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AN ONTO-NOMOLOGICAL THEORY OF MODALITY

ABSTRACT. This paper is dedicated to the formulation of a *restricted* theory of ontic modality (for example, I do not address questions that arise when modal operators interact with quantifiers, although some of the theoretical developments presented here certainly suggest such questions). As will be seen, notwithstanding its restrictions, the theory has a pleasing richness to it, as well as formal rigor and intuitive satisfactoriness. It also offers an unusual perspective on modality.

1. What Statements of Possibility and Necessity Mean

Besides a *semantical problem* there is also an *epistemological* and an *ontological problem* connected with modality: How can we justifiably determine which statements of necessity and possibility are true, and which false? What are the ontological requirements of modal theory? Answers to these questions I will leave almost totally implicit. (Draw your own conclusions.)

The semantical problem for ontic modalities can be formulated as the following question:

> What are the (necessary and sufficient) truth conditions of sentences with objective meaning having the forms "it is possible that A", "it is necessary that A," and "if A, then B"?

Here is an answer to this question in the form of an explicit definition:

$$\begin{aligned}
\Diamond^n(x) &:= \mathrm{S}(x) \wedge \neg\mathrm{P}(\mathrm{neg}(x), b^n),\\
\Box^n(x) &:= \mathrm{P}(x, b^n),\\
{}^n\!\!\rightarrow(x, y) &:= \mathrm{P}(y, \mathtt{conj}(b^n, x)),\\
\Diamond^n A &:= \Diamond^n(\mathtt{that}\ A),\\
\Box^n A &:= \Box^n(\mathtt{that}\ A),\\
B\ {}^n\!\!\rightarrow A &:= {}^n\!\!\rightarrow(\mathtt{that}\ B, \mathtt{that}\ A).
\end{aligned}$$

Of course, this, at the moment, is quite incomprehensible. So let me explain.

The predicate $\mathrm{P}(x, y)$ expresses that *the state of affairs x is an intensional part of the state of affairs y*. Hence $\forall x\forall y(\mathrm{P}(x, y) \supset \mathrm{S}(x) \wedge \mathrm{S}(y))$ is analytically true. What is meant by a state of affairs x being an intensional part of a state of affairs y can be effectively illustrated by an example: the state of affairs that this object has a surface is an intensional part of the state of affairs that this object is colored.

The notion of parthood — or its inverse: containment — in the intensional sense is readily comprehensible with respect to states of affairs (and also with respect to properties). No antecedent comprehension of the term "possible world" is necessary for grasping that notion, and there is no checking on possible worlds in applying that notion either positively or negatively. The concept of intensional parthood is more basic than the concept of possible world. For the sake of brevity, I will usually omit the modifier "intensional", and simply speak of a state of affairs being a part of another state of affairs (or of itself).

"b^n" designates *the Nth basis of necessity* (hence "b^1" designates the *first* basis of necessity, "b^2" the *second* basis of necessity, etc.). A basis of necessity is always some state of affairs or other. Hence $\mathrm{S}(b^n)$ is analytically true.

The functor "$\mathtt{conj}(x, y)$" designates *the conjunction of x and y*; for *states of affairs x* and y, their conjunction, $\mathtt{conj}(x, y)$, is identical with $\iota z\,[\mathrm{S}(z) \land \mathrm{P}(x, z) \land \mathrm{P}(y, z) \land \forall u(\mathrm{P}(x, u) \land \mathrm{P}(y, u) \supset \mathrm{P}(z, u))]$ ("the unique state of affairs of which x and y are parts, and which is a part of every state of affairs of which x and y are parts"). This definition does of course have a theoretical background: there are principles that guarantee that there is exactly one state of affairs that fulfills the defining predicate, given that x and y are states of affairs; this theoretical background will be stated shortly.

The functor "$\mathrm{neg}(x)$" designates *the negation of x*, and is defined as follows: $\mathrm{neg}(x) := \mathtt{CONJ}\ y(\mathrm{QA}(y) \land \neg \mathrm{P}(y, x))$ ("the conjunction of all quasi-atomic states of affairs that are not parts of x"). This definition will be elucidated shortly, when its theoretical background has been stated.

We are now in a position to grasp the content of the above series of definitions. The first three definitions define modal predicates, the three other definitions define the corresponding modal sentence connectives.

According to the first and fourth definition, it is possible *in the Nth sense* that A if, and only if, the negation of the state of affairs that A is not a part of *the Nth basis of necessity*. In other words: $\Diamond^n A \equiv \neg \mathrm{P}(\mathrm{neg}(\mathtt{that}\ A), b^n)$ is a logical truth (due to the first and fourth definition).

According to the second and fifth definition, it is necessary *in the Nth sense* that A if, and only if, the state of affairs that A is part of *the Nth basis of necessity*. In other words: $\Box^n A \equiv \mathrm{P}(\mathtt{that}\ A, b^n)$ is a logical truth (due to the second and fifth definition).

According to the third and sixth definition, "if B, then A" *in the Nth sense* is true if, and only if, the state of affairs that A is part of the conjunction of *the Nth basis of necessity* with the state of affairs that B. In other words: $B \,{}^{n}\!\!\rightarrow A \equiv \mathrm{P}(\mathtt{that}\ A, \mathtt{conj}(b^n, \mathtt{that}\ B))$ is a logical truth (due to the third and sixth definition).

Clearly, the above definitions do not fully specify modal concepts: *modalities*. For this, one needs to specify *a basis of necessity*. If we intend to speak about an *ontic* modality, then the corresponding basis of necessity needs to be specified *ontically*. But there certainly seems to be not only one ontically specifiable basis of necessity. For the time being, let me indicate two salient ontic bases of necessity:

b^1: the basis of logical (or conceptual) necessity — this is simply the minimal (or "tautological") state of affairs (for further clarification, see the next section).

b^2: the basis of nomological (or natural) necessity — this is the state of affairs which is the conjunction of all states of affairs that are laws of nature.

Given the above definitions, it is now clear that $\Diamond^1 A$ means as much as "it is logically possible that A", $\Diamond^2 A$ as much as "it is nomologically possible that A" (and correspondingly the intended sense of $\Box^1 A$ and $\Box^2 A$ is likewise clear). Even so, as long as the general theory of states of affairs that provides the background for the above definitions is not specified, the content of an assertion of $\Diamond^1 A$ remains vague, and the content of an assertion of $\Diamond^2 A$ remains vague even after that specification (which is provided in the next section). For giving precise content to $\Diamond^2 A$, one has to specify, in addition to the general theory of states of affairs, the concept of law of nature, *and* one has to specify which states of affairs are laws of nature in the sense of the specified concept of law of nature. These tasks generate considerable philosophical difficulties, which are just conspicuous aspects of a more general problem: *Can just any state of affairs be a basis of necessity, and if not, what distinguishes a state of affairs that can be a basis of necessity from a state of affairs that cannot?* This question, and to some extent the problem of nomological necessity, will be addressed in due course.

Remark. Note that the bases of necessity implicitly invoked in making assertions of the form "if A, then B" are singularly unstable, even if those assertions are meant ontically, and vary from context to context much more so than the bases of monadic modalities do; in fact, they may even vary in the same context (of utterance). This generates the illusion that inferences that are prima facie logically valid for "if A, then B" — for example, the transitivity-inference: *If A, then B. If B, then C. Therefore: If A, then C* — are, *enfin*, not logically valid for it. (The transitivity-inference will be logically valid for "if A, then B" if the same basis of necessity is employed for all three conditionals involved; the transitivity-inference can easily fail to be logically valid if the basis of necessity is allowed to vary.) □

2. The Mereology of States of Affairs

The semantics of modality outlined above (without the use of model theory, simply by presenting explicit definitions of modal terms) is incomplete without stating the ontological background theory that is connected with it. Here is this background theory:

(P0) $\quad \forall x \forall y(\mathrm{P}(x,y) \supset \mathrm{S}(x) \wedge \mathrm{S}(y))$,

(P1) $\quad \forall x \forall y \forall z(\mathrm{P}(x,y) \wedge \mathrm{P}(y,z) \supset \mathrm{P}(x,z))$,

(P2) $\quad \forall x(\mathrm{S}(x) \supset \mathrm{P}(x,x))$,

(P3) $\quad \forall x \forall y(\mathrm{P}(x,y) \wedge \mathrm{P}(y,x) \supset x = y)$,

(P4) $\quad \exists z\,[\,\mathrm{S}(z) \wedge \forall x(\mathrm{S}(x) \wedge A[x] \supset \mathrm{P}(x,z)) \wedge$
$\qquad \forall y(\mathrm{S}(y) \wedge \forall x(\mathrm{S}(x) \wedge A[x] \supset \mathrm{P}(x,y)) \supset \mathrm{P}(z,y))]$,

(P5) $\quad \forall z \forall z'(\mathrm{S}(z) \wedge \mathrm{S}(z') \wedge \forall x(\mathrm{QA}(x) \wedge \mathrm{P}(x,z) \supset \mathrm{P}(x,z')) \supset \mathrm{P}(z,z'))$,

(P6) $\quad \forall x\,[\,\mathrm{P}(x, \mathtt{CONJ}\; y A[y]) \wedge \neg \mathtt{M}(x) \supset$
$\qquad \exists k'(\mathrm{P}(k',x) \wedge \neg \mathtt{M}(k') \wedge \exists z(\mathrm{P}(k',z) \wedge A[z]))]$,

(P7) $\quad w^* \neq k^*$,

(P8) $\quad \mathrm{QC}(w^*)$,

(P9) $\quad w^* \neq t^*$,

(P10) $\quad A \equiv \mathrm{O}(\mathtt{that}\; A)$.

These eleven principles state the most basic principles of a mereology of states of affairs; they contain some defined terms which I will explain as I go through them.

The first four principles describe the basic properties of P as a relation of (proper or improper) intensional parthood between states of affairs. (Note that P3 formulates an identity criterion for states of affairs.)

P4 states that the states of affairs falling under an arbitrary description $A[x]$ have a smallest state of affairs that comprises them all. In other words (in view of P3), for any description $A[x]$, there is the state of affairs which is the conjunction of all states of affairs that satisfy $A[x]$. And accordingly we can formulate the following definition:

(D1) $\quad \mathtt{CONJ}\; x A[x] := \iota z\,[\,\mathrm{S}(z) \wedge \forall x(\mathrm{S}(x) \wedge A[x] \supset \mathrm{P}(x,z)) \wedge$
$\qquad \forall y(\mathrm{S}(y) \wedge \forall x(\mathrm{S}(x) \wedge A[x] \supset \mathrm{P}(x,y)) \supset \mathrm{P}(z,y))]$.

The fulfillment of the so-called *existence-condition*[1] for this definition (using the operator of definite description, ι) is guaranteed for all predicates $A[x]$ by P4. The fulfillment of the uniqueness-condition[2] for this definition is guaranteed for all predicates $A[x]$ by P3. Here are two prominent states of affairs that can be defined as conjunctions:

(D2) $\quad t^* := \mathtt{CONJ}\; x \neg \mathrm{S}(x)$,

(D3) $\quad k^* := \mathtt{CONJ}\; x\; \mathrm{S}(x)$.

[1] One should rather call it "the at-least-one-condition".

[2] One could also call it "the at-most-one-condition".

One can easily prove that $\mathtt{CONJ}\, x \neg S(x)$ is identical with $\iota z\,[S(z) \wedge \forall y(S(y) \supset P(z,y))]$ ("the state of affairs that is a part of every state of affairs"), and that $\mathtt{CONJ}\, x\, S(x)$ is identical with $\iota z\,[S(z) \wedge \forall x(S(x) \supset P(x,z))]$ ("the state of affairs of which every state of affairs is a part"). Thus, for obvious reasons, k^* can be called "the maximal [or total] state of affairs," and because intensional parthood is the ontological counterpart of logical implication, k^* can also be called, metaphorically, "the *self-contradictory* state of affairs". In turn, for obvious reasons, t^* can be called "the minimal state of affairs", and again because intensional parthood is the ontological counterpart of logical implication, a metaphorical designation of t^* is "the *tautological* state of affairs". It is useful to have the following defined predicates (although they provably apply only to one state of affairs: t^*, respectively k^*):

(D4) $\quad M(z) := S(z) \wedge \forall y(\, S(y) \supset P(z,y)),$

(D5) $\quad T(z) := S(z) \wedge \forall x(\, S(x) \supset P(x,z)).$

The fact (provable on the basis of P3) that there is only one "tautological" state of affairs, and only one "self-contradictory" one, reveals that the conception of states affairs *here* employed is *coarse-grained*. A coarse-grained conception of states of affairs is not adequate if the aim is to employ states of affairs as *meanings* of sentences that are either true or false; for if states of affairs are to be the meanings of such sentences, then, under the coarse-grained conception of states of affairs, all logically true sentences turn out to have the same meaning (i.e., t^*), and all logically false sentences turn out to have the same meaning too (i.e., k^*), which does not seem to be at all desirable. Nevertheless, although coarse-grained states of affairs cannot serve adequately as the meanings of true or false sentences, they *can* serve in a respectable semantical function: namely, as the *intensions* of true or false sentences, where intensions are taken to be rough approximations to meanings.[3] For the logic of *ontic* modalities, and quite generally for the purposes of a philosophical theory of *ontic* modalities, coarse-grained states of affairs are entirely sufficient; fine-grained states of affairs need not be considered.

The following predicate is useful to the point of being indispensable:

(D6) $\quad O(x) := S(x) \wedge A(x).$

In other words, *to obtain* (*to be the case, to be a fact*) is nothing else than *to be an actual state of affairs*. Hence it is clear what is meant by P10, and hence it is also clear which other prominent state of affairs is defined by the following definition:

(D7) $\quad w^* := \mathtt{CONJ}\, x\, O(x).$

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[3]Identity of meaning implies identity of intension, but not vice versa. Identity of intension implies identity of truth-value, but not vice versa.

It is *the world* in Wittgenstein's sense: the totality of all obtaining states of affairs. The import of principles P7 and P9 is now clear; according to them, the world is neither the minimal (or "tautological") nor the maximal (or "self-contradictory") state of affairs.

Note, furthermore, that the "small" conjunction of states of affairs can be defined on the basis of the "big":

(D8) $\quad \mathtt{conj}(x, y) := \mathtt{CONJ}\, v(v = x \vee v = y).$

As can easily be seen, $\mathtt{CONJ}\, v(v = x \vee v = y)$ is for all states of affairs x and y identical with $\iota z\,[\mathrm{S}(z) \wedge \mathrm{P}(x,z) \wedge \mathrm{P}(y,z) \wedge \forall u(\mathrm{P}(x,u) \wedge \mathrm{P}(y,u) \supset \mathrm{P}(z,u))]$, in other words: the conjunction of states of affairs x and y is the smallest state of affairs of which both x and y are parts.

Moving on to the next principle after P4, it becomes clear that P5 is an *atomistic* principle, once the predicate $\mathtt{QA}(x)$ is defined:

(D9) $\quad \mathtt{QA}(x) := \mathrm{S}(x) \wedge \forall y(\,\mathrm{P}(y,x) \supset y = x \vee \mathtt{M}(y)).$

According to D9, a *quasi-atomic state of affairs* is a state of affairs whose only proper part (if it has a proper part) is the minimal state of affairs (considering that $\forall y(\mathtt{M}(y) \equiv y = t^*)$). One does well to distinguish *quasi-atomic* states of affairs, *atomic* states of affairs — states of affairs that have no proper part — and *elemental* states of affairs — states of affairs that have exactly one proper part. It can easily be shown that all atomic or elemental states of affairs are quasi-atomic, and vice versa: that all quasi-atomic states of affairs are either atomic or elemental. In the mereology of states of affairs, there is exactly one atom: t^*, and thus, in the mereology of state of affairs, atomicity coincides with minimality. For this reason, I do not here introduce a formally defined predicate for expressing atomicity of states of affairs. But I do formally introduce a predicate for expressing elementalness of states of affairs:

(D10) $\quad \mathrm{EL}(x) := \mathtt{QA}(x) \wedge \neg\mathtt{M}(x).$

While there is exactly one atomic state of affairs, which is a consequence of P4 and P3, the above eleven principles do not determine the exact number of elemental states of affairs. Some mereologists have argued that entities that have the same proper parts must be identical — which, if correct, would imply that all elemental states of affairs are identical with each other, since they all have exactly one proper part, and the very same proper part, namely t^*. But the invoked identity-principle is false for *nonmaterial mereologies*, like the mereology of states of affairs, or the mereology of sets.[4] Thus there can easily be more than

[4]Its validity is also dubious for material mereologies: Since material atoms have no proper parts (in the relevant material sense), they all have the same proper parts, and hence, according to the

one elemental state of affairs, and in fact the eleven principles already require that there be at least two elemental states of affairs.

P5 states that states of affairs are, in a manner, *exhausted* by the quasi-atomic states of affairs that are parts of them: if all the quasi-atomic states of affairs that are parts of one state of affairs are also parts of the other, then this is already sufficient for concluding that the former state of affairs is itself a part of the latter. Equivalently, one can say that states of affairs are exhausted by the *elemental* states of affairs that are parts of them (for the quasi-atomic states of affairs nearly coincide with the elemental states of affairs, and the one state of affairs that blocks their coincidence — that is, t^* — is a part of every state of affair, and therefore does not make a difference regarding the exhaustion of states of affairs by parts of them that belong to a certain kind).

Moving on to the next principle after P5, P6 is a very important principle that regulates how the parts of a conjunction of states of affairs stand to the description (or the property) used in specifying that conjunction. P6 states that every nonminimal part of a conjunction of states of affairs has a nonminimal overlap with some state of affairs that satisfies the description used for specifying that conjunction. Thus stated, P6 is a perfectly evident principle for a mereology of states of affairs.

Given the principles P0–P6, one can prove: $\forall x(\mathrm{S}(x) \supset x = \mathtt{CONJ}\, y(\mathrm{QA}(y) \wedge \mathrm{P}(y, x))$, and $\forall x(\mathrm{S}(x) \supset x = \mathtt{CONJ}\, y(\mathrm{EL}(y) \wedge \mathrm{P}(y, x))$. In fact, as is suggested by the second theorem, the number of elemental states of affairs determines the total number of states of affairs according to the simple equation: $\mathrm{card}(\mathrm{S}) = 2^{\mathrm{card}(\mathrm{EL})}$ ("The cardinal number of states of affairs is 2 put to the power of the cardinal number of elemental states of affairs"). But for proving this the system has to be set-theoretically embedded.

Furthermore, given the principles P0–P6, it can be shown that if the functor of negation is defined as follows:

(D11) $\quad \mathrm{neg}(x) := \mathtt{CONJ}\, y(\mathrm{QA}(y) \wedge \neg \mathrm{P}(y, x))$,

then the general truths that one would expect from a negation of states of affairs become provable (for example, $\forall x(\mathrm{S}(x) \supset \mathrm{neg}(\mathrm{neg}(x)) = x)$, $\mathrm{neg}(t^*) = k^*$, $\forall x \forall y(\mathrm{S}(x) \wedge \mathrm{S}(y) \wedge \mathrm{neg}(x) = \mathrm{neg}(y) \supset x = y)$). It should not go unmentioned — as a further demonstration of the great definitional power of the proposed mereology of states of affairs — that the "big" disjunction can be defined on the basis of the "big" conjunction as follows:

(D12) $\quad \mathtt{DISJ}\, x A[x] := \mathtt{CONJ}\, y(\forall x(A[x] \supset \mathrm{P}(y, x))$.

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identity-principle at issue, all material atoms are identical with each other — a consequence which is simply absurd, *unless there are no material atoms*. If, on the other hand, material atoms are excepted from the identity-principle at issue, the obvious question is: why not make more exceptions? If material atoms can differ from each other although they all have no proper parts, why may not elemental states of affairs differ from each other although they all have the same proper part?

Then the "big" disjunction can be used for defining the "small" disjunction:

(D13) $\mathtt{disj}(x, y) := \mathtt{DISJ}\, v(v = x \vee v = y)$.

As can easily be seen, $\mathtt{DISJ}\, v(v = x \vee v = y)$ is for all states of affairs x and y identical with $\iota z\,[S(z) \wedge P(z, x) \wedge P(z, y) \wedge \forall u(P(u, x) \wedge P(u, y) \supset P(u, z))]$, in other words: the disjunction of states of affairs x and y is the largest state of affair that is a part both of x and of y. Given the definitions of the three functors $\mathtt{neg}(x)$, $\mathtt{conj}(x, y)$, and $\mathtt{disj}(x, y)$, all the well-known and less well-known Boolean principles that form the stock of truth-functional propositional logic can be proved on the basis of P0–P6.[5]

The one principle that has not yet been touched on is P8, which contains one more as yet undefined predicate: $QC(x)$. Here is its definition:

(D14) $QC(x) := S(x) \wedge \forall y(P(x, y) \supset x = y \vee T(y))$.

According to D14, a *quasi-complete state of affairs* is a state of affairs which is a proper part (if it is a proper part of anything) only of the total (or maximal) state of affairs (considering that $\forall y(T(y) \equiv y = k^*)$). The predicate $QC(x)$ is the counterpart of the predicate $QA(x)$. As the quasi-atomic states of affairs are divided into the one minimal state of affairs, t^*, and all the other quasi-atomic states of affairs: the elemental state of affairs, so the quasi-complete state of affairs are divided into the one maximal state of affairs, k^*, and all the other quasi-complete states of affairs: the *maximally consistent states of affairs*:

(D15) $MC(x) := QC(x) \wedge \neg T(x)$.

It can be shown that the maximally consistent states of affairs are precisely the negations of the elemental states of affairs (and therefore the elemental states of affairs precisely the negations of the maximally consistent states of affairs). It is easily verified on the basis of P8 and P7 that w^* — *the world* — is a maximally consistent state of affairs. Because of this fact, it is justified to call the states of affairs that are maximally consistent "possible worlds". The following remarkable theorems are provable on the basis of P0–P6: $\forall x \forall y[MC(x) \wedge S(y) \supset (P(y, x) \equiv \neg P(\mathtt{neg}(y), x))]$; $\forall x \forall y \forall z[MC(x) \wedge S(y) \wedge S(z) \supset (P(\mathtt{disj}(y, z), x) \equiv P(y, x) \vee P(z, x))]$.

This concludes my brief survey of the *basic* mereology of states of affairs. (It is explored in much greater detail in my book *Axiomatic Formal Ontology*.) Some philosophers would prefer that the present theory of states of affairs be

[5] To be precise: Every logically true formula $\phi[p_1, \ldots, p_n]$ of truth-functional propositional logic can be translated (in the obvious way) into a functional term $\phi^*[x_1, \ldots, x_n]$ of the mereology of states of affairs such that $\forall x_1 \ldots \forall x_n(S(x_1) \wedge \cdots \wedge S(x_n) \supset \phi^*[x_1, \ldots, x_n] = t^*)$ is provable on the basis of P0–P6.

not called "a mereology". There may be historical reasons for restricting the use of the term "mereology" to theoretical systems that deal with *individuals* and their part-relations, but there are no compelling systematical reasons for this restriction. The present theory of states of affairs is indeed a powerful Boolean algebra, but at the same time it is a *mereology*: a *nonmaterial* mereology, and a mereology that has *a minimal element*, indeed *exactly one minimal element*, which one might call "its center".

This *centered* or *Boolean* mereology of coarse-grained states of affairs requires *universes of states of affairs* that have a spindle-shaped structure, with k^* at one end and t^* at the other. If, for example, we put the number of elemental states of affairs at *three*,[6] and call the elemental states of affairs that we are considering "a", "b" and "c", then we obtain the universe U_3:

$$\begin{array}{c} [k^* :]abc \\ ab \;\; ac \;\; [w^* :]bc \\ a \;\; b \;\; c \\ t^* \end{array}$$

If $S(x)$ is interpreted to be true of exactly the eight elements in U_3 and if $P(x, y)$ is interpreted to hold true of exactly those ordered pairs $\langle x, y\rangle$ of elements in U_3 such that x is part of y (in the obvious sense suggested by the diagram), then this universe of states of affairs[7] fulfills the principles P0–P9.

There are smaller universes than U_3 that also fulfill P0–P9. In fact, three such universes are contained in U_3. One of them is U_2:

$$\begin{array}{c} [k^* :]ac \\ a \;\; [w^* :]c \\ t^* \end{array}$$

There is no smaller universe of states of affairs than U_2 that fulfills the principles P0–P9.

3. The Actuality of States of Affairs in the Mereology of States of Affairs

The stated mereology of states of affairs is basic, but it is far from complete. First of all, two principles for actuality have to be added. They are obviously

[6]This number is wildly unrealistic if we are talking about *reality*; but it may be entirely appropriate if we are talking about some very restricted "virtual reality", for example the reality of a very simple game.

[7]I omit, as obvious, any arrow that indicates that the state of affairs from which the arrow originates is a part of the state of affairs at which the arrow ends (with its head). Inserting the arrows would, by the way, also be misleading, because it would suggest that there are more states of affairs in the universe U_3 than are designated by the eight designators used. Note that the universe U_3 can be a small part of a much larger *universe of discourse*.

true, but nevertheless not provable on the basis of the principles already stated:

(P11) $\forall x \forall y(\mathrm{S}(x) \wedge \mathrm{A}(x) \wedge \mathrm{P}(y, x) \supset \mathrm{A}(y))$,

(P12) $\mathrm{A}(w^*)$.

P11 states that every intensional part of an actual state of affairs is itself actual (an actual *state of affairs*, according to P0); P12 states that *the world* is actual.

Given the mereology of states of affairs as it now stands, there is a sober truth about the nonactuality of states of affairs. It is this: If the number of elemental states of affairs is N (and the number of elemental state of affairs is $\geqslant 2$ according to the stated principles), then the number of nonactual (or nonobtaining) states of affairs is this: $2^N - 2^{N-1}$. And this implies that *actualism* about states of affairs — the doctrine that there are no nonactual (or nonexistent) states of affairs, that all states of affairs are facts — is not merely false, it is also *incoherent*.

But how is the equation $\mathrm{card}(\mathrm{S} \wedge \neg \mathrm{A}) = 2^{\mathrm{card(EL)}} - 2^{\mathrm{card(EL)}-1}$ obtained? Central for obtaining it is the following theorem:

$\forall x[\mathrm{S}(x) \supset (\mathrm{A}(x) \equiv \mathrm{P}(x, w^*))]$ —
"A state of affairs is actual if, and only if, it is a part of the world".

Proof. It is easily seen — given P3, P4, the definition of w^* and the definition of $\mathrm{O}(x)$ — that $\forall x[S(x) \supset (\mathrm{A}(x) \supset \mathrm{P}(x, w^*))]$ is true. It remains to be seen that $\forall x[\mathrm{S}(x) \supset (\mathrm{P}(x, w^*) \supset \mathrm{A}(x))]$ is also true. Suppose, therefore, $\mathrm{P}(x, w^*)$ (the additional supposition $\mathrm{S}(x)$ is not needed, since it follows from the supposition already made by P0). From this, it follows on the basis of P11 and P12: $\mathrm{A}(x)$, because we (provably) have $\mathrm{S}(w^*)$.

The number of nonactual states of affairs is obtained by subtracting from the number of all states of affairs — that is, from $2^{\mathrm{card(EL)}}$ — the number of all actual states of affairs. According to the theorem just proven, this number is the number of all states of affairs that are parts of w^*. How many states of affairs are there that are parts of w^*? For determining their number, one needs to determine the number of the elemental states of affairs that are parts of w^*. This number, as can be proven, is equal to $\mathrm{card(EL)} - 1$ (since it can be proven that there is exactly one elemental state of affairs that is not a part of w^*, namely, $\mathrm{neg}(w^*)$). Since every subset of the set of elemental states of affairs which are parts of w^* determines a different part of w^*, and vice versa, the number of parts of w^* is: $2^{\mathrm{card(EL)}-1}$. Hence one obtains that the number of nonactual states of affairs is: $2^{\mathrm{card(EL)}} - 2^{\mathrm{card(EL)}-1}$. □

Looking at the five principles that (disregarding P10, which has a special status) govern the actuality of states of affairs: P7, P8, P9, P11, and P12, one may well wonder which of them are true for conceptual reasons only, and which are indeed true, but not for conceptual reasons only. Incidentally, there can be no reasonable doubt about the purely conceptual nature of the truth of the principles P0–P6. Who finds this doubtful should consider that the truth of P0–P6 is compatible with there being only one state of affairs: the prodigious definitional power of the subsystem P0–P6 is combined with an utter ontological weakness. Not even $t^* \neq k^*$ can be deduced from these principles. This ontological weakness should go a long way towards removing doubts about the truth of P0–P6 being of a purely conceptual nature. *True*, one can deduce from these principles: $\exists x\ \mathrm{S}(x)$. But that seems to be a safe conceptual truth.

In my view, indeed, numbering-statements *of any kind* are not excluded from being conceptually true. And in fact, in my view, "card(S) $\geqslant N$" and "card(S) $= N'$" are conceptually true for whatever numbers N and N' they are true (therefore, since "card(S) $\geqslant 1$" is true — being deducible from true principles — it is also conceptually true; and since "card(S) $\geqslant 2$" is true as well — being deducible from true principles — it is conceptually true as well). The idea that true numbering-statements cannot be conceptually true is rooted in the positivistic prejudice that conceptual truths cannot say anything informative about the universe of discourse (also sometimes called "the world", which designation, however, is here reserved for something else), cannot say, for example, how many entities of a certain kind are (*at least* or *exactly*) in it.

But back to P7, P8, P9, P11, and P12. The only principle out of these five whose purely conceptual truth is, I think, *evident* is P11. The purely conceptual truth of the other four is not evident, and perhaps some of them are, though true, not conceptually true after all. But it seems clear (1) that k^* is purely for conceptual reasons a nonactual state of affairs, and (2) that t^* is purely for conceptual reasons an actual state of affairs (accepting (1) and (2) is, by the way, another way to establish that it is a conceptual truth that there are at least two states of affairs).[8] Moreover, one can plausibly hold (3) that the conjunction of all actual states of affairs is purely for conceptual reasons itself actual. (1) and (3) have the consequence that P12 and P7 are conceptual truths. (Unlike P12, P7 is

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[8]The propositions (1) and (2) can be shown to be true; they are, however, not immediately forced upon us.

The statement $\mathrm{A}(t^*)$ is on the basis of P0–P6, P11 equivalent with $\exists x(\mathrm{S}(x) \wedge \mathrm{A}(x))$. Is this *indubitably* a *conceptual* truth? Is it *indubitably* a *conceptual* absurdity to suppose that no state of affairs is actual? Yes, indeed, it is: Suppose $\neg\exists x(\mathrm{S}(x) \wedge \mathrm{A}(x))$; hence on the basis of the conceptually true principle P10: $\mathrm{O}(\mathtt{that}\ \neg\exists x(\mathrm{S}(x) \wedge \mathrm{A}(x)))$; hence by applying D6: $\mathrm{S}(\mathtt{that}\ \neg\exists x(\mathrm{S}(x) \wedge \mathrm{A}(x))) \wedge \mathrm{A}(\mathtt{that}\ \neg\exists x(\mathrm{S}(x) \wedge \mathrm{A}(x)))$; hence: $\exists x(\mathrm{S}(x) \wedge \mathrm{A}(x))$.

In turn, the statement $\neg\mathrm{A}(k^*)$ is on the basis of P0–P6, P11, P12 equivalent with $\exists x(\mathrm{S}(x) \wedge \neg\mathrm{A}(x))$, with $\neg\mathrm{P}(k^*, w^*)$, and with $w^* \neq k^*$. Is $\exists x(\mathrm{S}(x) \wedge \neg\mathrm{A}(x))$ *indubitably* a *conceptual* truth? Is it *indubitably* a *conceptual* absurdity to suppose that all states of affairs are actual? Yes, indeed, it is: Suppose $\forall x(\mathrm{S}(x) \supset \mathrm{A}(x))$; $\mathrm{S}(\mathtt{that}\ \exists x(\mathrm{S}(x) \wedge \neg\mathrm{A}(x)))$ is a conceptual truth (it is an instance of a conceptually true general principle stated in the next section: P13); hence, according to the supposition made: $\mathrm{A}(\mathtt{that}\ \exists x(\mathrm{S}(x) \wedge \neg\mathrm{A}(x)))$; hence by applying D6: $\mathrm{O}(\exists x(\mathrm{S}(x) \wedge \neg\mathrm{A}(x)))$; hence on the basis of P10: $\exists x(\mathrm{S}(x) \wedge \neg\mathrm{A}(x))$.

not an independent principle, since not only $\mathrm{A}(t^*)$ but also $\neg\,\mathrm{A}(k^*)$, and therefore $\neg\,\mathrm{P}(k^*, w^*)$, and consequently $w^* \neq k^*$, are theorems logically deducible from the rest of the given conceptually true principles; see footnote 8.) But we still have no hint why P8 and P9 should also be conceptual truths.

We can, however, make a case for P8 ⊃ P9 — i.e., $\mathrm{QC}(w^*) \supset w^* \neq t^*$ — being a conceptual truth. I have said above that "card(S) ⩾ N" is a conceptual truth *if* it is a truth. But "card(S) ⩾ 4" is certainly true; and therefore it is conceptually true. Now, from this conceptual truth ("card(S) ⩾ 4", or in other words: "$\exists^{\geqslant 4} x\, \mathrm{S}(x)$") it follows on the basis of the conceptually true principles P0–P6: $\mathrm{QC}(w^*) \supset w^* \neq t^*$. And therefore this latter statement is also a conceptual truth.

But this leaves it open whether (a) $w^* \neq t^*$ is a conceptual truth, and $\mathrm{QC}(w^*)$ is not, or whether (b) both sentences are conceptual truths, or whether (c) neither sentence is a conceptual truth. How one decides in this matter depends on one's theory of actuality, in particular, on one's theory of the actuality of states of affairs. One might, for example, hold that a certain quasi-complete state of affairs is actual for purely conceptual reasons. But personally I do not believe that this position is very plausible. In my view, not even $\exists y(\mathrm{S}(y) \wedge y \neq t^* \wedge \mathrm{A}(y))$ is a conceptual truth. But then $w^* \neq t^*$ (in view of $\mathrm{S}(w^*)$ and $\mathrm{A}(w^*)$ being conceptual truths) and $\mathrm{QC}(w^*)$ cannot be conceptual truths either.

4. Further Principles for "That"

In the previous section, I have added to the basic principles of the mereology of states of affairs, i.e., P0–P10, two further actuality-principles: P11 and P12. Thus there are now in addition to the seven *principles of intensional parthood*, P0–P6, five *principles of actuality*: P7, P8, P9, P11 and P12, of which P7 proved to be redundant — last but not least on the basis of the conceptual truth P10 (see footnote 8), which is, so far, the only principle for "that".

More principles than P10 are needed for "that":

(P13) $\mathrm{S}(\mathtt{that}\ A)$,

(P14) $\mathtt{that}\ \neg A = \mathtt{neg}(\mathtt{that}\ A)$,

(P15) $\mathtt{that}\,(A \wedge B) = \mathtt{conj}(\mathtt{that}\ A, \mathtt{that}\ B)$,

(P16) $\mathtt{that}\ \forall x A[x] = \mathtt{CONJ}\ y \exists x(y = \mathtt{that}\ A[x])$.

Remark. Using the definition $A \vee B := \neg(\neg A \wedge \neg B)$ one can prove on the basis of P14 and P15: $\mathtt{that}\,(A \vee B) = \mathtt{neg}(\mathtt{conj}(\mathtt{neg}(\mathtt{that}\ A), \mathtt{neg}(\mathtt{that}\ B))) = \mathtt{disj}(\mathtt{that}\ A, \mathtt{that}\ B)$. Using the definition $A \supset B := \neg A \vee B$ one can prove in addition: $\mathtt{that}\,(A \supset B) = \mathtt{disj}(\mathtt{neg}(\mathtt{that}\ A), \mathtt{that}\ B)$. Using the definition $\exists x A[x] := \neg \forall x \neg A[x]$, one can prove on the basis of P14 and P16: $\mathtt{that}\ \exists x A[x] = \mathtt{neg}(\mathtt{CONJ}\ y \exists x(y = \mathtt{neg}(\mathtt{that}\ A[x]))) = \mathtt{DISJ}\ y \exists x(y = \mathtt{that}\ A[x])$. □

These four principles and P10 — each of which is rather obvious in itself — are, in conjunction, consistent with the twelve principles *not concerning "that"* that have already been stated. Those twelve principles — i.e., P0–P9, P11, and P12 — are consistent in themselves, since they have a verifying model (see above, at the end of Section 2, the universe of states of affairs U_2; in order to accommodate also the principles P11 and P12, imagine circles — symbolizing the property of actuality — drawn around c and t^*, but not around ac and a). And the consistency of all stated principles follows, because it is easily seen that the principles P10 and P13–P16 can be proven on the basis of the principles P0–P9, P11 and P12 if the following definition of "that A" is adopted:

(D*) $\quad \mathtt{that}\ A := \mathtt{CONJ}\, y(y = y \wedge \neg A)$.

Thus, the universe U_2 is not only a verifying model for P0–P9, P11 and P12, it is also a verifying model for P0–P16. This series of principles, or in other words: this theory of states of affairs, has herewith been proven consistent. (Of that theory, P13–P16 are conceptual truths, just like all the other principles, excepting P8 and P9.)

The above definition of "that A" allows to deduce this pleasing result. But of course it is not a definition that captures the intended meaning of "that A," because, according to it, "that A" always designates either t^* or k^*, whatever true or false sentence A we are looking at. Interesting enough, though, what is sometimes thought to initiate all by itself an inflation of intensional entities: the introduction of "that" as a name-forming operator applicable to sentences, does not necessarily do any such thing.

But the equation "$\mathtt{that}\ A = \mathtt{CONJ}\, y(y = y \wedge \neg A)$" and the utter ontological restriction to which this equation gives rise (of the states of affairs that serve as intensions of sentences: according to it, all true sentences have the same intension, namely t^*, and all false sentences also the same intension, namely k^*)[9] *is not provable if* "that A" *is a basic (and not a defined) operator* — although one might think it to be provable according to an argument that is reminiscent of an argument of some notoriety, called "the slingshot":

Suppose A is a *true* sentence, hence (provably) ID1: $\iota x(x = b) = \iota x(A \wedge x = b)$, and (provably) ID2: $\mathtt{CONJ}\, y(y = y \wedge \neg A) = t^*$. Furthermore:

1. $A \equiv \iota x(x = b) = \iota x(A \wedge x = b)$ a (provable) logical truth[10]

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[9]Note that the intension of a statement that is designated by "A" (for A, substitute the statement) is designated by "that A," which, without further determinations, is an ambiguous designator that can also be taken to designate at least two other things: *naturally, the meaning* of the statement, and *artificially, the truth-value* of the statement.

[10]1 is a provable logical truth if the singular term b is appropriately chosen: b must be different from c^* — the singular term that serves as designating the artificial referent of all definite descriptions $\iota x A[x]$ for which $\exists^{=1} x A[x]$ is not true.

2. $\texttt{that}\ A = \texttt{that}\ (\iota x(x = b) = \iota x(A \wedge x = b))$ from 1 by EQU (see below)
3. $\texttt{that}\ A = \texttt{that}\ (\iota x(x = b) = \iota x(x = b))$ from 2 and ID1
4. $\texttt{that}\ (\iota x(x = b) = \iota x(x = b)) = t^*$ a plausible assumption
5. $\texttt{that}\ A = \texttt{CONJ}\ y(y = y \wedge \neg A)$ from 3, 4, and ID2

In this argument, the principle designated by "EQU", which is used in line 2, is the principle *that logically equivalent sentences have the same intensions.*

Suppose now that A is a *false* sentence, hence $\neg A$ is a true sentence, and therefore according to the above argument: $\texttt{that}\ \neg A = \texttt{CONJ}\ y(y = y \wedge A)$. But, according to P14, $\texttt{neg}(\texttt{that}\ A) = \texttt{that}\ \neg A$; and we also have: $\texttt{CONJ}\ y(y = y \wedge A) = t^* = \texttt{neg}(k^*) = \texttt{neg}(\texttt{CONJ}\ y(y = y \wedge \neg A))$ (because $\texttt{CONJ}\ y(y = y \wedge \neg A) = k^*$, since A, according to supposition, is false). Hence: $\texttt{neg}(\texttt{that}\ A) = \texttt{neg}(\texttt{CONJ}\ y(y = y \wedge \neg A))$. And therefore we obtain the same result as above: $\texttt{that}\ A = \texttt{CONJ}\ y(y = y \wedge \neg A)$.

This entire deduction of the equation corresponding to definition (D^*) is *almost* impeccable. EQU, indeed, is not deducible in the system P0–P16. But one can add the provability-rule EQU* to the proof-rules of the system: If $A \equiv B$ is logically provable, then $\texttt{that}\ A = \texttt{that}\ B$ is also logically provable.[11] This proof-rule serves the same purpose as EQU, and there is nothing at all wrong with it. There is also nothing at all wrong with the *plausible assumption* in line 4; it, too, is not deducible in the system P0–P16, but it can be added to that system without scruples.

Well, what, then, is wrong with the deduction?—The false step occurs in moving from 2 to 3 on the basis of ID1. But there is nothing whatever wrong with ID1, the problem is the implicitly applied inference-rule: *substitution of identicals*. This rule is not universally valid; there are occasions when its application leads from a true sentence to a false one, and above we have such an occasion: 2 and ID1 are true (2 purely logically, and ID1 on the basis of assumption), but 3 is *not* true—at least not for any sentence A that is not logically true. The rule of substitution of identicals is safe as long as identicals are not substituted for each other in contexts that are ruled by an occurrence of "that"; for the name-forming operator "that" creates *intensional contexts*—contexts in which the *topical* (or *contextual*) referential function of some singular terms can shear way from their *factual* referential function, creating an ambiguity that had better not be ignored. (Such singular terms are called "(referentially) unstable"; for more on this matter, see Section 9 below.)

P13–P16 and P10 are still not all the principles that are needed for "that". In Section 7 and Section 9 below, more "that"-principles will follow.

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[11] On the concept of (broadly) logical provability, see footnote 15.

5. The Number of States of Affairs

The ontological weakness of the mereology of states of affairs has still been preserved up to this point. No principle demanding a dramatically high number of states of affairs has so far been added, and the principles already assumed do merely require that there be at least four states of affairs: according to them, there need not be more states of affairs than this. From one point of view, this is as it should be, since the mereology of states of affairs should be applicable to finite artificial universes of states of affairs, with more or less restricted numbers of states of affairs (with 4, or 8, or 16, or 32, or 64 states of affairs, for example). If the mereology of states of affairs is to remain thus applicable (as a *multipurpose* abstract machine), then it must remain open to having any *principle of finite number* added to it that is appropriate to the occasion of application, i.e., some principle or other that looks like this: $\exists^{=N} x\ \mathrm{EL}(x)$ ("There are exactly N elemental states of affairs", where N is some natural number $\geqslant 2$).

From the *realistic* point of view, however — i.e., if the mereology of states of affairs is to decribe the *real* universe of states of affairs — there are only two envisageable *principles of the number of states of affairs* (via specifying the number of elemental states of affairs) that are not entirely arbitrary: (1) There is *exactly one* elemental state of affairs (in other words, there are *exactly two* states of affairs, namely, t^* and k^*). (2) There are *infinitely many* elemental states of affairs (in other words, there are *infinitely many* states of affairs). On the basis of the principles already stated (P0–P16), (1) has already been ruled out. This leaves us with (2) — every other envisageable principle of the number of states of affairs would be entirely arbitrary as an assertion about the *real* universe of states of affairs ((1) having been ruled out). And (2) can be formulated by employing the means of a first-order language, as follows:

(P17) $\exists^{\geqslant N} x\ \mathrm{EL}(x) \supset \exists^{\geqslant N+1} x\ \mathrm{EL}(x)$, for every natural number $N \geqslant 2$.[12]

6. Modal Bases and Modal Principles in the Mereology of States of Affairs

In Section 1 above, two bases of necessity were indicated: b^1 — the basis of logical necessity, and b^2 — the basis of nomological necessity. For characterizing the basis of logical necessity completely we merely need to add the principle:

[12] The initial assertion $\exists^{\geqslant 2} x\ \mathrm{EL}(x)$ is already provable. Note that the reference to natural numbers is not necessary but can be eliminated, since $\exists^{\geqslant N} x\ \mathrm{EL}(x)$ is definable without reference to natural numbers: $\exists^{\geqslant 1} x\ \mathrm{EL}(x) := \exists x\, \mathrm{EL}(x)$; $\exists^{\geqslant 2} x\ \mathrm{EL}(x) := \exists x \exists x'(\mathrm{EL}(x) \wedge \mathrm{EL}(x') \wedge x \neq x')$, $\exists^{\geqslant 3} x\ \mathrm{EL}(x) := \exists x \exists x' \exists x''(\mathrm{EL}(x) \wedge \mathrm{EL}(x') \wedge \mathrm{EL}(x'') \wedge x \neq x' \wedge x' \neq x'' \wedge x \neq x'')$; etc.

(P18) $\mathrm{S}(b^n) \wedge b^1 = t^*$.[13]

For characterizing the basis of nomological necessity completely, one would have to indicate exactly which states of affairs are laws of nature, which I will not do here (and who *can* do it?). But the following principles clearly hold for the basis of nomological necessity, independently of any specification of the laws of nature:

(P19) $\mathrm{P}(b^2, w^*)$,

(P20) $b^2 \neq t^*$,

(P21) $b^2 \neq w^*$.

In general we have: If $\mathrm{P}(b^n, w^*)$ is true of the basis of necessity b^n, then the (corresponding) necessity $\Box^n$ is *faithful to truth*, and vice versa; if $\mathrm{P}(b^n, w^*)$ is false of the basis b^n, then the necessity $\Box^n$ is *unfaithful to truth*, and vice versa. For necessities $\Box^n$ that are faithful to truth one can prove in the mereology of states of affairs: $\Box^n B \supset B$.

Proof. Assume $\Box^n B$, where $\Box^n$ is a necessity faithful to truth. Hence according to the definitions at the beginning of Section 1: $\mathrm{P}(\mathtt{that}\ B, b^n)$. Hence because of $\mathrm{P}(b^n, w^*)$ and P1: $\mathrm{P}(\mathtt{that}\ B, w^*)$. Hence because of $\forall x[\mathrm{S}(x) \supset (\mathrm{A}(x) \equiv \mathrm{P}(x, w^*))]$ and P13: $\mathrm{A}(\mathtt{that}\ B)$. Hence because of D6 and P13: $\mathrm{O}(\mathtt{that}\ B)$. Hence because of P10: B. $\Box$

One can also prove in the mereology of states of affairs: $\mathrm{P}(b^n, w^*) \equiv \forall x(\Box^n(x) \supset \mathrm{O}(x))$ (where $\forall x(\Box^n(x) \supset \mathrm{O}(x))$ is the obvious predicate-logical correlate of $\Box^n B \supset B$ if the latter is taken as a general schema). A question I leave open for the time being is whether one can deduce, in the mereology of states of affairs, $\mathrm{P}(b^n, w^*)$ from assuming $\Box^n B \supset B$ as a general schema (but see Section 9).

Note that the fact that a necessity is faithful to truth does not by itself mean that the necessity is ontic or alethic, for the basis which is an intensional part of w^*, making the corresponding necessity faithful to truth, may have been picked out according to criteria which are wholly or partly epistemic (for example). Nor does it seem necessary that every ontic necessity is faithful to truth.

A property of necessities which is weaker than *faithfulness to truth* (in view of the provable statement $\mathrm{P}(w^*, k^*)$, P3, P7) is *consistency*: If $b^n \neq k^*$ is true of the basis of necessity b^n, then the necessity $\Box^n$ is *a consistent necessity*, and vice

[13]Strictly speaking, $\mathrm{S}(b^n)$ does nothing to characterize b^1 beyond what is already asserted by $b^1 = t^*$ (since one can already prove $\mathrm{S}(t^*)$). But $\mathrm{S}(b^n)$ will be needed frequently (for bases of necessity b^n that are not described in any further way, so that their being states of affairs does not emerge as a consequence of their description), and it is better to assert it here explicitly (in as good as place as any other) than to make unacknowledged use of it.

versa; if $b^n \neq k^*$ is false of the basis b^n, then $\Box^n$ is *an inconsistent necessity*, and vice versa. For necessities $\Box^n$ that are consistent one can prove in the mereology of states of affairs: $\Box^n B \supset \neg\Box^n\neg B$.

PROOF. Assume $\Box^n B$, where $\Box^n$ is a consistent necessity. Hence: $\mathrm{P}(\mathtt{that}\ B, b^n)$. Assume also $\Box^n\neg B$. Hence: $\mathrm{P}(\mathtt{that}\ \neg B, b^n)$. Hence: $\mathrm{P}(\mathtt{neg}(\mathtt{that}\ B), b^n)$, according to P14. It is provable in the mereology of states of affairs: $\forall x\forall y(\mathrm{P}(x,y) \wedge \mathrm{P}(\mathtt{neg}(x), y) \equiv y = k^*)$. Hence: $b^n = k^*$—contradicting the consistency of $\Box^n$. Therefore: $\neg\Box^n\neg B$. □

One can also prove in the mereology of states of affairs: $b^n \neq k^* \equiv \forall x[\Box^n(x) \supset \neg\Box^n(\mathtt{neg}(x))]$ (where $\forall x[\Box^n(x) \supset \neg\Box^n(\mathtt{neg}(x))]$ is the obvious predicate-logical correlate of $\Box^n B \supset \neg\Box^n\neg B$ if the latter is taken as a general schema). *Another* question I leave open for the time being is whether one can deduce, in the mereology of state of affairs, $b^n \neq k^*$ from assuming $\Box^n B \supset \neg\Box^n\neg B$ as a general schema (but see again Section 9).

We also have in general: If $b^n \neq t^*$ is true of the basis of necessity b^n, then the necessity $\Box^n$ is *a contingent necessity*, and vice versa; if $b^n \neq t^*$ is false of the basis b^n, then $\Box^n$ is *a noncontingent necessity*, and vice versa. The designation "contingent necessity" is confusing, and still remains so when the gloss is added that what is meant by "contingent necessity" is *logically contingent* necessity. In what sense contingent necessities $\Box^n$ are called "(logically) contingent" becomes clear when one considers that the following is deducible in the mereology of states of affairs: $b^n \neq t^* \equiv \exists x(\Box^n(x) \wedge \neg\Box^1(x))$.

Finally we have in general: If $b^n \neq w^*$ is true of a basis of necessity b^n, then the necessity $\Box^n$ is a *proper necessity*, and vice versa; if $b^n \neq w^*$ is false of the basis b^n, then $\Box^n$ is an *improper necessity*, and vice versa. It is deducible in the mereology of states of affairs for every *consistent* necessity $\Box^n$: $b^n \neq w^* \equiv \exists x(\mathrm{O}(x) \wedge \neg\Box^n(x))$.

PROOF. Let $\Box^n$ be a consistent necessity. Assume $b^n = w^*$, and $\mathrm{O}(x)$. Hence according to D7, D1, P3 and P4: $\mathrm{P}(x, w^*)$. Hence $\mathrm{P}(x, b^n)$. Hence $\Box^n(x)$. Assume conversely: $\forall x(\mathrm{O}(x) \supset \Box^n(x))$. Hence because of $\mathrm{S}(w^*)$ (a consequence of D7, D1, P3 and P4), P12, D6: $\mathrm{O}(w^*)$. Hence: $\Box^n(w^*)$. Hence: $\mathrm{P}(w^*, b^n)$. According to P8: $\mathrm{QC}(w^*)$, and hence (according to D14): $w^* = b^n \vee \mathrm{T}(b^n)$. Since $\Box^n$ is a consistent necessity, we have: $b^n \neq k^*$, and therefore: $\neg\mathrm{T}(b^n)$. Hence: $b^n = w^*$. □

Since there are, according to P17, infinitely many states of affairs, there are infinitely many necessities faithful to truth, just as many necessities faithful to truth as there are bases for them. The extremes are marked, on the one side, by *logical necessity*, with the basis t^* $(= b^1)$, which is a proper and noncontingent necessity faithful to truth, and, on the other side, by *factuality*, with the basis w^*, which is an improper and contingent necessity faithful to truth. There are infinitely many proper and contingent necessities faithful to truth in between, *nomological necessity*, with the basis b^2, being one of them. Most of these necessities do not have a designation. The sequence $\Box^1, \Box^2, \Box^3, \Box^4, \Box^5, \Box^6, \ldots$ — even if prolonged to infinity — does not contain enough terms to give a designation to every proper and contingent necessity faithful to truth. (But there are certainly enough designations in the sequence for every necessity we specifically refer to here.) It is a matter of fact that we do not give much thought to most of the infinitely many proper and contingent necessities faithful to truth. But this should not mislead us into thinking that somehow they are not "real" necessities.

The *strength (or force) of a necessity* is inverse to the (intensional) strength of its basis. *Logical necessity* — having the absolutely weakest basis: the state of affairs t^* which has no intensional content — is the absolutely strongest necessity. *Factuality* — having the strongest basis which is faithful to truth (and which is *almost* the *absolutely* strongest basis): the state of affairs w^* — is the weakest necessity faithful to truth, so weak that it is quite rightfully called an improper necessity. We have the following correlation: $\Box^n$ is a stronger necessity than $\Box^m$ if, and only if, b^n is a proper intensional part of b^m. There are of course many pairs of necessities which are such that the basis of each pair-member does not contain the basis of the other pair-member: neither one of the paired necessities is stronger than the other.

An important fact should be noted. *General determinism* is sometimes asserted as the thesis that *every obtaining state of affairs is necessary*, where "is necessary" is thought to express a necessity $\Box^n$ which is faithful to truth (and therefore consistent) *and* has more force than "obtains". But as, a matter of fact, it cannot have more force. Who accepts $\forall x(O(x) \supset \Box^n(x))$ for a consistent necessity $\Box^n$ must, according to the theorem stated and proven above, also accept $b^n = w^*$, and therefore the necessity $\Box^n$ *is not* stronger than *factuality*, which is the weakest necessity faithful to truth. This shows that general determinism is an incoherent position: it intends to spread a *proper* necessity over more states of affairs than it can apply to, namely, over all obaining states of affairs.

Leibniz famously asserted that *general logical determinism* is true, i.e., that $\forall x(O(x) \supset \Box^1(x))$ is true. But if $\forall x(O(x) \supset \Box^1(x))$ is true, then, according to the theorem $b^1 \neq w^* \equiv \exists x(O(x) \wedge \neg\Box^1(x))$, $b^1 = w^*$ is also true ($\Box^1$ being a consistent necessity), or in other words: $w^* = t^*$ is true (since $b^1 = t^*$, according to P18) — contradicting P9. General logical determinism is, therefore, false — provably false in the mereology of states of affairs. Leibniz is, therefore, provably wrong. But it should be noted that P9 is one of the two principles of the mereology of states of affairs (the other being P8) which are not conceptually true. Leibniz, therefore, did not commit a logical mistake in asserting the truth of general logical determinism. It seems, however, that Leibniz would have endorsed $\mathtt{MC}(w^*)$—"the world is maximally consistent" (which is equivalent to the conjunction of P7: $w^* \neq k^*$, and P8: $\mathtt{QC}(w^*)$). And the only way to square $w^* = t^*$ (to which Leibniz is committed by being a logical determinist) with $\mathtt{MC}(w^*)$ is by assuming that there are less than four states of affairs (see the end of Section 3; $\mathtt{MC}(w^*)$ definitionally implies $\mathtt{QC}(w^*)$), and more than one state of affairs. Therefore, the only way to square $w^* = t^*$ with $\mathtt{MC}(w^*)$ is by assuming that *there are exactly two states of affairs* (because the mereology of states of affairs forbids that there are exactly three states of affairs): t^* ($= w^*$) and k^*. Leibniz, therefore, as a logical determinist and "maximal-consistentist," is committed to the view that there are only two states of affairs. It would be striking if there were any evidence in his works that he did believe this.

7. The Classical Modal Principles

We have already seen that for nomological necessity, $\square^2$, two classical modal principles are true: $\square B \supset B$, and $\square B \supset \neg\square\neg B$. (I leave out the index attached to "$\square$" if reference is made to some determinable necessity or other; I also leave out the index attached to "b" if reference is made to some determinable basis or other.) The same two principles are also true for logical necessity, $\square^1$.

What about the following other classical principles: (1) $\square(A \supset B) \supset (\square A \supset \square B)$, (2) $\square B \supset \square\square B$, (3) $\neg\square B \supset \square\neg\square B$? Let us see whether we need further principles for proving them, and which principles.

$\square(A \supset B) \supset (\square A \supset \square B)$ is provable without further assumptions.

Proof. Assume $\square(A \supset B)$, and assume $\square A$. Hence $\mathrm{P}(\mathtt{that}\,(A \supset B), b)$, and $\mathrm{P}(\mathtt{that}\ A, b)$. Hence: $\mathrm{P}(\mathtt{disj}(\mathtt{neg}(\mathtt{that}\ A), \mathtt{that}\ B), b)$, according to P14, P15, etc., and $\mathrm{P}(\mathtt{that}\ A, b)$. Hence $\mathrm{P}(\mathtt{that}\ B, b)$—because $\mathrm{S}(\mathtt{that}\ A)$ and $\mathrm{S}(\mathtt{that}\ B)$, according to P13, and because of the theorem $\forall x\forall y\forall z[\mathrm{S}(x) \wedge \mathrm{S}(y) \wedge \mathrm{P}(\mathtt{disj}(\mathtt{neg}(x), y), z) \wedge \mathrm{P}(x, z) \supset \mathrm{P}(y, z)]$. Hence $\square B$. $\square$

But the attempt to prove $\square B \supset \square\square B$ reveals that there is need of further principles: Assume $\square B$. Hence $\mathrm{P}(\mathtt{that}\ B, b)$ (b being the state of affairs which is the appropriate basis of necessity). From this one must derive: $\mathrm{P}(\mathtt{that}\ \square B, b)$, which, purely definitionally, implies $\square\square B$. The step from $\mathrm{P}(\mathtt{that}\ B, b)$ to $\mathrm{P}(\mathtt{that}\ \square B, b)$ is, by definition alone, equivalent to the step from $\mathrm{P}(\mathtt{that}\ B, b)$ to $\mathrm{P}(\mathtt{that}\ \mathrm{P}(\mathtt{that}\ B, b), b)$, which, in turn, is a consequence of the following general principle:

Principle A $\quad \forall x\forall y(\mathrm{P}(x, y) \supset \mathrm{P}(\mathtt{that}\ \mathrm{P}(x, y), y))$.

Let us now see which further principle is necessary for proving $\neg\Box B \supset \Box\neg\Box B$. It is this (which is easily found, given Principle A):

Principle B $\quad \forall x\forall y(\mathrm{S}(x) \wedge \mathrm{S}(y) \wedge \neg\mathrm{P}(x,y) \supset \mathrm{P}(\mathrm{neg}(\mathtt{that}\ \mathrm{P}(x,y)), y))$.

Both Principle A and Principle B are easily provable if one assumes the following stronger principle:

(P22) $$\forall x\forall y(\mathrm{P}(x,y) \supset \mathtt{that}\ \mathrm{P}(x,y) = t^*) \wedge \forall x\forall y(\neg\mathrm{P}(x,y) \supset \mathtt{that}\ \mathrm{P}(x,y) = k^*).$$

I adopt this latter principle, since a relationship of intensional parthood, or a negation of such a relationship, is clearly a matter of logical necessity.[14]

Another classical modal principle is not really a principle but a *provability-rule*: If B is logically provable,[15] then $\Box B$ is logically provable. On the basis of the definition of $\Box$, this inference-rule amounts to: If B is logically provable, then $\mathrm{P}(\mathtt{that}\ B, b)$ is logically provable (b being the state of affairs that is the appropriate basis of necessity). I put this, more compactly, into the following form:

(P23) $$\vdash B \Rightarrow\ \vdash \mathrm{P}(\mathtt{that}\ B, b).$$

The provability-rule $\vdash B \Rightarrow\ \vdash \Box B$ is an easy consequence of P23.

One should note that the modal propositional system S5, that is, classical truth-functional propositional logic (axiomatized in some way) *plus*

$\vdash \Box B \supset B$,

$\vdash \Box(A \supset B) \supset (\Box A \supset \Box B)$,

$\vdash \Box B \supset \Box\Box B$,

$\vdash \neg\Box B \supset \Box\neg\Box B$,

$\vdash B \Rightarrow\ \vdash \Box B$,

(where "$\vdash$" stands for "logically provable") has now been justified as *the* correct system of modal propositional logic for all necessities that (a) can be represented as founded on a basis of necessity, and that (b) are faithful to truth. The foundations of justification are the mereology of states of affairs and a conception

[14] An equivalent formulation of P22 is this: $\forall x\forall y(\mathrm{P}(x,y) \supset \Box^1 \mathrm{P}(x,y)) \wedge \forall x\forall y(\neg\mathrm{P}(x,y) \supset \Box^1 \neg\mathrm{P}(x,y))$.

[15] "Logically provable" means: *provable purely on the basis of principles and inference-rules that are conceptually (broadly logically, analytically) valid.* Thus, if P8 or P9 are necessary for deducing a certain theorem in the mereology of states of affairs, then that theorem is not logically provable — because P8 and P9 are not conceptually true (see Section 3).

of necessity that is expressed by the following two definitions: $\square(x) := \mathrm{P}(x, b)$, $\square A := \square(\mathtt{that}\ A)$. These definitions are expressive of a *basis-theory of necessity*, since "b" refers to a state of affairs which is *the basis* of the necessity concerned.

Remark. The basis-theory of necessity is not a theory of necessity that is applicable to all kinds of necessity. There are some necessities for which the analysis in terms of a basis of necessity is inadequate, for example, *epistemic necessity*: knowledge. What somebody knows is not what follows from his basis of knowledge (there is no such thing); it is what follows from his *basis of belief* and satisfies in addition several further conditions. Take the simplest concept of knowledge (which is implied by all other concepts of knowledge): *true conviction*. In this sense, what somebody knows is what follows from his basis of belief and is true. The principle $\neg\square B \supset \square\neg\square B$ is not valid for knowledge in this sense, for the following situation frequently obtains:

B is false, and therefore $\neg\square B$ is true; but (in this same situation) $\mathtt{that}\ B$ is an intensional part of the basis of belief of the person concerned, and therefore that person is convinced that B (i.e., $\square_c B$), hence convinced that she is convinced that B (i.e., $\square_c\square_c B$, according to P22), hence convinced that she knows that B (because of $\square_c\square_c B \supset \square_c(\square_c B \wedge B)$). Hence $\square\neg\square B$ must be false. (For if it were true, then the person concerned would know, and therefore be convinced, that she does not know that B. Hence the person concerned would be both convinced that she knows that B (as we have already seen) and convinced that she does not know that B — which cannot be.)

Note that conviction — or *doxastic necessity* — *can* very well be treated according to the basis-theory of necessity. Since one will not require that conviction be faithful to truth but only that it be consistent, $\square B \supset B$ needs to be replaced by $\square B \supset \neg\square\neg B$ in the above system; this replacement turns it into a system which defines a defensible modal propositional logic for — *dispositional, rational* — conviction. □

The theory of necessity advocated here is, moreover, an *onto-nomological theory of necessity*, since it is grounded in ontological laws for the realm of states of affairs. By being an onto-nomological theory of necessity, it is automatically also an adequate onto-nomological theory of possibility, since possibility can be adequately defined on the basis of necessity ($\Diamond A := \neg\square\neg A$) and is, therefore, completely determined by the latter concept. (But the onto-nomological theory of possibility can also be developed *independently* of the theory of necessity: on the basis of the mereology of states of affairs, and the definitions $\Diamond^n(x) := \mathrm{S}(x) \wedge \neg\,\mathrm{P}(\mathrm{neg}(x), b^n)$ and $\Diamond^n A := \Diamond^n(\mathtt{that}\ A)$.)

It needs a separate paper to handle the question how far *conditionals* (or *relational necessities*) can be treated within the mereology of states of affairs on the basis of the definitions ${}^n\!\!\to(x, y) := \mathrm{P}(y, \mathtt{conj}(b^n, x))$ and $B^n\!\!\to A := {}^n\!\!\to(\mathtt{that}\ B, \mathtt{that}\ A)$.

8. Necessity and Possible Worlds

The theory of modality developed here is a theory of modality in which the concept of possible world plays no essential role. Nevertheless, in view of the popularity of the possible-worlds-theory of modality, it will be good to see how that theory is derivable from, and therefore reducible to, the present theory.

We have seen above (in Section 2) that maximally consistent states of affairs can be regarded as possible worlds (and that there are exactly as many maximally consistent states of affairs as there are elemental states of affairs: *negation* is a function that maps elemental states of affairs onto maximally consistent states of affairs). This can only mean that *the possible worlds* (the entire set of them) *can be identified* with *the maximally consistent states of affairs* (the entire set of them). (It would hardly make sense to allow that the maximally consistent states of affairs are possible worlds, but to demand that there are in addition some other possible worlds that are not maximally consistent states of affairs.)

We need to define another important concept of the mereology of states of affairs:

(D16) $\quad \mathtt{O}(x,y) := \mathtt{P}(x,y).$

$\mathtt{O}(x,y)$ is read as "(the state of affairs) x obtains in (the state of affairs) y". One can then prove the following theorems:

$$\forall x[\,\mathtt{S}(x) \supset (\,\mathtt{O}(x) \equiv \mathtt{O}(x,w^*))],$$
$$\forall x[\,\mathtt{S}(x) \supset (\Box^n(x) \equiv \forall y(\mathtt{MC}(y) \wedge \mathtt{O}(b^n,y) \supset \mathtt{O}(x,y)))],$$
$$\Box^n A \equiv \forall y(\,\mathtt{MC}(y) \wedge \mathtt{O}(b^n,y) \supset \mathtt{O}(\mathtt{that}\ A,y)),$$
$$\forall x[\,\mathtt{S}(x) \supset (\Diamond^n(x) \equiv \exists y(\mathtt{MC}(y) \wedge \mathtt{O}(b^n,y) \wedge \mathtt{O}(x,y)))],$$
$$\Diamond^n A \equiv \exists y(\,\mathtt{MC}(y) \wedge \mathtt{O}(b^n,y) \wedge \mathtt{O}(\mathtt{that}\ A,y)).$$

The first theorem states that a state of affairs obtains (*simpliciter*) if, and only if, it obtains in *the world.* The second theorem states that a state of affairs is n-necessary if, and only if, it obtains in every possible world in which the basis of the n-necessity obtains. The third theorem is a corollary of the second theorem: for the sentence connective of necessity (not the predicate). The fourth theorem states that a state of affairs is n-possible if, and only if, it obtains in some possible world in which the basis of the n-necessity obtains. The fifth theorem is a corollary of the fourth theorem: for the sentence connective of possibility (not the predicate).

PROOF. As an example, here is the (not altogether easy) proof of the second theorem:

1. Suppose: $\mathrm{S}(x)$, $\Box^n(x)$, $\mathrm{MC}(y)$, $\mathrm{O}(b^n, y)$. Hence $\mathrm{P}(x, b^n)$ and $\mathrm{P}(b^n, y)$ (according to the definition of $\Box^n(x)$, and D16). Hence by P1: $\mathrm{P}(x, y)$, hence by D16: $\mathrm{O}(x, y)$.

2. Suppose: $\mathrm{S}(x), \forall y(\mathrm{MC}(y) \wedge \mathrm{O}(b^n, y) \supset \mathrm{O}(x, y))$. Hence (because of P3, P4, D1): $\mathrm{P}(x, \mathrm{CONJ}\ z\forall y(\mathrm{MC}(y) \wedge \mathrm{O}(b^n, y) \supset \mathrm{O}(z, y)))$. But $\mathrm{CONJ}\ z\forall y(\mathrm{MC}(y) \wedge \mathrm{O}(b^n, y) \supset \mathrm{O}(z, y)) = b^n$, because (applying D16) $\mathrm{CONJ}\ z\forall y(\mathrm{MC}(y) \wedge \mathrm{P}(b^n, y) \supset \mathrm{P}(z, y)) = b^n$. Therefore: $\mathrm{P}(x, b^n)$, and consequently: $\Box^n(x)$.

$\mathrm{CONJ}\ z\forall y(\mathrm{MC}(y) \wedge \mathrm{P}(b^n, y) \supset \mathrm{P}(z, y)) = b^n$ remains to be proved. It is a consequence of:

$$(\dagger) \qquad \mathrm{P}(b^n, \mathrm{CONJ}\ z\forall y(\mathrm{MC}(y) \wedge \mathrm{P}(b^n, y) \supset \mathrm{P}(z, y))),$$

$$(\ddagger) \qquad \mathrm{P}(\mathrm{CONJ}\ z\forall y(\mathrm{MC}(y) \wedge \mathrm{P}(b^n, y) \supset \mathrm{P}(z, y)), b^n),$$

according to P3.

($\dagger$) holds, because $\forall y(\mathrm{MC}(y) \wedge \mathrm{P}(b^n, y) \supset \mathrm{P}(b^n, y))$ and P4, P3, D1, P18.

For ($\ddagger$) assume $\mathrm{QA}(u)$, $\mathrm{P}(u, \mathrm{CONJ}\ z\forall y(\mathrm{MC}(y) \wedge \mathrm{P}(b^n, y) \supset \mathrm{P}(z, y)))$; what is in question is established according to P5 if $\mathrm{P}(u, b^n)$ can be deduced from this assumption.

If $\mathrm{M}(u)$, then $\mathrm{P}(u, b^n)$. ($\mathrm{S}(b^n)$ according to P18.)

If $\neg\mathrm{M}(u)$, then (according to P6): $\exists k'[\mathrm{P}(k', u) \wedge \neg\mathrm{M}(k') \wedge \exists z'(\mathrm{P}(k', z') \wedge \forall y(\mathrm{MC}(y) \wedge \mathrm{P}(b^n, y) \supset \mathrm{P}(z', y)))]$. Because of $\mathrm{QA}(u)$, $\mathrm{P}(k', u)$, $\neg\mathrm{M}(k')$: $k' = u$ (according to D9), and also $\mathrm{EL}(u)$ (according to D10). Hence: $\exists z'(\mathrm{P}(u, z') \wedge \forall y(\mathrm{MC}(y) \wedge \mathrm{P}(b^n, y) \supset \mathrm{P}(z', y)))$. Hence: $\exists z'(\mathrm{P}(u, z') \wedge \forall y(\mathrm{S}(y) \wedge \mathrm{EL}(\mathrm{neg}(y)) \wedge \neg\mathrm{P}(\mathrm{neg}(y), b^n) \supset \neg\mathrm{P}(\mathrm{neg}(y), z')))$.[16] Hence: $\exists z'(\mathrm{P}(u, z') \wedge \forall y(\mathrm{S}(y) \wedge \mathrm{EL}(\mathrm{neg}(y)) \wedge \mathrm{P}(\mathrm{neg}(y), z') \supset \mathrm{P}(\mathrm{neg}(y), b^n)))$. Hence: $\exists z'(\mathrm{P}(u, z') \wedge \forall y(\mathrm{S}(y) \wedge \mathrm{EL}(\mathrm{neg}(\mathrm{neg}(y))) \wedge \mathrm{P}(\mathrm{neg}(\mathrm{neg}(y)), z') \supset \mathrm{P}(\mathrm{neg}(\mathrm{neg}(y)), b^n)))$. Hence: $\exists z'(\mathrm{P}(u, z') \wedge \forall y(\mathrm{EL}(y) \wedge \mathrm{P}(y, z') \supset \mathrm{P}(y, b^n)))$.[17] Hence because of $\mathrm{EL}(u)$: $\mathrm{P}(u, b^n)$. $\Box$

Note that the conjunct $\mathrm{O}(b^n, y)$ drops out of the five theorems for $n = 1$, since $b^1 = t^*$ (P18) and $\forall y(\mathrm{S}(y) \supset \mathrm{O}(t^*, y))$. Thus one obtains, for example:

$$\forall x\,[\,\mathrm{S}(x) \supset (\Box^1(x) \equiv \forall y(\mathrm{MC}(y) \supset \mathrm{O}(x, y)))],$$

$$\forall x\,[\,\mathrm{S}(x) \supset (\Diamond^1(x) \equiv \exists y(\mathrm{MC}(y) \wedge \mathrm{O}(x, y)))].$$

The Leibnizian conception of modality emerges as a pair of theorems of the mereology of states of affairs (which theorems also show that Leibnizian necessity is logical necessity, Leibnizian possibility logical possibility). I add the

[16] The theorems employed are $\forall y[\mathrm{MC}(y) \supset \forall x(\mathrm{S}(x) \supset (\mathrm{P}(x, y) \equiv \neg\mathrm{P}(\mathrm{neg}(y), x)))]$ and $\forall y[\mathrm{MC}(y) \equiv \mathrm{EL}(\mathrm{neg}(y))]$. b^n is a state of affairs according to P18. The additional conjunct "$\mathrm{S}(y)$" is simply a definitional consequence of "$\mathrm{MC}(y)$".

[17] The theorem employed is $\forall y(\mathrm{S}(y) \supset \mathrm{neg}(\mathrm{neg}(y)) = y)$.

following two theorems that rather strikingly illuminate the relation between *modality* and *basis of modality*[18]:

$$b^n = \mathtt{CONJ}\, y\, \Box^n(y),$$
$$b^n = \mathtt{DISJ}\, y\, \Diamond^n(y).$$

9. Final Observations: Another "That" — Principle and the Trickiness of "w^*"

In Section 6, I left it open whether one can deduce, in the mereology of states of affairs, $\mathrm{P}(b^n, w^*)$ from assuming $\Box^n B \supset B$ as a general schema, and likewise whether one can deduce $b^n \neq k^*$ from assuming $\Box^n B \supset \neg\Box^n\neg B$ as a general schema. Let us try to make these deductions.

(1) Assume $\Box^n B \supset B$ as a general schema. Hence: $\Box^n\, \mathrm{O}(b^n) \supset \mathrm{O}(b^n)$. According to the definition of $\Box^n(x)$: $\Box^n(b^n)$, because of $\mathrm{P}(b^n, b^n)$, which in turn is true because of P18 and P2. Now, there is another very plausible "that"-principle — a principle complementing P22 and P23:

(P24) $\forall x(\mathrm{S}(x) \supset x = \mathtt{that}\ \mathrm{O}(x))$.

Applying P24, one obtains $\Box^n(\mathtt{that}\ \mathrm{O}(b^n))$ from $\Box^n(b^n)$, and therefore: $\Box^n\, \mathrm{O}(b^n)$. Hence, since we already have $\Box^n\, \mathrm{O}(b^n) \supset \mathrm{O}(b^n)$: $\mathrm{O}(b^n)$. Therefore: $\mathrm{P}(b^n, w^*)$ (according to $\forall x[\mathrm{S}(x) \supset (\mathrm{A}(x) \equiv \mathrm{P}(x, w^*))]$ and D6).

(2) Assume $\Box^n B \supset \neg\Box^n\neg B$ as a general schema. Hence $\Box^n\mathrm{O}(b^n) \supset \neg\Box^n\neg\mathrm{O}(b^n)$. Just as in (1), one obtains $\Box^n\, \mathrm{O}(b^n)$. Therefore: $\neg\Box^n\neg\, \mathrm{O}(b^n)$, and hence (by definition) $\neg\, \mathrm{P}(\mathtt{that}\ \neg\, \mathrm{O}(b^n), b^n)$, hence (applying P14) $\neg\, \mathrm{P}(\mathtt{neg}(\mathtt{that}\ \mathrm{O}(b^n)), b^n)$, hence (applying P24) $\neg\, \mathrm{P}(\mathtt{neg}(b^n), b^n)$, hence $b^n \neq k^*$ (because $\mathrm{P}(\mathtt{neg}(k^*), k^*)$).

The designator "w^*" has been defined above as "$\mathtt{CONJ}\ x\ \mathrm{O}(x)$" (D7). The designator "$t^*$", on the other hand, has been defined as "$\mathtt{CONJ}\ x \neg\, \mathrm{S}(x)$" (D2). Though the two definitions have a very similar structure, there is a striking difference between the two defined designators. Of "t^*," one would say that it could not, not even in principle, have designated a different state of affairs than it designates in fact. Of "w^*", however, one would indeed say that it could, in principle, have designated a different state of affairs than it designates in fact.[19] This is a consequence of the predicate "$\mathrm{O}(x)$" occurring in its definition: one would say that this predicate could have applied differently than it does in fact apply, because other states of affairs could have obtained (i.e., could have been actual) than in fact

[18] A basis of necessity b^n is likewise a *basis of possibility*, in short: it is a *basis of modality*.

[19] Note that these are *meta*-modal statements. They can, however, be represented in the object-language: $\forall y'(t^* \neq y' \supset \Box^1(t^* \neq y'))$, $\exists y'(w^* \neq y' \wedge \Diamond^1(w^* = y'))$.

obtain, and if they had, "w^*" would have designated a different state of affairs than it designates in fact.

This *instability* of "w^*" leads to certain problems:

(1′) Let y be an obtaining (therefore, according to D6, actual) state of affairs that is neither w^* nor t^*. Hence according to a previously proven theorem (in Section 3): $\mathrm{P}(y, w^*)$. Hence according to P22: $\mathtt{that}\ \mathrm{P}(y, w^*) = t^*$, and therefore: $\Box^1\, \mathrm{P}(y, w^*)$.

(2′) Since $y \neq t^*$, it follows according to P24: $\mathtt{that}\ \mathrm{O}(y) \neq t^*$. But it seems that we have the following identity: $\mathtt{that}\ \mathrm{O}(y) = \mathtt{that}\ \mathrm{P}(y, w^*)$. Therefore: $\mathtt{that}\ \mathrm{P}(y, w^*) \neq t^*$, and consequently: $\neg\Box^1\, \mathrm{P}(y, w^*)$.

We are confronted with a contradiction. It seems the only way of escaping from it — short of a modification of principles — is to reject the identity statement used in the second deduction, namely, the statement "$\mathtt{that}\ \mathrm{O}(y) = \mathtt{that}\ \mathrm{P}(y, w^*)$". But, unfortunately, this is not a statement that can easily be rejected: it has intuition on its side, and what is more important: *purely on the basis of principles that are conceptually true* one can prove $\forall y(\mathrm{O}(y) \equiv \mathrm{P}(y, w^*))$ in the mereology of states of affairs, and therefore also the particular case $\mathrm{O}(y) \equiv \mathrm{P}(y, w^*)$.

Remark. See the proof of $\forall y(\mathrm{S}(y) \supset (\mathrm{A}(y) \equiv \mathrm{P}(y, w^*)))$ in Section 3. One merely needs to additionally consider P0 and that "$\mathrm{O}(y)$" is defined as "$\mathrm{S}(y) \wedge \mathrm{A}(y)$" (according to D6) in order to obtain a proof of $\forall y(\mathrm{O}(y) \equiv \mathrm{P}(y, w^*))$ from the already proven theorem. No principle or inference-rule that is not conceptually (or broadly logically) valid is used in the proofs. □

Using the provability-rule EQU*: "If $A \equiv B$ is logically provable, then $\mathtt{that}\ A = \mathtt{that}\ B$ is also logically provable" (see Section 4), one obtains: $\mathtt{that}\ \mathrm{O}(y) = \mathtt{that}\ \mathrm{P}(y, w^*)$. There is no escaping this conclusion.

The real source of the contradiction derived above is not the statement "$\mathtt{that}\ \mathrm{O}(y) = \mathtt{that}\ \mathrm{P}(y, w^*)$" (which cannot be rejected, as has just been shown). The real source of the contradiction is an illicit step of inference, which is so unobtrusive as to be easily overlooked. In Section 4 an absurd conclusion was obtained by substituting, according to the rule of substitution of identicals, the term "$\iota x(A \wedge x = b)$", which is *unstable* for every sentence A that is neither conceptually true nor conceptually false,[20] into a "that"-context. Above, *another* absurd conclusion is obtained by substituting *another* unstable term — "w^*" — into *another* "that"-context. But not by applying the rule of substitution of identicals. Rather, the rule used in the second case is *universal instantiation*: from the first conjunct of P22 [$\forall x \forall y(\mathrm{P}(x, y) \supset \mathtt{that}\ \mathrm{P}(x, y) = t^*)$],

[20] If A is a sentence that is neither conceptually true nor conceptually false, then "$\iota x(A \wedge x = b)$" could have designated a different entity then it designates in fact: If A is true, then the factual referent of "$\iota x(A \wedge x = b)$" is b, but it could have been c^*; if A is false, then the factual referent of "$\iota x(A \wedge x = b)$" is c^*, but it could have been b. (Concerning c^*, see footnote 10.)

"$\mathrm{P}(y, w^*) \supset \mathtt{that}\ \mathrm{P}(y, w^*) = t^*$" is obtained (in (1′)) by universal instantiation. (And further: from the already established statement "$\mathrm{P}(y, w^*)$" and the newly won "$\mathrm{P}(y, w^*) \supset \mathtt{that}\ \mathrm{P}(y, w^*) = t^*$," one obtains by applying *modus ponens*: "$\mathtt{that}\ \mathrm{P}(y, w^*) = t^*$".) But universal instantiation, just like substitution of identicals, is not a universally valid inference-rule: it can lead from a true sentence to a false one when being applied in such a manner that substitution into a "that"-context occurs.[21]

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[21] Note that all modal operators Q — ontic, epistemic, or otherwise — have "that"-contexts connected to them, since they all can be defined on the basis of the corresponding predicate for states of affairs as follows: $\mathrm{Q}(B) := \mathrm{Q}(\mathtt{that}\ B)$. This is the reason why failures of the rules of substitution of identicals and of universal instantiation were first noticed in connection with substitutions into the scopes of modal operators.