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Transmittance through Aharonov-Bohm Rings: Signature of Spin-Charge Separation

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Abstract. We present numerical calculations concerning the phenomenon of spin-charge separation in low-dimensional strongly correlated electron systems. Specifically, we study the t-J model on a flux-threaded ring connected to semi-infinite free-electron leads. The transmittance through such an Aharonov-Bohm ring shows an interference pattern with characteristic sharp dips at certain flux values, determined by the filling, which are a direct consequence of spin-charge separation in a nanoscopic system.

Keywords: spin-charge separation, strongly interacting electrons, transmittance through quantum rings
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INTRODUCTION

It is well known that electronic correlations are strongly enhanced in low-dimensional systems. In particular, in one dimension (1D), the excitations are very well described by the Luttinger-liquid theory (LL) . One of the defining features of this model is the phemomenon of spin-charge separation (SCS), where the electronic charge and spin degrees of freedom can be described independently [1, 2].

Continuing progress in fabrication techniques, and the discovery of new materials of quasi-1D electronic character, have led in the last decade to a variety of experiments which seek evidence of SCS. Prominent examples of candidate materials include the organic Bechgaard and Fabre salts [3], molybdenum bronzes and chalcogenides [4], cuprate chain and ladder compounds [5], and also carbon nanotube systems [6]. The non-Fermi-liquid normal-state properties of high temperature superconductors have also led to attempts to trace their origin to the possible realization of SCS in strongly correlated electron systems in 2D [7]. There have been several previous approaches for the identification of SCS, including the analysis of non-universal power-law *I*-*V* characteristics [4], the search for characteristic dispersive features by angle-resolved photoemission spectroscopy (ARPES) [8], establishing a violation of the Wiedemann-Franz law [9], and analyzing spin and charge conductivities [8, 10]. The interpretation of experimental results has been considered ambiguous in some cases. However, a verification of SCS has been reported from ARPES data on $SrCuO_2$ [11].

Theoretical methods for detecting and visualizing SCS were proposed and demonstrated many years ago. For example, direct calculations of the real-time evolution of electronic wave packets in Hubbard rings revealed different velocities in the dispersion of spin and charge densities as an immediate consequence of SCS [12] (Fig. 1). These calculations considered a finite system of 16 sites with periodic boundary conditions and 2 initial particles, with a third particle added at time $t = 0$ in the form of a wave packet with an average crystal momentum of $\pi/2$. The local charge and spin densities

$$\begin{aligned}\rho_c(i,t) &= \langle\psi(t)|n_{i\uparrow}+n_{i\downarrow}|\psi(t)\rangle \\ \rho_s(i,t) &= \langle\psi(t)|n_{i\uparrow}-n_{i\downarrow}|\psi(t)\rangle \qquad (1)\end{aligned}$$

were calculated for subsequent times. By comparing the non-interacting ($U = 0$) and interacting ($U = 10t$) cases one observes clearly that, while in the former case both densities remain proportional, in the latter the charge and spin wave packets propagate with different velocities.

Recently, Kollath and coworkers have repeated this calculation for larger systems using the Density Matrix Renormalization Group (DMRG) technique[13], and observed distinct features of SCS in a model for one-dimensional cold Fermi gases in a harmonic trap. They also proposed quantitative estimates for an experimental observation of SCS in an array of atomic wires.

A different approach was adopted in Ref. [14], where the authors analyzed the transmission through Aharonov-Bohm (AB) rings. The motion of the electrons in the ring was described by a LL propagator, where different charge and spin velocities, respectively v_c and v_s, are included explicitly. With this assumption the flux-

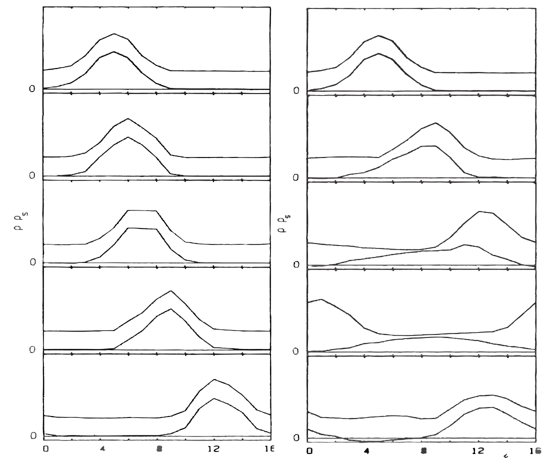

FIGURE 1. (a) Charge (top) and spin (bottom line) densities as a function of position i for different times in a 16-site ring. Time values from top to bottom time values are 0, 0.5, 1, and 2 (in units of the hopping t). (a) non-interacting ($U = 0$); b) strongly interacting ($U = 10t$) [12].

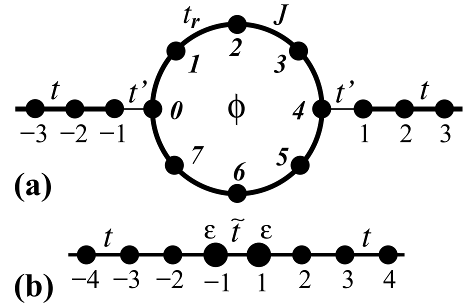

FIGURE 2. (a) Schematic representation of an interacting system on a ring connected by links t' to free-electron leads. (b) Effective model of two impurities in a single conducting chain derived in the limit of weak t'.

dependence of the transmission is no longer periodic only in multiples of a flux quantum $\Phi_0 = hc/e$, but instead new structures appear at fractional flux values which are determined by the ratio v_s/v_c. In essence, these structures arise because transmission requires the separated spin and charge degrees of freedom of an injected electron to recombine at the drain lead after traveling through the ring in the presence of the AB flux.

In a recent publication [15] we studied the robustness of the SCS phenomenon beyond LL theory, and proposed an experimental configuration which may serve as a clean and direct probe of SCS in finite systems. We analyzed the AB-ring transmission, focusing primarily on the t-J model as a prototypical interacting system relevant to artificially designed 1D nanostructures such as small rings of quantum dots. We found a clear reduction of the transmittance of such a device at magnetic fields corresponding to fractional values of the flux in units of the flux quantum. These are explained in terms of an analysis of the momentum quantum numbers of the spin and charge excitations, and of the orbital-flux-induced phase shifts accumulated on traversing the ring between the two contact leads. The flux-dependent interference pattern in the transmission through an AB ring is thus shown to be a valid and feasible tool for the unambiguous detection of certain signatures of SCS.

THE MODEL

The system in Fig. 2(a) has the Hamiltonian

$$H = H_{\text{leads}} + H_{\text{link}} + H_{\text{ring}}, \tag{2}$$

where

$$H_{\text{leads}} = -t \sum_{i=-\infty,\sigma}^{-1} a^{\dagger}_{i-1,\sigma} a_{i,\sigma} - t \sum_{i=1,\sigma}^{\infty} a^{\dagger}_{i,\sigma} a_{i+1,\sigma} + \text{H.c.} \tag{3}$$

describes free electrons in the left and right leads,

$$H_{\text{link}} = -t' \sum_{\sigma} (a^{\dagger}_{-1,\sigma} c_{0,\sigma} + a^{\dagger}_{1,\sigma} c_{L/2,\sigma} + \text{H.c.}) \tag{4}$$

describes the hopping of quasiparticles between leads ($a_{i,\sigma}$) and ring ($c_{l,\sigma}$), and

$$\begin{aligned} H_{\text{ring}} &= -eV_g \sum_{l,\sigma} c^{\dagger}_{l,\sigma} c_{l,\sigma} - t_r \sum_{l,\sigma} (c^{\dagger}_{l,\sigma} c_{l+1,\sigma} e^{-i\phi/L} \\ &\quad + \text{H.c.}) + J \sum_{l} \mathbf{S}_l \cdot \mathbf{S}_{l+1} \end{aligned} \tag{5}$$

describes the interacting electron system, where $\mathbf{S}_l = \sum_{\alpha\beta} c^{\dagger}_{l\alpha} \sigma_{\alpha\beta} c_{l\beta}$ is the spin at site l and implicit projection of the t_r term to single site occupancy is considered. The AB ring has length L, is threaded by flux ϕ ($\phi = 2\pi\Phi/\Phi_0$ and Φ is the total magnetic flux), and subject to an applied gate voltage V_g.

Following Ref. [14], the transmission from the left to the right lead can be calculated to second order in t' from an effective low-energy Hamiltonian H_{eff} for the system with an additional particle of energy ω and wave vector $\pm k$ in the left or the right lead. H_{eff} is equivalent to the one-particle Hamiltonian for the chain represented in Fig. 2(b), with effective energy $\varepsilon(\omega) = t'^2 G^{\text{R}}_{0,0}(\omega)$ for sites adjacent to the ring (-1 and 1 in Fig. 2(a)), and effective hopping $\tilde{t}(\omega) = t'^2 G^{\text{R}}_{0,L/2}(\omega)$ across the ring; $G^{\text{R}}_{i,j}(\omega)$ denotes the Green function of the isolated ring.

At zero temperature, the transmittance and conductance of the system may then be computed using the effective impurity problem. The transmittance $T(\omega)$ is given by [14]

$$T(\omega, V_g, \phi) = \frac{4t^2 \sin^2 k |\tilde{t}(\omega)|^2}{|[(\omega - \varepsilon(\omega) + te^{ik}]^2 - |\tilde{t}^2(\omega)||^2}, \tag{6}$$

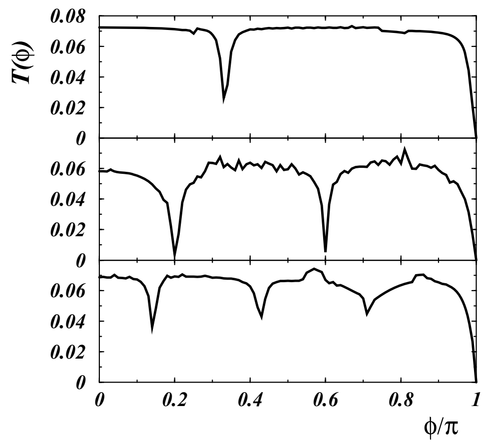

FIGURE 3. Transmittance as a function of flux for a t_r-J model with $J = 0.001t_r$, $t_r = t$, $t' = 0.3t$, and $L = 8$ sites. The filling of the ring is (a) $N+1 = 4$, (b) $N+1 = 6$, and (c) $N+1 = 8$. The transmission occurs through intermediate states with $N = 3$, 5, and 7 particles, respectively, which lead to minima at flux values $\phi_d \approx \pi(1-2n_s/N)$ (see text).

where $\omega = -2t\cos k$ is the tight-binding dispersion relation for the free electrons in the leads which are incident upon the impurities. These equations are exact for a non-interacting system.

From Eq. (6), $T(\omega, V_g)$ may be calculated from the Green functions of the isolated ring. We consider holes incident on a ring of L sites and $N+1$ electrons, obtaining the Green functions from the ground state of the ring [16] by numerical diagonalization and substituting these in Eq. (6). We fix the energy $\omega = 0$ to represent half-filled leads and explore the dependence of the transmittance on the threading flux.

NUMERICAL RESULTS

The transmittance of the ring, obtained by integration over the excitations in a small energy window, (which accounts for possible voltage fluctuations and temperature effects), at the Fermi level [14], is shown in Fig. 3 for different ring fillings. For injection of a hole in systems containing $N+1 = 4$, 6 or 8 electrons (Fig. 3(a)-(c)), the dynamical properties correspond to $N = 3$, 5, or 7 particles. The most striking result is the existence of dips at certain flux values, which constitute a clear signature of SCS.

The vanishing of the transmittance at flux $\phi = \pi$ is expected from negative interference of the components of the electron wave function traveling in the upper and lower halves of the ring. While the presence of the dip at $\phi = \pi$ is quite general, the origin of the other dips in the transmittance resides in the strongly correlated nature of the problem. We find numerically that the peaks are better defined for small interaction strengths ($J/t_r < 1/L$). In the $J = 0$ limit the model is equivalent to the infinite-U Hubbard model, and complete SCS takes place on all energy scales [17, 18].

Following the method of Ref. [19] for the ring with arbitrary flux, we construct spin wave functions which transform under the irreducible representations of the group C_N of cyclic permutations of the N spins of the L-site system. Each of these representations is labeled by a wave vector $k_s = 2\pi n_s/N$, where the integer n_s characterizes the spin wave function. For $J = 0$ each element of C_N commutes with H_{ring}. In each subspace of states whose spin wave function is characterized by the quantum number k_s, the problem may be mapped to that of a non-interacting, spinless system with effective flux [19]

$$\phi_{\text{eff}} = \phi + k_s = \phi + 2\pi n_s/N. \tag{7}$$

The total energy of any state of the ring becomes

$$E = -2t_r \sum_{l=1}^{N} \cos(k_l + \frac{\phi_{\text{eff}}}{L}),\ \ k_l = \frac{2\pi}{L} n_l, \tag{8}$$

where n_l and N are charge quantum numbers. Thus the dynamical charge properties are described completely by a spinless model. The SCS phenomenon enters in that the spin wave function modifies the effective flux seen by the charges. Hole injection in an N+1-particle system (or particle injection for an $N-1$-particle system) yields an intermediate N-particle state with a certain weight for each spin wave number k_s, before the hole (particle) is ejected through the other lead. Because the charge dynamics in the ring is determined by the flux $\phi_{\text{eff}}(k_s)$, states with spin wave numbers k_s do not contribute to the transport when $\phi_{\text{eff}}(k_s) = \pi$ because they interfere destructively (above). From Eq. (7) one therefore expects dips in the transmittance when $\phi = \phi_d$ with

$$\phi_d = \pi(1-2n_s/N). \tag{9}$$

If for given V_g the window includes the destructively interfering states, then $T(\phi)$ exhibits dips at flux ϕ_d. The dependence of the dip structures on the size of the window is shown in Fig. 4(a) for a system with $N+1 = 8$ particles. Although the integrated transmission decreases with window width, the principal features remain present, indicating the origin of the dips in destructive interference of levels very close to the Fermi energy.

Our numerical results indicate that the above reasoning remains valid for finite J, where SCS is incomplete at finite energies. In this case the loss of SCS at finite energies may be described as an intrinsic phenomenon related to the mixing of different charge subbands. This mixing is strongest at higher values of the flux [Fig. 4(b)].

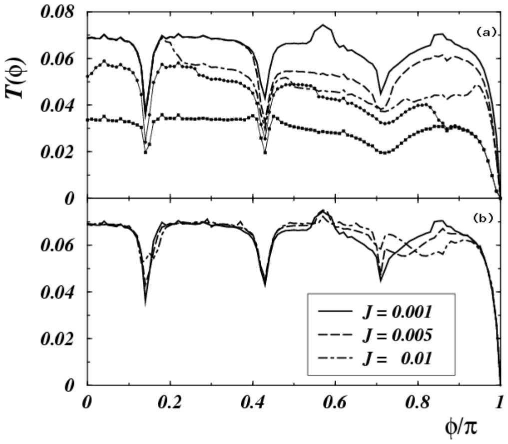

FIGURE 4. (a) Transmittance as a function of flux for a system of $L = 8$ with $J = 0.001t_r$, $N+1 = 8$ particles and different window sizes ranging from $0.6t$ to $0.15t$ (top to bottom) below the Fermi energy. (b) as (a) for a selection of values of J.

In the limit $U \to \infty$, v_s and v_c can be obtained from the ratio of the change in total energy and momentum when either n_s or one of the n_l is increased by one, using either Eqs. (7) and (8) or the Bethe-Ansatz equations of Ref. [17]. For the N-particle intermediate states relevant to our analysis, the effective spinless model has flux π and odd N, circumstances under which for spin quantum number n_s and flux $\phi_d(n_s)$ one obtains $v_s/v_c = 1/N$. This expression is then consistent with a qualitative understanding of the dips in Figs. 3(a)-(c) in terms of the charge and spin components of the injected particle performing different numbers of turns around the ring before reuniting at the drain lead. We stress, however, that in small t_r-J and Hubbard rings a distribution in effective velocities for the states involved in transmission processes is unavoidable at arbitrary flux values.

CONCLUDING REMARKS

In conclusion, we have studied the transmittance through AB rings of interacting electrons and shown that it provides a straighforward technique for the detection of SCS in small systems. The existence of transmission dips arising from non-trivial destructive interference effects at fractional values of the flux quantum Φ_0 is a robust signature of SCS. While the depths and widths of the dips vary with the system under consideration, their positions depend only on ring filling and weakly on interaction strengths. The experimental capability to construct systems exhibiting nanoscopic SCS exists already. An experimental realization of the AB system requires the design of artificial structures, such as rings of quantum dots, on the sub-μm scale; accessible laboratory fields will not permit AB experiments on molecular rings. The wide variety of quantum-dot assemblies synthesized in recent years [20] suggests that such structures are well within the compass of current nanofabrication technology [21].

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