Shot noise in spin pumps

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The long spin decoherence times measured for electrons in semiconductor quantum dots have spurred great experimental and theoretical efforts to access and control the spin degree of freedom of electrons localized in such systems, in the search of quantum phenomena that allowed to develop spin-based technologies (spintronics). Within this scope, double quantum dots (DQD) play a central role as realizations of two level systems [1] for which coherence can be controlled by time-dependent external fields [2].

A resonant external AC field may induce a spatial delocalization of electrons in the DQD so that coherent Rabi oscillations between states of each dot occur [3]. Together with a geometrical asymmetry, this induces a finite pump current even when no external bias is applied [4]. Based on this idea, spin pumping has been proposed for DQD where the spin degeneracy is lifted by means of a magnetic field [5]. If one allows up to two electrons in each QD and the asymmetry is introduced as a difference in the intradot Coulomb repulsions of each dot (for instance, $U_{\mathrm{R}} > U_{\mathrm{L}}$), one can delocalize an electron between the doubly occupied singlet states of each dot: $|\uparrow\downarrow,\uparrow\rangle$ and $|\uparrow,\uparrow\downarrow\rangle$. Then, choosing the chemical potential μ of the leads such that the Zeeman splitting allows the extraction of electrons with spin down polarization, a spin-polarized current is created.

In this work, we analyze the shot noise properties of such a spin pump, which gives us additional information not contained in the current [6]. Since the dynamics is governed by the electrons with a particular spin polarization (in this case, those with spin down), while those with opposite spin are not affected, one should expect a noise characteristics similar to that for up to two noninteracting electrons [7,8]. There, the states $|\uparrow,\downarrow\rangle$ and $|\uparrow\downarrow,\uparrow\downarrow\rangle$ play the role of the empty and doubly occupied states, respectively. However, the AC field induces photoassisted tunneling through the contact barriers [9–11], allowing the extraction of spin-up electrons. Thus, we have to consider the 16 basis states $|\phi_{\mathrm{L}},\phi_{\mathrm{R}}\rangle$, where $\phi_\ell = 0,\uparrow,\downarrow,\uparrow\downarrow$.

We consider a system that consists of two quantum dots weakly connected in series to two *unbiased* electron reservoirs by tunnel barriers. It is described by the Hamiltonian $\hat{H} = \hat{H}_{\mathrm{L}} + \hat{H}_{\mathrm{R}} + \hat{H}_{\mathrm{L}\Leftrightarrow\mathrm{R}} + \hat{H}_{\mathrm{leads}} + \hat{H}_{\mathrm{T}} + \hat{H}_{\mathrm{AC}}(t)$, where each dot is represented by $\hat{H}_{j=\{\mathrm{L,R}\}} = \sum_\sigma \varepsilon_{j\sigma}\hat{n}_{j\sigma} + U_j\hat{n}_{j\uparrow}\hat{n}_{j\downarrow}$ and the reservoirs: $\hat{H}_{\mathrm{leads}} = \sum_{l\varepsilon\{\mathrm{L,R}\}k\sigma}\varepsilon_{lk}\hat{d}^{\dagger}_{lk\sigma}\hat{d}_{lk\sigma}$. The energies $\varepsilon_{j\sigma}$ include the Zeeman splitting Δ_j stemming from the interaction with a magnetic fiel which breaks the spin degeneracy, such that the spin-up state becomes the ground state. Thus, $\varepsilon_{j\downarrow} = \varepsilon_{j\uparrow} + \Delta_j$. The QDs are weakly

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connected between them and to the leads through tunneling barriers: $\hat{H}_{L\Leftrightarrow R} = -t_{LR}\sum_\sigma \hat{c}^\dagger_{L\sigma}\hat{c}_{R\sigma} + \text{h.c.}$ and $\hat{H}_T = \sum_{l\varepsilon\{L,R\}k\sigma}(\gamma\hat{d}^\dagger_{lk\sigma}\hat{c}_{l\sigma} + \text{h.c.})$.

The external AC field is introduced as an oscillation with different phase in the gate voltages of the dots ($\hbar = e = 1$): $\hat{H}_{AC}(t) = \frac{1}{2}V_{AC}\sum_\sigma(\hat{n}_{L\sigma} - \hat{n}_{R\sigma})\cos\omega t$. By applying a unitary transformation to the Hamiltonian: $\hat{H}'(t) = \hat{U}(t)(\hat{H} - \mathrm{i}\partial_t)\hat{U}^\dagger(t)$, being $\hat{U}(t) = \exp\left[-\mathrm{i}\int_{t_0}^t \mathrm{d}t'\hat{H}_{AC}(t')\right]$, the time dependence is transferred from the energy levels to the tunneling couplings $\gamma'_j = \gamma_j\sum_\nu(-1)^\nu J_\nu(\alpha/2)\mathrm{e}^{\mathrm{i}\nu\omega t}$ and $t'_{LR} = t_{LR}\sum_\nu(-1)^\nu J_\nu(\alpha)\mathrm{e}^{\mathrm{i}\nu\omega t}$, indicating photo-assisted tunneling [11]. $J_\nu(\alpha)$ denotes the νth order Bessel function of the first kind and $\alpha = V_{AC}/\omega$ is the dimensionless AC field intensity. For instance, when the frequency of the AC field matches the energy difference between the states $|\uparrow\downarrow,\uparrow\rangle$ and $|\uparrow,\uparrow\downarrow\rangle$, the spin down electron is delocalized by the interaction with *one* photon and the interdot hopping strongly depends on the Bessel function of *first* order.

The electron dynamics of the DQD system is given by the Liouville equation $\dot{\rho}(t) = -\mathrm{i}[\hat{H}'(t), \rho(t)] = \mathscr{L}(t)\hat{\rho}(t)$, for the reduced density matrix operator $\hat{\rho} = \mathrm{tr}_{\mathrm{leads}}\hat{\chi}$, obtained by tracing all the reservoir degrees of freedom in the density operator of the whole system $\hat{\chi}$. Assuming Markov and Born approximations [12], we derive for the density matrix elements $\rho_{m'm}(t) = \langle m'|\hat{\rho}(t)|m\rangle$ the master equation [11]

$$\begin{aligned}\dot{\rho}_{m'm} = &-\mathrm{i}\langle m'|[\hat{H}_L + \hat{H}_R + \hat{H}_{L\Leftrightarrow R}(t), \hat{\rho}]|m\rangle \\ &+ \left.\sum_{n\neq m'}\Gamma_{m'n}\rho_{nn} - \sum_{n\neq m}\Gamma_{nm}\rho_{mm}\right)\delta_{m'm} \\ &- \Omega_{m'm}\rho_{m'm}(1-\delta_{m'm}).\end{aligned} \tag{1}$$

The tunneling rates through the contacts, Γ_{mn}, are affected by the AC field according to [11]

$$\Gamma_{mn} = \sum_{\nu=-\infty}^{\infty} J_\nu^2\left(\frac{\alpha}{2}\right)\xi_{mn}(\omega_{mn} + \nu\omega), \tag{2}$$

where

$$\xi_{mn}(\varepsilon) = \{f(\varepsilon)\delta_{N_m,N_n+1} + (1 - f(-\varepsilon))\delta_{N_m,N_n-1}\}\Gamma \tag{3}$$

are the usual tunneling rates obtained when photo-assisted tunneling through the contacts is neglected, $N_k = \sum_{j\sigma}\langle k|\hat{n}_{j\sigma}|k\rangle = \sum_j N_k^j$, $f(\varepsilon) = 1/(1 + \mathrm{e}^{(\varepsilon-\mu)\beta})$ is the Fermi distribution function, where $\beta = 1/k_B T$ and μ is the chemical potential of the leads, and $\Gamma = 2\pi|\gamma|^2$. The coupling to the contacts induces decoherence through the relation $\mathrm{Re}\,\Omega_{m'm} = \frac{1}{2}(\sum_{k\neq m'}\Gamma_{km'} + \sum_{k\neq m}\Gamma_{km})$.

Within the same approximation, one can derive for the current, defined as the time derivative of the charge in the right lead, the expression $I = \mathrm{tr}_{\mathrm{sys}}[(\mathscr{J} + -\mathscr{J}_-)\rho_0]$, where ρ_0 denotes the stationary solution of the master Eq. (1) and $\mathscr{J}_\pm$ are the superoperators describing the tunneling of an electron from the right dot to the right lead and back, respectively: $\mathscr{J}_\pm\rho = \sum_{m'm}\Gamma_{m'm}\rho_{mm}\delta_{N_m^R,N_{m'}^R\pm1}|m'\rangle\langle m'|$.

The noise will be characterized by the variance of the transported net charge which at long times grows linear in time, $\langle\Delta Q_R^2\rangle = St$. For its computation, we introduce the operator $\mathrm{tr}_{\mathrm{leads}}(N_R\chi)$ [8,13] which resembles the reduced density operator and obeys

$$\dot{\zeta}(t) = \mathscr{L}(t)\zeta(t) + (\mathscr{J}_+ - \mathscr{J}_-)\rho(t). \tag{4}$$

One can show that $\zeta(t)$ has a divergent component which is proportional to ρ_0 and does not contribute to the zero-frequency noise S [8]. Thus, S is fully determined by the traceless part $\zeta_1(t) = \zeta(t) - \rho_0\mathrm{tr}_{\mathrm{sys}}\zeta(t)$. In terms of ρ_0 and $\zeta_\perp$, the zero-frequency noise reads [8]

$$S = \mathrm{tr}_{\mathrm{sys}}[2(\mathscr{J}_+ - \mathscr{J}_-)\zeta_\perp + (\mathscr{J}_+ + \mathscr{J}_-)\rho_0]. \tag{5}$$

We integrate numerically the equations of motion (1) and (4) extracting the convergent component of $\zeta(t)$ until the stationary solution is reached. As expected, the current shows a resonance peak when the frequency of the AC field matches the frequency $\omega = \omega_0 = (\omega_{RL}^2 + 4t_{RL}^2)^{1/2}$, where ω_{RL} is the energy difference between the states $|\uparrow,\uparrow\downarrow\rangle$ and $|\uparrow\downarrow,\uparrow\rangle$, while the shot noise is reduced with respect to the Poissonian statistics (for which $S = I$). This behaviour is typical for resonant tunneling processes [14], cf. Fig. 1.

Most interesting is the dependence of I and S on the AC amplitude, because the photon absorption rate strongly depends on the field intensity and, eventually, one no longer finds an ideal spin pump behaviour [11]. Moreover, the dependence of the renormalized interdot tunneling on the Bessel function of first order, leads to dynamical localization [6,15] in the DQD that supresses the current, as shown in Fig. 2. On the other hand, photo-assisted tunneling through the right contact is not suppressed. This enhances the fluctuations of the electron number in the right lead with the consequence that the shot noise becomes larger while the net pump current remains

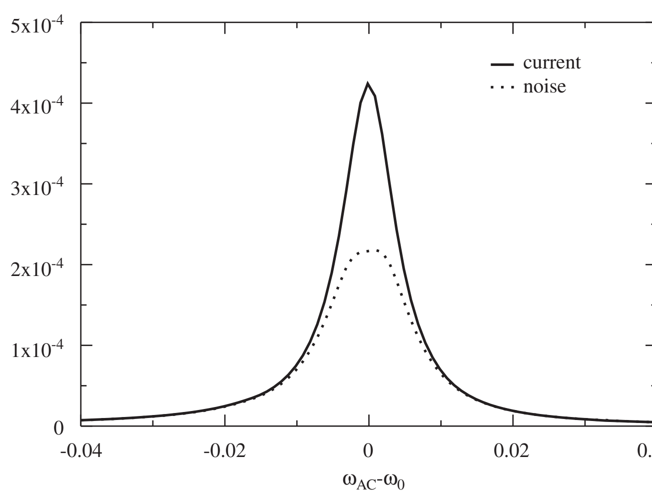

Fig. 1. Current and shot noise when tuning the frequency of the AC field for the driving amplitude intensity $V_{AC} = \omega_0$. Parameters (in meV): $\varepsilon_L = \varepsilon_R = 0.5$, $\Delta_z = 0.026$ (corresponding to a magnetic field $B_z \sim 1T$), $U_L = 1$, $U_R = 1.3$, $\mu = 1.81$, $t_{LR} = 0.005$, $\Gamma = 0.001$. For these parameters, the current is mostly spin-down polarized [11].

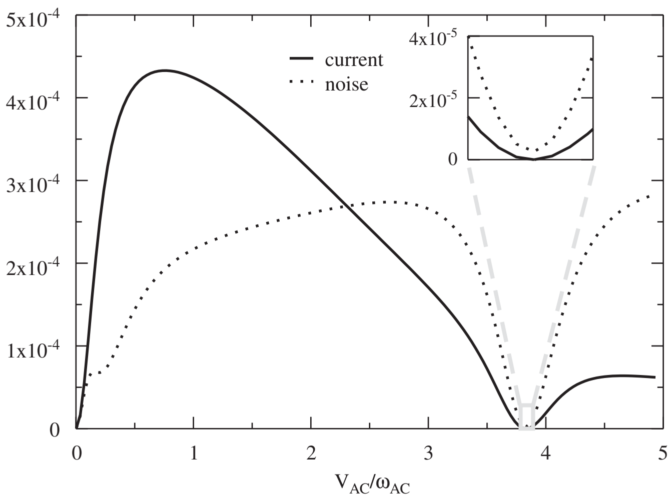

Fig. 2. Current and shot noise as a function of the driving amplitude for resonant driving, $\omega = \omega_0$. All other parameters are as in Fig. 1.

low. Consequently, one observes super-Poissonian noise characterized by $S > I$.

To summarize, we have shown that tuning the AC parameters (frequency and intensity) can significantly modify the pump current and its shot noise characteristics. In particular, we found super-Poissonian fluctuations for parameters that are in the vicinity of dynamical localization.

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References

[1] W.G. van der Wiel, et al., Rev. Mod. Phys. 75 (2003) 1.
[2] R. Hanson, et al., cond-mat/0610433.
[3] G. Platero, R. Aguado, Phys. Rep. 395 (2004) 1.
[4] C.A. Stafford, N.S. Wingreen, Phys. Rev. Lett. 76 (2006) 1916; B.L. Hazelzet, et al., Phys. Rev. B 63 (2001) 165313; F.J. Kaiser, et al., Eur. Phys. J. B 54 (2006) 201.
[5] E. Cota, R. Sanchez, et al., Phys. Rev. B 74 (2006) 035326.
[6] S. Kohler, et al., Phys. Rep. 406 (2005) 379.
[7] B. Elattari, S.A. Gurvitz, Phys. Lett. A 292 (2002) 289.
[8] F.J. Kaiser, S. Kohler, Ann. Phys. (Leipzig) 16 (2007) 702.
[9] P.K. Tien, J.P. Gordon, Phys. Rev. 129 (1963) 647.
[10] L.P. Kouwenhoven, et al., Phys. Rev. B 50 (1994) 2019.
[11] R. Sánchez, et al., Phys. Stat. Sol. (A) 203 (2006) 1154.
[12] K. Blum, Density Matrix Theory and Applications, Plenum, New York, 1996.
[13] T. Novotný, et al., Phys. Rev. Lett. 92 (2004) 248302.
[14] Ya.M. Blanter, M. Büttiker, Phys. Rep. 336 (2000) 1.
[15] F. Grossman, et al., Phys. Rev. Lett. 67 (1991) 516.