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INFINITELY MANY LEAF-WISE INTERSECTIONS ON COTANGENT BUNDLES

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Abstract. If the homology of the free loop space of a closed manifold B is infinite dimensional then generically there exist infinitely many leaf-wise intersection points for fiber-wise star-shaped hypersurfaces in T^*B.

1. Introduction

Let B be a closed manifold and $\Sigma \subset T^*B$ be a fiber-wise star-shaped hypersurface with respect to the standard Liouville vector field. Σ is foliated by the Reeb flow associated to the Liouville 1-form λ. We denote by L_x the leaf through $x \in \Sigma$. Let $\psi \in \mathrm{Ham}_c(T^*B)$ be in the space of Hamiltonian diffeomorphisms generated by compactly supported time dependent Hamiltonian functions. Then a leaf-wise intersection is a point $x \in \Sigma$ with the property $\psi(x) \in L_x$. The search for leaf-wise intersections was initiated by Moser in [Mos78] and pursued further in [Ban80, Hof90, EH89, Gin07, Dra08, AF08, Zil08, Gur09, Kan09]. A brief history of the search for leaf-wise intersections is given below.

We call Σ non-degenerate if Reeb orbits on Σ form a discrete set. A generic Σ is non-degenerate, see [CF09, Theorem B.1]. We denote by $\mathscr{L}_B$ the free loop space of B.

Theorem 1. *Let $\dim \mathrm{H}_*(\mathscr{L}_B) = \infty$. If $\dim B \geq 2$ and Σ is non-degenerate then for a generic $\psi \in \mathrm{Ham}_c(T^*B)$ there exist infinitely many leaf-wise intersections.*

Remark 1.1.

- To our knowledge all so far known existence results for leaf-wise intersections assert only finite lower bounds. Moreover, all known results make smallness assumptions on either the C^1 or Hofer norm of ψ.
- The assumption $\dim B \geq 2$ is necessary as the example $B = S^1$ shows.
- If $\pi_1(B)$ is finite then $\dim \mathrm{H}_*(\mathscr{L}_B) = \infty$ by a theorem of Vigué-Poirrier and Sullivan [VPS76]. If the number of conjugacy classes of $\pi_1(B)$ is infinite then $\dim \mathrm{H}_0(\mathscr{L}_B) = \infty$. Therefore, the only remaining case is if $\pi_1(B)$ is infinite but the number of conjugacy classes of $\pi_1(B)$ is finite.

1.1. **History of the problem and related results.** The problem addressed above is a special case of the leaf-wise coisotropic intersection problem. For that let $N \subset (M, \omega)$ be a coisotropic submanifold. Then N is foliated by isotropic leafs, see [MS98, Section 3.3]. The problem asks for a leaf L such that $\phi(L) \cap L \neq \emptyset$ for $\phi \in \mathrm{Ham}_c(M, \omega)$.

The first existence result was obtained by Moser in [Mos78] for simply connected M and C^1-small ϕ. This was later generalized by Banyaga [Ban80] to non-simply connected M.

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The C^1-smallness assumption was replaced by Hofer, Ekeland-Hofer in [Hof90],[EH89] for hypersurfaces of restricted contact type in $\mathbb{R}^{2n}$ by a much weaker smallness assumption, namely that the Hofer norm of ϕ is smaller than a certain symplectic capacity. Only recently, the result by Ekeland-Hofer was generalized in two different directions. It was extended by Dragnev [Dra08] to so-called "coisotropic submanifolds of contact type in $\mathbb{R}^{2n}$". Ginzburg [Gin07] generalized from restricted contact type in $\mathbb{R}^{2n}$ to restricted contact type in subcritical Stein manifolds. Moreover, examples by Ginzburg [Gin07] show that the Ekeland-Hofer result is a symplectic rigidity result, namely it becomes wrong for arbitrary hypersurfaces. In [AF08] the authors proved multiplicity results for restricted contact-type hypersurfaces. These were recently generalized by Kang in [Kan09]. Ziltener [Zil08] established multiplicity results in the special case of fibrations. Finally, Gurel [Gur09] obtained existence results for leaf-wise intersections for coisotropic submanifolds of restricted contact type.

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2. Leaf-wise intersections and Rabinowitz Floer homology

Let (M,ω) be a symplectic manifold and $f \in C^\infty(M)$ an autonomous Hamiltonian function. Since energy is preserved the hypersurface $\Sigma := f^{-1}(0)$ is invariant under the Hamiltonian flow ϕ_f^t of f. The Hamiltonian flow ϕ_f^t is generated by the Hamiltonian vector filed X_f which is uniquely defined by the equation $\omega(X_f,\cdot) = df$. If 0 is a regular value of f the hypersurface is a coisotropic submanifold which is foliated by 1-dimensional isotropic leaves, see [MS98, Section 3.3]. If we denote by L_x the leaf through $x \in \Sigma$ we have the equality

$$L_x = \bigcup_{t\in\mathbb{R}} \phi_f^t(x)\ . \tag{2.1}$$

Given a time-dependent Hamiltonian function $H : [0,1] \times M \longrightarrow \mathbb{R}$ with Hamiltonian flow ϕ_H^t we are interested in points $x \in \Sigma$ with the property

$$\phi_H^1(x) \in L_x\ . \tag{2.2}$$

This notion was introduced and studied by Moser in [Mos78]. Such points are called leaf-wise intersections. For a physical interpretation of leaf-wise intersections it is useful to think of the Hamiltonian H as a perturbation of the conservative Hamiltonian system ϕ_f^t. More dramatically one can think of H as an earthquake lasting from time $t = 0$ to $t = 1$. Without the earthquake the physical system propagates along a fixed leaf of Σ. Now we can ask whether the physical system survives the earthquake unharmed. This happens precisely if there exists a leaf-wise intersection. We refer to the article [Mos78] by Moser for further physical applications and examples.

Definition 2.1. A leaf-wise intersection $x \in \Sigma$ is called periodic if the leaf L_x is a closed orbit of the flow ϕ_f^t.

Definition 2.2. A pair $\mathfrak{M} = (F,H)$ of Hamiltonian functions $F,H : S^1 \times M \longrightarrow R$ is called a Moser pair if it satisfies

$$F(t,\cdot) = 0 \quad \forall t \in [\tfrac12, 1] \qquad \text{and} \qquad H(t,\cdot) = 0 \quad \forall t \in [0, \tfrac12]\ , \tag{2.3}$$

and F is of the form $F(t,x) = \rho(t)f(x)$ for some smooth map $\rho : S^1 \to S^1$ with $\int_0^1 \rho(t)dt = 1$ and $f : M \longrightarrow \mathbb{R}$.

Definition 2.3. We set

$$\mathcal{H} := \{H \in C^\infty(S^1 \times M) \mid H \text{ has compact support and } H(t,\cdot) = 0 \quad \forall t \in [0, \tfrac{1}{2}]\} \tag{2.4}$$

Remark 2.4. It's easy to see that the $\mathrm{Ham}(M,\omega) \equiv \{\phi_H^1 \mid H \in \mathcal{H}\}$, e.g.[AF08].

Let $(M, \omega = -d\lambda)$ be an exact symplectic manifold. Then for a Moser pair $\mathfrak{M} = (F,H)$ the perturbed Rabinowitz action functional is defined by

$$\begin{aligned} \mathcal{A}^{\mathfrak{M}} : \mathscr{L}_M \times \mathbb{R} &\longrightarrow \mathbb{R} \\ (v,\eta) &\mapsto \int_{S^1} v^*\lambda - \int_0^1 H(t,v)dt - \eta \int_0^1 F(t,v)dt \end{aligned} \tag{2.5}$$

where $\mathscr{L}_M := C^\infty(S^1, M)$. We recall that $\omega(X_F, \cdot) = dF(\cdot)$. Then a critical point (v,η) of $\mathcal{A}^{\mathfrak{M}}$ is a solution of

$$\left.\begin{aligned} \partial_t v = \eta X_F(t,v) + X_H(t,v) \\ \int_0^1 F(t,v)dt = 0 \end{aligned}\right\} \tag{2.6}$$

We observed in [AF08] that critical points of $\mathcal{A}^{\mathfrak{M}}$ give rise to leaf-wise intersections.

Proposition 2.5 ([AF08])**.** *Let (v,η) be a critical point of $\mathcal{A}^{\mathfrak{M}}$ then $x := v(\frac{1}{2}) \in f^{-1}(0)$ and*

$$\phi_H^1(x) \in L_x \tag{2.7}$$

thus, x is a leaf-wise intersection.

Moreover, the map $\mathrm{Crit}\mathcal{A}^{\mathfrak{M}} \to \{$leaf-wise intersections$\}$ is injective unless there exists a periodic leaf-wise intersection (see Definition 2.1).

Definition 2.6. A Moser pair $\mathfrak{M} = (F,H)$ is of contact-type if the following four conditions hold.

(1) 0 is a regular value of f.
(2) df has compact support.
(3) The hypersurface $f^{-1}(0)$ is a closed restricted contact type hypersurface of (M,λ).
(4) The Hamiltonian vector field X_f restricts to the Reeb vector field on $f^{-1}(0)$.

Remark 2.7. If $\Sigma \subset T^*B$ is a fiber-wise star-shaped hypersurface there exists a contact-type Moser pair $\mathfrak{M}$ with $\Sigma = f^{-1}(0)$.

Definition 2.8. A Moser pair $\mathfrak{M}$ is called regular if $\mathcal{A}^{\mathfrak{M}}$ is Morse.

We recall the following

Proposition 2.9 ([AF08])**.** *A generic contact-type Moser pair is regular.*

For a regular contact-type Moser pair $\mathfrak{M}$ on an exact symplectic manifold which is convex at infinity Rabinowitz Floer homology $\mathrm{RFH}_*(\mathfrak{M})$ is defined from the chain complex

$$\mathrm{RFC}_k(\mathfrak{M}) := \Big\{\xi = \sum_{\mu_{\mathrm{CZ}}(c)=k} \xi_c\, c \mid \#\{c \in \mathrm{Crit}\mathcal{A}^{\mathfrak{M}} \mid \xi_c \neq 0 \in \mathbb{Z}/2, \mathcal{A}^{\mathfrak{M}}(c) \geq \kappa\} < \infty\ \forall \kappa \in \mathbb{R}\Big\} \tag{2.8}$$

where the boundary operator is defined by counting gradient flow lines of $\mathcal{A}^{\mathfrak{M}}$ in the sense of Floer homology, see [CF09, AF08] for details. In particular, on cotangent bundles T^*B $\mathrm{RFH}_*(\mathfrak{M})$ is well-defined.

If the Moser pair is of the form $\mathfrak{M}=(F,0)$ then $\mathcal{A}^{\mathfrak{M}}$ is never Morse. But for a generic F the action functional $\mathcal{A}^{\mathfrak{M}}$ is Morse-Bott with critical manifold being the disjoint union of constant solutions of the form $(p,0)$, $p\in f^{-1}(0)$, and a family of circles corresponding to closed characteristics of ω on $f^{-1}(0)$.

Definition 2.10. A Moser pair is called weakly regular if it is of the form just described or if it is regular.

Remark 2.11. For weakly regular Moser pairs $\mathfrak{M}$ Rabinowitz Floer homology $\mathrm{RFH}_*(\mathfrak{M})$ can still be defined by taking the critical points of a Morse function on the critical manifolds as generators, see [CF09] for details.

Remark 2.12. We note that if we have two Moser pairs $\mathfrak{M}_0=(F_0,H_0)$ and $\mathfrak{M}_1=(F_1,H_1)$ associated to two fiber-wise star-shaped hypersurfaces Σ_0 and Σ_1 then they can be joint through a smooth family of Moser pairs $\mathfrak{M}^r=(F^r,H^r)$ such that the corresponding hypersurfaces Σ_r remain fiber-wise star-shaped. In particular, each $\mathfrak{M}^r$ is a contact-type Moser pair.

Let $\mathfrak{M}^r=(F^r,H^r)$, $r\in[0,1]$ be a smooth family of contact-type Moser pairs. We fix once for all a smooth function $\beta\in C^\infty(\mathbb{R},[0,1])$ satisfying $\beta(s)=0$ for $s\leq 0$, $\beta(s)=1$ for $s\geq 1$, and $0\leq\beta'\leq 2$. Then we set

$$F_s:=F^{\beta(s)},\ H_s:=H^{\beta(s)},\ \text{and}\ \mathfrak{M}_s:=(F_s,H_s) \tag{2.9}$$

for $s\in\mathbb{R}$. The corresponding s-dependent Rabinowitz action functional is

$$\mathcal{A}_s(v,\eta):=\int_{S^1}v^*\lambda-\int_0^1 H_s(t,v(t))dt-\eta\int_0^1 F_s(t,v(t))dt \tag{2.10}$$

It is used to define the standard continuation homomorphisms in Rabinowitz Floer homology, that is, given two weakly regular Moser pairs $\mathfrak{M}^0$ and $\mathfrak{M}^1$ there exist natural isomorphisms

$$m^{\mathfrak{M}^0}_{\mathfrak{M}^1}:\mathrm{RFH}_*(\mathfrak{M}^0)\longrightarrow\mathrm{RFH}_*(\mathfrak{M}^1)\,, \tag{2.11}$$

see [AF08] for details.

3. Proof of Theorem 1

Let (B,g) be a closed Riemannian manifold and S^*_gB the unit cotangent bundle with respect to g. Cutting off the function $\frac{1}{2}(||p||^2_g-1)$ outside a large compact subset of T^*B gives rise to a contact-type Moser pair $\mathfrak{M}_0=(F_0,0)$ for S^*_gB.

Remark 3.1. According to a Theorem by Abraham [Abr70] for a generic metric g the Moser pair $\mathfrak{M}_0=(F_0,0)$ is weakly regular. More precisely, every bumpy metric satisfies this condition.

We recall

Theorem 3.2. [CFO09, AS09] *For degrees* $*\neq 0,1$

$$\mathrm{RFH}_*(\mathfrak{M}_0)\cong\begin{cases}H_*(\mathscr{L}_B)\\ H^{-*+1}(\mathscr{L}_B)\end{cases} \tag{3.1}$$

Proof of Theorem 1. We fix a fiber-wise star-shaped hypersurface Σ and $\psi \in \mathrm{Ham}_c(T^*B)$. This gives rise to a Moser pair $\mathfrak{M} = (F, H)$. If Σ is non-degenerate and ψ is generic the perturbed Rabinowitz action functional $\mathcal{A}^{\mathfrak{M}}$ is Morse, see Proposition 2.9. Since Σ is fiber-wise star-shaped the Moser pair $\mathfrak{M}$ can be joined to $\mathfrak{M}_0$ through contact-type Moser pairs, see Remark 2.12. Thus, using the continuation isomorphism

$$m_{\mathfrak{M}}^{\mathfrak{M}_0} : \mathrm{RFH}_*(\mathfrak{M}_0) \longrightarrow \mathrm{RFH}_*(\mathfrak{M}) \tag{3.2}$$

we conclude that

$$\mathrm{RFH}_*(\mathfrak{M}) \cong \begin{cases} H_*(\mathscr{L}_B) \\ H^{-*+1}(\mathscr{L}_B) \end{cases} \tag{3.3}$$

Since we assume that $\dim \mathrm{H}_*(\mathscr{L}_B) = \infty$ we have $\dim \mathrm{RFH}_*(\mathfrak{M}) = \infty$ and therefore, the Morse function $\mathcal{A}^{\mathfrak{M}}$ has infinitely many critical points. Now, Proposition 2.5 implies that there exist infinitely many leaf-wise intersections or a period leaf-wise intersection. Thus, to prove Theorem 1 we need to exclude the latter for a generic $\psi \in \mathrm{Ham}_c(T^*B)$. That is, we need to make sure that for generic ψ the critical points of $\mathcal{A}^{\mathfrak{M}}$ do not intersect closed Reeb orbits. This is exactly the content of Theorem 3.3. □

We recall that a fiber-wise star-shaped hypersurface Σ is called non-degenerate if the set $\mathcal{R}$ of Reeb orbits on Σ form a discrete set. A generic Σ is non-degenerate, see [CF09, Theorem B.1].

Theorem 3.3. *Let $\Sigma = f^{-1}(0) \subset T^*B$ be a non-degenerate star-shaped hypersurface and $\mathfrak{M}_0 = (F_0, 0)$ be the corresponding weakly regular Moser pair. If $\dim B \geq 2$ then the set*

$$\mathcal{H}_\Sigma := \{H \in \mathcal{H} \mid \mathcal{A}^{(F_0,H)} \text{ is Morse and } \mathrm{im}\,(x) \cap \mathrm{im}\,(y) = \emptyset \quad \forall x \in \mathrm{Crit}\mathcal{A}^{(F_0,H)}, y \in \mathcal{R}\} \tag{3.4}$$

is generic in $\mathcal{H}$ (see Definition 2.3).

Proof. We set $M := T^*B$, $\mathscr{L} = W^{1,2}(S^1, M)$, and $\mathcal{H}^k := \{H \in C^k(S^1 \times M) \mid H(t, \cdot) = 0 \quad \forall t \in [0, \frac{1}{2}]\}$. Furthermore, we define the Banach space bundle $\mathcal{E} \longrightarrow \mathscr{L}$ by $\mathcal{E}_v = L^2(S^1, v^*TM)$. We consider the section $S : \mathscr{L} \times \mathbb{R} \times \mathcal{H}^k \longrightarrow \mathcal{E}^\vee \times \mathbb{R}$ defined by

$$S(v, \eta, H) := d\mathcal{A}^{(F_0,H)}(v, \eta)\ . \tag{3.5}$$

Its vertical differential $DS : T_{(v_0,\eta_0,H)}\mathscr{L} \times \mathbb{R} \times \mathcal{H}^k \longrightarrow \mathcal{E}^\vee_{(v_0,\eta_0,H)}$ at $(v_0, \eta_0, H) \in S^{-1}(0)$ is

$$DS_{(v_0,\eta_0,H)}[(\hat{v}, \hat{\eta}, \hat{H})] = \mathscr{H}_{\mathcal{A}^{(F_0,H)}}(v_0, \eta_0)\big[(\hat{v}, \hat{\eta}, \hat{H})\,;\, \bullet\big] + \int_0^1 \hat{H}(t, v_0)dt \tag{3.6}$$

where $\mathscr{H}_{\mathcal{A}^{(F_0,H)}}$ is the Hessian of $\mathcal{A}^{(F_0,H)}$. In [AF08] we proved the following.

Proposition 3.4. *The operator $DS_{(v_0,\eta_0,H)}$ is surjective for $(v_0, \eta_0, H) \in S^{-1}(0)$. In fact, $DS_{(v_0,\eta_0,H)}$ is surjective when restricted to the space*

$$\mathcal{V} := \{(\hat{v}, \hat{\eta}, \hat{H}) \in T_{(v_0,\eta_0,H)}\mathscr{L} \times \mathbb{R} \times \mathcal{H}^k \mid \hat{v}(\tfrac{1}{2}) = 0\}\ . \tag{3.7}$$

Thus, by the implicit function theorem the universal moduli space

$$\mathcal{M} := S^{-1}(0) \tag{3.8}$$

is a smooth Banach manifold. We consider the projection $\Pi : \mathcal{M} \longrightarrow \mathcal{H}^k$. Then $\mathcal{A}^{(F_0,H)}$ is Morse if and only if H is a regular value of Π, which by the theorem of Sard-Smale form a generic set (for k large enough). Moreover, the Morse condition is C^k-open. Thus, for functions in an open and dense subset of $\mathcal{H}^k$ the functional $\mathcal{A}^{(F_0,H)}$ is Morse.

Next we define the evaluation map

$$\begin{aligned} \mathrm{ev}: \mathcal{M} &\longrightarrow \Sigma \\ (v_0, \eta_0, H) &\mapsto v_0(\tfrac{1}{2}) \end{aligned} \tag{3.9}$$

From Proposition 3.4 together with Lemma 3.5 below it follows that the evaluation map $\mathrm{ev}_H := \mathrm{ev}(\cdot,\cdot,H) : \mathrm{Crit}\mathcal{A}^{(F_0,H)} \longrightarrow \Sigma$ is a submersion for a generic choice of H. Thus, the preimage of the one dimensional set $\mathcal{R}^\tau := \{\text{Reeb orbits with period } \leq \tau\}$ under ev_H doesn't intersect $\mathrm{Crit}\mathcal{A}^{(F_0,H)}$ using that $\dim T^*B \geq 4$. Therefore, the set

$$\mathcal{H}_\Sigma^n := \{H \in \mathcal{H}^n \mid \mathcal{A}^{(F_0,H)} \text{ is Morse and } \mathrm{im}\,(x) \cap \mathrm{im}\,(y) = \emptyset \quad \forall x \in \mathrm{Crit}\mathcal{A}^{(F_0,H)}, y \in \mathcal{R}^n\} \tag{3.10}$$

is generic in $\mathcal{H}$ for all $n \in \mathbb{N}$. Now, the set $\mathcal{H}_\Sigma$ is a countable intersection of the sets $\mathcal{H}_\Sigma^n$, $n \in \mathbb{N}$. This proves the assertion of Theorem 3.3. □

We learned the following Lemma from Dietmar Salamon.

Lemma 3.5. Let $\mathcal{E} \longrightarrow \mathcal{B}$ be a Banach bundle and $s : \mathcal{B} \longrightarrow \mathcal{E}$ a smooth section. Moreover, let $\phi : \mathcal{B} \longrightarrow N$ be a smooth map into the Banach manifold N. We fix a point $x \in s^{-1}(0) \subset \mathcal{B}$ and set $K := \ker d\phi(x) \subset T_x\mathcal{B}$ and assume the following two conditions.

(1) The vertical differential $Ds|_K : K \longrightarrow \mathcal{E}_x$ is surjective.
(2) $d\phi(x) : T_x\mathcal{B} \longrightarrow T_{\phi(x)}N$ is surjective.

Then $d\phi(x)|_{\ker Ds(x)} : \ker Ds(x) \longrightarrow T_{\phi(x)}N$ is surjective.

For convenience we provide a proof here.

Proof. We fix $\xi \in T_{\phi(x)}N$. Condition (2) implies that there exists $\eta \in T_x\mathcal{B}$ satisfying $d\phi(x)\eta = \xi$. Condition (1) implies that there exists $\zeta \in K \subset T_x\mathcal{B}$ satisfying $Ds(x)\zeta = Ds(x)\eta$. We set $\tau := \eta - \zeta$ and compute

$$Ds(x)\tau = Ds(x)\eta - Ds(x)\zeta = 0 \tag{3.11}$$

thus, $\tau \in \ker Ds(x)$. Moreover,

$$d\phi(x)\tau = d\phi(x)\eta - \underbrace{d\phi(x)\zeta}_{=0} = d\phi(x)\eta = \xi \tag{3.12}$$

proving the Lemma. □

References

[Abr70] R. Abraham, *Bumpy metrics*, Global Analysis (Proc. Sympos. Pure Math., Vol. XIV, Berkeley, Calif., 1968), Amer. Math. Soc., Providence, R.I., 1970, pp. 1–3.

[AF08] P. Albers and U. Frauenfelder, *Leaf-wise intersections and Rabinowitz Floer homology*, 2008, arXiv:0810.3845.

[AS09] A. Abbondandolo and M. Schwarz, *Estimates and computations in Rabinowitz-Floer homology*, 2009, arXiv:0907.1976.

[Ban80] A. Banyaga, *On fixed points of symplectic maps*, Invent. Math. **56** (1980), no. 3, 215–229.

[CF09] K. Cieliebak and U. Frauenfelder, *A Floer homology for exact contact embeddings*, Pacific J. Math. **293** (2009), no. 2, 251–316.

[CFO09] K. Cieliebak, U. Frauenfelder, and A. Oancea, *Rabinowitz Floer homology and symplectic homology*, 2009, arXiv:0903.0768, to appear in Annales Scientifiques de L'ENS.

[Dra08] D. L. Dragnev, *Symplectic rigidity, symplectic fixed points, and global perturbations of Hamiltonian systems*, Comm. Pure Appl. Math. **61** (2008), no. 3, 346–370.

[EH89] I. Ekeland and H. Hofer, *Two symplectic fixed-point theorems with applications to Hamiltonian dynamics*, J. Math. Pures Appl. (9) **68** (1989), no. 4, 467–489 (1990).

[Gin07] V. L. Ginzburg, *Coisotropic intersections*, Duke Math. J. **140** (2007), no. 1, 111–163.
[Gur09] B. Gurel, *Leafwise Coisotropic Intersections*, 2009, arXiv:0905.4139, to appear in IMRN.
[Hof90] H. Hofer, *On the topological properties of symplectic maps*, Proc. Roy. Soc. Edinburgh Sect. A **115** (1990), no. 1-2, 25–38.
[Kan09] J. Kang, *Existence of leafwise intersection points in the unrestricted case*, 2009, arXiv:0910.2369.
[Mos78] J. Moser, *A fixed point theorem in symplectic geometry*, Acta Math. **141** (1978), no. 1–2, 17–34.
[MS98] D. McDuff and D. Salamon, *Introduction to symplectic topology*, second ed., Oxford Mathematical Monographs, The Clarendon Press Oxford University Press, New York, 1998.
[VPS76] M. Vigué-Poirrier and D. Sullivan, *The homology theory of the closed geodesic problem*, J. Differential Geometry **11** (1976), no. 4, 633–644.
[Zil08] F. Ziltener, *Coisotropic Submanifolds, Leafwise Fixed Points, and Presymplectic Embeddings*, 2008, arXiv:0811.3715, to appear in Journal of Symplectic Geometry.

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