_{\mathrm{R}}$. Then in the right lead, one is effectively replacing *hot* electrons (with energy $\varepsilon>\mu$) with *cold* electrons ($\varepsilon<\mu$). According to this principle, the right electrode is being *cooled* at the expense of heating the left electrode. This mechanism, which relies on the properties of coherent electron transport, may be viewed as the basis of a quantum refrigerator.[14] Under suitable conditions, the two dominant transport mechanisms may cancel each other, yielding a vanishing electric current which prevents electrode charging.

II. HEAT PUMP

The classification of electrons as hot or cold depending on whether its energy is above or below the chemical potential in its electrode is based on the property that the entropy variation in an infinitesimal process is given by $TdS=dU-\mu dN$. For independent electrons, this translates into $TdS=(\varepsilon-\mu)dN$, where ε is the energy of the electrons being added ($dN>0$) or removed ($dN<0$). In a transport context, the entropy and temperature variation rates are determined by the many electron scattering processes continuously taking place at the interface. We always refer to the equilibrium entropy eventually reached in the reservoir for the new values of the conserved quantities energy and particle number.

Since we are ultimately more interested in reducing the temperature than the entropy, it is important to note that their variations are not necessarily proportional to each other. One finds $C_V dT=(\varepsilon-\sigma)dN$, where C_V is the heat capacity and $\sigma\equiv\mu-T(\partial\mu/\partial T)_n$, with n the particle density. In the most interesting case where the total electron number remains invariant on average ($\dot{N}=0$), the *total* entropy and temperature variations are proportional to each other. In the following, we present results for the rate of entropy variation, knowing that it amounts to temperature variation in the most interesting case of constant electron number. Specifically, we compute the heat production rate in lead $\ell=\mathrm{L,R}$ (Refs. 15–18):

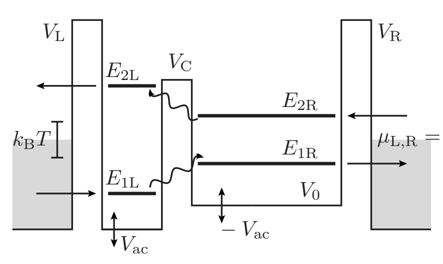

FIG. 1. Asymmetric double-well heterostructure used for electron ac transport calculations. Energy levels are symmetrically placed around the common Fermi level. Dominant transmission processes contributing to cooling are shown: in lead R *hot* electrons are replaced with *cold* electrons, all within a range $\sim k_BT$ around μ.

$$\dot{Q}_\ell = \sum_q (\varepsilon_q - \mu_\ell)\dot{N}_{\ell q}, \tag{1}$$

$N_{\ell q}$ and ε_q being the electron number and energy of state q in electrode ℓ of chemical potential $\mu_\ell = \mu$.

Our goal is to understand the ac thermal transport properties of quantum-well heterostructures, where the electron potential in the perpendicular z direction has the piecewise constant form shown in Fig. 1, while it is uniform in the parallel xy plane. In such a delocalized system, the independent-electron approximation is generally adequate. The bottom of the right well oscillates as $V=V_0+V_{ac}\cos(\Omega t)$, while the left well operates in phase opposition with the same amplitude and frequency. To better focus on the main physical aspects, we analyze first transport through a single channel, later discussing the effect of many channels.

Electron transport properties can be described in terms of scattering probabilities. Within a single-channel picture, the electric current flowing into lead R under ac driving is given by[19]

$$\dot{N}_R = \frac{1}{h}\sum_{k=-\infty}^{\infty}\int d\varepsilon[T_{RL}^{(k)}(\varepsilon)f_L(\varepsilon) - T_{LR}^{(k)}(\varepsilon)f_R(\varepsilon)], \tag{2}$$

where $f_\ell(\varepsilon)$ is the Fermi distribution in lead ℓ and $T_{\ell\ell'}^{(k)}(\varepsilon)$ is the probability for an electron to be transmitted from lead ℓ' to lead ℓ while its energy changes from ε to $\varepsilon + k\hbar\Omega$, k being an integer number. Likewise, it can be shown that Eq. (1) leads to

$$\dot{Q}_R = \frac{1}{h}\sum_{k=-\infty}^{\infty}\int d\varepsilon[(\mu_R-\varepsilon)T_{LR}^{(k)}(\varepsilon)f_R(\varepsilon) + (\varepsilon + k\hbar\Omega - \mu_R)T_{RL}^{(k)}(\varepsilon)f_L(\varepsilon) + k\hbar\Omega R_{RR}^{(k)}(\varepsilon)f_R(\varepsilon)], \tag{3}$$

where $R_{RR}^{(k)}(\varepsilon)$ is the probability that an electron is reflected in lead R from energy ε to $\varepsilon + k\hbar\Omega$. Invoking time-reversal symmetry and the monotonicity of $f_R(\varepsilon)$, it can be proven that inelastic reflection always contributes to heating. Therefore, any possible refrigeration of lead R relying on the transmission scheme depicted in Fig. 1 must be efficient enough to overcome the heating due to inelastic reflection. The electron scattering probabilities are calculated exactly following the transfer-matrix method.[20]

In Fig. 2, we present numerical results for the heat production rate at lead R. The well lengths are 40 and 80 nm; the heights of the barriers are $V_L = V_R = 60$ meV and $V_C = 30$ meV, measured with respect to the bottom of the conduction band; and their widths are 4 and 5 nm, respectively. The difference between the bottoms of the two wells is $V_0 = 1.5$ meV and the effective electron mass is $m^* = 0.07m_e$. This results in $E_{2R} - E_{1R} = 3.4$ meV, as determined, e.g., by the dc transmission characteristics. The structure parameters have been chosen such that $\hbar\Omega = 1.94$ meV coincides with $E_{2L} - E_{2R}$ and $E_{1R} - E_{1L}$. We take μ to lie halfway between E_{1R} and E_{2R}. Clearly, the most negative heat production occurs for $eV_{ac}/\hbar\Omega \sim 0.2$. This results from a combination of nonlinearity, which yields a V_{ac}^2 dependence for small V_{ac}, and the increase of reflection heating [see Fig. 2(b)] reinforced by the suppression of electron transmission through the dominant single-photon channels as $eV_{ac}/\hbar\Omega$ approaches the first zero of the first-order Bessel function.[21] The result is that $|\dot{Q}_R|$ goes through a maximum for a moderate value of $eV_{ac}/\hbar\Omega$.

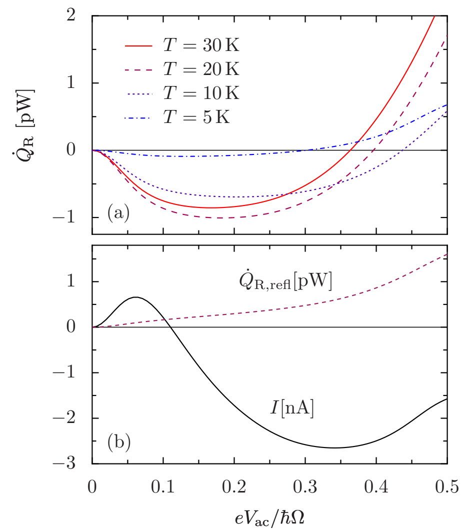

FIG. 2. (Color online) (a) Heat production rate in lead R for the structure of Fig. 1 as a function of the driving amplitude eV_{ac} for various lead temperatures and $\hbar\Omega = 1.94$ meV. See main text for details. (b) For $T = 20$ K, heating contributions from inelastically reflected electrons (solid line) and total electric current (line) are shown.

Another interesting feature is that, as a function of T, the cooling rate is maximized for $T \sim 20$ K, which is roughly $(E_{2R} - E_{1R})/2$. If the temperature is too low, the level 2R is empty and 1R is full, which inhibits the exchange of electrons. If it is too high, the cooling rate saturates as T_R increases, and even decreases slightly because T_L (here equal to T_R) also increases. Later we argue more generally that cooling is optimized when not only $(E_{2R} - E_{1R})/2$ but also $\Gamma/2$ (the half-width of the transmitting channels) is of order $k_B T_R$. Here, $\Gamma/2 \sim 0.2$ meV, noticeably smaller than $k_B T_R$.

The potentially most interesting scenario is that where cooling takes place while the net electric current is zero (in a classical context, see Ref. 22). That this is not generally the case can be inferred from the inset of Fig. 2. If we fix the structure and driving parameters, then the chemical potential and the temperatures are left as the independent variables. If μ is adjusted to satisfy the constraint $\dot{N}_R = 0$, the cooling rate $\dot{Q}_R$ becomes a unique function of T_L and T_R. Figure 3 shows the resulting cooling rate as a function of T_R for several values of T_L. Remarkably, we observe that the heat production in R can be negative even for $T_L > T_R$. We conclude that it is technically possible to extract heat from the cold reservoir and pump it to the hot reservoir with a vanishing net electric current. Thermodynamically, such a refrigeration process requires external work, which here amounts to $2\hbar\Omega$ per useful scattering event and is provided by the classical ac source. In practice, inelastic reflection will further reduce the efficiency.

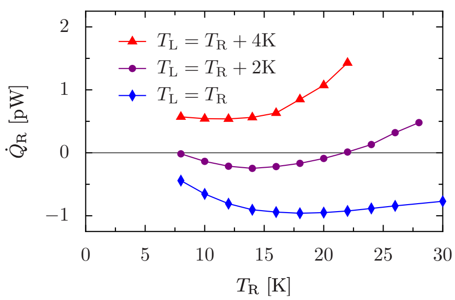

FIG. 3. (Color online) Cooling rate of the R electrode as a function of its temperature T_R for several values of $T_L \geqslant T_R$, with μ adjusted to yield zero electric current, for $\hbar\Omega = 1.94$ meV and $eV_{ac}/\hbar\Omega = 0.2$.

In a three-dimensional (3D) context, one must generalize Eq. (3) to include a sum over transverse modes while replacing μ_R with $\mu_R - \hbar^2\kappa^2/2m^*$, where each channel is characterized by its parallel wave vector $\vec{\kappa}$. For fixed T_L and T_R, $\dot{Q}_R$ remains negative within a finite range of μ values (not shown). This suggests that, after summing the contributions from the many transverse channels, global cooling is still possible in a suitably designed 3D interface.

III. QUANTUM LIMIT

Once we have proven that it is, in principle, possible to pump heat from a cold to a hot reservoir by coherent control of electron transmission, it is natural to ask whether there is any fundamental limit to the maximum cooling rate per quantum channel which would play a role analogous to the quantum of electric or thermal conductance (e^2/h and $\pi^2 k_B^2 T/3h$, respectively). It seems evident that the maximum cooling rate should be achieved in an ideal setup where a metal at temperature T is connected through a totally transparent interface to another metal at the same chemical potential but at zero temperature. The result is the quantum of cooling power:

$$C_Q \equiv |\dot{Q}|_{\max} = \frac{2}{h}\int_0^\infty d\varepsilon \varepsilon f(\varepsilon) = \frac{\pi^2}{6}\frac{k_B^2 T^2}{h}, \qquad (4)$$

where $f(\varepsilon) \equiv [\exp(\varepsilon/k_B T)+1]^{-1}$ and $\pi^2 k_B^2/6h = 473$ fW K^{-2}. Following information theory arguments, a similar result can be derived.[23,24] Differentiation of Eq. (4) yields the quantum of thermal conductance. Invoking only time-reversal symmetry and unitarity, Eq. (3) can be shown to satisfy (with $T_R = T$)

$$\dot{Q}_R \geqslant -C_Q, \qquad (5)$$

for arbitrary electrodes (including $\mu_L \neq \mu_R$) and driving parameters, thus confirming that C_Q is an upper bound to the cooling rate.

The quantum limit may be intuitively understood as follows: $k_B T$ is the maximum amount of heat that can be carried away in an elementary process. Such processes take place at a rate $\sim|\dot{Q}|/k_B T$, which cannot exceed $h/k_B T$ if one is to avoid effective heating caused by energy uncertainty. This results in $|\dot{Q}| \lesssim k_B^2 T^2/h$, as given more precisely in Eq. (4). This argument suggests that $k_B^2 T^2/h$ is also a quantum limit for the cooling rate per active degree of freedom (with characteristic energy scale $\ll k_B T$) when cooling acts on the volume instead of through the surface, as, e.g., in laser cooling.[6,25]

The question that naturally arises is whether in a setup like that of Fig. 1 it is possible to approach the quantum limit. This problem can be explored analytically within a simple model. We neglect reflection heating and assume that electron transmission is dominated by two one-photon inelastic channels, or pipelines[19] (see Fig. 1), named $\tau_u(\varepsilon) \equiv T_{LR}^{(1)}(\varepsilon)$ and $\tau_d(\varepsilon) \equiv T_{LR}^{(-1)}(\varepsilon) = T_{RL}^{(1)}(\varepsilon - \hbar\Omega)$, which peak at energies E_{2R} and E_{1R}, respectively, always satisfying the unitarity requirement $\tau_u + \tau_d < 1$. We take the energy origin at the middle point $(E_{1R}+E_{2R})/2$, so that $E_{2R} = -E_{1R} \equiv \varepsilon_0 > 0$. Electrons entering the scattering region from R with initial energy $\pm\varepsilon_0$ will be transmitted with final energy $\varepsilon_0 \pm \hbar\Omega$ through the upper (lower) channel. If we assume the pipelines to be symmetric, $\tau_u(\varepsilon) = \tau_d(-\varepsilon) \equiv \tau(\varepsilon)$, and $\mu = 0$, we obtain $\dot{N}_R = 0$ and

$$\dot{Q}_R = -\frac{2}{h}\int d\varepsilon \varepsilon [f_R(\varepsilon) - f_L(\varepsilon + \hbar\Omega)]\tau(\varepsilon). \qquad (6)$$

We note that at zero temperature, Eq. (6) only yields heating, as should be expected.

If we take $\tau(\varepsilon)$ to be a Lorentzian of width Γ centered around ε_0, some complications arise due to its slow decay for large $|\varepsilon - \varepsilon_0|$. For instance, for large enough Ω, we always find heating $\dot{Q}_R \propto \ln \Omega$. On the other hand, for small Ω, $\dot{Q}_R < 0$ if and only if $T_L < T_R$. We conclude that, in the interesting case $T_L > T_R$, cooling of the R electrode can only occur within a finite range of Ω values. This range shrinks to zero for T_L large.

An interesting question is whether, given two electrodes with $T_L > T_R$, it is always possible to design an ac resonance structure yielding $\dot{Q}_R < 0$, and whether $|\dot{Q}_R|$ can ever approach the quantum limit. In Eq. (6), $g(\varepsilon) \equiv \varepsilon[f_R(\varepsilon) - f_L(\varepsilon + \hbar\Omega)] > 0$ only in the interval $0 < \varepsilon < \bar{\varepsilon} \equiv \hbar\Omega T_R/(T_L - T_R)$. For $T_L \to T_R$, we have $\bar{\varepsilon} \to \infty$; however, the integrand decays exponentially on a scale $\sim k_B T$ after having peaked at $\varepsilon \sim k_B T$. Therefore, cooling comes effectively from the interval $0 < \varepsilon < \varepsilon_1$, where $\varepsilon_1 \equiv \min\{\bar{\varepsilon}, 2k_B\bar{T}\}$ and $\bar{T} \equiv (T_L + T_R)/2$. To potentiate the contribution from that segment, we may design $\tau(\varepsilon)$ to be centered at $\varepsilon_0 \simeq \varepsilon_1/2$. If $\Gamma \to 0$, $\dot{Q}_R$ is guaranteed to become negative, although with a vanishing magnitude $|\dot{Q}_R| \propto \Gamma$. A typical optimal value is $\Gamma \sim \varepsilon_1$. We conclude that the cooling rate is maximized for $\varepsilon_0 \sim \Gamma/2 \sim \varepsilon_1/2$. The peak at $k_B T \sim \varepsilon_0$ for $T_L = T_R$ is confirmed by the lowest curve of Fig. 3.

If $\tau(\varepsilon)$ decays sufficiently fast away from the region where $g(\varepsilon) > 0$, one may estimate $|\dot{Q}_R| \sim (2/h)\varepsilon_1 \tau_{\max} g_{\max}$. For $T_L \to T_R$, we have both ε_1 and $g_{\max}$ of order $k_B T$, assuming $\hbar\Omega \gg k_B T$. In those conditions, $|\dot{Q}_R| \sim C_Q$ provided $\tau_{\max}$ is close to unity. By contrast, the cooling rate cannot approach the quantum limit if T_L grows substantially above T_R or if $\tau(\varepsilon)$ decays slowly, like in a Lorentzian resonance, since then the contribution from $g(\varepsilon)<0$ cannot be neglected.

Careful inspection of Eq. (6) reveals that $\dot{Q}_R$ increases monotonically as T_R decreases. Thus, if we start cooling the R electrode, $\dot{Q}_R$ begins to increase until it eventually becomes zero. At that point, no further cooling is possible. We have reached the lowest possible temperature for the refrigeration process defined by $\tau(\varepsilon)$ and Ω. We are limited by the lack of sufficient energy resolution: when $k_B T_R$ becomes small compared with the linewidth Γ, no heat pumping is possible for $T_L > T_R$.

IV. DISCUSSION

The quantum refrigerator which we have investigated may be viewed as a realization of Maxwell's demon[26] as it selectively lets hot electrons out while it only lets cold electrons in. The required work is provided by the external ac source which, combined with the spatial asymmetry of the structure, rectifies electron motion. The work might also be extracted from a hot Ohmic resistor.[27] Alternative schemes to that of Fig. 1 are, of course, possible: One may design two superlattices, each of them having two narrow bandwidths yielding a similar level distribution. A potential advantage of such a device would be that, away from resonance, transmission would decay fast. Thus it would show interesting features such as cooling for arbitrarily large Ω and, as discussed above, the guaranteed existence of a driving structure that brings the cooling rate close to the quantum limit. A nonresonant mechanical mismatch at the interface would prevent phonons from short-circuiting electron cooling during operation close to such a limit.

In conclusion, we have identified a mechanism for nonadiabatically pumping heat from a cold to a hot electron reservoir, which is based on the coherent control of electron ac transport and which can operate at zero average electric current. On the basis of electron transport considerations, the quantum of cooling power C_Q has been shown to be an upper bound to the cooling rate per quantum channel. We have investigated the case of Lorentzian resonances, where approaching the quantum limit is generally not feasible. We have noted, however, that with sharper resonances it is always possible to design a driven interface that provides cooling at a rate close to the quantum limit.

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