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Bell-state generation in circuit QED via Landau-Zener tunneling

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Abstract.
A qubit may undergo Landau-Zener transitions due to its coupling to one or several quantum harmonic oscillators. First we show that for a qubit coupled to one oscillator, Landau-Zener transitions can be used for single-photon generation and for the controllable creation of qubit-oscillator entanglement, with state-of-the-art circuit QED as a promising realization. Second, for a qubit coupled to two cavities, we show that Landau-Zener sweeps of the qubit are well suited for the robust creation of entangled cavity states, in particular symmetric Bell states, with the qubit acting as the entanglement mediator. Finally, for a qubit coupled to an environment or bath we propose to employ dissipative Landau-Zener sweeps of the qubit for the detection of bath properties. At the heart of our proposals lies the calculation of the *exact* Landau-Zener transition probability for the qubit, by summing all orders of the corresponding series in time-dependent perturbation theory.

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INTRODUCTION

Entanglement is a purely quantum mechanical property of multipartite systems. A system is entangled if its quantum state cannot be described as a direct product of states of its subsystems. It is measurable in terms of nonclassical correlations of the subsystems. Many efforts exist to make use of entanglement in quantum information processing (QIP) [1]. In this paper we propose to create entanglement in circuit cavity quantum electrodynamics (QED). We consider creation of entanglement between a superconducting qubit and the circuit analogue of an optical cavity and how to generate single photons in the latter [2]. We also demonstrate how two spatially separated circuit cavities can be entangled by letting a superconducting qubit undergo a Landau-Zener (LZ) sweep [3]. This will be a robust method to create Bell states in two-cavity circuit QED. Furthermore, we discuss how LZ sweeps of a qubit in circuit QED or elsewhere can give useful information on the qubit's dissipative environment [4, 5].

In optical cavity QED, atoms (qubits) become entangled with optical cavity modes (oscillators). Two optical-cavity modes can be entangled by adiabatic passage of an atom through one or more cavities [6–8]. It seems technologically challenging to scale up optical cavity QED to many qubits, as would be required for useful QIP. Recently, the

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field of circuit QED has emerged in which analogues of cavity QED have been realized with superconducting qubits and quantum harmonic oscillators [9–12]. Superconducting circuits are promising because of their potential scalability and because many of their parameters are highly tunable. The route proposed here to scale up present-day circuit QED is the circuit analogue of optical two-cavity QED or N-cavity QED [7, 8].

One method to manipulate the state of a qubit is to use Landau-Zener sweeps [13]. Recently, LZ transitions have been observed in superconducting qubits [14–17]. In this paper we concentrate on quantum state manipulation in multi-cavity circuit QED via Landau-Zener sweeps of a qubit. Bit flips in the qubit can take place even in the absence of a direct coupling between the qubit levels, induced instead by the coupling to the oscillators. We studied the semiclassical limit of this model before [18]. Here we focus on the situation in which the oscillators start in their ground states. We show how single photons and symmetric Bell states can be created in two circuit oscillators. Decisive advantage of our proposal is that qubit-oscillator interaction strengths are static, in contrast to standard cavity QED where precise dynamical control is required.

MODEL FOR LZ SWEEPS IN QUBIT-OSCILLATOR SYSTEMS

We consider the time-dependent Hamiltonian for a LZ sweep in a multi-level system

$$H(t) = \sum_a (\varepsilon_a + \frac{1}{2}vt)|a\rangle\langle a| + \sum_b (\varepsilon_b - \frac{1}{2}vt)|b\rangle\langle b| + \sum_{a,b}(X_{ab}|a\rangle\langle b| + X^*_{ab}|b\rangle\langle a|). \quad (1)$$

We assume that all diabatic states $|a\rangle$, $|b\rangle$ are mutually orthogonal and that $v > 0$. In the limit $t \to \pm\infty$, the states $|a\rangle$, $|b\rangle$ become eigenstates of the Hamiltonian (1). The off-diagonal part of the Hamiltonian is such that it only couples states of different groups while states within one group are uncoupled. For a nondegenerate initial state $|a\rangle$ a generalized Landau-Zener transition formula can be derived [5]

$$P_{a\to a} = \exp\Big(-\frac{2\pi\langle a|X^2|a\rangle}{\hbar v}\Big), \quad (2)$$

where the operator X is the interaction term in Eq. (1). The more specific Hamiltonian

$$H(t) = \frac{vt}{2}\sigma_z + \frac{\Delta}{2}\sigma_x + (\cos\theta\sigma_x + \sin\theta\sigma z)\sum_k \gamma_k(b_k + b_k^\dagger) + \sum_k \hbar\Omega_k b_k^\dagger b_k \quad (3)$$

describes a qubit with internal coupling Δ undergoing a LZ sweep with speed v. The qubit has a rather general coupling to N harmonic oscillators. If the system starts in the ground state (i.e. with qubit in state $|\uparrow\rangle$), then LZ formula (2) takes the form [4, 5]

$$P_{\uparrow\to\uparrow}(\infty) = 1 - P_{\uparrow\to\downarrow}(\infty) = \exp\Big(-\frac{\pi W^2}{2\hbar v}\Big); \qquad W^2 = (\Delta - E_0\sin\theta\cos\theta)^2 + S\sin^2\theta. \quad (4)$$

$P_{\uparrow\to\uparrow}(\infty)$ describes the probability for the qubit to end up "up". Here $E_0 = 4E$, with $E = \sum_k \gamma_k^2/(\hbar\Omega_k)$ the reorganization energy of the oscillators and $S = 4\sum_k \gamma_k^2$ their integrated spectral density. Below we consider one, two, and infinitely many oscillators.

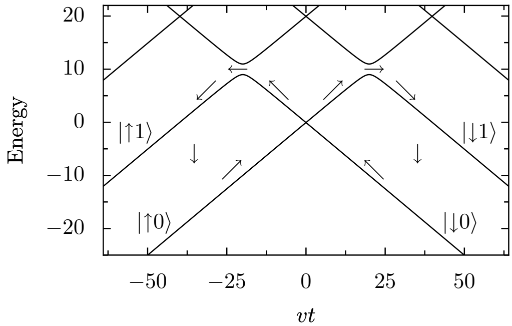

FIGURE 1. Adiabatic eigenstates during LZ sweep of a qubit coupled to an oscillator. Shown is the four-step LZcycle for single-photon generation in circuit QED: The first step is single-photon generation in the cavity via the adiabatic LZ transition $|\uparrow 0\rangle \rightarrow |\downarrow 1\rangle$, brought about by switching the qubit energy sufficiently slowly. Second, the photon is released from the cavity via the (controlled) cavity decay $|\downarrow 1\rangle \rightarrow |\downarrow 0\rangle$. In the third step, another individual photon is generated via the reverse LZ sweep $|\downarrow 0\rangle \rightarrow |\uparrow 1\rangle$. Fourth and finally, a further photon decay completes the cycle. This cycle can be repeated.

QUBIT-OSCILLATOR ENTANGLEMENT

We first consider LZ transitions in the standard cavity QED model of one qubit coupled to one oscillator, with $H(t)$ as in Eq. (3) with $\cos\theta = 1$ and $\Delta = 0$. Since we start out in the ground state $|\uparrow, 0\rangle$ and the Hamiltonian in Eq. (3) correlates every creation or annihilation of a photon with a qubit flip, the dynamics is restricted to the states $|\uparrow, 2n\rangle$ and $|\downarrow, 2n+1\rangle$. Remarkably, by a dynamical selection rule ("*no-go-up theorem*") [2, 5], of the states $|\uparrow, 2n\rangle$ only $|\uparrow, 0\rangle$ stays occupied. The final state can be written as

$$|\Psi(\infty)\rangle = \sqrt{1 - P_{\uparrow\rightarrow\downarrow}(\infty)}\,|\uparrow 0\rangle + \sqrt{P_{\uparrow\rightarrow\downarrow}(\infty)}\,\big(c_1|\downarrow 1\rangle + c_3|\downarrow 3\rangle + \ldots\big), \tag{5}$$

where $P_{\uparrow\rightarrow\downarrow}(\infty) = 1 - \exp(-2\pi\gamma^2/\hbar v)$ and $|c_1|^2 + |c_3|^2 + \ldots = 1$. Qubit and oscillator end up fully entangled. By measuring the qubit in state $|\downarrow\rangle$, a highly nonclassical oscillator state is produced in which only odd-photon states are occupied. In circuit QED [12] and in optical cavity QED one is always in the situation $\gamma \ll \hbar\Omega$, in which case $c_1 \approx 1$. Hence one can control via the sweep velocity v the final state to be any superposition of $|\uparrow 0\rangle$ and $|\downarrow 1\rangle$. In the adiabatic limit $v\hbar/\gamma^2 \ll 1$, the final state becomes $|\downarrow 1\rangle$. This is an important result: triggered by a Landau-Zener sweep of the qubit, exactly one photon can be created in the cavity. In an experiment, the photon will subsequently leak out of the cavity.

By exploiting these two processes, we propose a four-step LZ cycle for single-photon generation in circuit QED, as sketched in Fig. 1. This simple and robust scheme for repeated photon generation via Landau-Zener cycles could be implemented in circuit QED, where the atom-cavity coupling remains at a constant and high value and where qubits are highly tunable so that LZ sweeps can be made from minus to plus an "atomic" frequency, and back [2].

OSCILLATOR-OSCILLATOR ENTANGLEMENT

Now consider the situation that the qubit is coupled to *two* cavities instead of one, with identical resonance frequencies $\Omega_{1,2} = \Omega$ and qubit-oscillator strengths $\gamma_{1,2} = \gamma$. (For $\Omega_1 \neq \Omega_2$, see [3].) We will now show how the two cavity oscillators become entangled by a Landau-Zener sweep of the qubit. We will again assume $\Delta = 0$, so that all bit flips in the qubit are caused by interactions with the oscillators. The Hamiltonian becomes

$$H = \frac{vt}{2}\sigma_z + \gamma\sigma_x(b_1 + b_1^\dagger + b_2 + b_2^\dagger) + \hbar\Omega\left(b_1^\dagger b_1 + b_2^\dagger b_2\right). \tag{6}$$

We will assume that this system starts in the ground state $|\uparrow 0_1 0_2\rangle$. The general result (4) then implies that the probability for the qubit to end up in the state $|\downarrow\rangle$ equals

$$P_{\uparrow\rightarrow\downarrow}(\infty) = 1 - P_{\uparrow\rightarrow\uparrow}(\infty) = 1 - \mathrm{e}^{-2\pi(\gamma^2+\gamma^2)/\hbar v}. \tag{7}$$

This exact result follows without making a rotating-wave approximation and by taking the full Hilbert space of the two oscillators into account. The absence of any frequency dependence in Eq. (7) is therefore quite surprising.

In the following we are interested in the properties of the final qubit-two-oscillator state $|\Psi(\infty)\rangle$. Let us now make the realistic assumption $\hbar\Omega \gg \gamma$: then level crossings that are important for the final state only occur around the times when the qubit energy vt is resonant with the oscillator energies $\hbar\Omega$. There essentially only three qubit-oscillator states play a role in the dynamics: the initial zero-photon state $|\uparrow 00\rangle$ and the two one-photon states $|\downarrow 10\rangle$ and $|\downarrow 01\rangle$. In practice, the final state can therefore be written as

$$|\Psi(\infty)\rangle = \sqrt{P_{\uparrow\rightarrow\uparrow}(\infty)}\,|\uparrow 00\rangle + \sqrt{P_{\uparrow\rightarrow\downarrow}(\infty)}\,\left(s_{10}|\downarrow 10\rangle + s_{01}|\downarrow 01\rangle\right), \tag{8}$$

with probabilities $P_{\uparrow\rightarrow\uparrow}(\infty)$ and $P_{\uparrow\rightarrow\downarrow}(\infty)$ given in Eq. (7) and with general complex coefficients s that are only constrained by $|s_{10}|^2 + |s_{01}|^2 = 1$ to ensure state normalization. The qubit-two-oscillator system has an extra symmetry that we will now exploit in the analysis of the LZ dynamics. Let us first go back to the Hamiltonian (6) and not yet make the rotating-wave approximation. We introduce the new operators $b_\pm = (b_1 \pm b_2)/\sqrt{2}$, which have standard bosonic commutation relations. Both creation operators $b_\pm^\dagger$ create a single photon with equal probability in the first or the second oscillator: $b_\pm^\dagger|0_+0_-\rangle = (|1_1 0_2\rangle \pm |0_1 1_2\rangle)/\sqrt{2}$. Instead of describing oscillator states in a local basis, one can write a general two-oscillator state in terms of creation operators $b_\pm^\dagger$ acting on the ground state $|0_+0_-\rangle$. In terms of the $b_\pm^\dagger$, the Hamiltonian (6) becomes

$$H = \frac{vt}{2}\sigma_z + \sqrt{2}\gamma\sigma_x(b_+ + b_+^\dagger) + \hbar\Omega\left(b_+^\dagger b_+ + b_-^\dagger b_-\right). \tag{9}$$

Note that the qubit is fully decoupled from the antisymmetric operators $b_-^{(\dagger)}$. Consequently, for degenerate oscillator frequencies we find back the mathematical problem for a qubit coupled to *one* oscillator, which we already studied above. Now assume as before that the initial state is $|\uparrow 0_1 0_2\rangle = |\uparrow 0_+ 0_-\rangle$. In our new representation, only the

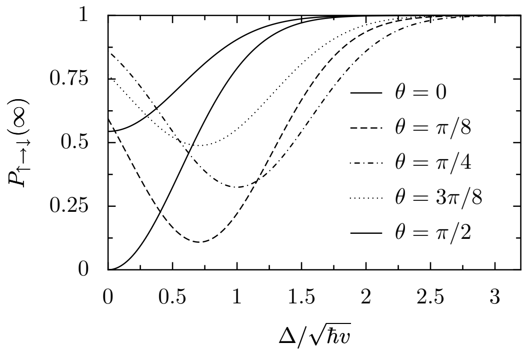

FIGURE 2. Final transition probability $P_{\uparrow\to\downarrow}(\infty)$ as a function of intrinsic interaction Δ, for several values of the coupling angle θ, based on Eq. (4). Bath quantities E_0 and S can be identified by varying Δ, which together provide information on the spectral density of the bath. Here: $E_0 = 2.0\sqrt{\hbar v}$ and $S = 0.8\hbar v$.

two states $|\uparrow 0_+0_-\rangle$ and $|\downarrow 1_+0_-\rangle$ will play a role in the dynamics. In this representation the LZ transition probabilities can be calculated with the S-matrix for the standard LZ two-level LZ problem [2, 3, 18]. Transformed back to the local basis, for the qubit coupled to degenerate oscillators starting in $|\uparrow 0_1 0_2\rangle$ we find the final state

$$|\Psi(\infty)\rangle = \sqrt{q}|\uparrow 0_1 0_2\rangle - \sqrt{1-q}e^{i\chi}\left(\frac{|\downarrow 1_1 0_2\rangle + |\downarrow 0_1 1_2\rangle}{\sqrt{2}}\right),$$

where $q = \exp(-2\pi\eta)$ with adiabaticity parameter $\eta = (\sqrt{2}\gamma)^2/\hbar v$, and χ is the Stokes phase. Clearly, if the qubit is finally measured in state $|\downarrow\rangle$, then the two oscillators end up in the symmetric Bell state $(|1_1 0_2\rangle + |0_1 1_2\rangle)/\sqrt{2}$, a two-particle entangled state important for QIP. Furthermore, for producing other final two-oscillator states one could use instead detuned oscillators that give rise to two independent LZ transitions [3].

PROPERTIES AND USE OF DISSIPATIVE LZ TRANSITIONS

For a qubit coupled to infinitely many oscillators, the Hamiltonian (3) in fact describes a LZ sweep in a dissipative system. Formula (4) then gives the exact LZ transition probability for qubit coupled to a bath at zero temperature. The reorganization energy E and the integrated spectral density S can then be given in terms of an integral over a continuous spectral density [4, 5]. In particular, for a qubit subject to pure dephasing (i.e. $\sin\theta = 0$), the formula gives $P_{\uparrow\to\uparrow}(\infty) = \exp(-\pi\Delta^2/(2\hbar v))$, identical to the standard LZ transition probability of an isolated qubit! This remarkable result settles a controversy about dissipative LZ transition probabilities [19–23]. It even holds universally, irrespective of the nature of the bath [5]. In general, because of the nonmonotonic dependence of $P_{\uparrow\to\uparrow}(\infty)$ in (4) on the internal interaction Δ, we find that dissipative LZ sweeps in a qubit can be used to detect the bath properties E and S by varying Δ [4, 5], see Fig. 2.

CONCLUSIONS

LZ sweeps of superconducting qubits are a robust way to produce single photons in circuit QED and to create symmetric Bell states in circuit oscillators. LZ transition probabilities are insensitive to dephasing at zero temperature. For qubits in a dissipative environment, LZ sweeps of can be a valuable tool to characterize the environment.

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