Campisi et al. Reply: logarithmic oscillators: ideal Hamiltonian thermostats

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Campisi *et al.* Reply: The logarithmic oscillator possesses a spectacular property: its heat capacity is infinite, hence it can lead a second system to Gibbs equilibrium by means of weak interactions [1]. The criticism of Meléndez *et al.* is that this can neither be implemented in simulations nor in experiments due to the length and time scales involved [2]. That our method can be employed in computer simulations is an incontrovertible fact that both we (see Figs. 2 and 3 in Ref. [1]) and, as well, the authors of the Comment (see Figs. 1 and 3 of Ref. [3]) have convincingly demonstrated with the number of particles ranging from $N = 1$ to $N = 18$. In Table I below we show that this is also experimentally feasible.

The table reports data referring to a one-dimensional (1D) implementation in which N Rb atoms ($m = 85.4678$ amu) move in a 1D box, and collide with themselves and with a particle subject to the potential $\varphi_b(x) = T \ln\sqrt{1 + x^2/b^2}$. This setup can be implemented with current cold-atom physics technology [4]. In the first column we have the number f of degrees of freedom of the system. In the second column we give the accuracy with which the actual distribution $p(v)$ of the absolute value v of any of the f velocities approximates the target Maxwell distribution $p_\beta(v) = (\pi/2\beta m)^{-1/2} e^{-\beta m v^2/2}$. In Fig. 3 of our Letter [1], the red solid line is $p(v)$ and the black dashed line is $p_\beta(v)$. The accuracy is here calculated as the Kolmogorov-Smirnov distance $H_{\mathrm{KS}}[p|p_\beta] = \max_u |\int_0^u dv[p(v) - p_\beta(v)]|$ [5]. In the third and fourth columns we have, respectively, the corresponding ratio E_{tot}/T, and trap lengths calculated as $L = 2b\sqrt{e^{2E_{\mathrm{tot}}/T} - 1} \simeq 2be^{E_{\mathrm{tot}}/T}$, with a cutoff length of $b = 10^{-8}$ m [6]. In the fifth column we report the corresponding collision times $\tau = L/N\bar{v}$ where $\bar{v} = \sqrt{k_B T/m}$ is the average velocity. Following Meléndez *et al.* we use $T = 1$ K. The upper part of the table is for fixed f and varying accuracy H_{KS}. It shows how the length and time scales vary accordingly. The lower part is for fixed accuracy H_{KS} and varying f. In agreement with our estimate [1], the ratio E_{tot}/T scales approximately as $E_{\mathrm{tot}}/T \sim f/2$. Note that, accordingly, the box size scales exponentially with f, i.e., $L \sim 2be^{f/2}$, which, as stressed in our Letter, limits the applicability to small systems [1]. As the table clearly shows, for f sufficiently small, good accuracies can be achieved with experimentally accessible length and time scales [7].

We also respond to the second criticism raised in the Comment. Since in our method the temperature appears as a parameter in the Hamiltonian, one can use it to study the response to a temporally varying temperature, using the theory of fluctuations in time-dependent Hamiltonians [1,8]. The authors of the comment instead studied the issue of a spatially varying temperature. Not only is this second criticism not pertinent to our Letter, but it is also neither sufficiently documented nor conclusive [9].

TABLE I. Length (L) and time (τ) scales of possible implementations of logarithmic oscillators as thermostats for cold Rb atoms confined into a 1D trap, depending on the number $f = N$ of atoms and the required accuracy H_{KS}.

f	H_{KS}	E_{tot}/T	L [m]	τ [sec]
20	0.005	16.45	2.78724×10^{-1}	1.41295×10^{-3}
20	0.01	14.8	5.35289×10^{-2}	2.71358×10^{-4}
20	0.02	13.1	9.77885×10^{-3}	4.95726×10^{-5}
20	0.03	11.9	2.94533×10^{-3}	1.4931×10^{-5}
20	0.04	11.05	1.25888×10^{-3}	6.38172×10^{-6}
10	0.02	7.75	4.64314×10^{-5}	4.70756×10^{-7}
20	0.02	13.1	9.77885×10^{-3}	4.95726×10^{-5}
30	0.02	18.1	1.45131×10^{0}	4.90482×10^{-3}
40	0.02	23.1	2.15393×10^{2}	5.45955×10^{-1}
50	0.02	28	2.89251×10^{4}	5.86529×10^{1}

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[5] R. J. Barlow, *Statistics: A Guide to the Use of Statistical Methods in the Physical Sciences* (Wiley, New York, 1989).
[6] The choice $b = \sigma = 10^{-10}$ m of Meléndez *et al.* is presently too small to be experimentally achievable.
[7] In clear contrast, the choice of 26 particles in three dimensions, i.e., $f = 3 \times 26 = 78$, used in the Comment, yields, due to the exponential growth, astronomical length and time scales. Such a choice evidently violates our criterion of "smallness" [1].
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