Skyrmion–Anti-Skyrmion Pair Creation by in-Plane Currents

Martin Stier,[1,*] Wolfgang Häusler,[1,2] Thore Posske,[1] Gregor Gurski,[1] and Michael Thorwart[1]
[1]I. Institut für Theoretische Physik, Universität Hamburg, Jungiusstraße 9, 20355 Hamburg, Germany
[2]Institut für Physik, Universität Augsburg, 86135 Augsburg, Germany

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Magnetic Skyrmions can be considered as localized vortexlike spin textures which are topologically protected in continuous systems. Because of their stability, their small size, and the possibility to move them by low electric currents, they are promising candidates for spintronic devices. Without changing the topological charge, it is possible to create Skyrmion–anti-Skyrmion pairs. We derive a Skyrmion equation of motion which reveals how spin-polarized charge currents create Skyrmion–anti-Skyrmion pairs. It allows us to identify general prerequisites for the pair creation process. We corroborate these general principles by numerical simulations. On a lattice, where the concept of topological protection has to be replaced by that of a finite energy barrier, the anti-Skyrmion partner of the pairs is annihilated and only the Skyrmion survives. This eventually changes the total Skyrmion number and yields a new way of creating and controlling Skyrmions.

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Magnetic Skyrmions (SKs) are vortexlike localized magnetization configurations [1,2] which have been predicted [3–5] before they were discovered experimentally [6–9] in magnetic layers with a strong spin-orbit interaction [10,11]. Despite their potentially small size [12,13], their thermodynamic stability is considerably strong [7,9,14]. This is due to the particular magnetic configuration which can be characterized by a total topological charge or SK number Q. This number is an integer and cannot be changed continuously [15,16]. This feature protects magnetic SKs against typical drawbacks of solid state systems such as disorder or imperfect fabrication [10,17]. Together with the property of easy repositioning by rather tiny in-plane electrical currents [18–22], this makes single SKs attractive candidates for racetrack memory devices [18,23–27]. Creation of SKs has been reported in the vicinity of notches [19], by circular currents [28], by geometrical constraints [29], or by sweeping the external magnetic field [30]. Controlled creation and annihilation of individual SKs has been demonstrated [31].

In this Letter, we derive a SK equation of motion which reveals the details of the process of how the total topological charge Q changes by an applied in-plane current. We find that this happens in two steps. First, a Skyrmion–anti-Skyrmion (SK–ASK) pair is created [30] initiated by small spatial fluctuations of the magnetization. Pair creation does not alter Q, since the SK and the ASK have equal topological charge of opposite sign, respectively. Because of the external current, the SK and ASK get spatially further separated. The SK equation of motion reveals the relevant terms at work which are not captured by the common Thiele approximation [32,33]. Finally, the ASK, being no stable solution for a given Zeeman field and a Dzyaloshinsky-Moriya interaction, decays due to Gilbert damping. It is this second step which is ultimately responsible for changing Q and which crucially relies on dissipation. All general findings are confirmed by numerical simulations. Recently, SK–ASK pair creation by in-plane currents in systems without Dzyaloshinsky-Moriya interaction has been reported [34]. Moreover, SK creation by in-plane currents [35] or spin-orbit torques [36] has been observed experimentally.

The two-dimensional magnetization configuration $\mathbf{M}(x,y,t)$ of a single current-driven SK evolves in time according to the extended Landau-Lifshitz-Gilbert (LLG) equation [37–40]

$$\partial_t \mathbf{n} = -\mathbf{n}\times\mathbf{B}_{\text{eff}} + \alpha\mathbf{n}\times\partial_t\mathbf{n} + (\mathbf{v}_s\cdot\nabla)\mathbf{n} - \beta\mathbf{n}\times(\mathbf{v}_s\cdot\nabla)\mathbf{n}, \quad (1)$$

where $\mathbf{n} = \mathbf{M}/|\mathbf{M}| = \mathbf{n}(x,y,t)$ is a normalized vector field. All interactions of the Hamiltonian H describing the system are contained in the effective field $\mathbf{B}_{\text{eff}} = -\partial H/\partial\mathbf{n}$. Below, in Eq. (8), we specify the Hamiltonian for a lattice model, but its detailed form is not relevant for the following general findings. $\mathbf{B}_{\text{eff}}$ contains the gyromagnetic ratio and we set $\hbar = 1$. Further parameters are the Gilbert damping constant α and the nonadiabaticity parameter β. Here, we focus on the impact of spin-polarized electric currents $\mathbf{v}_s = pa^3\mathbf{I}_c/(2e)$ [41] flowing in the magnetic plane with spin polarization p and lattice constant a, proportional to a charge current density $\mathbf{I}_c$. Then, we define the topological charge density

$$q(x,y,t) = \mathbf{n}\cdot[(\hat{\mathbf{v}}\cdot\nabla)\mathbf{n}\times(\hat{\mathbf{v}}_\perp\cdot\nabla)\mathbf{n}], \quad (2)$$

and the total topological charge

$$Q = Q(t) = \frac{1}{4\pi}\int dx\,dy\,q(x,y,t), \qquad Q \in \mathbb{Z}. \qquad (3)$$

In fact, this homotopy invariant completely determines the topological properties of SKs even though it does not specify, e.g., the vorticity of a SK (ASK) without further definitions [15,16,42]. In this Letter, however, the magnetic background will be fixed in such a way that $Q > 0$ ($Q < 0$) refers to Skyrmions (anti-Skyrmions) [30]. For convenience, we take the direction of the spin current $\mathbf{v}_s$ as a reference direction, $\hat{\mathbf{v}} = \mathbf{v}_s/|\mathbf{v}_s|$ and $\hat{\mathbf{v}}_\perp = \hat{\mathbf{z}} \times \hat{\mathbf{v}}$. While the topological invariant Q is conserved in time at low energies, the time evolution of $q(x,y,t)$ describes the current-induced local motion of SKs. In particular, as discussed below, it also describes the generation or annihilation of SK–ASK pairs.

To reveal the SK–ASK pair creation mechanism, we decompose the effective field according to

$$\mathbf{B}_{\mathrm{eff}} = b_{/\!/}\mathbf{n} + b_{\perp 1}(\mathbf{v}_s\cdot\nabla)\mathbf{n} + b_{\perp 2}\mathbf{n}\times(\mathbf{v}_s\cdot\nabla)\mathbf{n}. \quad (4)$$

By combining Eqs. (1), (2), and (4), we readily obtain the SK equation of motion

$$\partial_t q = -\nabla\cdot(\mathbf{j}^{(1)}_{\mathrm{SK}} + \mathbf{j}^{(2)}_{\mathrm{SK}}), \qquad (5)$$

with the SK current densities

$$\mathbf{j}^{(1)}_{\mathrm{SK}} = -j_1 q\mathbf{v}_s, \qquad (6a)$$

$$\mathbf{j}^{(2)}_{\mathrm{SK}} = j_2\{[(\hat{\mathbf{v}}\cdot\nabla)\mathbf{n}\cdot(\hat{\mathbf{v}}_\perp\cdot\nabla)\mathbf{n}]\mathbf{v}_s \qquad (6b)$$

$$- [(\hat{\mathbf{v}}\cdot\nabla)\mathbf{n}]^2\mathbf{v}_\perp\}, \qquad (6c)$$

with contributions parallel ($\propto\mathbf{v}_s$) and perpendicular ($\propto\mathbf{v}_\perp \equiv \hat{\mathbf{z}}\times\mathbf{v}_s$) to the current. The coefficients read

$$j_1 = [1 + \alpha\beta + \alpha b_{\perp 1} + b_{\perp 2}]/(1+\alpha^2), \qquad (7a)$$

$$j_2 = [\alpha - \beta - b_{\perp 1} + \alpha b_{\perp 2}]/(1+\alpha^2). \qquad (7b)$$

The SK equation of motion (5) resembles a continuity equation [43] which connects the topological charge density q with the SK current density. We note, however, that conservation of Q in Eq. (3) in the present case is not a consequence of Noether's theorem, albeit conserved quantities may still exist for Eq. (5) [44] under continuous variation of $\mathbf{n}$ [27].

The physical meaning of $\mathbf{j}^{(1)}_{\mathrm{SK}}$ and $\mathbf{j}^{(2)}_{\mathrm{SK}}$ becomes apparent when we consider SKs in the steady state where $\partial_t\mathbf{n} = 0$, and thus, $\partial_t q = 0$. For not too large charge current densities, no major structural changes of the magnetization occur and $\mathbf{B}_{\mathrm{eff}}$ remains parallel to $\mathbf{n}$. Then, the perpendicular components $b_{\perp 1} = b_{\perp 2}$ vanish and the coefficients j_1 and j_2 in Eq. (5) are constant. A special case occurs when $\alpha = \beta$, which implies that $j_1 = 1$, $j_2 = 0$. Then, Eq. (5) can be solved by a Galilean transformation [45] $\mathbf{x} \to \mathbf{x} - \mathbf{v}_{\mathrm{SK}}t$ and the Skyrmion moves undistorted with the velocity $\mathbf{v}_{\mathrm{SK}} = -\mathbf{v}_s$. This motivates us to call $\mathbf{j}^{(1)}_{\mathrm{SK}}$ a SK current density. When $\alpha \neq \beta$ (but still assuming $b_{\perp 1} = b_{\perp 2} = 0$), j_2 becomes nonzero. Then, we may rewrite Eqs. (6b) and (6c) in the form $\mathbf{j}^{(2)}_{\mathrm{SK}} = -j_2 q(\eta_{/\!/}\mathbf{v}_s + \eta_\perp\mathbf{v}_\perp)$, with the coefficients $\eta_{/\!/} = (\hat{\mathbf{v}}\cdot\nabla)\mathbf{n}\cdot(\hat{\mathbf{v}}_\perp\cdot\nabla)\mathbf{n}/q$ and $\eta_\perp = -[(\hat{\mathbf{v}}\cdot\nabla)\mathbf{n}]^2/q$. The term $\propto\eta_{/\!/}$ only adds a contribution to $\mathbf{j}^{(1)}_{\mathrm{SK}}$ (though with a dependence on the vector field $\mathbf{n}$) to drive the topological charge density along $\pm\mathbf{v}_s$. Crucial is the term $\propto\eta_\perp$. First, it points perpendicularly to the externally applied current, and second, it drives negative and positive topological charge densities in opposite directions, as it changes sign under the inversion $q \to -q$. This is essentially the SK Hall effect [15,27,46–49], but for arbitrary topological charge density. Therefore, we identify the contribution (6c) as being responsible for separating negative from positive topological charge density resulting in a common SK–ASK pair. Actually, this process can be expected to be a common scenario in real materials for sufficiently strong external currents. The only further prerequisites are $\alpha - \beta \neq 0$ and small spatial fluctuations of $q(x,y,t)$, which also imply finite gradients $(\hat{\mathbf{v}}\cdot\nabla)\mathbf{n}$ and $(\hat{\mathbf{v}}_\perp\cdot\nabla)\mathbf{n}$ and, thus, a finite $\mathbf{j}^{(2)}_{\mathrm{SK}}$. A finite gradient $(\hat{\mathbf{v}}_\perp\cdot\nabla)\mathbf{n}$ is, strictly speaking, not necessary for a nonvanishing SK current density $\mathbf{j}^{(2)}_{\mathrm{SK}}$ [cf. Eq. (6c)]. Nevertheless, it is important for a finite divergence $\nabla\cdot\mathbf{j}^{(2)}_{\mathrm{SK}} \neq 0$. Only, in this case, the Skyrmion current cannot be gauged away and is physically relevant. Ultimately, a SK–ASK pair is formed out of these fluctuations. We note, in passing, that the detailed motion of ASKs is typically more complicated than that of SKs, since commonly, an isolated ASK is not a stationary solution and, thus, already for $\mathbf{v}_s = 0$, $\mathbf{B}_{\mathrm{eff}}$ is clearly not parallel to $\mathbf{n}$, which implies that $b_\perp \neq 0$.

In the following, we illustrate these general principles for a concrete model realized by the Hamiltonian [20]

$$H = -J\sum_{\mathbf{r}}\mathbf{n}_{\mathbf{r}}\cdot(\mathbf{n}_{\mathbf{r}+\mathbf{e}_x} + \mathbf{n}_{\mathbf{r}+\mathbf{e}_y}) - \sum_{\mathbf{r}}\mathbf{B}_{\mathbf{r}}\cdot\mathbf{n}_{\mathbf{r}} - D\sum_{\mathbf{r}}[(\mathbf{n}_{\mathbf{r}}\times\mathbf{n}_{\mathbf{r}+\mathbf{e}_x})\cdot\mathbf{e}_x + (\mathbf{n}_{\mathbf{r}}\times\mathbf{n}_{\mathbf{r}+\mathbf{e}_y})\cdot\mathbf{e}_y], \qquad (8)$$

defined on lattice sites $\mathbf{r}$ in two dimensions. J is the exchange interaction and D the Dzyaloshinsky-Moriya interaction (DMI) strength. We use the values $J = 1$ meV, $D/J = 0.18$ reported for MnSi [20]. Here, we only discuss a bulk DMI which stabilizes Bloch SKs. Yet, we have also verified our findings for systems with an interfacial DMI which stabilizes Néel SKs [50,51]. No qualitative modifications occur. Experimental values for α and β typically cover a broad range, e.g., from $\alpha = 0.03$ to 0.3 in CoPt [52,53] or $\beta = 0.02$ to 0.12 in Permalloy [54,55]. Below, we use typical values in this range. In the numerical simulations, we use a $L_x \times L_y = 160 \times 160$ square lattice with periodic boundary conditions. For convenience, we translate $v_s = pa^3 I_c/(2e)$ to a charge current density I_c by assuming full polarization $p = 1$ and a lattice constant $a = 0.5$ nm. Depending on the magnitude of the external Zeeman field $\mathbf{B}$, either a helical phase, a SK lattice, or the ferromagnetic (field polarized) phase is the ground state [20,30]. A field $\mathbf{B} = (0, 0, B_z) = -0.03J\hat{\mathbf{z}}$ is, in fact, strong enough to align all magnetic moments, $\mathbf{n}(x, y, t) \equiv -\hat{\mathbf{z}}$. Then, $q(x, y, t)$ remains zero everywhere and, according to Eqs. (5) and (6), for all times, since $\mathbf{j}_{\mathrm{SK}}^{(1,2)} = 0$, even at nonzero applied current densities.

To realize at least a small initial nonzero topological charge density q, we add a tiny modulation to the magnetic field pointing in the y direction, i.e., $B_y = b_0[\sin(2\pi x/L_x) + \sin(2\pi y/L_y)]$ and $b_0 = B_z/100$. As a matter of fact, the precise form of the initial inhomogeneous magnetization is of minor importance. The time evolution of the system is calculated by solving the extended LLG Eq. (1) by standard advanced numerical methods.

Starting from the fully field polarized state $\mathbf{n}(x, y) \equiv -\hat{\mathbf{z}}$, first, we let the system accommodate to the additional B_y field at zero external current. After this initial equilibration, we switch on the current at $t = 0$ and calculate $q(x, y, t)$ at every time step. A movie of this evolution is available in the Supplemental Material [56] while a selection of snapshots of q is shown in Fig. 1. Initially, the very small amplitude b_0 of B_y generates a tiny seed topological charge density of both positive and negative sign with an overall $Q = 0$. Gradually, under the influence of the external current, SK–ASK pairs begin to form with growing magnitudes of q. Consistent with our theoretical prediction, the SK and ASK centers separate in the y direction, perpendicular to the external current. After its full development, since it is unstable, the ASK disappears on a time scale $\propto 1/\alpha$. Thereby, its diameter shrinks relatively quickly, eventually below the lattice constant. At this moment, $Q(t)$ abruptly changes by 1. As the evolution of SK–ASK pairs is interfered by the relatively short life time of the ASK, we further illustrate the details of this process by an additional movie [56] where we set the DMI to zero. Then, neither the SK nor the ASK is energetically preferred and the full SK–ASK pair evolves in time as recently reported in Ref. [34].

The scenario of SK creation is demonstrated further in Fig. 2, where we show the time-dependence of $Q(t)$. Over large time spans, the total topological charge takes an integer value, while, occasionally, $Q(t)$ jumps to the next integer within a short transition time. These transitions are accompanied by sudden rises of the total negative topological charge $Q_{\mathrm{ASK}}(t) = (1/4\pi)\int_{q<0} dxdyq(x, y, t)$, a quantity that we define by integrating over regions with negative $q(x, y, t)$ only. During the times when $Q(t)$ stays integer, $Q_{\mathrm{ASK}}(t)$ may decrease gradually with time.

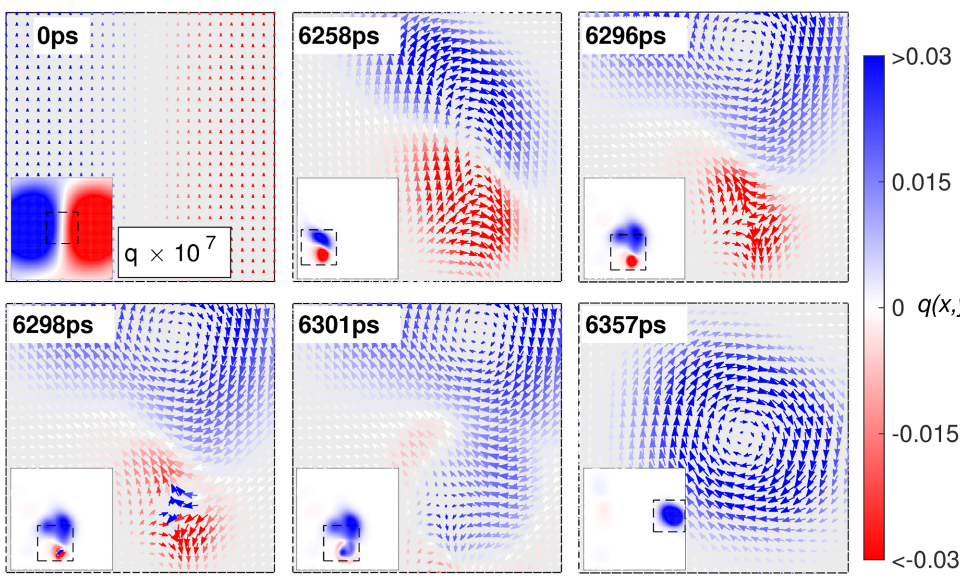

FIG. 1. Snapshots of the topological charge density q (insets) and the magnetic texture (arrows) in a magnified section (marked by the dashed rectangles within the insets) at the times as indicated. The initial topological charge adopted by the applied inhomogeneous magnetic field is tiny (see q for $t = 0$ which is multiplied by 10^7 for clarity). Because of these fluctuations, the SK–ASK pair is created by a current in the x direction by separating positive and negative topological charge density perpendicular to the current direction (here, in the y direction). The ASK is eventually destroyed around $t \approx 6300$ ps and only the SK survives. Parameters are chosen as $I_c = 7.7 \times 10^{11}$ A/m^2, $\alpha = 0.25$ and $\beta = 0$. Color code refers to topological charge density q always.

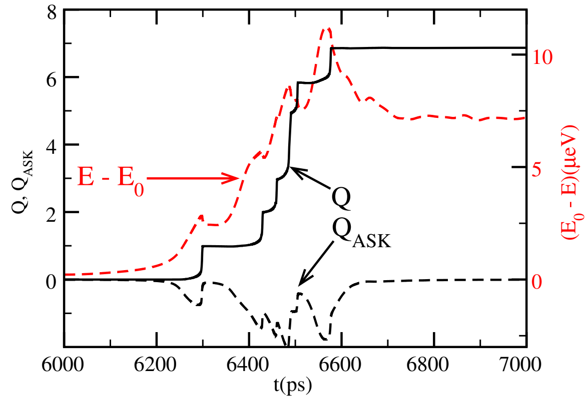

FIG. 2. Black solid line: Time dependence of the total topological charge $Q(t)$. Black dashed line: Time dependence of the total negative charge defined as $Q_{\mathrm{ASK}}(t) = (1/4\pi)\int_{q<0} \mathrm{d}x\mathrm{d}y q(x,y,t)$ stemming from negative q only. Note that Q_{ASK} does not need to be an integer and that the restriction to lattice points imposes some small, unimportant ambiguity on the precise determination of $q(x,y,t)$. Red dashed line: Time dependence of the energy in reference to the initial energy, $E - E_0 \equiv E(t) - E(t=0)$, per lattice site. Before every Q jump, Q_{ASK} gradually decreases, accompanied by an increase of the energy which eventually is taken from the external current. Parameters as in Fig. 1.

This indicates the gradual creation of SK–ASK pairs, their growth, and their spatial separation, before the finally isolated, but unstable ASK annihilates during a time much shorter than the duration of its creation, as described above. This initial gradual evolution of the first SK–ASK pair due to a weak inhomogeneous Zeeman field is clearly seen in Fig. 2. On the other hand, as soon as a finite number of SKs exists (after 6300 ps in Fig. 2), their intrinsic inhomogeneous magnetization suffices to facilitate further creation of SK–ASK pairs in their surroundings, even at a homogeneous Zeeman field as we have convinced ourselves independently.

Since the system starts close to the ferromagnetic ground state, the SK creation costs energy, which is provided by the external current. Figure 2 confirms the connection between the increase of the energy and the negative SK density.

The duration of the SK creation can be quantified by the time τ which we define as the time span from the onset of the current flow till the creation of the first SK. This creation time is a combination of the time τ_{pair} needed to form a sufficiently large SK–ASK pair and the annihilation time τ_{ASK} of the ASK. Since both processes happen at least partially simultaneously, the resulting τ is not a direct sum of both. Still, $\tau_{\mathrm{ASK}} \ll \tau_{\mathrm{pair}}$ such that we can safely take $\tau \approx \tau_{\mathrm{pair}}$. Since we attribute the creation of SK–ASK pairs to the existence of a finite $\mathbf{j}_{\mathrm{SK}}^{(2)}$, we expect SKs to be created faster when the magnitude of $\mathbf{j}_{\mathrm{SK}}^{(2)}$ is larger. From Eqs. (6b), (6c), and (7b) we find $|\mathbf{j}_{\mathrm{SK}}^{(2)}| \propto (\alpha - \beta)I_c$ in the limit of vanishing $b_{\perp 1}$ and $b_{\perp 2}$. In Fig. 3, this relation between τ and $\mathbf{j}_{\mathrm{SK}}^{(2)}$ is confirmed by the numerical results. Indeed, τ depends on $|\alpha - \beta|$ and I_c. In particular, no SKs can be created when $\alpha = \beta$, which implies that the dissipative current is essential for the charge current-induced SK creation. Still, finite creation times appear in an experimentally relevant parameter regime. Finally, we note that, even though we have chosen a particular seed magnetic field to create topological charge density fluctuations, their precise origin is not important. In fact, only an inhomogeneous $q(x,y)$, besides, $\alpha \neq \beta$ and $I_c \neq 0$, is necessary for a nonzero $\mathbf{j}_{\mathrm{SK}}^{(2)}$. Thus, a multitude of ways are eligible for creating such fluctuations, for example by local fields, material modification, or by temperature. On the other hand, a change of Q will often be undesirable in distinct setups. Then, $\mathbf{j}_{\mathrm{SK}}^{(2)}$ contributions to Eqs. (6) should be suppressed by a proper choice of the material with a small $|\alpha - \beta|$, or by avoiding magnetization fluctuations, apart from simply working in the low current regime.

In this Letter, we have established the Skyrmion equation of motion by combining the general definition of the Skyrmion density and the extended Landau-Lifshitz-Gilbert equation. We define Skyrmion current densities that conserve the total topological charge of a sample. In the presence of an in-plane spin-polarized current, we identify terms that give rise to a simple movement of Skyrmions against the externally applied current. Other contributions

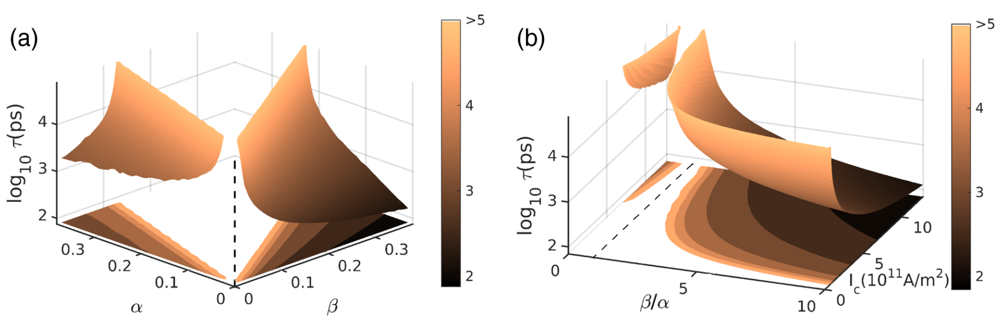

FIG. 3. Decadic logarithm of the SK creation time τ in dependence of (a) the Gilbert damping constant α and the nonadiabaticity parameter β for $I_c = 10^{12}$ A/m^2, and, (b) the ratio β/α and the charge current density I_c for $\alpha = 0.05$. Finite creation times are never achieved at $\beta = \alpha$ (dashed lines).

to Skyrmion current densities that we identify explicitly drive the separation of positive Skyrmion density from negative anti-Skyrmion density perpendicular to the applied current. These latter contributions eventually cause the creation of Skyrmion–anti-Skyrmion pairs, already out of very small magnetic inhomogeneities. The theoretical predictions are corroborated by numerical simulations and applied to systems with bulk and interfacial DMI.

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*Corresponding author.
mstier@physnet.uni-hamburg.de

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[56] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.118.267203 for movies of Skyrmion-pair evolution and detailed spin structures.