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Process Semantics of Petri Nets over Partial Algebra

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Abstract. "Petri nets are monoids" is the title and the central idea of the paper [7]. It provides an algebraic approach to define both nets and their processes as terms. A crucial assumption for this concept is that arbitrary concurrent composition of processes is defined, which holds true for place/transition Petri nets where places can hold arbitrarily many tokens.

A decade earlier, [10] presented a similar concept for elementary Petri nets, i.e. nets where no place can ever carry more than one token. Since markings of elementary Petri nets cannot be added arbitrarily, concurrent composition is defined as a partial operation.

The present papers provides a general approach to process term semantics. Terms are equipped with the minimal necessary information to determine if two process terms can be composed concurrently. Applying the approach to elementary nets yields a concept very similar to the one in [10].

The second result of this paper states that the semantics based on process terms agrees with the classical partial-order process semantics for elementary net systems. More precisely, we provide a syntactic equivalence notion for process terms and a bijection from according equivalence classes of process terms to isomorphism classes of partially ordered processes. This result slightly generalizes a similar observation given in [11].

1 Introduction

One of the main advantages of Petri nets is their capability to express true concurrency in a very natural way. Thus, Petri nets offer not only sequential semantics, which correspond to classical marking graphs, but also process semantics. Processes express possible runs of a system, in which independent transitions can occur concurrently. ¿From the very beginning of Petri net theory processes were based on partial order between net elements.

In [7] it is observed that place/transition nets can be understood as graphs whose vertices are multisets of places, and transitions are arcs with sources and targets given by their pre-multisets and post-multisets. Reflexive arcs represent markings. By multiset addition one can generate the concurrent marking graph

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from a net. For example, using composition of a reflexive arc, given by a multiset X, and an arc representing a single transiton t with pre-multiset $pre(t)$ and post-multiset $post(t)$, the arc $X + t$ changes the marking $M = X + pre(t)$ to the marking $M' = X + post(t) = M - pre(t) + post(t)$. Addition of non-reflexive arcs represents their concurrent occurrence. Concatenating graph arcs with corresponding target and source yields a representation of processes, which again can be composed concurrently or sequentially. Thus, process terms of a Petri net are obtained in a very easy way using only few production rules. Since the sum is defined for each pair of processes, this approach does not allow to express situations in which processes interfere and therefore cannot occur concurrently.

Many classes of Petri nets do not allow arbitrary concurrent composition of processes. For example, in processes of elementary net systems, no two conditions representing tokens on the same place can be concurrent. Hence, for example, no process can run concurrently with a copy of itself. A similar observation holds for nets with capacity limitations. Also inhibitor arcs and read arcs [4,8] restrict the possible concurrent composition of processes.

The aim of our work is to develop a unifying general framework to solve the problem of concurrent process composition in a conceptual way. Therefore, we employ *Petri nets over partial algebra*, defined in [5,6] as a unifying concept for Petri nets with modified occurrence rules. We claim that this approach is also suitable as a basis for process construction of different classes of Petri nets where dependencies between processes that restrict concurrent composition are taken into consideration.

In this paper, we show that Petri nets over partial algebra are suitable to define processes and their concurrent composition for elementary net systems and one-safe nets. Technically, we equip processes with the necessary information used to decide whether they are independent. We show that the minimal necessary information basically consists of the set of places associated to conditions of a process. This result coincides with the observation of [10], where also the set of involved places was used to define when two process terms can be concurrently composed.

In order to justify the algebraic approach introduced in [7] it was shown in [3] that process terms of place/transition nets from [7] are equivalent to processes based on partial order defined in [1]. In a similar fashion we show in this paper that process terms of elementary nets defined using partial algebra correspond to the usual processes of elementary nets based on partial order. Usually, processes are only defined for contact-free Petri nets, where the causal order between events is always generated by the flow of a token between the corresponding transitions. Our definition of elementary nets does not generally assume contact-freeness, i.e. we also consider situations where a transition can only occur after another transition because otherwise some place would carry two tokens at the intermediate marking. The usual way to cope with contacts is to introduce complement places in nets and according complement conditions in processes (see e.g. [9]). Our result is also based on this approach for general elementary nets, generalizing a similar result of [11] for contact-free elementary nets.

After basic definitions in Section 2, Petri nets over partial algebras (shortly PG nets) are defined in Section 3. In particular, it is shown how terms for processes are constructed within this algebra and how independency between terms is defined. An equivalence relation between such terms is introduced, which in fact is a congruence with respect to the partial operations used for the construction of the terms. In Section 4 elementary nets with corresponding PG nets are defined. Furthermore the notion of processes of elementary nets based on partial order is recalled, and a concurrent composition and concatenation of such processes is introduced. In Section 5 we present the main result of the paper, namely we show the one to one correspondence between isomorphism classes of processes of an elementary net and congruence classes of process terms of the corresponding PG net.

2 Basic Definitions

We use $\mathbb{N}$ to denote the nonnegative integers. Given two arbitrary sets A and B, the symbol B^A denotes the set of all functions from A to B. Given a function f from A to B and a subset C of A we write $f|_C$ to denote the restriction of f to the set C. The symbol 2^A denotes the power set of a set A. The set of all multi-sets over a set A is denoted by $\mathbb{N}^A$. Given a binary relation $R \subseteq A \times A$ over a set A, the symbol R^+ denotes the transitive closure of R.

Definition 1. *A partial groupoid is an ordered tuple $\mathcal{H} = (H, dom_{\dot{+}}, \dot{+})$ where H is the carrier of $\mathcal{H}$, $dom_{\dot{+}} \subseteq H \times H$ is the domain of $\dot{+}$, and $\dot{+} : dom_{\dot{+}} \to H$ is the partial operation of $\mathcal{H}$.*

Definition 2. *We say that a partial groupoid $\mathcal{H} = (H, dom_{\dot{+}}, \dot{+})$ can be embedded into a commutative monoid if there exists a commutative monoid $(H', +)$ such that $H \subseteq H'$ and the operation $+$ restricted to $dom_{\dot{+}}$ is equal to the partial operation $\dot{+}$. The monoid $(H', +)$ is called the embedding of $\mathcal{H}$.*

In the rest of the paper we will consider only partial groupoids $(H, dom_{\dot{+}}, \dot{+})$ which can be embedded into a commutative monoid, and moreover fulfil the following conditions:

- The relation $dom_{\dot{+}}$ is symmetric.
- $\forall a, b, c \in H : ((a \dot{+} b, c) \in dom_{\dot{+}} \Rightarrow (a, c), (b, c) \in dom_{\dot{+}})$.

We use the operation $\dot{+}$ to express concurrent composition of processes. As motivated in the introduction, not each pair of processes can be composed, hence $\dot{+}$ is a partial operation. $dom_{\dot{+}}$ contains the pairs of processes which are independent and can be composed. Obviously, this relation should be symmetric. The second requirement states that whenever the concurrent composition of two processes a and b is independent from c then both a and b are independent from c.

The partial groupoid $(H, dom_{\dot{+}}, \dot{+})$ is extended to the partial groupoid $(2^H, \{dom_{\dot{+}}\}, \{\dot{+}\})$ such that

- $\{dom_{\dot{+}}\} = \{(X, Y) \in 2^H \times 2^H | X \times Y \subseteq dom_{\dot{+}}\}$.
- $X\{\dot{+}\}Y = \{x \dot{+} y | x \in X \wedge y \in Y\}$.

We will use more than one partial operations on the same carrier. Therefore the following definition: A partial algebra is a set (called carrier) together with a couple of partial operations on this set (with possibly different arity). Given a partial algebra with carrier X, an equivalence $\sim$ on X is a *congruence* if for every n-ary partial operation op ($n \in \mathbb{N}$): If $a_1 \sim b_1, \ldots, a_n \sim b_n$, $(a_1, \ldots, a_n) \in dom_{op}$ and $(b_1, \ldots, b_n) \in dom_{op}$, then $op(a_1, \ldots, a_n) \sim op(b_1, \ldots, b_n)$. If moreover $a_1 \sim b_1, \ldots, a_n \sim b_n$ and $(a_1, \ldots, a_n) \in dom_{op}$ imply $(b_1, \ldots, b_n) \in dom_{op}$ then the congruence $\sim$ is said to be *closed.* Thus, a congruence is an equivalence preserving all operations of a partial algebra, while a closed congruence moreover preserves the domains of the operations. Recall that the intersection of two congruences is again a congruence. Given a binary relation on X, there always exists the least congruence containing this relation. In general, the same does not hold for closed congruences. Given a partial algebra $\mathcal{X}$ with carrier X and a closed congruence $\sim$ on $\mathcal{X}$, we write as usual, $[x]_\sim = \{y \in X | x \sim y\}$ and $X/_\sim = \bigcup_{x \in X}[x]_\sim$. The natural homomorphism $h : X \to X/_\sim$ w.r.t. $\sim$ is given by $h(x) = [x]_\sim$. Given a subset of $A \subseteq X$, we write $[A]_\sim = \bigcup_{a \in A}[a]_\sim$. A closed congruence $\sim$ defines the partial algebra $\mathcal{X}/_\sim$ with an n-ary partial operation $op/_\sim$ defined for each n-ary partial operation $op : dom_{op} \to X$ of $\mathcal{X}$ as follows: $dom_{op/_\sim} = \{([a_1]_\sim, \ldots, [a_n]_\sim) | (a_1, \ldots a_n) \in dom_{op}\}$ and, for each $(a_1, \ldots, a_n) \in dom_{op}$, $op([a_1]_\sim, \ldots, [a_n]_\sim) = [op(a_1, \ldots a_n)]_\sim$. The partial algebra $\mathcal{X}/_\sim$ is called factor algebra of $\mathcal{X}$ with respect to the closed congruence $\sim$.

3 Process Terms of Petri Nets over Partial Algebras

Definition 3. *A graph is a quadruple $(H, T, pre, post)$, where H is a set of vertices, T is a set of arcs and $pre, post : T \to H$ are source and target functions, respectively.*

The formal definition of Petri nets over partial algebra was introduced in [5] and extended in [6].

Definition 4. *Given a partial groupoid $(H, dom_{\dot{+}}, \dot{+})$, a graph $N = (H, T, pre, post)$ is called a* Petri net over the partial groupoid $(H, dom_{\dot{+}}, \dot{+})$ *(shortly a PG net).*

We write $t : a \to b \in N$ to denote that $t \in T, pre(t) = a, post(t) = b$.

Elements of H are called states or markings of the net, elements of T are transitions, and $pre, post$ denote sets of pre-conditions and post-conditions.

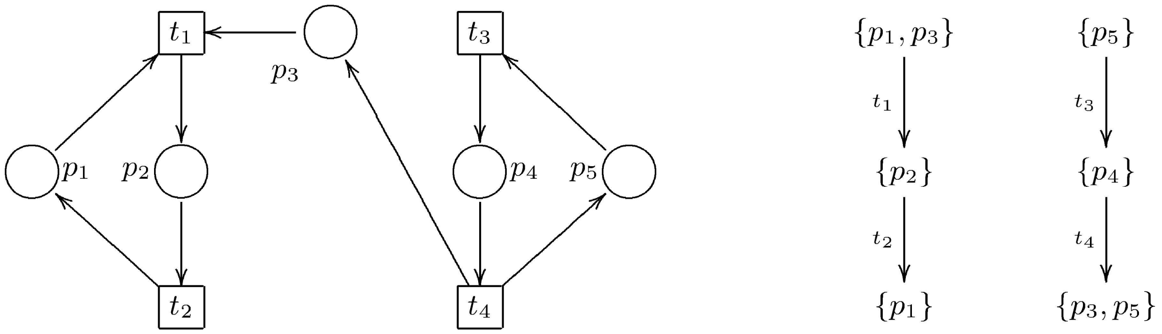

Fig. 1. An elementary net and the corresponding net over the partial groupoid $(2^{\{p_1,\dots,p_5\}}, dom_{\uplus}, \uplus)$ with $dom_{\uplus} = \{(M, M')|M \cap M' = \emptyset\}$ and $\uplus = \cup|_{dom_{\uplus}}$.

We consider elementary nets as elementary net systems [9] with arbitrary initial marking. Figure 1 illustrates the definition of a PG net and its relation to standard terminology of Petri nets.

In this paper we omit the definition of the enabling and firing rule for single transitions of PG nets, but rather directly define their process term semantics. The treatment of different enabling and firing rules and their relationships is discussed in [5,6].

To build process terms of a PG net, we need to have information about all states reachable in a process in order to decide whether the process is independent from another process. So we also have to consider an independence relation between states. We call two processes independent if their respective state spaces X and Y are independent, which means that every state $x \in X$ is independent from every state $y \in Y$. Given two independent processes with state spaces X and Y, the state space Z of the process derived from the concurrent composition $\dot{+}$ of the two processes is defined by $Z = X\{\dot{+}\}Y$.

Storing the set of all states which could be reached during a process can cause exponential growth and therefore it is not feasible. Fortunately, this exponential information is not necessary in the case of elementary nets, as will be shown in the next section. In general, for deriving a more compact information we can use any equivalence $\cong \in 2^H \times 2^H$ that is a closed congruence with respect to the operations $\{\dot{+}\}$ (concurrent composition) and $\cup$ (sequential composition). Equivalence classes of the greatest (and hence coarsest) closed congruence represent the minimal information assigned to process terms necessary for concurrent composition. This congruence is unique [2].

Thus, the process semantics of a PG net is a graph generated from PG net by reflexive, additive and concatenative closure where addition respects partiality of state independence and concatenation respects equality of target and source.

Definition 5. *Let $(H, dom_{\dot{+}}, \dot{+})$ be a partial groupoid, $\cong \in 2^H \times 2^H$ be the greatest closed congruence of the partial algebra $X = (2^H, dom_{\{\dot{+}\}}, \{\dot{+}\}, \cup)$ and $supp : X \to X/_{\cong}$ be the natural homomorphism. Given a PG net $N = (H, T, pre, post)$ over $(H, dom_{\dot{+}}, \dot{+})$, the process term semantics of N is the graph*

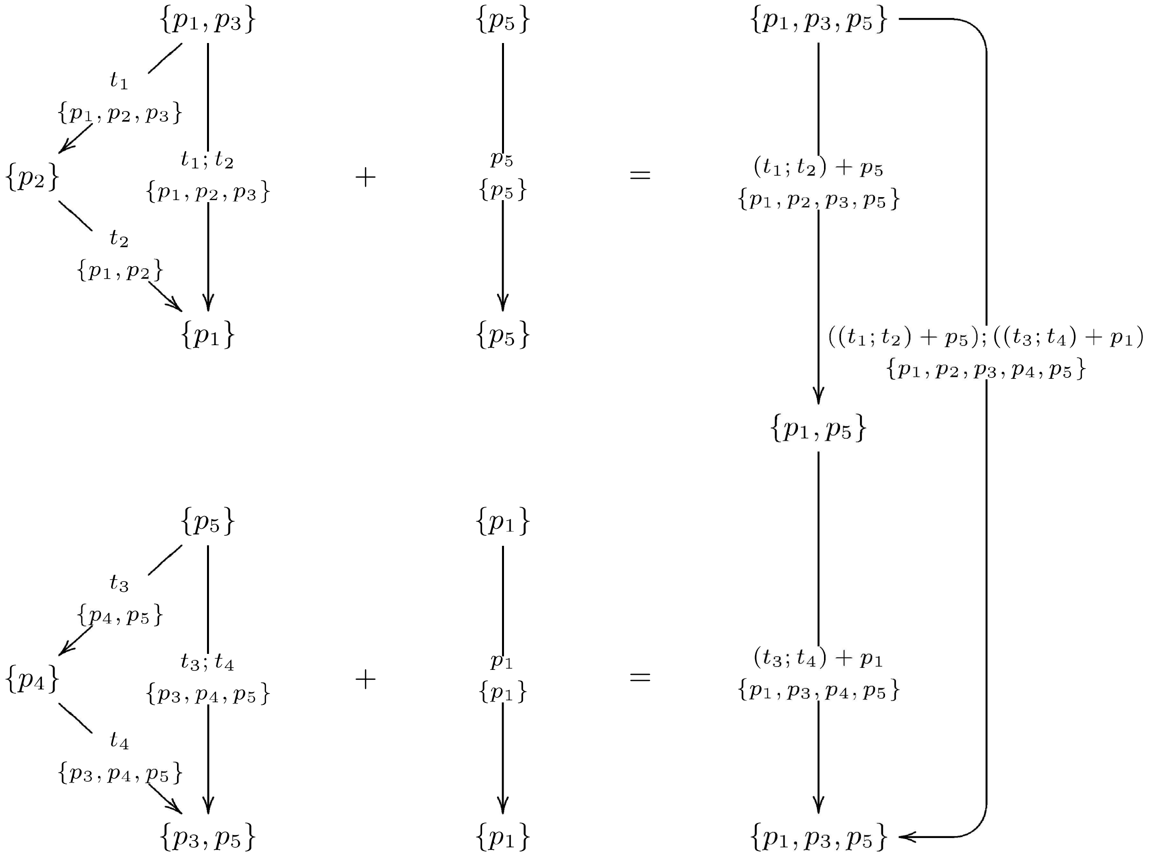

Fig. 2. Deriving a process term of the PG net from Figure 1

$\mathcal{P}(N) = (H, T_\mathcal{P}, pre_\mathcal{P}, post_\mathcal{P})$ together with a function $s : T_\mathcal{P} \rightarrow X/_\cong$. The elements of $T_\mathcal{P}$ (the arrows of the graph) are called process terms. $T_\mathcal{P}$, $pre_\mathcal{P}$, $post_\mathcal{P}$ and s are defined inductively by the following production rules, where $\alpha : a \longrightarrow b \in \mathcal{P}(N)$ denotes that $\alpha \in T_\mathcal{P}, pre_\mathcal{P}(\alpha) = a, post_\mathcal{P}(\alpha) = b$:

$$\frac{a \in H}{a : a \longrightarrow a \in \mathcal{P}(N), s(a) = supp(\{a\})}$$

$$\frac{t \in T}{t : pre(t) \rightarrow post(t) \in \mathcal{P}(N), s(t) = supp(\{pre(t), post(t)\})}$$

$$\frac{\alpha : a \rightarrow b, A \in \mathcal{P}(N) \wedge \beta : c \rightarrow d, B \in \mathcal{P}(N) \wedge (A, B) \in dom_{\dot{+}}/_\cong}{(\alpha + \beta) : a \dot{+} c \longrightarrow b \dot{+} d \in \mathcal{P}(N), s(\alpha + \beta) = A \dot{+} /_\cong B}$$

$$\frac{\alpha : a \rightarrow b, A \in \mathcal{P}(N) \wedge \beta : b \rightarrow c, B \in \mathcal{P}(\mathcal{N})}{(\alpha; \beta) : a \longrightarrow c \in \mathcal{P}(N), s(\alpha; \beta) = A \cup /_\cong B}$$

These rules define partial binary operations, called concurrent composition (+) *and* concatenation (;) *of process terms.*

Examples for constructing process terms are shown in Figures 2 and 3.

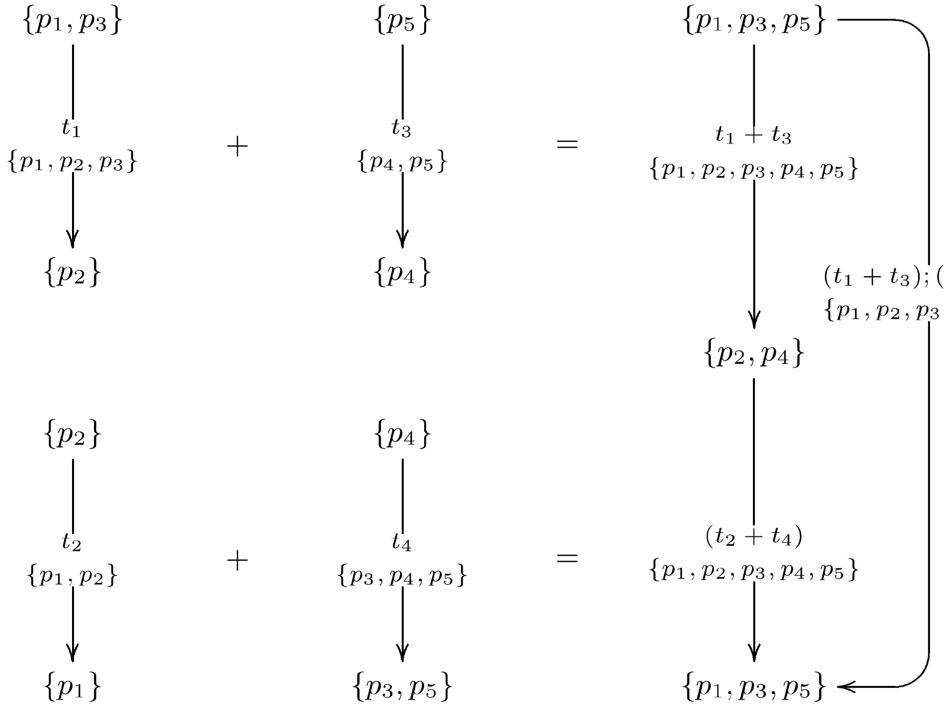

Fig. 3. Deriving another process term of PG net from Figure 1.

3.1 Equivalence of Process Terms

We now identify process terms by an equivalence relation $\sim_t$ which preserves the operations $+$ and ;. Formally we define a congruence on $T_{\mathcal{P}}$ with respect to $+$ and ;. Let $\sim_t$ be the least congruence on $T_{\mathcal{P}}$ with respect to $+$ and ; given the following axioms: Let $a, b \in H$ and $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be process terms with associated function s.

(1) $(\alpha_1 + \alpha_2) \sim_t (\alpha_2 + \alpha_1)$, whenever $+$ is defined for α_1 and α_2 .
(2) $((\alpha_1; \alpha_2); \alpha_3) \sim_t (\alpha_1; (\alpha_2; \alpha_3))$, whenever these terms are defined.
(3) $((\alpha_1 + \alpha_2) + \alpha_3) \sim_t (\alpha_1 + (\alpha_2 + \alpha_3))$, whenever these terms are defined.
(4) $\alpha = ((\alpha_1 + \alpha_2); (\alpha_3 + \alpha_4)) \sim_t \beta = ((\alpha_1; \alpha_3) + (\alpha_2; \alpha_4))$, whenever these terms are defined and $s(\alpha) = s(\beta)$.
(5) $(\alpha_1; post_{\mathcal{P}}(\alpha_1)) \sim_t \alpha_1 \sim_t (pre_{\mathcal{P}}(\alpha_1); \alpha_1)$.
(6) $a \dotplus b \sim_t (a + b)$ whenever these terms are defined.
(7) $\alpha + a \sim_t \alpha$ whenever the left term is defined, $pre_{\mathcal{P}}(\alpha) \dotplus a = pre_{\mathcal{P}}(\alpha)$ and $post_{\mathcal{P}}(\alpha) \dotplus a = post_{\mathcal{P}}(\alpha)$.

Axiom (1) represents commutativity of concurrent composition, axioms (2) and (3) associativity of concurrent composition and concatenation of process terms, axiom (4) distributivity whenever both terms have the same information about states, axiom (5) states that elements of H are partial neutral elements with respect to ;, axiom (6) expresses that composition of these neutral elements is congruent to the neutral element constructed from their composition, and finally axiom (7) states that elements of H which are neutral to source and target of a term are neutral to the term itself.

Remark 1. Observe that for any two equivalent process terms $\alpha_1 \sim_t \alpha_2$, we have $pre_{\mathcal{P}}(\alpha_1) = pre_{\mathcal{P}}(\alpha_2)$ and $post_{\mathcal{P}}(\alpha_1) = post_{\mathcal{P}}(\alpha_2)$. Moreover $\alpha_1 \sim_t \alpha_2$ implies $s(\alpha_1) = s(\alpha_2)$. Thus, by construction of process terms, the congruence $\sim_t$ is a closed congruence.

The process term $((t_1;t_2)+p_5);((t_3;t_4)+p_1)$ from Figure 2 and the process term $(t_1+t_3);(t_2+t_4)$ from Figure 3 are congruent:

$$\begin{aligned}
((t_1;t_2)+p_5);((t_3;t_4)+p_1) &\overset{(4),(5)}{\sim_t} ((t_1{+}p_5);(t_2{+}p_5));((t_3{+}p_1);(t_4{+}p_1)) \\
&\overset{(1),(2),(4)}{\sim_t} (t_1+p_5);((t_2;p1)+(p_5;t_3));(t_4+p_1) \\
&\overset{(1),(5)}{\sim_t} (t_1+p_5);(t_3+t_2);(t_4+p_1) \\
&\overset{(5)}{\sim_t} (t_1+p_5);((t_3;p_4)+(p_2;t_2));(t_4+p_1) \\
&\overset{(4)}{\sim_t} (t_1+p_5);((t_3+p_2);(t_2+p_4));(t_4+p_1) \\
&\overset{(1),(2),(4),(5)}{\sim_t} (t_1+t_3);(t_2+t_4).
\end{aligned}$$

4 Elementary Nets

In this section we define elementary nets and some useful notations. Elementary nets can be considered as elementary net systems with arbitrary initial marking. For a given marking, we use complement places to assure contact-free behavior.

Definition 6. *(Power set with distinct union) Given a finite set P, let $(2^P, dom_{\uplus}, \uplus)$ be the partial groupoid defined by*

$$dom_{\uplus} = \{(A,B) \in 2^P \times 2^P | A \cap B = \emptyset\}$$

and $\uplus = \cup|_{dom_{\uplus}}$. Denoting $H = 2^P$, we define the mapping $supp : 2^H \to H$, $supp(A) = \bigcup_{a \in A} a$.

To define process terms for algebraic elementary nets we have to find the greatest closed congruence on $(2^H, \{\dotplus\}, dom_{\{\dotplus\}}, \cup)$.

We show that the mapping *supp* is (isomorphic to) the natural homomorphism w.r.t. the gratest closed congruence on $(2^H, \{\dotplus\}, dom_{\{\dotplus\}}, \cup)$.

Lemma 1. *The relation $\cong \subseteq 2^H \times 2^H$ defined by $A \cong B \iff supp(A) = supp(B)$ is a closed congruence on $(2^H, dom_{\{\dotplus\}}, \{\dotplus\}, \cup)$*

Proof. Straightforward observation.

Lemma 2. *The closed congruence $\cong \subseteq 2^H \times 2^H$ is the greatest closed congruence on $(2^H, dom_{\{\dotplus\}}, \{\dotplus\}, \cup)$.*

Proof. We will show that any congruence $\approx$ such that $\cong$ is a proper subset of $\approx$ is not closed. Assume there are $A, B \in 2^H$ such that $A \approx B$ but $A \ncong B$. Then $supp(A) \neq supp(B)$.

We construct a set $C \in 2^H$ such that $(A, C) \in dom_{\{+\}}$ but $(B, C) \notin dom_{\{+\}}$ or vice versa (which implies that $\approx$ is not closed). Denoting $supp(A) = a$ and $supp(B) = b$ we obtain $a \neq b$.

Without loss of generality we can assume $b \backslash a \neq \emptyset$. Set $C = \{c\}$ with $c = b \backslash a$. Then $c \cap a = \emptyset$, but $c \cap b \neq \emptyset$, i.e. $(A, C) \in dom_{\{+\}}$, but $(B, C) \notin dom_{\{+\}}$.

Now we are prepared to define elementary nets using our formalism.

Definition 7. *Given a finite set P (of places), a PG net $AEN = (2^P, T, pre, post)$ over a partial groupoid $(2^P, dom_{\uplus}, \uplus)$, is called an algebraic elementary net. Its process term semantics is given by $\mathcal{P}(AEN)$.*

In order to justify our approach to Petri nets and their processes defined by terms we show that the process semantics of classical elementary nets, as defined *e.g.* in [9], essentially coincides with the above formalism. Let us first recall basic definitions of elementary nets and their process semantics based on partial orders.

Definition 8. *An elementary net is a triple $EN = (P, T, F)$, where P and T are disjoint finite sets of places and transitions and $F \subseteq (P \times T) \cup (T \times P)$ is a (flow) relation such that*

(1) $\forall t \in T \exists p, q \in P : (p, t), (t, q) \in F$, and
(2) $\forall t \in T \forall p, q \in P : (p, t), (t, q) \in F \Rightarrow p \neq q$.

A marked elementary net is a tuple $MEN = (EN, M_0)$, where $EN = (P, T, F)$ is an elementary net and $M_0 \subseteq P$ is an initial marking.

Given an element $x \in P \cup T$, the set ${}^\bullet x = \{y | (y, x) \in F\}$ is called pre-set of x and the set $x^\bullet = \{y | (x, y) \in F\}$ is called post-set of x. An element x satisfying ${}^\bullet x = x^\bullet = \emptyset$ is called isolated (by definition, only places can be isolated).

Definition 9. *Given an elementary net $EN = (P, T, F)$, the corresponding algebraic elementary net $AEN = (2^P, T, pre, post)$ is defined by $pre(t) = {}^\bullet t$ and $post(t) = t^\bullet$ for each $t \in T$.*

Figure 1 shows an elementary net with corresponding algebraic elementary net.

In this paper we omit the definition of firing rule of classical elementary nets. It may be found *e.g.* in [9]. A detailed discussion about different possibilities of enabling and firing rules of elementary nets (also when understood as PG nets) can be found in [6]. Thus, we approach directly the definition of processes of classical elementary nets.

Definition 10. *A process net is an elementary net $N = (P_N, T_N, F_N)$ with unbranched places (i.e. $\forall p \in P_N : |{}^\bullet p|, |p^\bullet| \leq 1$) which is acyclic (i.e. $\forall x \in P_N \cup T_N : (x, x) \notin F_N^+$).*

Definition 11. *Given a process net $N = (P_N, T_N, F_N)$, the partial order F_N^+ generates relations $\mathbf{co}, \mathbf{li} \subset (P_N \cup T_N) \times (P_N \cup T_N)$, defined by*

(1) $\mathbf{co} = \{(x, y) | (x, y), (y, x) \notin F_N^+\}$.
(2) $\mathbf{li} = \{(x, y) | (x, y) \notin \mathbf{co} \vee x = y\}$.

A set $CO_N \subseteq P_N$ satisfying $\forall x, y \in CO_N : (x, y) \in \mathbf{co}$ is called co-set. A slice of N is a maximal co-set. The initial and final slice are given by

(3) $^\circ N = \{p \in P_N |\ \nexists t \in T_N : (t, p) \in F_N\}$.
(4) $N^\circ = \{p \in P_N |\ \nexists t \in T_N : (p, t) \in F_N\}$.

The past and future of a slice S_N of N is defined by

(5) $^{\rightarrow}S_N = \{x \in P_N \cup T_N | \exists p \in S_N : (x, p) \in F_N^+ \vee x = p\}$,
(6) $S_N^{\rightarrow} = \{x \in P_N \cup T_N | \exists p \in S_N : (p, x) \in F_N^+ \vee x = p\}$.

Processes of elementary nets are only defined for so called contact-free marked elementary nets (for more details see e.g. [9]). In this paper, processes of a marked net which is not contact free are studied through processes of a contact-free marked net, obtained by a so called complement construction. By $\overline{MEN}$ (see [9]) we denote the net constructed from a net MEN by adding some so-called co-places. In [9] the set of co-places depends on the initial marking. Using a small simplification (which doesn't change process semantics, but only adds some unnecessary co-places) we will define a net $\overline{EN}$ by adding a co-place for each place $p \in P$. This net is contact-free for all possible initial markings of $\overline{EN}$, where an initial marking $\overline{M_0}$ of $\overline{EN}$ is constructed from an initial marking M_0 of EN by adding all co-places of places to $\overline{M_0}$ which are not in M_0.

Definition 12. *Given an elementary net $EN = (P, T, F)$, let C be a set satisfying $|C| = |P|$ and $C \cap (P \cup T) = \emptyset$, and let $c : P \to C$ be an arbitrary bijection. Let $\overline{EN} = (\overline{P}, \overline{T}, \overline{F})$ be the elementary net defined by*

– $\overline{P} = P \cup C$,
– $\overline{T} = T$ *and*
– $\overline{F} = F \cup \{((c(p), t) | (t, p) \in F\} \cup \{(t, c(p)) | (p, t) \in F\}$.

Given a marked elementary net $MEN = (EN, M_0)$, define

$$\overline{M_0} = M_0 \cup \{c(p) | p \in P \wedge p \notin M_0\} \text{ and } \overline{MEN} = (\overline{EN}, \overline{M_0}).$$

Note that, given an elementary net EN, the construction of $\overline{EN}$ is unique up to isomorphism.

A process of a marked elementary net MEN is now defined via the associated elementary net $\overline{MEN}$:

Definition 13. *Let $EN = (P, T, F)$ be an elementary net and $M_0 \subseteq P$ be a marking. A process N of $MEN = (EN, M_0)$ is a tuple (P_N, T_N, F_N, Φ_N), where (P_N, T_N, F_N) is a process net and $\Phi_N : (P_N \cup T_N) \to (\overline{P} \cup \overline{T})$ is a mapping satisfying*

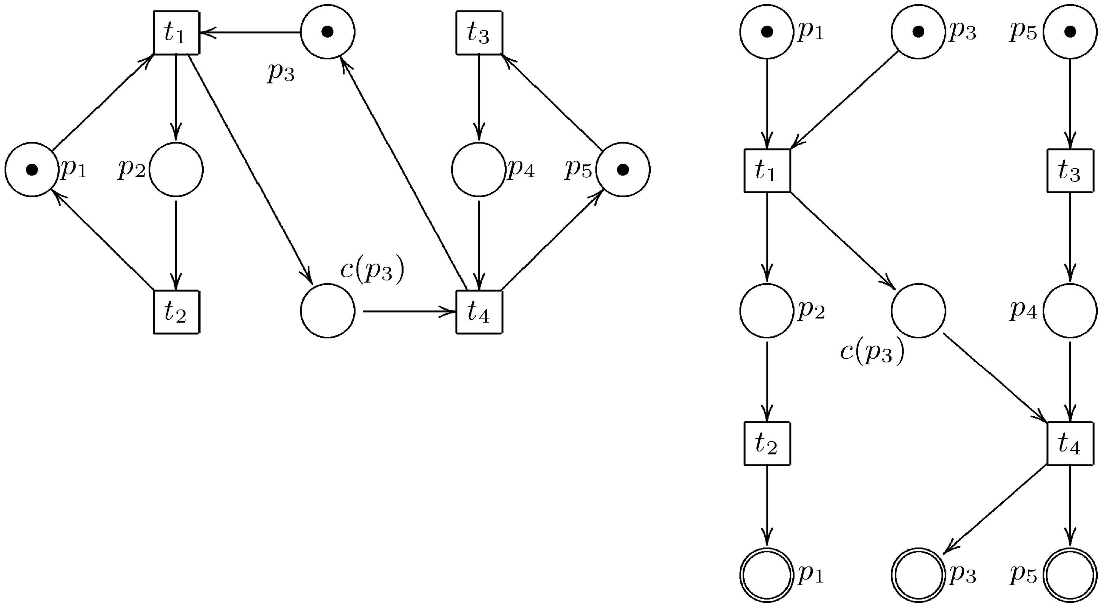

Fig. 4. A marked contact-free net where elements of the initial slice are marked and elements of the final slice are depicted by double-line circles.

(1) No isolated place of N is mapped by Φ_N to a co-place of $\overline{EN}$.
(2) $\Phi_N|_{{}^\circ N}$ is injective.
(3) $\Phi_N({}^\circ N) \cap P = M_0$ and $\Phi_N({}^\circ N) \subseteq \overline{M_0}$.
(4) $\forall t \in T_N : \Phi_N|_{{}^\bullet t}$ and $\Phi_N|_{t^\bullet}$ are injective, and
(5) $\forall t \in T_N : \Phi_N({}^\bullet t) = {}^\bullet(\Phi_N(t))$ and $\Phi_N(t^\bullet) = (\Phi_N(t))^\bullet$,

where the ${}^\bullet$-notation refers to $\overline{EN}$. Let $\mathcal{P}(EN, M_0)$ be the set of all processes of the marked elementary net $MEN = (EN, M_0)$. By $\mathcal{P}(EN) = \bigcup_{M_0 \subseteq P} \mathcal{P}(EN, M_0)$ we denote the set of all processes of an elementary net EN.

Note that the properties of the definition imply that $\Phi_N(P_N) \subseteq \overline{P}$ and $\Phi_N(T_N) \subseteq \overline{T}$. Moreover, Φ_N is injective on co-sets (see [9]).

We will not distinguish isomorphic processes of an elementary net.

The above definition of processes differs from the one defined in [9] since we have no isolated places which are mapped by Φ_N to co-places. Figure 4 shows a process of the elementary net from Figure 1.

We now define elementary processes according to the elementary process terms of the corresponding algebraic elementary net and the production rules.

Remark 2. Let $EN = (P, T, F)$ be an elementary net and let $\overline{EN} = (\overline{P}, \overline{T}, \overline{F})$.

(a) Let $M \subseteq P$ be a marking of EN. Then

$$N(M) := (M, \emptyset, \emptyset, id_M)$$

is a process of EN called elementary process associated to M.

(b) Let $t \in T$ be a transition of EN. Then

$$N(t) := (\,{}^{\bullet}t \cup t^{\bullet}, \{t\}, \{(p,t) :\ p \in {}^{\bullet}t\} \cup \{(t,p) :\ p \in t^{\bullet}\}, id_{{}^{\bullet}t \cup t^{\bullet} \cup \{t\}}),$$

where ${}^{\bullet}t, t^{\bullet}$ are defined w.r.t. $\overline{EN}$, is a process of EN, called elementary process of t.

(c) Let $N_i := (P_i, T_i, F_i, \Phi_i)$, $i = 1, 2$, be two processes of EN with disjoint sets of places and transitions, such that $\Phi_1(P_1) \cap \Phi_2(P_2) = \emptyset$. Then

$$N_1 + N_2 := (P_1 \cup P_2, T_1 \cup T_2, F_1 \cup F_2, \Phi),$$

where $\Phi|_{N_1} = \Phi_1$ and $\Phi|_{N_2} = \Phi_2$, is a process of EN, called the sum of the processes N_1 and N_2.

(d) Let $N_i := (P_i, T_i, F_i, \Phi_i)$, $i = 1, 2$, be two processes of EN with disjoint sets of places and transitions, such that $\Phi_1(N_1^{\circ}) \cap P = \Phi_2({}^{\circ}N_2) \cap P$. Define the interface $Int(N_1, N_2) \subseteq \overline{P}$ of the two processes N_1 and N_2 by

$$Int(N_1, N_2) := \Phi_1(N_1^{\circ}) \cap \Phi_2({}^{\circ}N_2).$$

Define $P_2' := P_2 \setminus \{p \in {}^{\circ}N_2 | \Phi_2(p) \in Int(N_1, N_2)\}$
and $F_2' := F_2 \cap ((T_2 \cup P_2') \times (T_2 \cup P_2'))$. Then
$N_1; N_2 := (P_1 \cup P_2', T_1 \cup T_2, F_1 \cup F_2' \cup \{(p_1, t_2) | p_1 \in N_1^{\circ} \wedge (\exists p_2 \in {}^{\circ}N_2 : \Phi_1(p_1) = \Phi_2(p_2) \wedge (p_2, t_2) \in F_2)\}, \Phi)$, where $\Phi|_{N_1} = \Phi_1$ and $\Phi|_{N_2} = \Phi_2$, is a process of EN, called the concatenation of the processes N_1 and N_2.

The Figures 5-7 illustrate the construction of the process of Figure 4 from elementary processes using the above rules (a)-(d).

Lemma 3. *Let $N_i = (P_i, T_i, F_i, \Phi_i)$, $i = 1, 2$, be processes of $EN = (P, T, F)$ satisfying $(\Phi_1(P_1) \cap \Phi_2(P_2)) \cap P = \emptyset$. Then*

$$\Phi_1(P_1) \cap \Phi_2(P_2) = \emptyset.$$

Proof. No isolated place $p_i \in P$ is mapped by Φ_i to a co-place $c(p) \in c(P)$. Hence, if $\Phi_i(p_i) = c(p) \in c(P)$ then there exists a transition $t_i \in T_i$ such that $(p_i, t_i) \in F_i \vee (t_i, p_i) \in F_i)$. Without loss of generality, let $(p_i, t_i) \in F_i$. Then, from the definition of $\overline{F}$ and the definition of processes (5), there exists a place $p_i' \in P_i$ satisfying $\Phi_i(p_i') = p$.

5 Relationship between Process Terms and Processes of Elementary Nets

This section contains the main result of the paper: The set of processes defined via process terms is identical with the set of classical processes of elementary nets.

In the sequel, let AEN be the algebraic elementary net corresponding to an elementary net $EN = (P, T, F)$. We are going to construct inductively processes $N_\alpha = (P_\alpha, T_\alpha, F_\alpha, \Phi_\alpha)$ of EN, associated to process terms $\alpha : a \to b \in \mathcal{P}(AEN)$ with information $s(\alpha)$ (according to the 4 steps of the construction of process terms). These processes enjoy the following properties:

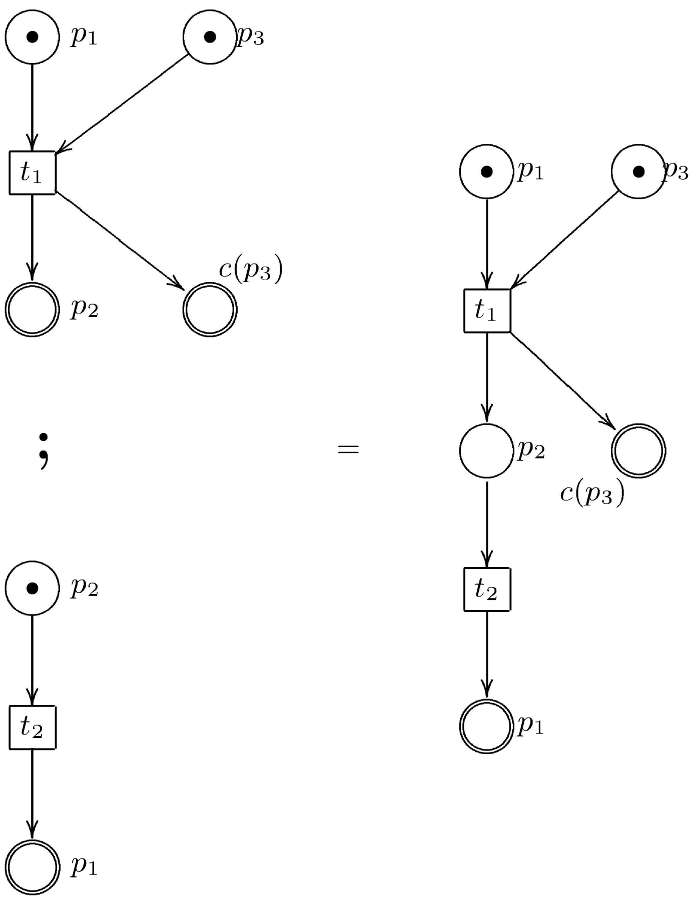

Fig. 5. Concatenation of the processes $N(t_1)$ and $N(t_2)$ for the net from Figure 1

(1) $\Phi_\alpha(^\circ N_\alpha) \cap P = a$ and $\Phi_\alpha(N_\alpha^\circ) \cap P = b$.
(2) $\Phi_\alpha(P_\alpha) \cap P = s(\alpha)$.

(a) Let $\alpha = M : M \to M, s(\alpha) = M$ be the reflexive process term of a marking $M \subseteq P$ of EN. Define

$$N_\alpha := N(M),$$

which is, according to Remark 2 (a), a process of EN.
Clearly properties (1) and (2) hold for $N(\alpha)$.

(b) Let $\alpha = t : pre(t) \to post(t)$ with $s(\alpha) = supp(\{pre(t), post(t)\})$ be the process term generated by a transition $t \in T$. Define

$$N_\alpha := N(t),$$

which is a process of EN according to Remark 2 (b).
Properties (1) and (2) follow from $^\bullet t \cap P = pre(t)$ and $t^\bullet \cap P = post(t)$.

(c) Let α_1, α_2 be process terms with information $s(\alpha_1), s(\alpha_2)$, such that $\alpha = \alpha_1 + \alpha_2, s(\alpha) = s(\alpha_1) \uplus s(\alpha_2)$ is a defined process term. We define a process

$$N_\alpha := N_{\alpha_1} + N_{\alpha_2}.$$

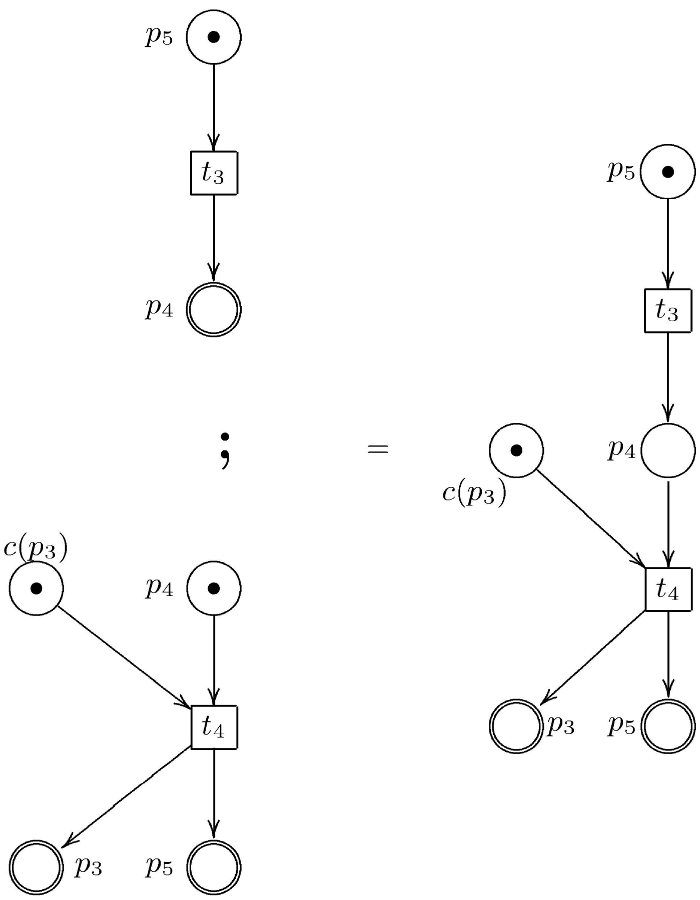

Fig. 6. Concatenation of the processes $N(t_3)$ and $N(t_4)$ for the net from Figure 1.

This is possible according to Remark 2 (c), because
(i) N_{α_1} and N_{α_2} have disjoint sets of places and transitions, and
(ii) $\Phi_{\alpha_1}(P_{\alpha_1}) \cap \Phi_{\alpha_2}(P_{\alpha_2}) = \emptyset$, where $N_{\alpha_i} = (P_{\alpha_i}, T_{\alpha_i}, F_{\alpha_i}, \Phi_{\alpha_i})$ $(i = 1, 2)$.
Condition (i) can always be achieved by appropriate renaming.
Condition (ii) follows from the fact that $\alpha_1+\alpha_2$ is defined, *i.e.* $s(\alpha_1) \cap s(\alpha_2) = \emptyset$. Property (2), which fulfilled for processes N_1 and N_2 according to the second condition of induction, implies $(\Phi_1(P_1) \cap \Phi_2(P_2)) \cap P = \emptyset$. Now (ii) follows from Lemma 3.
Obviously properties (1) and (2) are fulfilled.

(d) Let α_1, α_2, be process terms with information $s(\alpha_1), s(\alpha_2)$ such that $\alpha := \alpha_1; \alpha_2, s(\alpha) := s(\alpha_1) \cup s(\alpha_2)$ is a defined process term. We define a process

$$N(\alpha) = N(\alpha_1); N(\alpha_2).$$

This is possible according to Remark 2 (d), because $\Phi_{\alpha_1}(N^\circ_{\alpha_1}) \cap P = \Phi_{\alpha_2}({}^\circ N_{\alpha_2}) \cap P$, by property (1) and $post_{\mathcal{P}}(\alpha_1) = pre_{\mathcal{P}}(\alpha_2)$.
For the new process N_α, property (1) is obvious. We have $\Phi_\alpha(P_\alpha) \cap P = \Phi_\alpha(P_{\alpha_1} \cup (P_{\alpha_2} \setminus \{p_2 \in^\circ N_{\alpha_2} | \Phi_\alpha(p_2) \in Int(N_{\alpha_1}, N_{\alpha_2})\})) \cap P$. Property (2) follows from $Int(N_{\alpha_1}, N_{\alpha_2}) \subset \Phi_\alpha(P_{\alpha_1})$.

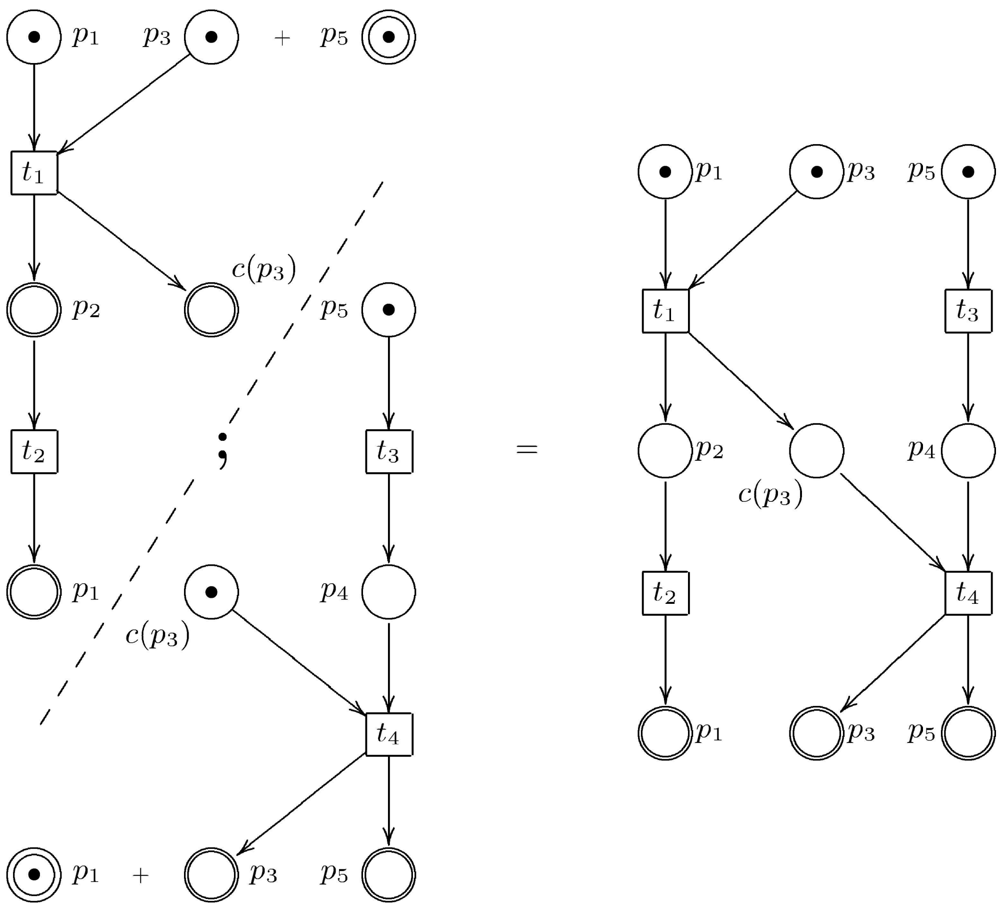

Fig. 7. The process from Figure 4 constructed from elementary processes $((N(t_1); N(t_2)) + N(p_5)); ((N(t_3); N(t_4)) + N(p_1))$.

Since the order of construction steps of N_α from elementary parts of a process term α is given by the parenthesis in α, there is a unique process net associated to a process term.

Definition 14. *Let $\tau : T_{\mathcal{P}} \to \mathcal{P}(EN)$ be the mapping defined by $\tau(\alpha) := N_\alpha$.*

Observe that the process given in Figure 4 equals the process $\tau(\alpha)$, where α is the process term from Figure 2. The constructions of α (Figure 2) and $\tau(\alpha)$ (Figures 5-7) are analogous.

Lemma 4. *Let $N = (P_N, T_N, F_N, \Phi_N)$ be a process of EN and $t_1, t_2 \in T_N$ with $(t_1, t_2) \in \mathbf{co} \wedge t_1 \neq t_2$. Then $\Phi_N(t_1) + \Phi_N(t_2)$ is a defined process term.*

Proof. $(t_1, t_2) \in \mathbf{co}$ implies that ${}^\bullet t_1 \cup {}^\bullet t_2$ and $t_1^\bullet \cup t_2^\bullet$ are co-sets. Since Φ_N is injective on slices, $\Phi_N({}^\bullet t_1) \cap \Phi_N({}^\bullet t_2) = \Phi_N(t_1^\bullet) \cap \Phi_N(t_2^\bullet) = \emptyset$. Assume a place p in $\Phi_N({}^\bullet t_1) \cap \Phi_N(t_2^\bullet)$ or $\Phi_N({}^\bullet t_2) \cap \Phi_N(t_1^\bullet)$. Without loss of generality let $p \in \Phi_N({}^\bullet t_1) \cap \Phi_N(t_2^\bullet)$. Then there are places $p_1 \in {}^\bullet t_1$ and $p_2 \in t_2^\bullet$ such that $\Phi_N(p_1) = \Phi_N(p_2)$. Then either $(p_1, p_2) \in \mathbf{co} \wedge p_1 \neq p_2$ (which contradicts the injectivity of Φ_N on co-sets) or $(p_2, p_1) \in F^+ \vee p_1 = p_2$ (which contradicts

$(t_1, t_2) \in \mathbf{co}$) or $(p_1, p_2) \in F^+$, which implies $(t_1, t_2) \in F^+$ because places are unbranched, what is again a contradiction..

Remark 3. (a) Given process terms α_i, $i = 1, \ldots, 4$ of AEN, whenever terms $\alpha = ((\alpha_1 + \alpha_2); (\alpha_3 + \alpha_4))$ and $\beta = ((\alpha_1; \alpha_3) + (\alpha_2; \alpha_4))$ are defined then $s(\alpha) = s(\beta)$.

(b) For any two process terms α_1 and α_2 such that $\alpha_1 + \alpha_2$ is defined we have $\alpha_1 + \alpha_2 \sim_t (\alpha_1; post(\alpha_1)) + (pre(\alpha_2); \alpha_2) \sim_t (\alpha_1 + pre(\alpha_2)); (\alpha_2 + post(\alpha_1))$ and analogously $\alpha_1 + \alpha_2 \sim_t (\alpha_2 + pre(\alpha_1)); (\alpha_1 + post(\alpha_2))$.

(c) If $(\alpha_1; \alpha_2) + M$ is defined, M being a marking, then

$$(\alpha_1; \alpha_2) + M \sim_t (\alpha_1 + M); (\alpha_2 + M).$$

Theorem 1. *The mapping $\tau : T_{\mathcal{P}} \to \mathcal{P}(EN)$ is surjective.*

Proof. Let $N = (P_N, T_N, F_N, \Phi_N)$ be a process of EN. We inductively construct a process term α with $N_\alpha = N$:

(i) Set $\alpha_0 = \Phi_N({}^\circ N) : \Phi_N({}^\circ N) \cap P \to \Phi_N({}^\circ N) \cap P$ with $s(\alpha_0) = \Phi_N({}^\circ N)$. We have $N(\alpha_0) = N|_{{}^\circ N}$

(ii) Assume we have constructed process terms $\alpha_0, \ldots, \alpha_{n-1}$, such that $\alpha_0; \ldots; \alpha_{n-1} : \Phi_N({}^\circ N) \cap P \to \Phi_N(N_{n-1}) \cap P, s(\alpha_0) \cup \ldots \cup s(\alpha_{n-1})$ is a process term, N_{n-1} is a slice of N ($N_0 := {}^\circ N$) and $N(\alpha_0; \ldots; \alpha_{n-1}) = (P_N \cap {}^{\rightarrow}N_{n-1}, T_N \cap {}^{\rightarrow}N_{n-1}, F_N \cap ({}^{\rightarrow}N_{n-1} \times {}^{\rightarrow}N_{n-1}), \Phi_N|_{{}^{\rightarrow}N_{n-1}})$. Take all transitions $t_1, \ldots, t_k \in T_N$ with ${}^\bullet t_i \subseteq N_{n-1}$, $i = 1, \ldots, k$, and define
$N' := N_{n-1} \setminus ({}^\bullet t_1 \cup \ldots \cup {}^\bullet t_k)$,
$N_n := N' \cup t_1^\bullet \cup \ldots \cup t_k^\bullet$,
$\alpha_n := (\Phi_N(N') \cap P) + \Phi_N(t_1) + \ldots + \Phi_N(t_k)$.
Clearly, α_n is well-defined, N_n is a slice of N,

$$\alpha_0; \ldots; \alpha_n : \Phi_N({}^\circ N) \to \Phi_N(N_n)$$

is a process term with information $s(\alpha_0) \cup \ldots \cup s(\alpha_n)$ and

$$N(\alpha_0; \ldots; \alpha_n) = (P_N \cap {}^{\rightarrow}N_n, T_N \cap {}^{\rightarrow}N_n, F_N \cap ({}^{\rightarrow}N_n \times {}^{\rightarrow}N_n), \Phi_N|_{{}^{\rightarrow}N_n}).$$

(iii) Let $m \in \mathbb{N}$, such that $N_m = N^\circ$.
Then $\alpha := \alpha_0; \ldots; \alpha_m : \Phi_N({}^\circ N) \cap P \to \Phi_N(N^\circ) \cap P$ is a process term with $N(\alpha) = N$.

Corollary 1. *Every process $N \in \mathcal{P}(EN)$ can be inductively constructed from elementary processes using partial operations + and ; as defined in Remark 2.*

The method of maximal steps used in the proof of Theorem 1 is illustrated in Figure 8.

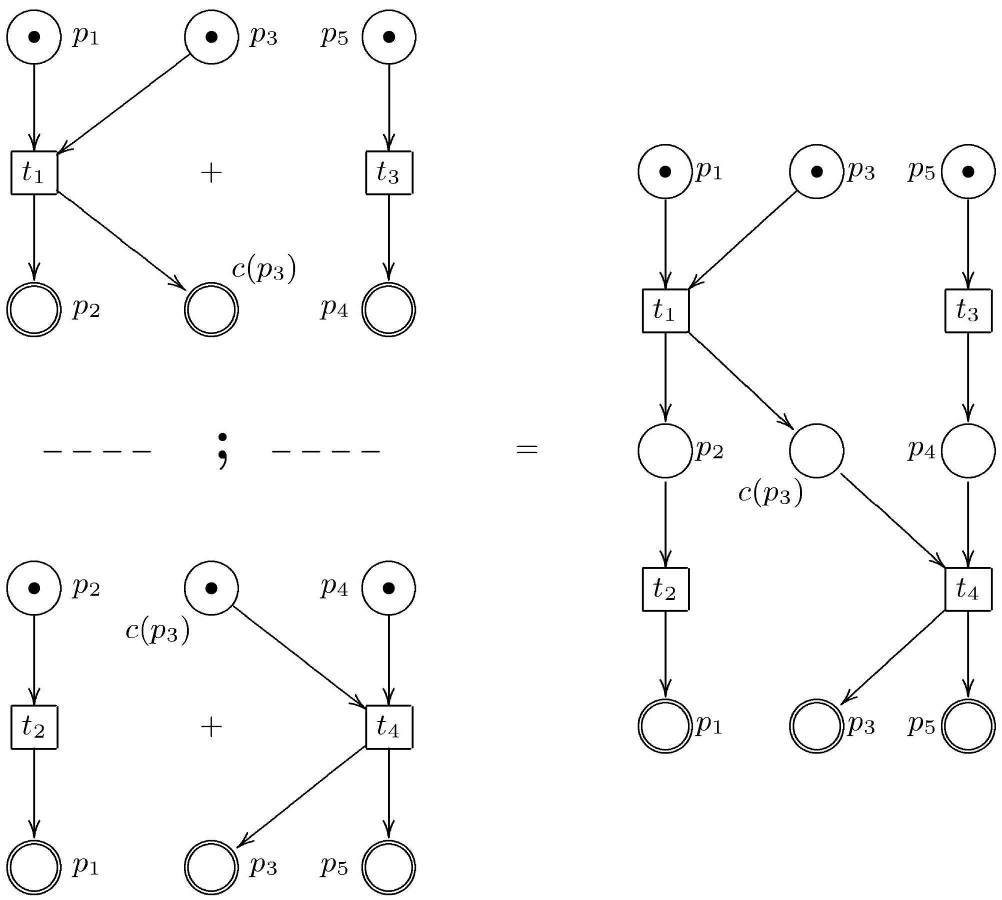

Fig. 8. Constructing the process from Figure 4 by concatenating $N(t_1) + N(t_2)$ and $N(t_3) + N(t_4)$ using the maximal step method.

Theorem 2. *For two process terms $\alpha, \beta \in \mathcal{P}(\mathcal{N})$, $\alpha \sim_t \beta$ implies that $\tau(\alpha)$ and $\tau(\beta)$ are isomorphic.*

Proof. It is sufficient to show the proposition for every (of the seven) construction rule of $\sim_t$:

(1) $N(\alpha_1 + \alpha_2) = N(\alpha_2 + \alpha_1)$ is obvious.
(2) $N(\alpha_1; \alpha_2); N(\alpha_3) = N(\alpha_1); N(\alpha_2; \alpha_3)$ is obvious.
(3) $N(\alpha_1 + \alpha_2) + N(\alpha_3) = N(\alpha_1) + N(\alpha_2 + \alpha_3)$ is obvious.
(4) We have to show, that $(N(\alpha_1) + N(\alpha_2)); (N(\alpha_3) + N(\alpha_4)) = (N(\alpha_1); N(\alpha_3)) + (N(\alpha_2); N(\alpha_4))$. Let $N_1 = (P_1, T_1, F_1, \Phi_1)$ be the process on the left side and $N_2 = (P_2, T_2, F_2, \Phi_2)$ be the process on the right side.
Clearly $T_1 = T(\alpha_1) \cup \ldots \cup T(\alpha_4) = T_2$.
Further we have $P_1 = [P(\alpha_1) \cup P(\alpha_2)] \cup$
$[(P(\alpha_3) \cup P(\alpha_4)) \setminus \{p' \in {}^\circ(N(\alpha_3) + N(\alpha_4)) | \Phi_3(p') \in P \vee \Phi_4(p') \in P\})] =$
$[P(\alpha_1) \cup P(\alpha_2)] \cup$
$[(P(\alpha_3) \setminus \{p' \in {}^\circ N(\alpha_3) | \Phi_3(p') \in P\}) \cup P(\alpha_4)) \setminus \{p' \in^\circ N(\alpha_4) | \Phi_4(p') \in P\})]$.

Because N_2 is defined, this equals
$[P(\alpha_1) \cup (P(\alpha_3) \setminus \{p' \in^\circ N(\alpha_3) | \Phi_3(p') \in P\})] \cup$
$[P(\alpha_2) \cup (P(\alpha_4) \setminus \{p' \in^\circ N(\alpha_4) | \Phi_4(p') \in P\})] = P_2$.
Since the flow relation and labeling of composed processes are constructed by restriction from the original flow relations and labelings, $F_1 = F_2$ and $\Phi_1 = \Phi_2$ follow immediately.

(5) $N(\alpha); N(post_{\mathcal{P}}(\alpha)) = N(\alpha) = N(pre_{\mathcal{P}}(\alpha)); N(\alpha)$ is obvious.
(6) Obvious.
(7) For elementary nets it suffices to consider the case $a = \emptyset$. Its proof is obvious.

Theorem 3. *If, for two process terms $\alpha, \beta \in T_{\mathcal{P}}$, $\tau(\alpha)$ and $\tau(\beta)$ are isomorphic, then $\alpha \sim_t \beta$.*

Proof. Without loss of generality let α and β be process terms with $N(\alpha) = N(\beta) = N = (P_N, T_N, F_N, \Phi_N)$ and $\gamma = \gamma_1; \ldots; \gamma_m$ be the process term constructed from the process N in the proof of Theorem 1 by considering maximal steps. Then γ_i is of the form

$$\gamma_i = \Phi_N(t_1^i) + \ldots + \Phi_N(t_{k_i}^i) + \Phi_N(M^i),$$

$t_j^i \in T_N$ and $M_i \subseteq P_N$, $i = 1, \ldots, m$, $j = 1, \ldots, k_i$. We show that α is equivalent to γ. By symmetry, the same holds for β, and we are done.

According to Remark 3, we assume without loss of generality that α is of the form

$$\alpha = \Phi_N(t_1) + (\Phi_N(M_1) \cap P); \ldots; \Phi_N(t_k) + (\Phi_N(M_k) \cap P)$$

with transitions $t_i \in T_N$ and subsets $M_i \subseteq P_N$, $i = 1, \ldots, k$. We will use shorthands $\alpha = t_1; \ldots; t_k$, and ignore the sets M_i, because they are determined by the definition of the concatenation of process terms. Clearly, α and γ 'contain' the same transitions, i.e. $\{t_1, \ldots, t_k\} = \{t_1^1, \ldots, t_{k_1}^1, \ldots, t_1^m, \ldots, t_{k_m}^m\}$.

Assume $t_i = t_1^1$ for an $i \geq 2$. It suffices to prove

$$t_1; \ldots; t_i \sim_t t_1; \ldots; t_i; t_{i-1} \sim_t \ldots \sim_t t_i; t_1; \ldots; t_{i-1},$$

because firstly the same procedure applied to $t_2^1, \ldots, t_{k_1}^1$ provides $t_1; \ldots; t_k \sim_t \gamma_1; \ldots$ (3), and secondly this procedure applied to $\gamma_2, \ldots, \gamma_m$ finishes the theorem. In fact, it even is enough to show that we can exchange t_i and t_{i-1} in α. A sufficient condition is that $\Phi_N(t_i) + \Phi_N(t_{i-1})$ is a defined process term.

We have to distinguish two cases: If $t_{i-1} = t_k^1$ for some $k \in \{2, \ldots, k_1\}$, $\Phi_N(t_i) + \Phi_N(t_{i-1})$ is defined according to the process term γ. The other possibility is $t_{i-1} = t_k^l$ for an $l \in \{2, \ldots, m\}$ and $k \in \{1, \ldots, k_l\}$. By construction of the process N_α from α follows $(t_{i-1}, t_i) \in F_N^+$ or $t_{i-1} \,\mathbf{co}\, t_i$. On the other hand, by construction of γ follows either $(t_i, t_{i-1}) \in F_N^+$ or $t_i \,\mathbf{co}\, t_{i-1}$. It follows $t_{i-1} \,\mathbf{co}\, t_i$. By Lemma 4, $\Phi_N(t_i) + \Phi_N(t_{i-1})$ is defined.

Figures 7 and 8 illustrate that the equivalent terms from Figures 5 and 6 are mapped by τ onto the same process.

Finally, looking at the definition of τ, we can state our main result for elementary nets, which now follows easily from the previous theorems.

Theorem 4. *Given any elementary net EN, there exists a one-to-one correspondence between the (isomorphism classes of) processes $\mathcal{P}(EN)$ of the elementary net EN and the $\sim_t$-congruence classes of process terms $T_{\mathcal{P}}$ of the corresponding algebraic elementary net AEN. This correspondence preserves source, target and information about states of processes and process terms, as well as concurrent composition and concatenation of processes (congruence classes of process terms).*

Remark 4. Let us rephrase Theorem 4 using terminology from partial algebra [2]: Given a process term $\alpha \in T_{\mathcal{P}}$, the congruence class $[\alpha]_{\sim_t} \in [T_{\mathcal{P}}]_{\sim_t}$ corresponds to the process $\tau(\alpha) = N \in \mathcal{P}(EN)$ such that source and target are preserved, *i.e.* $\Phi({}^\circ N) \cap P = pre_{\mathcal{P}}(\alpha)$, $\Phi(N^\circ) \cap P = post_{\mathcal{P}}(\alpha)$, and information about states is preserved, *i.e.* $\Phi(P_N) \cap P = s(\alpha)$. Moreover, denoting by $T_{\mathcal{P}}$ the partial algebra of process terms with concurrent composition and concatenation as defined in Definition 5, and by $\mathcal{P}(EN)$ the partial algebra of (isomorphism classes of) net processes with concurrent composition and concatenation as defined in Remark 2, the factor algebra $T_{\mathcal{P}}/_{\sim_t}$ is isomorphic to the partial algebra $\mathcal{P}(EN)$, (*i.e.* τ is a surjective closed homomorphism between $T_{\mathcal{P}}$ and $\mathcal{P}(EN)$.

6 Conclusion

This paper has shown that concepts of partial algebra are capable to define a syntactic process semantics of elementary nets which precisely distinguishes those runs that are also obtained by partially ordered process nets. Elementary nets can be viewed as place/transition nets with a restricted occurrence rule: In case of a contact situation, a transition is not enabled. In a more general setting, we claim that partial algebra is the suitable tool to define true-concurrency semantics for arbitrary restrictions of the occurrence rule, such as capacity restrictions, inhibitor arcs, read arcs, as suggested in [5]. We are currently working on a generalization of the results of this paper to Petri nets with arbitrarily restricted occurrence rule.

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