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# Sharp increase of microwave absorption due to shaking of Josephson vortices in BiSCCO superconductor

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We study microwave (9.36 GHz) dissipation in the layered, highly anisotropic superconductors Bi2212 and Bi2223 subjected to a constant magnetic field parallel to the layers. The signal has a characteristic magnetic field dependence, increasing linearly at small fields, reaching a maximum, and decreasing at large fields. We demonstrate experimentally that the microwave absorption is strongly enhanced by application of a low frequency ac magnetic field parallel to the static field. The results are interpreted in the framework of Josephson phase electrodynamics in layered superconductors. In particular, the enhancement is explained as a result of the depinning of the Josephson vortices by the ac field shaking. The theoretical dc field dependence of the microwave absorption for  $H_{dc} \gg H_{ac}$  is in good agreement with experiment. The magnitude of the microwave signal increases rapidly with the anisotropy of the superconductor.

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## I. INTRODUCTION

The vortex state in layered cuprate superconductors has a highly complex phase diagram in the temperature–magnetic field–field orientation space.<sup>1</sup> Relatively simple vortex structures are formed when the magnetic field is directed along one of the principal axes of the layered structure. A dc magnetic field applied perpendicular to the layers penetrates inside the superconductor in the form of pancake vortices. A dc magnetic field directed parallel to the layers penetrates inside the superconductor in the form of Josephson vortices (JVs).<sup>2</sup> The JVs do not have normal cores, but have wide nonlinear cores, concentrated between the central layers. They form a triangular lattice, stretched along the layers, resulting in stacks aligned along the  $c$  direction and widely separated in the in-plane direction. The experimental observation of JVs has drawn great interest both in basic and applied physics. Most of these investigations are devoted to static properties of the JVs, while their dynamics has been less studied.<sup>3</sup>

Experimentally, microwave absorption in a dc magnetic field oriented parallel to the copper-oxide layers was observed a long time ago<sup>4–6</sup> and probed the JV dynamics and pinning. The signal due to their interaction with the magnetic component of the microwave field is typically weak. However, as we demonstrate in the present work, the microwave absorption is greatly enhanced by application of an ac magnetic field collinear to the dc field. The physical reason for the signal amplification is that the ac field shakes pinned fluxons. As a result, depinned JVs near the surface of the superconductor are able to absorb energy. The conclusion is supported by comparison of the present investigation with a detailed investigation carried out by Enriquez *et al.*,<sup>5</sup> where the microwave dissipation in single crystals of Bi2212 was measured as a function of dc field parallel or close to the  $a$ - $b$  plane, but without application of an ac field.

The experimental method of induced microwave dissipation by ac magnetic field<sup>7</sup> (IMDACMF) is applied to study the behavior of the fluxons in bulk single crystals of three high  $T_c$  compounds with different degrees of anisotropy; moderately anisotropic  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  (YBCO), a strongly anisotropic optimally doped  $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_8$  (Bi2223) crystal with  $T_c=90$  K, and extremely anisotropic crystals  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  (Bi2212). This allows us to study the dependence of the effect on anisotropy. It turns out that the signal intensity increases with increasing anisotropy. In addition, it is shown by tilting the dc magnetic field with respect to the  $a$ - $b$  plane that the effect is solely due to JV. We present a theoretical explanation of the microwave dissipation in the presence of an ac field based on a standard Josephson electrodynamics of a layered type II superconductor.

## II. EXPERIMENT

### A. Geometry and setup

Details of the experimental setup have been described elsewhere.<sup>7</sup> It consists of a Bruker ELEXSYS continuous-wave electron spin resonance spectrometer working at X-band frequency (9.36 GHz) and a digital oscilloscope. A microwave source feeds a rectangular H102 cavity with an optimally doped Bi2212 crystal ( $1 \times 1 \times 0.1$  mm<sup>3</sup>) placed in the center, where only the microwave magnetic field is present. The sample, whose temperature could be varied down to liquid helium temperature using a He continuous flow cryostat (Oxford Instruments), is exposed to collinear dc and ac (100 kHz) magnetic fields parallel to the  $a$ - $b$  plane. The microwave magnetic field is applied parallel to the  $a$ - $b$  plane as well, but perpendicular to the dc magnetic field (see Fig. 1). The ac magnetic field induces a signal proportional to the microwave losses in the microwave power reflected

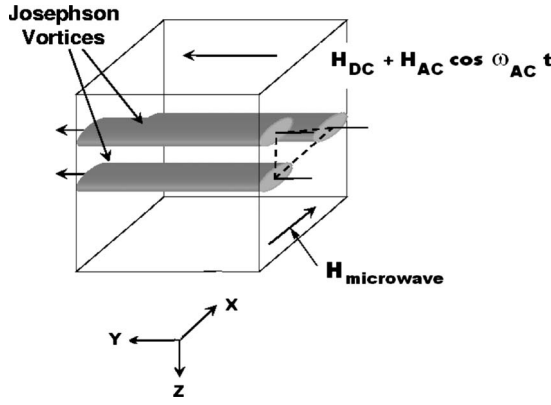


FIG. 1. Geometry of the problem. The  $xy$  plane corresponds to the  $ab$  plane of the cuprate superconductors, i.e., it is parallel to the CuO layers.

from the cavity. It is rectified by a diode that feeds either the lock-in detector of the spectrometer or the digital oscilloscope. The lock-in detector gives the signal intensity and phase of the first harmonic only, while the oscilloscope displays also higher harmonics within its bandwidth. Under certain restrictions specified below, the setup measures the derivative of the power absorption with respect to the dc magnetic field.<sup>6</sup>

The experiments were performed using the zero-field cooling procedure, namely, the sample was cooled from above  $T_c$  down to the required temperature at zero dc and ac fields. Then the ac or the dc field was set to a predetermined value, and the dc or the ac magnetic field, respectively, was increased in steps. The signal was registered at each field value. The reason for increasing the second field in steps and not continuously was that not only the signal intensity but also the signal phase at the ac frequency varied as a function of the variables. Thus, when measuring with the lock-in detector, it was necessary to determine the phase at each measurement. Here, the phase values are stated with respect to the phase of the external ac field  $H_{ac}$ . The method of aligning the crystal with an accuracy of less than  $2^\circ$  is described in a previous work.<sup>7</sup> Comparing the results obtained by the lock-in detector and the oscilloscope provided a full picture including the linearity of the response to the ac field. Similar experiments were performed with optimally doped YBCO and Bi2223.

### B. Experimental results

Microwave absorption power  $P$  of a material generally depends on external parameters such as temperature and magnetic field. The technique used in the present investigation is a modification of the standard microwave absorption technique,<sup>5</sup> in which the difference  $P(H) - P(H=0)$  is generally measured (circles in Fig. 2). The external ac field induces modulation of the absorption power with period  $2\pi/\omega_{ac}$  of order  $10 \mu\text{s}$ ,

$$P(t) = P(H_{dc} - H_{ac} \sin(\omega_{ac} t)). \quad (1)$$

The time dependence of the intensity is shown in Fig. 3 for a

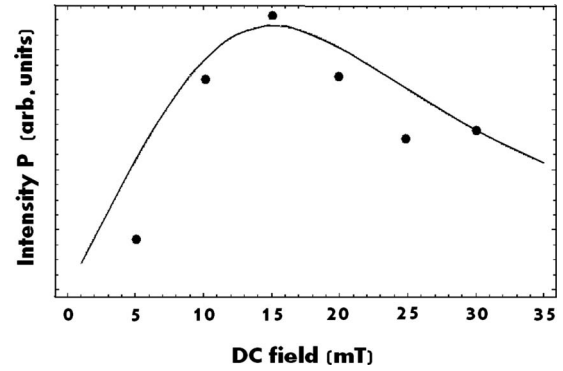


FIG. 2. Microwave absorption intensity without the ac component as function of  $H_{dc}$ . Equation (36) is fitted to the experiment of Ref. 5.

temperature of 40 K. The ac field was fixed at 1.2 mT, while the dc field was increased between 6 and 50 mT. One clearly distinguishes three regimes, see a schematic picture in Fig. 4(a). At small fields, the response is linear. Around  $H_{dc} = 12 \text{ mT} \equiv H_0$ , the response becomes nonlinear, exhibiting the second harmonic of the ac frequency (see the second and third curves in Fig. 3), while at higher fields it becomes linear again, having acquired, however, a phase shift of  $180^\circ$ . Therefore,  $H_0$  separates the two linear regions.

This behavior can be qualitatively explained without invoking the detailed model presented below. Assume the dynamics of the JV is reversible (on the ac time scale of the order of  $10 \mu\text{s}$ ) and  $P(H)$  has a typical shape of linear increase crossing over into a slower decrease at maximum  $H_{\text{max}}$  (solid line in Fig. 2). If the derivative  $P'(H) \equiv \frac{dP(H)}{dH}$  is sufficiently large at  $H_{dc}$  (positive or negative), one can expand the absorption power  $P(t)$  around  $H_{dc}$ :

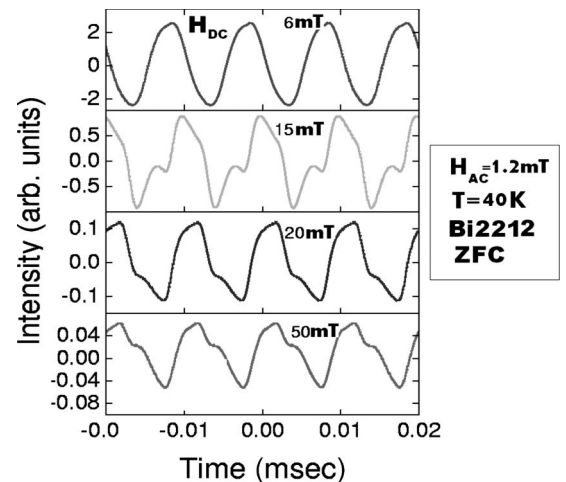


FIG. 3. Time dependence of the microwave absorption intensity on the microsecond scale (a typical period of the ac field). As the magnetic field increases, the response changes from linear to nonlinear and back to linear with a phase difference of  $180^\circ$ . The signal at 50 mT still exhibits clear contributions of the second harmonic of the ac frequency.

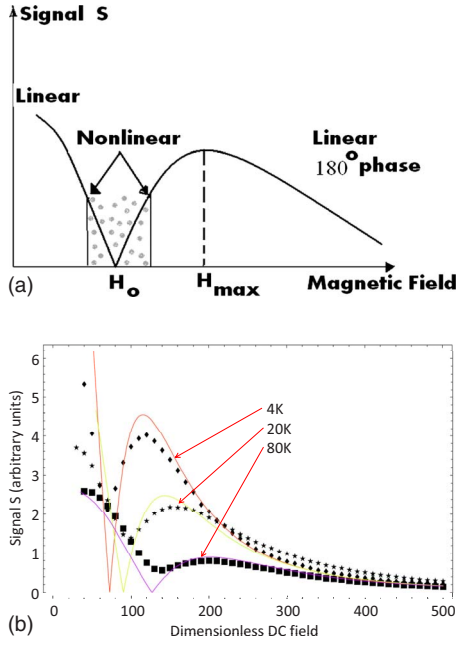


FIG. 4. (Color online) Dependence of the absorption signal on dc magnetic field. (a) Schematic plot of the peak to peak signal as a function of the dc magnetic field in the limit of vanishing ac field amplitude. Two linear regions and a nonlinear region are shown. (b) Comparison of the approximate expression Eq. (40) (solid lines) with experimental data for three temperatures  $T=4, 20$ , and  $80$  K. The theoretical fitting parameters  $a/10^9$  and  $k/10^6$  are (105,3), (150,8.4), and (110,16) correspondingly. The ac magnetic field is held at  $H_{ac}=1.2$  mT.

$$P(t) = P(H_{dc}) - P'(H_{dc})H_{ac} \sin(\omega_{ac}t). \quad (2)$$

The expected time dependence is similar to the one given in the top and bottom curves in Fig. 3, respectively. However, close to  $H_0$ , the first derivative is small, and one should consider the second derivative

$$P(t) = P(H_{dc}) + \frac{1}{2}P''(H_{dc})[H_{ac} \sin(\omega_{ac}t)]^2. \quad (3)$$

It is in this region of fields that the second harmonic appears, see Fig. 4(a).

Note that due to the lock-in technique, the measured quantity is not the power  $P$ , but rather the electronically detected peak to peak difference

$$S = \max_t P - \min_t P. \quad (4)$$

Note that if  $H_{ac}$  is sufficiently small and  $|P'(H)|$  is not small, the time dependence of the power is sinusoidal, and one can approximate the signal by

$$S = 2|P'(H)|H_{ac}. \quad (5)$$

Otherwise, Eq. (5) should be replaced by the nonlinear relation

$$S = \frac{1}{2}|P''(H)|H_{ac}^2. \quad (6)$$

The signal  $S$  as function of  $H_{dc}$  for  $H_{ac}=1.2$  mT is shown for various temperatures in Fig. 4(b). It exhibits a dip at a certain field, which will be identified with  $H_0(T)$ , followed by a maximum at  $H=H_{max}$ , and finally decreases to zero asymptotically. The maximal value of the signal  $S(H_{max})$  decreases with temperature. The location of the dip weakly depends on temperature. The dependence of these features on anisotropy was also studied using three different high  $T_c$  materials and will be discussed later. These features will be explained theoretically next.

### III. THEORY

#### A. Basic equations and separation of time scales

In a strongly type II superconductor like Bi2212, for fields  $H \ll H_{c2}(T)$ , one can use the London approximation with an order parameter in which the absolute value is “frozen” and the superconducting (Josephson) phase is the only degree of freedom  $\psi(\mathbf{r}) = \psi_0 \exp[i\phi(\mathbf{r})]$ . We assume also that the temperature is not too close to  $T_c$  and the magnetic field is not strong enough to reduce significantly the order parameter. Moreover, thermal fluctuations on the mesoscopic scale are neglected, retaining only “microscopic” thermal fluctuations which make the penetration depth temperature dependent, that is,  $\lambda_c(T), \lambda_{ab}(T) \propto (1 - T/T_c)^{-1/2}$ . We assume that the supercurrent is concentrated mainly in very thin bilayers separated by a distance  $s$ . The coordinate perpendicular to layers, therefore, becomes discrete:  $z = ns$  (see Fig. 1). Let us first consider the set of equations neglecting both bulk and surface pinning.

The set of equations<sup>3,8</sup> in the geometry of the experiment involves the  $x$  component of magnetic induction in the  $n$ th layer  $B_n(x, y)$  and the gauge-invariant phase difference between adjacent layers  $\theta_n = \phi_{n+1} - \phi_n - (2\pi s/\Phi_0)A_z$  (here,  $\Phi_0$  is the unit flux and  $A_z$  is the vector potential):

$$\left( \frac{4\pi\sigma_{ab}}{c^2} \frac{\partial}{\partial t} + \lambda_{ab}^{-2} \right) \left( \frac{\Phi_0}{2\pi s} \frac{\partial \theta_n}{\partial x} - B_n \right) = - \frac{B_{n+1} + B_{n-1} - 2B_n}{s^2}, \quad (7)$$

$$\frac{\Phi_0 \sigma_c}{2\pi c s j_J} \frac{\partial \theta_n}{\partial t} + \sin \theta_n + \mu(x) \sin \theta_n + \frac{1}{\omega_p^2} \frac{\partial^2 \theta_n}{\partial t^2} - \frac{c}{4\pi j_J} \frac{\partial B_n}{\partial x} = J_s, \quad (8)$$

where

$$D_z = \text{Re}[D_{sh} e^{i\omega_{sh}t} + D_{mw} e^{i\omega_{mw}t}], \quad (9)$$

$$J_s = \frac{1}{4\pi j_J} \frac{\partial D_z}{\partial t} = J_s^{mw} + J_s^{sh}. \quad (10)$$

Frequency  $\omega_{sh}$  and  $J_s^{sh}$  are identical to parameters of external ac signal.  $D_z$  is external electric field including both the “shaking frequency” and the microwave frequency  $\omega_{sh} \ll \omega_{mw}$ ;  $\mu(x)$  is a pinning potential (see Ref. 9). The electric

field is linked to the phase differences by the Josephson relation

$$E_z = \frac{\Phi_0}{2\pi s c} \frac{\partial \theta_n}{\partial t}. \quad (11)$$

The coefficients  $\sigma_{ab}$  and  $\sigma_c$  are conductivities, the plasma frequency is  $\omega_p = c/\epsilon_c^{1/2}\lambda_c$  with  $\epsilon_c$  being the dielectric constant, and the Josephson current density

$$j_J = \frac{c\Phi_0}{8\pi^2 s \lambda_c^2}. \quad (12)$$

The equations are supplemented by the boundary conditions ( $\mathbf{H}_{sh} \perp \mathbf{H}_{mw}$ ) (see Fig. 1)

$$H_y = H_{0y} + \text{Re}[H_{sh} e^{i\omega_{sh} t}], \quad (13)$$

$$\left( \frac{\partial \theta_n}{\partial x} \right)_b = \frac{2\pi s}{\Phi_0} H_y \quad (14)$$

including both the dc and the time dependent parts consistent with Eq. (9).

Using dimensionless variables  $x \rightarrow x/\gamma s; t \rightarrow \omega_p t$ , fields  $b_n = B_n 2\pi \gamma \lambda_{ab}^2 / \Phi_0$ , current density  $j_s = J_s 8\pi^2 \gamma^2 s \lambda_{ab}^2 / c \Phi_0$ , and parameters ( $\gamma = \lambda_c / \lambda_{ab}$  is the anisotropy parameter)

$$\nu_c = 4\pi \sigma_c / \epsilon_c \omega_p, \quad \nu_{ab} = 4\pi \sigma_{ab} \lambda_{ab}^2 \omega_p / c^2, \quad l = \lambda_{ab} / s, \quad (15)$$

the equations and the boundary conditions become

$$\left( b_{n+1} + b_{n-1} - 2b_n - \frac{b_n}{l^2} \right) + \frac{\partial \theta_n}{\partial x} + \nu_{ab} \frac{\partial}{\partial t} \left( \frac{\partial \theta_n}{\partial x} - \frac{b_n}{l^2} \right) = 0, \quad (16)$$

$$\frac{\partial^2 \theta_n}{\partial t^2} + \nu_c \frac{\partial \theta_n}{\partial t} + \sin \theta_n + \mu(x) \sin \theta_n - \frac{\partial b_n}{\partial x} = j_s, \quad (17)$$

$$\mathbf{h}_b = \mathbf{h}_{dc} + \text{Re}[\mathbf{h}_{sh} e^{i\omega_{sh} t}], \quad (18)$$

$$\left( \frac{\partial \theta_n}{\partial x} \right)_b = \mathbf{h}_b. \quad (19)$$

Since there is *no microwave magnetic field in the y direction*, the high frequency component cannot participate in formation of the JV structure. Hence, only the dc component of the magnetic field is essential for the JV structure, while the interaction with the microwave current  $j_s^{mw}$  governs the JV dynamics only. In the absence of an ac field, the static JV structure oscillates with the external microwave frequency and a low frequency signal directed parallel to the dc field cannot essentially change this behavior. However, this ac component becomes essentially important due to spatial defects in the layered structure. If the JVs in equilibrium are pinned by the defect potential, the JVs lose their mobility and the microwave absorption is decreased. On the other hand, a low frequency current driving the vortices in the pinning potential profile by the Lorentz force (ac) can release them from the pinning traps and restore their mobility. This

mechanism (shaking) can lead to drastic enhancement of the microwave absorption power.

In order to study this phenomenon formally, one has to perform an average procedure of the exact equations over the fast oscillations, solve the problem for low frequency, and afterward take into account the external high frequency driving force. The set of equations responsible for a quasistatic JV structure<sup>10</sup> is

$$\frac{\partial \theta_n}{\partial x} - \frac{b_n}{l^2} = 0, \quad (20)$$

$$\frac{\partial^2 \theta_n}{\partial t^2} + \nu_c \frac{\partial \theta_n}{\partial t} + \sin \theta_n + \mu(x) \sin \theta_n - \frac{\partial b_n}{\partial x} = j_s^{sh}. \quad (21)$$

The boundary condition becomes

$$\mathbf{h}_y = \mathbf{h}_{0y} + \text{Re}[\mathbf{h}_{sh} e^{i\omega_{sh} t}], \quad (22)$$

$$\left( \frac{\partial \theta_n}{\partial x} \right)_b = \mathbf{h}_{0y} + \text{Re}[\mathbf{h}_{sh} e^{i\omega_{sh} t}]. \quad (23)$$

One observes that naturally the slow component equations are independent of microwave, but can affect the microwave absorption in two ways: via reduction of the bulk or surface pinning and by a significant thermal heating due to shaking in a wider ac skin depth, causing a temperature gradient. In fact, shaking is incorporated phenomenologically via pinning strength and surface temperature.

### B. Static Josephson vortex lattice

Let us first consider the dc structure determined by Eqs. (20) and (21), neglecting the ac component and pinning and dissipation terms. Since the static problem is the same for any  $n$  and  $y$ , the problem becomes one dimensional,

$$\frac{d^2 \theta^{st}}{dx^2} = \sin \theta^{st},$$

and is governed by the Lawrence-Doniach-Gibbs energy:

$$G = dx \left[ \frac{1}{2} \left( \frac{d\theta^{st}}{dx} \right)^2 + (1 - \cos \theta^{st}) \right] - 2\pi N h_{dc}. \quad (24)$$

Integer  $N$  in Eq. (24) counts the number of the phase slips across the length  $L$  in which the magnetic field creates  $N$  equally separated solitons. A general solution of the static equations (20) and (21) is

$$\theta^{st}(x) = 2am \left[ \sqrt{\frac{E-1}{2}} x + F \left( \frac{\theta^{st}(0)}{2}, -\frac{2}{E-1} \right), -\frac{2}{E-1} \right]. \quad (25)$$

Here,  $am$  and  $F$  are elliptical functions.<sup>11</sup> The period of the vortex structure can be found by minimization of the Gibbs energy ( $dG/da=0$ , where  $a$  is the inter-JV distance), while the external magnetic field at the edges of the sample



is assumed to be constant. The equation determining the integration constant  $E$  is derived after some algebra as

$$\int_0^{2\pi} \sqrt{E - \cos \theta} d\theta = \sqrt{2} \pi h_{dc}. \quad (26)$$

This expression was solved numerically to determine  $\theta^{st}(x)$  for a given  $h_{dc}$ .

### C. Pinning potential and spring constant $K$

In a weak magnetic field when the JV are well separated, the pinning of an individual JV is independent of the pinning of others. In this case, the structural defects in Eqs. (8), (21), and (17) can be treated by the McLaughlin-Scott<sup>12</sup> perturbation theory. Considering, for simplicity, a single structural defect in the form  $\mu(x) = \mu \delta(x)$ , one obtains that the equilibrium position of a pinned JV  $x_0(j_s^{sh})$  slowly oscillates as  $j_s^{sh}(t)$ . If the amplitude  $j_s^{sh}(t)$  exceeds some critical value  $j_c$ , the Josephson vortices become free.<sup>13</sup>

Going back to the exact equations, one obtains for a small but rapidly oscillating vortex displacement  $u$  from the “adiabatic” Josephson vortex position (in dimension units)

$$\rho_J \frac{d^2 u}{dt^2} + \eta_J \frac{du}{dt} + K(j_s^{sh}, \mu) u = \frac{\Phi_0}{c} j_s^{mw}, \quad (27)$$

where the driven force in the right side is responsible for the JV interaction with the microwave current.

Here,  $\rho_J$  and  $\eta_J$  are the mass and the viscosity of the JV calculated by Koshelev using the static configurations, Eq. (25), for well separated JV:<sup>3</sup>

$$\eta_J = \frac{\varepsilon_c \omega_p \Phi_0^2}{\pi (4\pi c s)^2 \gamma} (C_c \nu_c + C_{ab} \nu_{ab}), \quad \rho_J = \frac{\varepsilon_c C_c \Phi_0^2}{\pi (4\pi c s)^2 \gamma}. \quad (28)$$

(The anisotropy parameter  $\gamma = \lambda_c / \lambda_{ab} \sim 500$  for BSCCO,  $C_c = 9.0$  and  $C_{ab} = 2.4$ .)

The pinning constant  $K$  plays the role of a spring constant for the JV in the potential well. In a more realistic case when structural defects are randomly distributed, the spring constant is defined by the maximum value of the pinning potential. If the magnetic field is increased, then the collective pinning effects become dominant.<sup>9</sup> However, even in this case, the JV dynamics obeys the same equation with a phenomenologically defined parameter  $K$  (Labush constant). In fact, JV interaction with a set of spatial defects is described by a harmonic force of strength  $K$  in this approach. This parameter is reduced by the low frequency component of the magnetic field (the shaking effect).

### D. Microwave absorption for $\omega_{mw} \ll \omega_p$

Consider a superconductor in the vortex state carrying an ac supercurrent  $J_s = \exp(-i\omega_{mw}t)$  along the  $c$  axis. One obtains from the Maxwell equations

$$E_z = -\frac{4\pi\lambda_c^2}{c^2} i\omega_{mw} J_s - \frac{B_y}{c} i\omega_{mw} u \quad (29)$$

the oscillation amplitude  $u$  of a single vortex which can be found from Eq. (27):

$$(-\rho_J \omega_{mw}^2 - i\eta_J \omega_{mw} + K)u = \frac{\Phi_0}{c} J_s. \quad (30)$$

Neglecting  $\sigma_c$ , one obtains<sup>3</sup> the inverse dielectric function

$$\frac{\varepsilon_c}{\varepsilon_c(h, \omega)} = \frac{\omega^2}{\omega^2 - \left(1 - \frac{2\pi h}{C_c \omega^2 - K + iC_{ab} \nu_{ab} \omega}\right)^{-1}}, \quad (31)$$

where  $\omega = \omega_{mw} / \omega_p$ . The power absorption is

$$P = -\text{Im} \left[ \frac{\varepsilon_c}{\varepsilon_c(h, \omega)} \right] = \omega^3 C_{ab} \nu_{ab} \frac{2\pi h}{[\omega^2(-C_c - 2\pi h) + K]^2 + [C_{ab} \nu_{ab} \omega]^2}. \quad (32)$$

At small fields, it linearly grows with  $h$

$$P \approx \omega^3 C_{ab} \nu_{ab} \frac{2\pi h}{(K - \omega^2 C_c)^2 + (C_{ab} \nu_{ab} \omega)^2}. \quad (33)$$

For large fields when the JVs overlap and the equations can be linearized, one obtains<sup>3</sup>

$$\frac{\varepsilon_c}{\varepsilon_c(h, \omega)} = -\frac{\omega^2 h^2 (2\omega^2 - h^2/2 + i\nu_{ab} \omega h^2/2)}{-\omega^2 h^2 (2\omega^2 - h^2/2) + 4\omega^2 + i\omega \nu_{ab} h^2 (1 - \omega^2 h^2/2)}. \quad (34)$$

Asymptotically, at large  $h$ , the resulting power absorption is

$$P \approx 2\nu_{ab} \frac{1}{h^2 \omega}. \quad (35)$$

An interpolation formula combining the two limits,

$$P = a \frac{h}{k + h^3}, \quad (36)$$

depends on two parameters only:

$$k = \frac{(K - \omega^2 C_c)^2}{2\pi \omega^4 C_{ab}} + \frac{C_{ab} \nu_{ab}^2}{2\pi \omega^2} \quad (37)$$

and

$$a = 2\nu_{ab} \omega^{-1} = \frac{8\pi \sigma_{ab} \lambda_{ab}^2}{\omega_{mw} \lambda_c^2 \varepsilon_c}. \quad (38)$$

This approximate expression is used in fits resulting in theoretical curves in Figs. 1 and 3(b).

### E. Effects of shaking of Josephson vortex: Depinning and the Joule heat

Generally, there are two major effects which impact the account of microwave absorption. The first is the reduction of the pinning constant  $K$  due to the fact that shaking force is applied over times sufficiently long for a vortex to escape the

surface or bulk pinning potential. The second is the Joule heat generation in the skin depth  $\sim(\omega_{sh})^{-1/2}$ , which is much wider than that for the microwave radiation. The heat generation causes a temperature gradient with higher value near the surface. This, in turn, results in the reduction of surface barriers along with other thermal effects. While comparing with experiment in the next section, we take into account the dependence of  $K$  on both amplitude and frequency of the shaking field phenomenologically.

#### IV. COMPARISON WITH EXPERIMENT

Figure 4(b) shows the temperature evolution of the signal intensity  $S$  in Bi2212 defined in Eq. (4) as a function of dc magnetic field in the superconducting regime at temperatures 4, 20, and 80 K with an ac field  $H_{ac}=1.2$  mT, significantly smaller than the applied dc field  $H_{dc}$ . The dependence on  $H_{dc}$  is explained by the positive and negative derivatives of the power at small and large fields, respectively [see Figs. 2 and 4(a)]. In the region in which the derivative  $\frac{dP}{dH}$  is zero, the signal (peak to peak difference) does not actually vanish, but rather acquires a small value due to the fact that  $H_{ac}$  is finite (due to limited physical significance of the effect of finite  $H_{ac}$  amplitude, we still used the limit of vanishing  $H_{ac}$  in the fits). At higher fields, the function  $S(H)$  reaches a maximum. This maximum is shifted to lower fields with decreasing temperatures [see Fig. 4(b)].

##### A. Theoretical fit to data

In Fig. 2, we analyze with the help of Eq. (36) the data of an early experiment<sup>5</sup> in which shaking was absent and one measures directly  $P(h)-P(h=0)$ . The maximum absorption in this experiment occurs at  $h_0=(k/2)^{1/3}$  with the value

$$P_{\max} = a \frac{1}{3(k/2)^{2/3}}. \quad (39)$$

The data can be fitted well and exhibited the correct asymptotic behavior both at small and large fields. The signal is small, suggesting that pinning dominates,  $k \simeq \frac{K^2}{2\pi\omega^4 C_{ab}}$ . In our experiments, the peak to peak difference is measured, corresponding in the limit of small amplitude of the ac component of the magnetic field to the derivative with respect to the dc field,

$$S = |P'| = a \frac{|k - 2h^3|}{(k + h^3)^2}. \quad (40)$$

As was mentioned in Sec. II B, this formula is correct everywhere, except in a region near  $P'=0$  at  $h_0=(k/2)^{1/3}$  [shaded area in Fig. 4(a)]. At small field (but, of course, still much larger than  $h_{ac}$ ), the signal is the largest  $S(h=0)=a/k$ , while the local maximum appears at  $h_{\max}=(2k)^{1/3}$ . The maximal value is  $S(h_{\max})=\frac{a}{3k}$ . If pinning is reduced by shaking, so that the spring constant  $K$  can be neglected, one gets

$$h_{\max} = \left( \frac{C_{ab}\nu_{ab}^2}{\pi\omega^2} \right)^{1/3} \propto T^{2\alpha/3}, \quad (41)$$

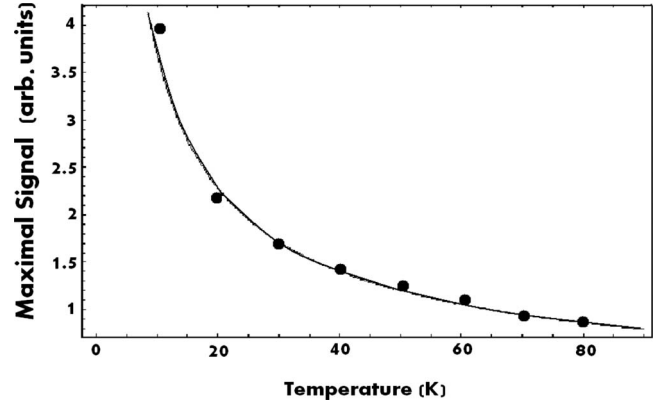


FIG. 5. Dependence of the maximum in signal on temperature for the ac field  $H_{ac}=1.2$  mT. The fit according to Eq. (42) determines the exponent  $\alpha=0.7$ .

$$S(h_{\max}) = \frac{4\pi\omega}{3C_{ab}\nu_{ab}} = \frac{\varepsilon_c\lambda_c^2\omega_{mw}}{3\lambda_{ab}^2C_{ab}\sigma_{ab}} \propto \gamma^2 T^{-\alpha}. \quad (42)$$

The fits in Fig. 4(b) all use the same value of  $a$ , while values of  $k$  depend on temperature. The approximate simple formula neglects  $h_{ac}$  compared with  $h_{dc}$ , the fact that leads to the vanishing of the signal at  $h_0$ . For finite  $h_{ac}$ , the intensity never vanishes. Results for the maximal signal as function of temperature are presented in Fig. 5 and are fitted by  $\alpha=0.7$  according to Eq. (42).

##### B. Anisotropy effect

To find the dependence of the microwave dissipation on the interlayer coupling, we performed the same set of experiments on Bi2223, whose anisotropy is lower than that in Bi2212.<sup>14</sup> Several Bi2223 crystals were investigated, whose  $T_c$  varied between 110 and 105 K. The signal decreases when the anisotropy parameter decreases, although quantitative comparison with Eq. (42), for example, cannot be made at present. Consistently, the signal in YBCO is very weak and cannot be reliably measured. Note that the direction of the dc magnetic field (parallel to the layers) is important. If the field is tilted away from this direction, segments of fluxons become Abrikosov (pancake vortices in highly anisotropic compounds) vortices rather than the JV discussed above. In this case, the depinning by shaking is ineffective. This was established experimentally by measuring the microwave absorption with field tilted in different angles. The signal sharply decreases as the tilt angle increases.

#### V. DISCUSSION AND CONCLUSIONS

It was demonstrated that the addition of the low frequency ac magnetic field collinear to the dc field applied in the  $a$ - $b$  plane of high  $T_c$  superconductors greatly enhances the microwave absorption signal. Physically, this is explained by the effect of depinning of the JVs by “shaking” due to the ac component. In the present case, this method is applied to enhance the signal due to the Josephson vortices rather than the Abrikosov vortices.<sup>15</sup> Therefore, the experimental

method of IMDACMF can be effectively applied to study the behavior of the JVs in strongly anisotropic superconductors. Three bulk single crystals of high  $T_c$  compounds with different degrees of anisotropy, a moderately anisotropic YBCO, a strongly anisotropic optimally doped Bi2212 crystal, and extremely anisotropic Bi2223 crystals, were investigated. The enhancement due to shaking is more pronounced in more anisotropic materials and depends on temperature. We present a theoretical explanation of the microwave dissipation in the presence of an ac field based on Josephson electrodynamics in a layered type II superconductor. It provides not only a fit to the old results on microwave dissipation without ac field,<sup>5</sup> but is in good agreement with the ac experiments. The time dependence of the microwave absorption intensity exhibits a nonlinear behavior in a range of dc fields near a certain field  $H_0$  independent of the shaking. Well above this field, the response is linear, but acquires a 180° phase. These results allow one to obtain both the dynamic characteristics of the JV lattices and their pinning. In particular, Eqs. (41) and (42) allow us to measure the temperature dependence of the JV lattice parameters. Some of this information cannot be obtained by other methods. In particular, the method might be useful to study the lock-in transition in layered superconductors.<sup>16</sup> This transition should be accom-

panied by a significant drop in the microwave response.

The present work investigating the interaction of JV moving through the superconducting surface with an electromagnetic field (microwave field in the present work) can be, in principle, extended to the full electromagnetic spectrum up to submillimeter wavelength (and even shorter wavelength), where a large number of investigations have been carried out.<sup>17</sup> It can show the effect of the dynamics of JV on the full spectrum of electromagnetic absorption.

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