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Juyeon Yi, Peter Talkner, Yong Woon Kim

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Single-temperature quantum engine without feedback control

Juyeon Yi,[1] Peter Talkner,[2,*] and Yong Woon Kim[3,*]
[1]Department of Physics, Pusan National University, Busan 46241, South Korea
[2]Institut für Physik, Universität Augsburg, Universitätsstraße 1, 86159 Augsburg, Germany
[3]Graduate School of Nanoscience and Technology, Korea Advanced Institute of Science and Technology, Daejeon 34141, South Korea
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A cyclically working quantum-mechanical engine that operates at a single temperature is proposed. Its energy input is delivered by a quantum measurement. The functioning of the engine does not require any feedback control. We analyze work, heat, and the efficiency of the engine for the case of a working substance that is governed by the laws of quantum mechanics and that can be adiabatically compressed and expanded. The obtained general expressions are exemplified for a spin in an adiabatically changing magnetic field and a particle moving in a potential with slowly changing shape.

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I. INTRODUCTION

An engine converts some form of energy into mechanical work in a cyclic process that can be repeated at whim. An important example is a heat engine designed to utilize the energy exchange between heat reservoirs at different temperatures [1–3]. The research of heat engines has a long-standing history, going back to the industrial revolution in the 18th century. Recently, motivated by Feynman's quote, "There's plenty of room at the bottom" [4], interest in the working principles of engines functioning on mesoscopic and also on molecular and atomic scales, as well as in their designs and optimization, has grown substantially. A major challenge for the understanding of small-scale engines is to cope with the presence of unavoidable random noise in the form of thermal fluctuations [5–7]. At sufficiently low temperatures, quantum effects such as coherence and quantum noise may also become relevant. Related questions that have been discussed in the literature range from whether quantum effects are of any influence on the performance of an engine at all, to whether quantum effects deteriorate or might even improve the performance of an engine [8–18].

According to the second law of thermodynamics, energy conversion of heat into work cannot be perfectly efficient [2]. A finite amount of unconsumed energy must be dissipated into a low-temperature heat reservoir in order to restore the initial state and complete a cyclic process. Therefore, heat engines operating at a single temperature do not exist. This is also a significant consequence of the fluctuation theorem [19,20] stating that for a cyclic process with only one temperature involved, the average of work *done on* the system $\langle W\rangle$ cannot be negative (for a review, see [21]).

An engine operating at a single temperature does still exist if one allows a Maxwell demon to help, or, in other words, if a feedback control is active, as in a Szilard engine [22,23]. The seeming contradiction to the second law can be resolved if the feedback control mechanism is included in the dynamics of the system constituting the engine [24]. On a more formal level, it can be understood in terms of the information gain $I > 0$ as the result of a measurement that is part of the feedback control [25]. The information gain leads to a modified, negative lower bound of the average work done by the system as $\langle W\rangle \geqslant -k_B T I$ [26,27]. This idea has been applied to classical [28–30] and quantum systems [31–33].

In this paper, we propose a cyclically working quantum engine at a single temperature without feedback control. An essential ingredient of the engine protocol is a quantum measurement performed on the working substance of the engine. The result of this measurement is ignored and therefore cannot trigger any control. This measurement would be ineffective for an engine working according to the laws of classical physics. In a quantum system, however, a measurement imposes a change of the state and consequently an increase of the energy of the system, which is the working substance in the present context.

This paper is organized as follows: Section II is devoted to specifying the relevant steps and processes that make up the cycles of the proposed engine. In Sec. III, we analyze work and heat generated by the engine cycle, and we discuss the results. The findings of Sec. III are exemplified in Sec. IV, and finally Sec. V concludes our paper.

II. ENGINE CYCLE

The working substance of the engine is described by a Hamiltonian $H(\lambda)$ depending on a parameter λ. At the beginning of any cycle, the parameter assumes the value λ_i, and the working substance is in a canonical equilibrium state at the temperature T ($k_B T = \beta^{-1}$). Starting with this state $\mathbf{0}$, a cycle consists of two adiabatic processes, AP I and AP II, interrupted by the measurement, QM, and a final thermalization step, T. This series of "strokes" is sketched as follows:

$$\mathbf{0} \overset{\text{AP I}}{\Longrightarrow} \mathbf{1} \overset{\text{QM}}{\Longrightarrow} \mathbf{2} \overset{\text{AP II}}{\Longrightarrow} \mathbf{3} \overset{\text{T}}{\Longrightarrow} \mathbf{0}. \tag{1}$$

The first stroke AP I is an adiabatic compression caused by a sufficiently slow parameter change from λ_i to λ_f. Under the assumption that the energy levels of the Hamiltonian $H(\lambda)$ do not cross anywhere as a function of the parameter λ, in this stroke the occupation probabilities of the energy branches do not change [36]. This stroke is required to lead to an increasing level spacing. In this sense, it corresponds to a compression of the working substance.

*Authors to whom all correspondence should be addressed: peter.talkner@physik.uni-augsburg.de; y.w.kim@kaist.ac.kr

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While keeping the Hamiltonian $H(\lambda_f)$ fixed, a quantum measurement (QM) of an observable that does not commute with $H(\lambda_f)$ is performed on the working substance. The measurement causes a state change that will be considered as instantaneous. This state change implies a redistribution of the occupation probabilities of the energy eigenstates.

Subsequent to the measurement stroke QM, during AP II the system undergoes an adiabatic expansion by means of a slow parameter change from λ_f back to λ_i. During this stroke, the occupation probabilities of the energy levels stay constant at those values reached immediately after the measurement.

Finally, while the parameter is kept at λ_f, the system is brought into weak contact with a thermal reservoir at the initial temperature T. After a sufficiently long time has elapsed, the system again reaches the initial equilibrium state, and the cycle can be resumed with the adiabatic process I.

III. WORK AND HEAT

In any thermodynamic process, the energy change of a system can be decomposed into work and heat. A change of energy is considered as work if it is caused by the variation of an externally controlled parameter (λ in our case), and as heat if it results from contact of the system with its environment. We adopt the convention to consider both heat and work as positive if the energy of the working substance increases. According to this definition, the energy changes caused by the adiabatic processes AP I and AP II must be classified as work, while in the other two strokes QM and T heat is exchanged. For the measurement stroke and for the thermalization, a measurement apparatus and a heat bath represent the respective environments with which energy is exchanged. We now demonstrate that work can be done within a complete cycle, as described above.

During the stroke AP I the working substance is thermally isolated and its dynamics is governed by a slowly changing Hamiltonian $H(\lambda) = \sum_n E_n(\lambda)|n;\lambda\rangle\langle n;\lambda|$, where $E_n(\lambda)$ and $|n;\lambda\rangle$ are the corresponding eigenvalues and eigenstates, respectively. Here we assume that the eigenvalues $E_n(\lambda)$ are not degenerate for all considered values of λ. Further, by assumption, the working substance initially resides in a canonical equilibrium state. Hence, the population of its energy levels is determined by

$$p_n^{\mathrm{eq}}(\lambda_i) = e^{-\beta E_n(\lambda_i)}/Z, \tag{2}$$

where $Z = \sum_n e^{-\beta E_n(\lambda_i)}$ is the canonical partition function. In the course of the adiabatic stroke, the occupation probabilities of the energy eigenstates remain unchanged at $p_n^{\mathrm{eq}}(\lambda_i)$ such that the state of the working substance at the time when the parameter λ is reached is given by the following density matrix:

$$\rho_I(\lambda) = \sum_n p_n^{\mathrm{eq}}(\lambda_i)|n;\lambda\rangle\langle n;\lambda|. \tag{3}$$

The work in this process is given by the differences of the energies $E_n(\lambda_f) - E_n(\lambda_i)$ occurring with the probability $p_n^{\mathrm{eq}}(\lambda_i)$. Consequently, the average work W_I in the stroke AP I becomes

$$W_I = \sum_n [E_n(\lambda_f) - E_n(\lambda_i)]p_n^{\mathrm{eq}}(\lambda_i). \tag{4}$$

In passing we note that, in contrast to the specification of work done in an arbitrary protocol requiring two energy measurements [34,35], here, due to the adiabaticity of the stroke, a single energy measurement, which could be performed at any instant of time during the stroke, suffices.

In the next stroke, a measurement of an observable A with the eigenvalues a_α as possible results is performed. We consider the class of minimally disturbing generalized measurements, which can be characterized by Hermitian measurement operators $M_\alpha = M_\alpha^\dagger$ satisfying $\sum_\alpha M_\alpha^2 = \mathbb{1}$ [37,38]. In a nonselective measurement, the post-measurement state assumes the form

$$\rho_{\mathrm{PM}} = \sum_\alpha M_\alpha \rho_I(\lambda_f) M_\alpha. \tag{5}$$

For M_α agreeing with the projection operators onto the eigenspaces of the observable A, the standard result of a projective measurement is recovered, but more general measurement schemes can be described in this way. One may wonder about any information change made by QM. If one uses the von-Neumann negentropy to quantify information, $I = \mathrm{Tr}\rho \ln \rho$, one obtains for the change of information caused by the quantum measurement

$$\Delta I = \mathrm{Tr}\rho_{\mathrm{PM}} \ln \rho_{\mathrm{PM}} - \mathrm{Tr}\rho_I(\lambda_f) \ln \rho_I(\lambda_f). \tag{6}$$

However, in contrast to a Szilard engine, which uses the information gain to control the engine, in the present setup the information change is not further exploited. While the negentropy stays constant during the subsequent stroke AP II, it is reset to its initial value by the final thermalization stroke T.

Of vital importance in our setting, however, is the energy change caused by the measurement. From the form (5) of the post-measurement state, one obtains the expression $p(m,n) = T_{m,n} p_n^{\mathrm{eq}}(\lambda_i)$ for the joint probability $p(m,n)$ of finding the eigenstate with label n before and the one with label m after the measurement. Here the transition probability $T_{m,n}$ is given by

$$T_{m,n} \equiv \sum_\alpha |\langle n;\lambda_f|M_\alpha|m;\lambda_f\rangle|^2. \tag{7}$$

Hence, the average energy change Q_M of the working substance caused by the measurement becomes

$$\begin{aligned} Q_M &= \sum_{m,n} [E_m(\lambda_f) - E_n(\lambda_f)]T_{m,n} p_n^{\mathrm{eq}}(\lambda_i) \\ &= \frac{1}{2}\sum_{m,n} [E_m(\lambda_f) - E_n(\lambda_f)]T_{m,n}\left[p_n^{\mathrm{eq}}(\lambda_i) - p_m^{\mathrm{eq}}(\lambda_i)\right] \geqslant 0, \end{aligned} \tag{8}$$

where the second equality is implied by the following properties of the transition matrix [39]:

$$\sum_m T_{n,m} = 1, \quad T_{n,m} = T_{m,n}. \tag{9}$$

Here, the first property follows from the normalization of the measurement operators M_α; the second one is a consequence of the fact that the measurement operators are supposed to be Hermitian. It is evident from the expression of the second line in Eq. (8) that Q_M cannot become negative because $T_{n,m} \geqslant 0$, and the probabilities $p_n^{\mathrm{eq}}(\lambda_i)$ decrease with increasing energies $E_n(\lambda_f)$. Therefore, the amount of heat transferred from the measurement apparatus to the working substance is always positive. In a way, it acts as the hot reservoir of a heat engine. Using the expression (7) for the transition probability, one can write the measurement heat Q_M as

$$Q_M = \sum_n \langle n;\lambda_f|H_M(\lambda_f) - H(\lambda_f)|n;\lambda_f\rangle p_n^{\mathrm{eq}}(\lambda_i), \qquad (10)$$

where $H_M(\lambda_f) = \sum_\alpha M_\alpha H(\lambda_f) M_\alpha$. For measurement operators M_α commuting with $H(\lambda_f)$, one finds $H_M(\lambda_f) = H(\lambda_f)$, and consequently the energy supplied by the measurement is only different from zero if the measurement operators M_α do not commute with the Hamiltonian.

The second adiabatic stroke AP II reverts the first one in changing the parameter from λ_f back to the initial value λ_i. Analogously to the argument leading to Eq. (4), the work W_{II} done by the working substance is given by

$$W_{\mathrm{II}} = \sum_n [E_n(\lambda_i) - E_n(\lambda_f)] p_n^{\mathrm{PM}}, \qquad (11)$$

where p_n^{PM} denotes the probability of finding the nth eigenstate in the post-measurement state (5). It is given by

$$p_n^{\mathrm{PM}} \equiv \langle n;\lambda|\rho_{\mathrm{PM}}|n;\lambda\rangle = \sum_m p_m^{\mathrm{eq}}(\lambda_i) T_{m,n}. \qquad (12)$$

The energy change of the working substance in the final stroke T is caused by contact with a heat bath at temperature T. Its average, therefore, is a heat, which we denote by Q_T. It can be expressed as

$$Q_T = \sum_n E_n(\lambda_i)\left[p_n^{\mathrm{eq}}(\lambda_i) - p_n^{\mathrm{PM}}\right] \leqslant 0. \qquad (13)$$

Along the same lines of arguments leading to the positive sign of Q_M, one finds that Q_T is negative and hence energy is flowing from the working substance into the heat bath.

Finally, we determine the total average work W done by the system as the sum of W_{I} and W_{II}, which are given by Eqs. (4) and (11), respectively. This sum can be expressed as

$$W = \frac{1}{2}\sum_{n,m}\left(\Delta^f_{m,n} - \Delta^i_{m,n}\right) T_{m,n}\left[p_m^{\mathrm{eq}}(\lambda_i) - p_n^{\mathrm{eq}}(\lambda_i)\right], \qquad (14)$$

where $\Delta^\alpha_{m,n}$ denotes the level distance between the mth and the nth energy eigenvalues of the Hamiltonian $H(\lambda_\alpha)$ for $\alpha = i,f$ as given by

$$\Delta^\alpha_{m,n} \equiv E_m(\lambda_\alpha) - E_n(\lambda_\alpha), \quad \alpha = i,f. \qquad (15)$$

To determine the sign of the total work, we consider separately pairs of indices n and m leading to different signs of $\Delta^i_{n,m}$. If n and m are such that $\Delta^i_{m,n} > 0$, then the level distance grows because of the compression in going from λ_i to λ_f, and hence $\Delta^f_{m,n} \geqslant \Delta^i_{m,n}$. Because of the monotonic decrease of the canonical probability $p_k^{\mathrm{eq}}(\lambda_i)$ with increasing energy $E_k(\lambda_i)$, the difference $p_m^{\mathrm{eq}}(\lambda_i) - p_n^{\mathrm{eq}}(\lambda_i)$ is negative; taking into account the positivity of the transition probabilities $T_{m,n}$, all contributions to the right-hand side of Eq. (14) with $\Delta^i_{m,n} > 0$ are negative. Similarly, $\Delta^i_{m,n} \leqslant 0$ implies $\Delta^f_{m,n} \leqslant \Delta^i_{m,n}$ and $p_m^{\mathrm{eq}}(\lambda_i) - p_n^{\mathrm{eq}}(\lambda_i) \geqslant 0$, also leading with $T_{m,n} \geqslant 0$ to a nonpositive contribution to the total work.

Summarizing, we note that within a cycle as sketched in (1), part of the energy Q_M injected by a nonselective, minimally disturbing measurement can be extracted as work W by means of adiabatic processes. The remaining energy Q_T is dumped as heat into a reservoir at the temperature T. The efficiency of the engine is given by

$$\eta = \frac{-W}{Q_M} = 1 - \frac{\sum_{m,n}\Delta^i_{m,n} T_{m,n} p_n^{\mathrm{eq}}(\lambda_i)}{\sum_{m,n}\Delta^f_{m,n} T_{m,n} p_n^{\mathrm{eq}}(\lambda_i)}. \qquad (16)$$

Hence, for an adiabatic compression and expansion in the strokes AP I and APII, respectively, the efficiency is positive and less than 1, in agreement with the first and second law of thermodynamics. In the particular case of uniform compression described by $\Delta^i_{m,n} = \gamma \Delta^f_{m,n}$, with γ being less than 1 and independent of m and n, the efficiency depends solely on the compression factor γ as

$$\eta = 1 - \gamma. \qquad (17)$$

We expect that such an engine will be characterized by smaller but still positive efficiency if the parameter λ is varied at a finite speed rather than adiabatically. A detailed discussion of this issue will be presented elsewhere.

IV. EXAMPLES

We illustrate our findings by two specific examples.

A. Spin 1/2 as a working substance

In the first example, we choose a spin 1/2 in an external magnetic field as the working substance, which hence is governed by the Hamiltonian

$$H(B) = -\mu_B B \sigma_z, \qquad (18)$$

where μ_B is the Bohr magneton, and σ_z is the z component of the Pauli spin matrices. The magnetic field B, which is supposed to point in the z direction, plays the role of the external parameter λ changing in the AP I stroke from $B_0 > 0$ to $B_1 > B_0$ and later in the AP II stroke back again to B_0. The energy eigenvalues of $H(B)$ are $E_\pm(B) = \mp\mu_B B$ in the spin-up (+) and the spin-down (−) state, respectively. The initial populations of these states are specified by the canonical probabilities $p_\pm^{\mathrm{eq}}(B_0) = e^{\pm\beta\mu_B B_0}/Z$, where the partition function is given by $Z = 2\cosh(\beta\mu_B B_0)$. The measurement stroke QM is done as a projective measurement of the spin-component σ_x. It is hence characterized by the measurement operators $M_\pm = (1 \pm \sigma_x)/2$ yielding the transition probability $T_{\pm,\pm} = 1/2$ between all pairs of states as well as uniform post-measurement probabilities $p_\pm^{\mathrm{PM}} = 1/2$. Due to the uniform population of the energy eigenstates after the σ_x measurement, the work done in the AP II stroke vanishes, and the total work is given by that of AP I, which, with Eq. (4), yields

$$W = W_I = \mu_B(B_0 - B_1)\tanh(\beta\mu_B B_0) < 0. \qquad (19)$$

The amount of heat supplied to the system in the measurement stroke follows from Eq. (8) to read

$$Q_M = \mu_B B_1 \tanh(\beta\mu_B B_0). \qquad (20)$$

From Eq. (13), the heat dumped to the thermal reservoir results as

$$Q_T = -\mu_B B_0 \tanh(\beta \mu_B B_0). \tag{21}$$

Finally, the efficiency is given by

$$\eta = 1 - \frac{B_0}{B_1}. \tag{22}$$

This result is in accordance with Eq. (17) because the ratio of the initial and final magnetic field determines the compression factor, i.e., $\gamma = B_0/B_1$.

B. Single particle as a working substance

In the second example, the working substance consists of a particle of mass m moving in a one-dimensional confining potential $V(\hat{x},\lambda)$. Its Hamiltonian hence is given by

$$H(\lambda) = \frac{\hat{p}^2}{2m} + V(\hat{x};\lambda), \tag{23}$$

where $\hat{p}$ and $\hat{x}$ are the momentum and the position operator, respectively. The form of the potential $V(x,\lambda)$ (with x being an eigenvalue of $\hat{x}$) can be controlled by the parameter λ.

We mention as particular cases a free particle in a box of linear size λ, which is described by $V_{\text{box}}(x,\lambda) = 0$ for $x \in (0,\lambda)$ and $V_{\text{box}}(x,\lambda) = \infty$ for $x \notin (0,\lambda)$, and a particle in a harmonic potential $V_h(x,\lambda) = \lambda x^2/2$ with curvature λ. In both cases, a change of λ from λ_i to λ_f as performed in AP I corresponds to a uniform compression with the compression factor $\gamma_{\text{box}} = (\lambda_f/\lambda_i)^2$ for the particle in a box and $\gamma_h = \lambda_i/\lambda_f$ for the harmonic potential, provided $\gamma < 1$.

For engines with single-particle working substances, we specify the QM stroke as a Gaussian position measurement that is characterized by the Hermitian measurement operator $M_\alpha = (2\pi\sigma^2)^{-1/4} e^{-(\hat{x}-\alpha)^2/(4\sigma^2)}$, where α is the measured position and σ^2 is the variance of the measurement apparatus characterizing its precision. Note that the measurement operators are properly normalized according to $\int_{-\infty}^{\infty} d\alpha M_\alpha^2 = \mathbb{1}$ and that in the limit $\sigma^2 \to 0$ a projective position measurement is approached. Because α is a continuous variable, the summation in the normalization of M_α becomes an integral. To determine the energy input Q_M caused by the measurement, we consider the difference $H_M(\lambda_f) - H(\lambda_f)$, which enters the expression (10) for Q_M. Using the normalization of the measurement operators, this difference can be written as $H_M(\lambda_f) - H(\lambda) = \int d\alpha\, M_\alpha[H(\lambda_f), M_\alpha]$, where $[\cdot,\cdot]$ denotes the commutator. Because M_α is a function of the position operator $\hat{x}$ but not of the momentum, only the kinetic part of the Hamiltonian contributes. The resulting commutator can be evaluated for the Gaussian M_α, and it is found, after some algebra, to yield $H_M(\lambda_f) - H(\lambda_f) = \hbar^2/(8m\sigma^2)\mathbb{1}$. With Eq. (10), we reach the expression for the energy delivered by the measurement,

$$Q_M = \frac{\hbar^2}{8m\sigma^2}. \tag{24}$$

It is a remarkable fact that this result is independent of any detail of the potential and also independent of the temperature of the initial state of the working substance. The amount of energy delivered by the measurement only depends on the mass of the particle and the variance of the Gaussian position measurement apparatus. It diverges in the limit of a projective measurement. For a uniform compression, the total work W results as

$$W = -(1-\gamma)\frac{\hbar^2}{8m\sigma^2}, \tag{25}$$

because then the efficiency is given by $\eta = 1 - \gamma$ according to Eq. (17).

V. CONCLUSIONS

We demonstrated that a nonselective, minimally disturbing measurement of any observable that does not commute with the Hamiltonian governing the dynamics of the system at the time of the measurement increases the energy of a quantum system. Because this energy gain is caused by contact with a measurement apparatus, which itself is a quantum system, it can be counted as heat. This is in accordance with earlier observations for particular model systems that repeated measurements may heat up the system to reach infinite temperature [40–43]. The amount of energy Q_M delivered in a single measurement depends on the so-called operation ϕ, characterizing the post-measurement state $\rho^{\text{PM}} = \phi_M(\rho) = \sum_\alpha M_\alpha \rho M_\alpha$ written in terms of the normalized measurement operators M_α. It can be expressed as the difference of the energy average in the post-measurement state and the state ρ immediately before the measurement yielding

$$Q_M = \text{Tr} H \phi_M(\rho) - \text{Tr} H\rho; \tag{26}$$

see also Eq. (10). The positivity of the injected energy is a consequence of the symmetry of the transition matrix imposed by a minimally disturbing measurement and the decay of the Boltzmann weights with increasing energy.

Here we analyzed a cyclic process that works similarly to a heat engine, with the only difference being that the hot heat bath is replaced by a measurement. For the work strokes adiabatic compression and expansion processes are considered. No feedback mechanism is implemented. We found that the total work is negative, meaning that a part of the heat delivered by the measurement can be extracted as work. In general, the efficiency of such an engine as given by Eq. (16) depends on the temperature of the heat bath and the details of the eigenenergies in the initial expanded and in the final compressed state. For a uniform compression, the efficiency simplifies to a mere function of the compression factor.

For the working substance, any quantum system can be employed that can be compressed and expanded in terms of an externally controllable parameter λ. As special examples, we considered a spin 1/2 in an external magnetic field that works as the controllable parameter and a particle in a deformable, confining potential.

In the present paper, we considered only the averages of work and heat. For a full understanding of the proposed type of engines, the full statistics of heat and work caused by the thermal fluctuations of the heat bath and by the intrinsic quantum nature of the working substance will be relevant.

The use of a minimally disturbing measurement as an energy input can be combined with traditional elements of heat engines such as feedback-control [44,45], or it can be used as a boost of a conventional nanoengine with two or more temperature baths. This opens a wide variety of future investigations and potential applications of quantum measurements.

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[38] It is interesting to note that the Jarzynski equality continues to hold in the presence of a minimally destructive measurement during a force protocol [46,47].
[39] Note that the symmetry of the transition matrix is a consequence of the assumption that the measurements are minimally disturbing, meaning that $M_\alpha = M_\alpha^\dagger$. For other types of measurements, the positivity of the energy change caused by the measurement need not be positive.
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