INTRODUCTION TO THE PAPERS OF WG 3: ALGEBRAIC THINKING

María C. Cañadas[a], Thérèse Dooley[b], Jeremy Hodgen[c], Reinhard Oldenburg[d]

[a]University of Granada, [b]Dublin City University,
[c]King's College London, [d]Frankfurt University

In CERME7, WG 3 "Algebraic thinking" continued the work carried out in previous CERME conferences (Ainley, Bagni, Hefendehl-Hebeker, & Lagrange, 2009).

The 13 papers were considered in four themes:

The transition to algebraic symbolisation

Caspi and Sfard investigate the discourse of 7th grade Israeli students as they move from informal meta-arithmetic toward formal algebra. By examining a historical example, they show how students' discourse, whilst informal and ambiguous, contains some algebra-like features, not normally found in everyday discourse. Dooley examines a group of primary pupils in Ireland aged 9-11 years. She uses the epistemic actions of recognising, building-with and constructing to analyse and describe the development of algebraic reasoning amongst the pupils. She argues that in some case the use of "vague" language facilitated this development. Drawing on a design science approach, Gerhard uses interviews with secondary students in Germany to exemplify the use of an analytic tool examining the transition from arithmetic to algebra. She argues that it is important to distinguish the transition from arithmetical to algebraic thinking and that from numbers to variables. Pytlak analyses a child's solution to a matchstick sequence task drawing on a wider study of primary children in Poland. She demonstrates how relatively sophisticated algebraic thinking can be achieved with geometric and numeric approaches but without the use of symbols.

Equations and symbolisation

In an intervention study of 135 primary children in Cyprus, Alexandrou-Leonidou and Philippou found that the children were capable of developing the dual meaning of the equal sign. This understanding, in turn, enabled the children to solve equations in multiple representation formats. By conducting a survey of 113 students in Turkey, Didiş, Baş and Erbaş examine students understandings and errors in relation to the solving quadratic equations. Their findings add further weight to the literature highlighting the ubiquity and problems of a purely instrumental, or procedural, understanding.

Technology

Drawing both on the historical development of mathematics and on examples of Italian students, Chiappini demonstrates how AlNuSet software can enable students to overcome crucial epistemological obstacles in the move from arithmetic to algebra, specifically negative numbers and the equivalence of different algebraic forms.

Hewitt discusses the work of a group of 9-10 year olds in England as they engaged with formal algebra for the first time using the software Grid Algebra. He outlines six perspectives from the literature on algebraic activity and uses these to analyse the students' activity in order to examine what constitutes algebraic activity.

Working in Italy, Maffei and Mariotti use Aplusix CAS to examine the interplay between different representations of algebra: standard (symbolic) representation, tree representation and natural language. They demonstrate that natural language has a dual role as a representation in itself and in describing the other representations. Nobre, Amado, Carreira and da Ponte show how a generic spreadsheet, Excel, can enable students to engage with algebraic structure without the need for algebraic symbolisation. Indeed, the three Grade 8 Portuguese students, were able to model and solve a complex problem involving simultaneous equalities and inequalities.

Generalisation

A. Barbosa reports on her analysis of the strategies used by 54 Portuguese students in 6th Grade working on generalization tasks as they participated in an intervention study. Students achieved better results with near generalisation than with far generalisation problems. Reporting on a survey of 359 Spanish Secondary students, Cañadas, Castro and Castro outline the different approaches to generalisation adopted. They find that students use graphical approaches infrequently and generally only when the problem was presented graphically. Chua and Hoyles discuss differences in the generalisation strategies used by 13 year old students in Singapore from the Express (higher attaining) course and from the Normal course. Express students were more flexible, adopting a numerical approach for a linear problem, but using a constructive approach for a quadratic problem.

GENERAL REFLECTIONS

Algebraic thinking is a "mature" domain within mathematics education research (Kieran, 2006). Indeed, alongside multiplicative reasoning, algebra is perhaps the most extensively researched area in mathematics education. The papers and posters reflect this and all the papers and posters drew on this body of research. Unsurprisingly given this research history, there were many aspects of consensus across the group, but there were also significant differences.

Points of consensus

In relation to the practice of teaching and learning algebraic thinking, there was general agreement that:

- Doing algebraic thinking provides considerable insight into school mathematics, but translating these insights into general classroom practice is not straightforward.
- Classrooms around Europe and elsewhere tend to be dominated by procedures and manipulation. Skemp's (1976) seminal work is still of considerable relevance.
- The promise of technology has largely yet to be realised in most classrooms.
- There are many approaches to algebra and learners should acquire many ways to look at and work with algebra.

All participants agreed on the importance of multiple perspectives, of talk and discourse, of rich tasks and of children's existing and naïve (mis-)understandings [1].

Reflecting the three plenary lectures at CERME-7, key overarching issues in the group discussions included a recognition of the importance of the teacher (*Sierpinska*), the importance of pupils experiencing "surprise" (*Hannula*) and the relationship between arithmetic and algebra (*Mariotti*).

Points of difference

The issue of "early algebra" and the relationship to / transition from arithmetic continues to be a thorny one, which generated much debate. The question as to whether there is a clear cognitive gap between (generalised) arithmetic and algebra remains an open one. Similarly, there was disagreement on whether there exists one *best* or *ideal* learning trajectory or whether there are several *good-enough* learning trajectories or whether learning is inevitably somewhat idiosyncratic. An international conference inevitably (and usefully) highlights issues of language and meaning. Working Group 3 was no exception. For example, whilst all agreed on the importance of talk and discourse, some participants preferred the more general term of "talk" and others preferred the more specific and theory-laden "discourse". Related to this, theory was used differently by different participants. Some opted for a pragmatic use of theory to solve and illuminate research problems as and when they occurred. Others attempted to draw synergies between different theoretical approaches in order to inform research.

ISSUES FOR FUTURE RESEARCH

We have already noted the concern with early algebra. Whilst this concern in part reflects a current theme in the literature (Kaput, Carraher & Blanton, 2007), it also responds to the policy context in which some countries (such as Portugal) are introducing algebra earlier. This policy imperative highlights several important issues for Working Group 3 and CERME more generally. Re-contextualisation – the translation of "existing" knowledge into new settings and contexts - is a valid and important field of study and we note that the replication of existing research has been somewhat undervalued in mathematics education as a field generally. However, in re-contextualising or replicating existing work, researchers need to demonstrate the contribution they make to the field as a whole through stronger literature reviews.

The issue of translating research knowledge into practice in general was a concern for almost all participants. The mismatch between what can be achieved in experimental settings and the general practice in the majority of classrooms is a serious concern. So, for example, in considering how to realise the potential of technology, the group discussed how technology can help children do something that they would not otherwise do and then how teaching can enable children to understanding "independent of" technology. Similarly, the group identified a need for further research into understanding group dynamics specific to algebraic thinking.

LOOKING FORWARD TO CERME-8

Finally, in looking forward to CERME-8, the group discussed ways of continuing and extending existing studies by:

- Identifying research collaborations with a view to replicating studies in different national / cultural contexts.
- Reporting follow-on studies to CERME-7 papers and posters.
- Examining the same research problem / dataset using different theoretical lenses and methodologies.

We hope that the majority of the participants will return the CERME-8.

NOTES

1. However, the issue of children's understandings was conceptualised differently with some using the notion of misconceptions and others rejecting this as too cognitive.

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