The restricted three-body problem and holomorphic curves

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Chapter 1

Introduction

1.1 The Birkhoff conjecture

The study of the restricted three body problem has a long history. Nevertheless many questions still remain to be solved. We quote from page 328 of Birkhoff's inspiring essay [38]

> "This state of affairs seems to me to make it probable that the restricted problem of three bodies admit of reduction to the transformation of a discoid into itself as long as there is a closed oval of zero velocity about J(upiter), and that in consequence there always exists at least one direct periodic orbit of simple type."

Translated into modern language Birkhoff asks if below the first critical energy value in each bounded component of the restricted three body problem there exists a disk-like global surface of section. The Poincaré return map associated to the disk-like global surface of section then has by Brouwer's translation theorem at least one fixed point. These fixed points give rise to direct periodic orbits.

What do we know a century after Birkhoff published his essay about his question? Unfortunately to the authors' knowledge we still cannot answer the question by Birkhoff with hundred percent certainty in the affirmative. Nevertheless the modern methods of symplectic geometry make it quite likely that the answer to this century old question can be found in the near future. The purpose of these notes is to make young ambitious researchers familiar with the main players of Birkhoff's question, namely the restricted three body problem and disk-like global surfaces of section and to introduce them to the modern techniques of symplectic geometry which reduce Birkhoff's question to questions about symplectic embeddings and systolic geometry.

Through the introduction of holomorphic curves into symplectic topology Gromov revolutionized the subject [104]. Hofer [119] and Hofer, Wysocki, and Zehnder [121, 123, 127] later extended the theory of holomorphic curves to the case of symplectizations of contact manifolds and discovered in [125] that disk-like global surfaces of section can be constructed with the help of holomorphic curves. The theory of Hofer, Wysocki, and Zehnder was later refined by Siefring with the discovery of his intersection number and by Hryniewicz with the introduction of fast finite energy planes. These theories allow us to find sufficient and necessary conditions for a periodic orbit to bound a disk-like global surface of section. In [9] the moon was "contacted", i.e., it was shown that for energies below and slightly above the first critical value the bounded components of the energy hypersurface of the regularized restricted three body problem are of contact type. This result enables us to apply holomorphic curve techniques to the restricted three body problem.

The restricted three body problem studies the dynamics of a massless body attracted by two massive bodies according to Newton's law of gravitation. For example the "massless body" could be imagined as a satellite and the two massive bodies are the earth and the moon. Another option is to think of the "massless body" as the moon and the two massive bodies are the sun and the earth. Or one could think of the "massless body" as the planet Tatooine attracted by the two stars Tatoo I and Tatoo II as in the Star Wars saga. In fact when the planet Tatooine in the Star Wars saga first appeared nobody knew if such planets exist in reality. Due to amazing progress in the search for exoplanets in the last decades we now know that such worlds with two suns exist, see for example [73].

1.2 The power of holomorphic curves

The question about the existence of a global surface of section is a question about all orbits not just periodic ones. However, holomorphic curves seem to confirm Poincaré's fantastic insight [204] that periodic orbits in some sense are the "skeleton" of the dynamics. The power of the technology of holomorphic curves lies in the fact that they reduce the question about existence of global surfaces of section to questions about periodic orbits.

Not every periodic orbit can bind a disk-like global surface of section. Indeed, if a periodic orbit binds a disk-like global surface of section, it necessarily has to satisfy some topological conditions. The mere fact that it is the boundary of an embedded disk implies that it is unknotted. Another obstruction is that it necessarily has to be linked to every other periodic orbit. A more subtle obstruction which comes from contact topology is that its self-linking number has to be -1. Now suppose that the dynamics can be interpreted as the Reeb flow of a starshaped hypersurface $\Sigma \subset \mathbb{C}^2$, which we know to hold true in the case of interest for Birkhoff's conjecture [9]. Then results of Hryniewicz and Salomão [134, 135] show that these necessary conditions are basically sufficient as well.

The idea behind this result which goes back to the ground breaking work of Hofer, Wysocki, and Zehnder [125] is the following. One considers finite energy planes in the symplectization $\Sigma \times \mathbb{R}$. These are finite energy solutions of a holomorphic curve equation for maps from the plane to the symplectization. Note that Σ has three dimensions, therefore the dimension of $\Sigma \times \mathbb{R}$ is four, while holomorphic planes are two-dimensional. A pair of two-dimensional objects in a four dimensional space generically intersects in isolated points. For holomorphic curves the local intersection index is positive. On the other hand to define an intersection number for holomorphic planes that is invariant under homotopies is not an easy task, because a plane is noncompact and one has to worry about intersections at infinity. In his celebrated approach to the Weinstein conjecture [119] Hofer already noted that the finiteness of energy for a plane guarantees that asymptotically the projection of a finite energy plane to Σ converges to a periodic Reeb orbit. If one fixes the asymptotics Siefring [219] managed to define an intersection number for finite energy planes which has all the properties one would expect for an intersection number from the closed case. In particular, it is homotopy invariant and moreover, in view of positivity of intersection for holomorphic curves, its vanishing guarantees that two curves with different image do not intersect at all. Even more is true. If Siefring's intersection number for two finite energy planes vanishes, then even their projections to Σ do not intersect unless the images coincide.

One now fixes a periodic Reeb orbit and considers the moduli space of finite energy planes asymptotic to this fixed periodic orbit. There are various symmetries on this moduli space. First there are the reparametrizations of maps from the plane to $\Sigma \times \mathbb{R}$ and moreover, there is an $\mathbb{R}$-action which comes from the obvious $\mathbb{R}$-action on the second factor of the target. If one considers the moduli space modulo these symmetries, the dimension of the space becomes two less than the Conley-Zehnder index of the asymptotic Reeb orbit. In particular, if the Conley-Zehnder index is three, the dimension of the moduli space is one. Moreover, due to some automatic transversality miracle it can be shown in this case that the moduli space is actually a manifold, so that it is a disjoint union of circles and intervals. In case the Conley-Zehnder index is bigger than three Hryniewicz [132] found a way to associate a subspace to the space of finite energy planes which after dividing out the symmetries becomes a one-dimensional manifold. Namely, he discovered the notion of "fast finite energy" planes, namely finite energy planes which have a fast exponential asymptotic decay, where the decay rate is chosen in such a way to guarantee that the projection of the finite energy plane to Σ is an immersion transverse to the Reeb vector field.

Now one considers the moduli space modulo symmetries of fast finite energy planes to a fixed asymptotic Reeb orbit γ with vanishing Siefring self-intersection number. By the properties of Siefring's intersection number the projections of these fast finite energy planes to Σ do not intersect and therefore build a local foliation of Σ. To get a global foliation one has to understand the compactness properties of this moduli space. A compactification of the moduli space of finite energy planes is provided by the SFT-compactness theorem [42]. In the case of fast finite energy planes with vanishing Siefring self-intersection number SFT-compactness implies that if the moduli space is noncompact a sequence of fast finite energy planes has to converge to a negatively punctured fast finite energy plane and at these punctures the projection of the punctured fast finite energy plane converges to periodic Reeb orbits which are unlinked to the Reeb orbit γ. In particular, if γ is linked to every other periodic Reeb orbit this scenario cannot occur and therefore the moduli space of fast finite energy planes has to be compact. In case it is not empty, we get an open book decomposition of Σ whose binding is the periodic Reeb orbit γ. Each page of this open book corresponds to the projection of a fast finite energy plane to Σ. It turns out that each page is a global surface of section for the Reeb flow.

1.3 Systolic inequalities and symplectic embeddings

As we have seen in the previous section in order to prove compactness of the moduli space of fast finite energy planes of vanishing Siefring self-intersection number asymptotic to the Reeb orbit γ one needs to make sure that γ is linked to every other periodic Reeb orbit. However, looking at the SFT-compactness theorem more closely reveals that if the moduli space is noncompact and therefore it contains a negatively punctured fast finite energy plane in its closure, then the periodic orbits at the negative punctures satisfy additional conditions

(i) The periods of the periodic orbits at the negative punctures are less than the period of γ.

(ii) At least one periodic orbit at a negative puncture orbit has Conley-Zehnder index less than or equal to 2.

In particular, we deduce from (i) that if γ has the shortest period among all periodic orbits, compactness is guaranteed. The periodic orbit of smallest period is referred to as the *systole*. Hence the Birkhoff conjecture prompts the following question, which if answered positively, actually would imply the Birkhoff conjecture with the help of holomorphic curve techniques.

Question: *Does the retrograde periodic orbit represent the systole?*

We point out, that we only expect a positive answer to the question for energy values below the first critical one. Indeed, above the first critical value a periodic orbit bifurcates out of the critical point known as Lyapunov orbit, which at least for energies slightly above the first critical value represents the systole.

In Riemannian geometry the systole is just the shortest geodesic and there is a huge literature on the subject, see for example [36]. The introduction of systolic question to the contact world is rather new and started with the work of Álvarez Paiva and Balacheff [13]. How much the Riemannian world and the contact world can differ was recently realized through the discoveries of Abbondandolo, Bramham, Hryniewicz, and Salomão [2]. The work of Lee [157] represents first steps to approach the systole in Hill's lunar problem via Symplectic homology.

In view of (ii) a different approach to the Birkhoff conjecture is to rule out periodic orbits of Conley-Zehnder index 2. The following definition is due to Hofer, Wysocki, and Zehnder [125]

Definition 1.3.1 *A starshaped hypersurface in* $\Sigma \subset \mathbb{C}^2$ *is called* dynamically convex *if the Conley-Zehnder indices of all periodic Reeb orbits are at least 3.*

Armed with this notion we can now ask

Question: *Is the restricted three body problem dynamically convex?*

Again this question only applies to energies below the first critical value. Apart from the fact that the dynamics of the restricted three body problem just below the first critical value can be interpreted as the Reeb flow of a starshaped hypersurface [9] the culprit is again the Lyapunov orbit which bifurcates out of the critical point. This periodic orbit has Conley-Zehnder index 2!

Only in rare cases dynamical convexity can be checked directly. Indeed, first finding all periodic orbits and then compute their Conley-Zehnder indices is in general not doable. However, Hofer, Wysocki, and Zehnder [125] found a much more tractable criterion.

Theorem [Hofer, Wysocki, Zehnder]: *A strictly convex hypersurface* $\Sigma \subset \mathbb{C}^2$ *is dynamically convex.*

It is interesting to note that while dynamical convexity is a symplectic notion, the concept of convexity is not. Indeed, it can well happen that a non-convex starshaped hypersurface admits a different symplectic embedding which is convex. On the other hand the theorem of Hofer, Wysocki, and Zehnder tells us that a starshaped hypersurface in the complex plane which has a periodic orbit of Conley-Zehnder index less than three does not admit a convex symplectic embedding at all. To our knowledge the following problem is still open.

Question: *Does there exist a dynamically convex starshaped hypersurface* $\Sigma \subset \mathbb{C}^2$ *which does not admit a convex symplectic embedding at all?*

The question about symplectic embeddings was one of the driving forces for the development of symplectic topology. Starting with Gromov's celebrated non-squeezing theorem [104] the question which symplectic manifolds can be symplectically embedded into each other is a highly active area of research, see for example [213, 214] for a survey.

Moreover, the question about convex symplectic embeddings and the systole are not unrelated. Indeed, it is known that in the convex case the Hofer-Zehnder capacity coincides with the systole [130], and obtaining estimates for the systole in the convex case is strongly related to the famous Viterbo conjecture [236, 198].

What do we know about dynamical convexity for the restricted three body problem. For energies below the first critical value the Levi-Civita regularization provides us with an embedding into $\mathbb{C}^2$ and it was shown in [9] that the corresponding hypersurface is starshaped. In view of the Theorem of Hofer, Wysocki, and Zehnder the natural question is, if this embedding is maybe even convex. In fact this is sometimes true. It was shown in [6] that for small mass ratios around the small mass the embedding is convex and therefore dynamical convexity holds. On the other hand this fails around the heavy body, even in the limit when the mass of the light body is zero. This limit is the rotating Kepler problem, namely the Kepler problem in rotating coordinates. In fact it was noted in [7] that the Levi-Civita embedding of the rotating Kepler problem is not convex. On the other hand in the same paper, the Conley-Zehnder indices of all periodic orbits were computed and the outcome was that the rotating Kepler problem is dynamically convex. The question arose if the rotating Kepler problem might provide an example of a dynamical convex starshaped hypersurface in $\mathbb{C}^2$ which does not admit a convex symplectic embedding. This turned out to be wrong as well. In fact in [93] a convex embedding for the rotating Kepler problem was found which used a combination of the Ligon-Schaaf and Levi-Civita embedding.

1.4 Beyond the Birkhoff conjecture

Birkhoff's assumption that there is a closed oval of zero velocity around the massive bodies precludes a satellite from traveling from the earth to the moon. This assumption of Birkhoff is equivalent to the satellite having an energy below the first critical value. Although we basically concentrate us in this monograph on this case, our motivation is actually to go above the first critical value. The binding orbit of the global surface of section that Birkhoff has in mind is the retrograde periodic orbit. Now it is impossible that the retrograde periodic orbit binds a global surface of section above the first critical point. The reason is that from the critical point a new periodic orbit bifurcates which is known as the Lyapunov orbit. The retrograde periodic orbit and the Lyapunov orbit are unlinked and therefore the retrograde orbit cannot bind a global surface of section anymore. Here is what we expect to happen. The holomorphic curves giving rise to global surfaces of section below the first critical value break precisely at the Lyapunov orbit and give rise to a finite energy foliation similar as in [129]. Interesting research in this direction was carried out in [83, 202]. If this picture is correct, then it follows from the theory developed in [129] that the stable and unstable manifold of the Lyapunov orbit intersect. For small mass ratios this phenomenon was first noted by Conley [59] and McGehee [173], and this fact turned out to have fantastic applications to space mission design.

The traditional way from the earth to the moon uses the Hohmann transfer [131]. This transfer uses two engine impulses; one at the beginning to bring the spacecraft to the transfer orbit and one at the end to stop. This method only takes advantage of the dynamics of the two body problem which have been well known since the times of Kepler. The Hohmann transfer was for example successfully applied for the Apollo Moon landings. It is fast, but one of the drawbacks is that it uses a lot of fuel. Not just at the beginning but also at the end of the journey to stop. Especially the fuel one needs to stop the rocket has to be carried during its whole journey which is very expensive. There is also some danger. Namely if the stopping does not work out properly the rocket either crashes into the moon or just flies by. It was Conley [58, 59, 60] who first propagated the idea to use the dynamics of the restricted three body problem to find low energy transit orbits to the moon. In this spirit the spacecraft ISEE-3 (International Sun-Earth Explorer-3) was brought in 1978 to a halo orbit at a Lagrange point of the earth-sun system using three-body trajectories [79, 80]. It was Belbruno who found the first realistic low energy transit orbit from an orbit around the earth to an orbit around the moon [31, 32]. Although many people at first were skeptical about these new methods, the rescue of the Japanese Hiten mission [34] impressively proved that chaotic motion can be applied to real mission design. In January 1990 the Japanese launched their first lunar mission. There were two robotic spacecrafts involved in this mission: MUSES-A (later on renamed Hiten) and MUSES-B (later on renamed Hagoromo). MUSES-B was supposed to go to the moon while MUSES-A was to stay on an orbit around the earth as a communication relay with MUSES-B. Unfortunately, contact with MUSES-B was lost and only MUSES-A remained. Since MUSES-A was never supposed to go to the moon it had only little fuel, too little to travel to the moon on a standard route. Nevertheless, on October 2, 1991, Hiten successfully arrived at the moon after a long travel which took advantage of chaotic motion in the earth-moon-satellite system as well as in the sun-earth-satellite system. The many ups and downs in this thrilling story can be nicely read in Belbruno's book *Fly me to the moon* [33]. While the traditional space mission design is referred to as *patched conics*, namely patched two body problems whose solutions are given by conic sections, the new approach to space mission design pioneered by Belbruno can be thought of as *patched restricted three body problems*. We refer the reader to the beautiful book by Koon, Lo, Marsden, and Ross [151] for more information on this new paradigm in space mission design.

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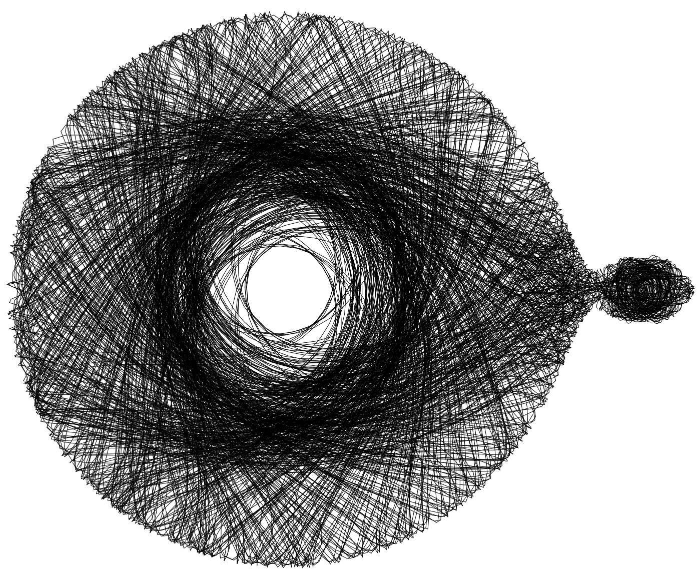

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