



What drives resource prices? A qualitative review with recommendations for further development of the Hotelling model

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What drives resource prices? A qualitative review with recommendations for further development of the Hotelling model

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Abstract This paper examines which factors have an impact on the price of finite resources. Firstly, empirical and theoretical approaches used to study the development of resource prices are considered. These drivers are then studied, individually as well as altogether, starting with the Hotelling model. Implications for the future price of finite resources are subsequently reviewed, and theoretical hypotheses derived. These are then tested by means of a qualitative review based on empirical findings. This paper demonstrates how current approaches have limited applicability in determining which drivers impact the price of finite resources. One of the main reasons is identified as the Hotelling's model's lack of accounting for growth in demand for finite resources.

Keywords Finite resources · Resource price drivers · Hotelling's rule · Qualitative review

JEL Q31 · L71 · L72 · E30

Introduction

Finite resources (e.g., oil, coal, metals, and minerals) were formed over millions of years and do not renew themselves with sufficient speed for modern consumption. An adequate supply of finite (energy and non-energy) raw materials is important for developed industrial regions. In this context, the price of raw materials and its future development are of great social and economic interest. To this end, a large number of studies has been conducted on this topic. A basic distinction

can be made between the two most common research methods: The first approach tries to explain the development of commodity prices based on a theoretical model. On the other hand, there is a broad line of research using empirical studies to examine price changes of raw materials. Recently, new trends affecting raw materials prices can be observed. As new additional (mainly empirical) studies are available, an upto-date qualitative review is required. Our paper tries to bridge this current research gap and also attempts to integrate both theoretical models and empirical research.

In order to draw conclusions about an adequate supply of finite resources, it seems reasonable to focus primarily on raw material prices. Price, and especially its changes over time, can be seen as an aggregated indicator for a raw materials availability. However, it must be pointed out that some factors are not considered in the pricing of a commodity. For example, negative environmental impacts related to the degradation of raw materials – as well as other external effects – are not taken into account in the price of commodities. Furthermore, market imperfections due to government influence (taxes, subsidies, trade restrictions) might exist for specific raw materials. Nor can it be assumed that an efficient market exists solely for raw materials with low trading volumes. Thus, different distortions might exist, which would affect the "true" price of a raw material. However, since the price of a good is the only objective criterion in economics, it should play a center role. Using various practical and scientific methods, an attempt is made in this paper to better understand the influences and issues which impact raw material pricing.

To provide an overview of relevant literature, we first examine the development of theoretical model approaches, followed by a discussion of fundamental empirical studies, and ending with the literature review, where existing metanalyzes are introduced. Classic economic theory has always included studies on the scarcity and effect of resources on

general production factors. As long ago as 1798, Malthus demonstrates in a qualitative study that an exponential growth in population, accompanied by limited availability of the production factor, would lead to a shortage of nutrients in the soil in the long term. Ricardo, on the other hand, (1817, section 5.21) relaxes the assumption of absolute scarcity, assuming there would be a "decreased rate of production" from land available for food production, but still comes to the conclusion that the growth of a society would come to a standstill in the absence of resources. Presenting a neo-classical perspective, Gray (1914) refers to Ricardo several times in his article "Rent Under the Assumption of Exhaustibility", including his views on the extraction of finite resources. By looking at a single individual mine, Gray arrives to the same general conclusion as Hotelling (1931) in his basic model of raw material supply: in the case of no extraction costs, the price of a finite resource grows in line with the rate of interest (Krautkraemer 1998, p. 2066). In contrast, Pindyck (1978) concludes from his theoretical analysis that raw materials prices follow a U-shaped curve. He surmises that an initial price drop is the result of initial discovery of large quantity of raw materials, but that later price trends change direction as these materials are extracted and exploration efforts are both undertaken and successful. In summary, it is clear on the basis of theoretical research alone, that only partial assumptions can be made. Furthermore, at first glance it does not appear possible to draw firm conclusions about actual price movements for finite resources.

Published empirical studies examining resource price movements are often fundamentally contradictory in their conclusions. For example, an examination of the prices for finite resources during the period of 1940 to 1976 suggests that raw material prices do increase, but that the amount by which they should increase differs greatly, depending upon which modeling approach is taken (Berck and Roberts 1996). However, raw materials prices can also change on a largely random basis (from 1922 to 1974) and are described by McRae (1978) as a "random walk". In contrast, Smith (1979) suggests that, in the period examined from 1900 to 1973, price trends for natural resources were predominantly negative, while Slade (1982) in her much-referenced study concludes that the price of many raw materials during the period 1870 to 1978 initially went down only to go up again, and that overall prices follow a U-shaped curve. Lee et al. (2006) determine from a study of finite resource prices from 1870 to 1990 that prices for finite resources are "stationary around deterministic trends with structural breaks".

Moreover, a number of meta-analyses examining the development of commodity prices and their drivers are based on individual studies of the Hotelling model. By comparing four different tests, Chermak and Patrick (2002) determine that two of these tests confirm the validity of Hotelling's theory, while the other two come to the opposite conclusion. In an extensive

meta-analysis of empirical studies based on Hotelling, Slade and Thille (2009) conclude that it is not possible to confirm the general validity of the basic Hotelling model. Krautkraemer (1998) starts his qualitative review article with an overview of the basic model of Hotelling and gradually expands on the concept by introducing the drivers 'exploration', 'capital investment and capacity constraints', 'ore quality' and 'market imperfections'. He goes on to explain—largely non-formally—the effects of these drivers on price development of a finite resource; however, clear statements about the effects of the individual drivers are not made.

Such qualitative reviews serve as a basis for further investigation of commodity price developments. Nevertheless, existing qualitative reviews mostly stem from studies testing the basic Hotelling model, or extend them only in terms of single drivers (Chermak and Patrick 2002; Slade and Thille 2009). Probably due to this limitation, these studies usually come to the conclusion that the validity of the (basic) model cannot be confirmed. Although Krautkraemer (1998) presents well derived starting points for an extension of the basic Hotelling model, his proposals are of limited use, since their implications are derived in mainly qualitative manner — despite the fact that the article was published quite some time ago.

Overall, it can be seen from this short overview of relevant literature that it is not possible to draw a generally valid conclusion on price trends for finite resources from existing literature alone. In order to find a more valid and reliable answer to the question "What drives resource prices?" we therefore need to take a more comprehensive, theoretical approach. Therefore as part of this study we will analyze results of established and current studies, while taking into consideration current developments in the raw materials markets, in order to identify points for which it is possible to find valid answers and those points for which answers cannot be found, or only to a limited extent.

Motivated by weak results provided by existing qualitative reviews of Hotelling's basic model and its already existing extensions, we expand Hotelling's model with a simultaneous approach, including additional drivers such as backstop technology and recycling. In addition, drivers integrating changes to the demand side are incorporated into the model. Last but not least, we introduce an approach to distinguish between short- and the long-term effects of different drivers on the evolution of commodity prices.

To identify relevant literature, the following databases were used: "ABI/Inform Complete" (ProQuest), "Business Source Premier" (EBSCO), "Econlit (EBSCO)" and "Google Scholar". On one hand, we used the keywords (determinants of) resource prices, resource price development, commodity/resource price drivers, finite resources, and Hotelling's model/rule, both individually and in combination. On the other hand, we also used other articles which were automatically

suggested by the databases after an appropriate article had been identified ("recommended articles"). Based on the references of these articles, we could manually identify other papers related to our research. Depending on specific content, articles were either used in the introduction where the scientific state of the art is compiled, or were added to the middle part of the paper, where the theory-based hypotheses are tested. As far as possible, we used articles which directly refer to Hotelling's model, in order to test the theory-based hypotheses. Due to the variety of articles directly or indirectly relating to raw material price drivers and the development of commodity prices, it was necessary to perform a reasonable selection to obtain an appropriate number of relevant articles.

The structure of this paper is as follows: Firstly, the principles of the Hotelling model are presented, followed by approaches designed to expand the model into a more comprehensive overall model. Deductions are made from the Hotelling model, which are then further expanded by cet. par. consideration of the determinants extraction costs, technical progress, degradation, market power and risk. How these drivers can be simultaneously incorporated into the model is demonstrated, and the necessity of incorporating growing demand into the Hotelling model is discussed, along with the attendant difficulties and the need for additional research.

The basic Hotelling model – a theoretical approach to the price development of finite resources

As the model proposed by Hotelling (1931) presents the theoretical basis for this paper's more detailed examination of drivers of resource prices, we should first look at his underlying assumptions. Hotelling assumes a fixed start point in time t=0 and a quantity based on the known stock of raw materials \overline{y} , at which point raw materials suppliers, who each have an identical cost structure, maximize their net present value $G(y_t)$ by selling raw materials. Assuming a polypolistic market environment, an end point in time T for the extraction of the stock, complete information, reliability of decision-making, a lack of state interference, and the absence of extraction costs, then discrete modeling gives the following profit function, in which p(t) stands for the price of the resource at point in time t, y(t) for the quantity sold in t, and γ for the constant discount rate (cp. Levhari and Liviatan 1977; Devarajan and Fisher 1981)

$$G(y_t) = \sum_{t=0}^{T} \frac{p_t y_t}{(1+\gamma)^t}.$$
 (I)

In order to solve the equation – with the constraint of the finiteness of the resource –Lagrange's approach can be

applied. Therefore, the Lagrange function can be written as follows

$$L(y,t) = \sum_{t=0}^{T} \frac{p_t y_t}{(1+\gamma)^t} + \lambda \quad \overline{y} - \sum_{t=0}^{T} y_t$$
(II)

Subsequently, the corresponding system of equations and the constraint can be prepared. By solving this set of functions and transforming the resulting equation, the price path of a finite resource can be derived as

$$p_t = p_0 (1 + \gamma)^t \tag{III}$$

Hotelling's rule in its original form states that the price p of a finite resource must rise with the rate of interest γ and is synonymous with the statement that the prices of finite resources rise exponentially (see Appendix 1 for a detailed derivation of the price path of the basic Hotelling model). Based on the Hotelling rule, the long-term price path of a finite resource is graphically illustrated in Fig. 1:

Raw materials suppliers who wish to maximize their profits are therefore not concerned whether they sell their raw materials at the point in t_0 at the price p_0 , or at a later point in time t at the higher price p_t . We can therefore construct the following theory-based hypothesis from the basic Hotelling model:

Hypothesis 1 The later a raw material is extracted, the higher its price.

A study of empirical literature that examines development of raw materials prices and relates them to Hotelling's rule reveals no clear picture: in investigating the market for old growth timber (which can be considered a de facto finite resource due to its extremely slow growth) Livernois et al. (2006, p. 183) consider their results to be probably the clearest confirmation of the basic Hotelling model in literature to date. Nevertheless, they do show difficulties in the application of

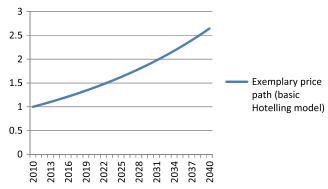


Fig. 1 Exemplary price path in accordance with the basic Hotelling model

Hotelling's model, allowing Slade and Thille (2009) to conclude that "the Hotelling model is not rejected." In contrast, Farrow (1985), who also tested the Hotelling model, concludes that the results reject the hypothesis that the data is consistent with the theoretical model. In his empirical study of resource prices, Heal (1981) discovers it is not possible to confirm the hypothesis of rising resource prices, which underlies Hotelling's rule. Also Tilton (1999), while examining the price development of metals, identifies a continuing downward trend of prices. In contrast, Cuddington (1999), investigating long-term price trends of 26 raw materials between 1900 and 1983, finds declining price trends for only five raw materials. Five other commodities showed a rising price trend, while the remaining 16 commodity prices were trendless. In summary, empirical studies show a mixed picture, leading to the statement that the price of finite resources does not consistently rise in the long run. Thus, hypothesis 1 cannot be confirmed.

Extensions of Hotelling's basic model

As the Hotelling model in its original form simply provided "some basic implications for how the finite availability of a non-renewable resource affects the resource price" (Krautkraemer 1998, p. 2065), the following sections show how the restrictive assumptions of the basic Hotelling model described above can be relaxed (compare Fisher 1981 and Tilton 2002 for similar approaches). The resulting changes to the price path will be presented thusly. These will initially be discussed in a cet. par. examination and then using a combination of drivers. Economic interpretations will also be explored and illustrated using a numerical example, before further general theory-based hypotheses are put forward and tested.

Extraction costs

Extraction costs are not considered in the basic model. If we include these in the Hotelling formula with the designation c, we obtain the following adjusted price path (cp. Krautkraemer 1998)

$$p_t = p_0 (1 + \gamma)^t - c_0 (1 + \gamma)^t + c_t.$$
 (IV)

With a positive discount rate γ , $c_0(1+\gamma)^t \ge c_0$ always applies due to $(1+\gamma)^t > 0$, resulting in a constant downward price path due to positive extraction costs $c_0 > 0$. The resource price therefore rises cet. par. *more slowly* than in the original case without extraction costs (large arrow). Based on the fact that the raw materials price is relatively lower in the future than it is

in time T, the adjusted price path then has the following course, which deviates from the basic model (see Fig. 2).

The original model will now be extended to an analysis of the market side. The impact of this hypothetical consideration to the price path is an abrupt increase of the price at time t=0. This ad hoc effect stems from the fact that each resource provider, who sets a price based on its marginal cost in a polypolistic market structure, must increase the price of its mined resource by the level of resource extraction costs c (small arrow in Fig. 2).

If we follow the reasoning of Slade (1982), extraction costs can be broken down by technical progress on the one hand and degradation (i.e., the diminishing quality of ore over time; cp. Gaudet 2007) on the other. Both drivers and their differentiated effects on the price path are considered in more detail below.

Technical progress and degradation

Slade (1982) maintains that extraction costs are negatively dependent on technology-driven, exogenous improvements to extraction methods, and positively dependent on endogenous degradation, which leads us to an expansion of the basic Hotelling model. If these concepts are incorporated into the Hotelling formula, with technical progress represented by tp and degradation by dg, this results in the following induced deltas to the original cost trend

$$-\Delta c(tp)_t = tp_0(1+\gamma)^t - tp_t \tag{V}$$

and

$$-\Delta c(dg)_t = -dg_0(1+\gamma)^t + dg_t.$$
 (VI)

As there is no publicly-available valid quantification of these drivers, their impact on the price path is presented

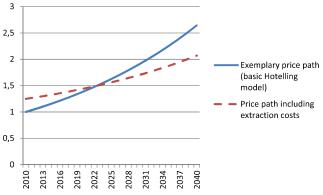


Fig. 2 Exemplary price path in accordance with the basic Hotelling model including extraction costs

simply as an example in Fig. 3: While the impact of degradation is therefore similar to that of extraction costs, as shown above, in that it pushes the price path, technical progress has the opposite effect (cp. large arrows).

Considering the impact of technological progress, a short-term effect may also occur, which moves the price path down at time t=0 (cp. small arrow in Fig. 3). This is the case when a novel mining method is available, which abruptly gives the opportunity to mine a resource in a cheaper way.

In contrast, degradation is a slow and plannable process. Thus, there is no short-term effect and no price jump is expected.

On the basis of the considerations presented here, we can construct the following hypotheses and relate them to empirical observations:

Hypothesis 2 Technical progress influences the price development of a finite resource.

If we test this hypothesis against the results found in empirical literature, we can see the following: Lasserre and Ouellette (1988) conclude from their examination of the effects of technical progress on the extraction of asbestos in Canada that this progress had a negative effect on the price path and led to a 76 % reduction in prices during the period observed from 1953 to 1982. In their study on the effect of technical progress in the years 1967 to 1990, Cuddington and Moss (2001, p. 1144) discover a significant effect on reducing exploration costs for natural gas and crude oil (though the significance of the effect was less marked for crude oil) and find, for example, that exploration costs for natural gas would have risen circa 22 % per annum, had they not been reduced by the effect of technical progress, to the actually observed figure of around 2.7 %. A further study on the effects of technical progress was carried out by Schmitz (2005), who measured the increase in "total factor productivity" (which includes labor productivity and capital productivity) in the iron ore industry in Canada and the USA and finds that during

the period 1981–1995 this factor went up by 51 %. These empirical findings lead to the result that hypothesis 2 can be confirmed.

As already depicted at the beginning of this section, we additionally aim to study the effects of degradation. To investigate this driver, the following hypothesis can be derived:

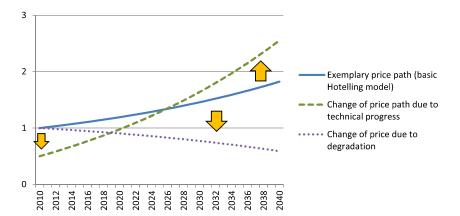
Hypothesis 3 Degradation negatively influences the price development of a finite resource.

In his theoretical model examining global nickel prices, Stollery (1983) argues that degradation (sometimes referred to as 'depletion' or 'reduction of ore quality' in literature) represents an important determinant of price and that the validity of the basic Hotelling model significantly improves by relaxing the assumption of a "fixed homogeneous reserve stock" (p. 163 f.). Using the example of asbestos extraction in Canada, Lasserre and Ouellette (1988) empirically examined both the effects of technical progress (as described in the last section) and the influence of degradation on the example of asbestos extraction in Canada. They come to the conclusion that in the period of 1953–1982, this resulted in a 63 % reduction in the productivity factor. Nevertheless, as they do not refer to the impact on the price, their study is of minor relevance for our purpose. Since suitable empirical studies are not available, it is difficult to test this hypothesis. Therefore, we cannot make a meaningful statement about its validity.

Combined examination of technical progress and degradation

Both literature on theoretical models and empirical studies agree that technical progress and degradation affect extraction costs in opposite directions. As no conclusion has been drawn as to which of the two drivers has the strongest impact on extraction costs, and therefore which over time produces a generally rising or falling price path, the following hypothesis has been set up and tested using empirical analysis:

Fig. 3 Example of effect of technical progress and degradation



Hypothesis 4 Extraction costs influence the price development of a finite resource.

In their study on the extraction costs of a range of different raw materials during the period of 1870-1957, Barnett and Morse (1963) observe a clear fall in extraction costs for minerals, with these overall costs falling steadily by more than 75 %. In contrast, Slade (1982) conducted an empirical study on raw material price development between 1870 and 1978 and came to the conclusion – as discussed in the introduction – that eleven of the twelve raw materials prices studied have a U-shaped curve. She identifies technical progress as the first predominant driver that initially leads to falling prices, but in time this is overcome by the reduction in ore quality and ensuing price increases. Miller and Upton (1985) apply an adaption of Hotelling's rule with regard to extraction cost. They study oil- and gas-producing companies in the USA and find evidence for the empirical validity of the theoretical model. According to the study by Lasserre and Ouellette (1988), with a figure of 76 %, technical progress has a stronger price-lowering effect than the price increases caused by degradation, which leads to an overall net price lowering of 13 %. In his study of copper prices, Buñuel (2002) comes to a more differentiated conclusion about the respective strengths of the two drivers, saying that whereas until the end of the 1930s technical progress had been the dominating driver and hence led to a drop in prices, this changed to a positive trend in the years 1938-1976, and then once again experienced a clear downturn in the years that followed.

Due to unavailability of extraction costs, several studies directly examine mine production and productivity. Ahumada and Cornejo (2014) focus on eight raw materials (including aluminum, copper, gold and petroleum) and find significant price dependencies. Using the example of oil, Chai et al. (2011) and Kaufmann et al. (2004) also detect this tendency. As a result, when considering the effects of extraction costs on the price of finite resources, we can assume hypothesis 4 to be confirmed.

Market power

Another driver that influences the price of finite resources is the structure of the supply-side market. Unlike in previous models based on perfect competition, in extreme cases a monopoly can arise, where raw materials are extracted by a single producer who can then set prices and control supply in order to maximize profits. In contrast to a polypoly situation, the holder of the monopoly can vary the quantities supplied and affect the price trend, which can be shown as

$$p_{(Monopoly)(t)} = p_0(y_0)(1+\gamma)^t + y_0 \frac{\partial p_0(y)}{\partial y_0} (1+\gamma)^t - y(t) \frac{\partial p(y,t)}{\partial y(t)}, \quad (VII)$$

where the price function p(y,t) must satisfy certain conditions (cf. Krautkraemer 1998). The effects of a monopoly are graphically shown in Fig. 4, where it is compared to the polypoly situation discussed in section 2.

Based on theoretical model considerations it is clear that in a monopoly situation the price path is cet. par. flatter than in a competitive situation and hence the extraction of resources occurs more slowly (cp. large arrow in Fig. 4). This can be explained by the fact that $\frac{\partial p_n}{\partial y_n}$ (i.e., the slope of a normal demand function) is always negative and $(1+\gamma)^t$ is always greater than one (due to a positive interest rate). Hence the absolute value of $y_0 \frac{\partial p_0}{\partial y_0} (1 + \gamma)^t$ is always greater than $y(t) \frac{\partial p(t)}{\partial v(t)}$ (Stiglitz 1976, p. 657). In this case, if one compares a polypolistic market structure and a market structure affected by market power, a short-term effect also occurs: An ad hoc increase in market power – for example, because of the merger of companies or caused by the formation of a cartel – leads to a sudden price increase. Its effect is shown by the little arrow in Fig. 4. If one examines mining of finite resource and its trading, oligopolistic market structures are often observed (e.g., OPEC in the oil market; Achzet et al. 2011). In this case it should be assumed that the correct price path lies in the range between the extremes of polypoly and monopoly.

If we integrate market power into the basic Hotelling model, then hypothesis 5 can be asserted and studied in relation to how it corresponds with empirical studies.

Hypothesis 5 Supplier-side market power influences the price development of a finite resource.

When reviewing the literature on the development of raw materials prices in relation to Hotelling's rule, it becomes clear

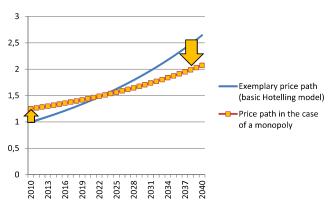


Fig. 4 Exemplary price path (basic Hotelling model) vs. monopoly situation

that firm empirical studies on the effects of monopolistic market power which establish a relationship with Hotelling's rule are few and far between (Devarajan and Fisher 1981, p. 68). This is due to the difficulty of finding a valid data set to separate the effects of market power on price paths. Nevertheless, theoretical studies provide plenty of evidence for the validity of hypothesis 5 (e.g., Lewis et al. 1979, p. 227, which refer to the Stiglitz and Solow models, among others).

Rather than being limited to a purely monopoly market structure, studies of oligopolies are now more focused. In this context, studies are related to oil price and investigate the influence of the OPEC cartel (and the related production concentration) on oil price. Bentzen (2007) therefore focusses on the time slot 1988–2004, Chevillon and Rifflart (2008) relate to the years 1988–2006, Gallo et al. (2010) covers the period 1982–2006. Independently from each other and the time period analyzed, they find a significant influence of the OPEC cartel on oil prices.

Thus, it can be deduced that market structure has an influence on the price of finite resources in the case of oligopolies. Although no direct statement can be derived in the case of a monopoly – caused by a lack of suitable empirical studies – it can be assumed that monopolies (as they constitute an even more extreme form of market power in comparison to oligopolies) also have an impact on the price of finite resources. Based on the presented studies and our additional remarks, we therefore argue that Hypothesis 5 can be confirmed.

Risk

In line with the assumptions presented in the introduction, it is henceforth assumed that raw materials suppliers have all the information they require and are in a position to make their extraction decisions under certainty. As this simplified view does not correspond to the uncertainties that plague decisionmaking situations in the real world, the following will show how the basic Hotelling model can be expanded to include risk drivers and how to approach implications of this expansion. In order to achieve this, the interest rate γ used by the basic model must be varied so that it contains a risk premium alongside the risk-free market rate of interest and thus present a risk-adjusted discount rate γ_{ra} (Copeland et al. 2005, p. 157) with the proviso that $\gamma_{ra} > \gamma$. In mining projects, raw material risk represents the main form of risk for the whole project, so only this aspect is considered in the following discussion. Applying an assumption, cet. par., results in the risk-adjusted price path

$$p_t = p_o (1 + \gamma_{ra})^t. (VIII)$$

Different risk drivers can be considered to determine the specific risk related to a finite resource. They can be identified

as imponderables relating to the producing countries and concentration, along with political risks and the future importance of this light metal. Based on this, we now use an exemplary risk-adjusted (= higher) interest rate γ_{ra} instead of the real interest rate used in the basic model. This has a clear effect on the predicted trends in raw materials prices based on the theoretical model.

Both the description and the chart show how even a small adjustment to the interest rate have a major effect on the price path. As the risk-adjusted discount rate is higher than the interest rate used in the basic model, the price path is steeper than in the original scenario. Unlike the above-discussed cases of mining costs, technological progress and market power (where short-term and long-term effects are distinguishable) and unlike the case of degradation (only characterized by a long-term effect), in case of risk both the previously shown long-term effects and an additional short-term effect change of the price path in the same direction. This ad hoc effect can be explained by the fact that increasing interest rates lead to an increase in mine operators' capital cost. As a result, all mines not having a positive net present value after the increase of interest rates drop out. Consequently, there is a price jump at the beginning of the period, shown by the small arrow in Fig. 5.

From this, it can be deduced that the resource is extracted more rapidly, which leads on to the following assumption:

Hypothesis 6 The higher the level of raw material risk, the steeper the price path will be.

Many empirical studies are based on the interest rate used in the Hotelling model. For example, Svedberg and Tilton (2006) used the example of copper (during the period 1870–2000) to show that the normally-applied real interest rate, such as one based on the US consumer price index or the US producer price index, overestimates the effect of actual inflation by 0.9 % to over 2.0 % p.a., and hence the interest rate used in the basic Hotelling model must also be adjusted. Livernois et al. (2006) tested Hotelling's rule using the example of tropical

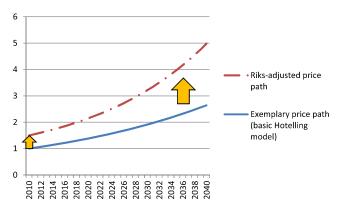


Fig. 5 Example of effect of risk-adjustment

timber, and come to the conclusion that theoretical and empirical observation are only congruent if the interest rate is adjusted to 8.6 %, whereby this figure can be construed as a scarcity rent and therefore not as a risk premium in the sense of hypothesis 6. Young and Ryan (1996) expanded the Hotelling model with regard to risk, and in their study of various metal raw materials establish positive risk premiums for copper and zinc. Although this study looks at the possibility of incorporating raw materials risk into the Hotelling model, no inferences were drawn about the validity of the hypothesis. And as theoretical literature also provides contradictory conclusions in relation to the effects of risk on the extraction of raw materials (Farzin 1984, p. 841; Gibert 1979, p. 47; Kumar 2002, p. 852; Pindyck 1980, p. 1203, p. 1217) based on papers that have been published to date, it is not possible to either prove or disprove it. This is partly due to the fact that this analysis of the interest rate does not distinguish between the proportion that is purely inflation-related, the risk-free interest rate, and an appropriate risk premium.

Backstop technology and recycling

So called backstop technologies bs as well as recycling re can be identified as additional drivers having an impact on the price development of finite resources. Since the following demonstrates that both drivers have similar effects on the basic Hotelling model, they are discussed together in this section.

A backstop technology can be defined as an ultimate technology resting on a very abundant resource base (Nordhaus et al. 1973, p. 532). In contrast to the basic Hotelling model, a price increase comes to an end as soon as the use of a backstop technology is technically feasible and economically viable (Heal 1976). This leads to an adjustment of the price path, now extended to the backstop technology *bs*

$$p_t = \min(p_0(1+\gamma)^t; p_{bs}). \tag{IX}$$

If it is possible to recycle the complete amount of a finite resource required for new products from unneeded old products, the costs resulting from this are cet. par. the maximum price limit. Thus, assuming that the entire demand for a finite resource can be met by recycling, this recycling driver also leads to a cap on the price path and thus is identical to a backstop technology in its long-term effect (see large arrow in Fig. 6).

Additionally, the short-term impact of a mandatory recycling, if it is introduced ad hoc, e.g., due to legal requirements in 2018, is also shown in Fig. 6. As the raw material reenter the production cycle in this case, a hypothetical drop in cost can be observed, as the market side is now additionally taken into account. The extent of the decline, however, is dependent on the amount of recycling. On the one hand, this

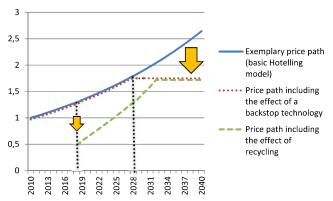


Fig. 6 Example of the effect of a recycling or backstop technology

effect (its mode of action is opposite to the above illustrated case, including mining costs) causes an immediate price drop (cp. small arrow). On the other hand, the price path hereafter rises more steeply than in the original case.¹

In contrast, if only a very small amount of the resource requirement can be met by recycling, this has no effect on the price path: In this case recycling (and the resulting amount of raw material recovered) can be modeled as a newly developed mine, and has no influence on the price (analogous to the case of atomistic market structure discussed above). However, if a larger part of the demand can be met, it is likely that this cet. par. weakens the increase of the price path. As a result, the price path – depending on the amount of raw material provided by recycling – now runs between the two extreme forms shown in Fig. 6. Analogous effects can be identified if a backstop technology does not have an *immediate* effect, but is rather characterized by a *flow* of innovations related to *gradual* price effects. Based on these considerations, the following two hypotheses can be derived.

Hypothesis 7 refers to the backstop technology and can be formulated so follow:

Hypothesis 7 If a backstop technology exists, the price of an affected finite resource does not exceed the price of the backstop technology.

Although the effect of a backstop technology on the price of finite resources has been studied in several articles focused on theoretical models (Heal 1976, p 373; Dasgupta and Stiglitz 1981, p. 88; Krautkraemer 1986), there are no empirical studies available on how backstop technologies have affected the Hotelling price path in the past. Nevertheless, some forward-looking studies describe the effects of a backstop

¹ In contrast, the following case would be possible as well: If, because of a legal requirement, mandatory recycling of a non-renewable resource is introduced and the cost of recycling is *higher* than the cost of mining the resource, this would cause a hypothetical price jump *upwards*.

technology. For example, Yaksic and Tilton (2009) state that the extraction of lithium from seawater can be seen as a backstop technology. Based on their study on price and quantity structure, they come to the conclusion that the price of lithium carbonate even in the worst scenario will never rise to about U.S. \$ 10 per pound in the long term, which – compared to the baseline in 2008 – corresponds to a maximum possible price increase of a factor of 10 (Yaksic and Tilton 2009, p. 190). Starting at this price level, the backstop technology (i.e., extraction from seawater) comes into use for unlimited quantities.

Also focused on a future model, Chakravorty et al. (1997) study backstop substitutes for finite resources and draw attention to the production of electricity. They conclude that conventional energy sources such as coal, oil and natural gas will be replaced within an observation period of about 20 years by photovoltaic technologies, since these new technologies can be seen as a backstop technology due to their decreasing cost over time (Chakravorty et al. 1997, p. 1218).

While both of these studies suggest that the price path is capped if a backstop technology is available and thus support hypothesis 7, the following observation suggests rejecting it: Since the year 1925 the so-called Fischer-Tropsch (FT) process has been available to produce oil from coal at an industrial scale (Dry 2002). Since the availability of coal—unlike oil—is secured for centuries, this process is a backstop technology limiting the maximum oil price to the expense of the FT process. The process has been already used profitably at a price of about \$ 40 (Bloomberg 2009). In contrast, oil price moves far beyond this price level and is also characterized by strong volatility (Narayan and Narayan 2007). This contradicts the theoretical model considerations to backstop technologies and does not support hypothesis 7.

On the basis of the existing empirical literature and the examples presented above, it is therefore not possible to confirm hypothesis 7, nor to reject it.

Regarding the effect of recycling, the following hypothesis can be formulated and examined:

Hypothesis 8 The higher the recycling rate of a finite resource, the flatter is its price path.

Beginning in the early 1970s, the trade-off between economic growth and environmental quality has been scientifically studied (d'Arge 1971; Skinner 1989). Against this background, first theoretical model approaches have been developed with the aim of reducing negative impacts of economic growth on the environment by recycling and to reduce the costs resulting from pollution (Weinstein and Zeckhauser 1974; Hoel 1978, p. 232; Huhtala 1999). In his empirical study of effects of recycling on development of commodities prices, Di Vita recognizes the common problem that "data on recycled waste prices are not easy to find" (Di Vita 2007, p. 148). He therefore merely refers to Berglund and Söderholm

(2003), who empirically examine the relation between economic growth and recycling using the example of waste paper, but cannot derive any conclusions about the effect of recycling on the resource price. Since the 1940s, the rate of paper recycling has constantly increased (Berglund and Söderholm 2003, p. 434) and has reached a worldwide share of 56 % (as of 2009). In some developed countries (such as Sweden and Germany) it is over 80 % (The Swedish Forest Industries Federation 2011, p. 50). In contrast, if we examine the development of the price, an enormous volatility can be observed on one hand (e.g., a sharp rise in cardboard prices by a factor of 6 between January 1994 and April 1995, which is even exceeded by the increase in prices for newspaper paper in the same period of time (Ackerman and Gallagher 2002, p. 278)). On the other hand, however, it is not possible to make a statement about the impact of rising recycling rates on the development of the price path. As a flattening of the price path is not evident, it is not possible to confirm hypothesis 8. In another comparative study, we will discuss the recycling of mass-produced metals. Since the mid-1970s the rate of metal recycling has been rising steadily (for example, in Germany from 33 % to a current value of 57 % (WVM 2010, p. 3). An investigation of metal prices, however, shows that a flattening of the price path is not recognizable. Instead, only high volatility can be observed, which is also independent of the increase in the recycling rate. Rather, the prices of the relevant metals have each increased (cp. e.g., London Metal Exchange 2012) and hypothesis 8 cannot be confirmed by the investigation of metals either. In contrast, Alonso (2010) comes to an opposite result. As part of her case study, she examines the market for platinum and cobalt and reaches the conclusion that recycling stabilizes the price in a market with rapidly-growing demand. Summarizing the empirical results of recycling: hypothesis 8 cannot be confirmed in every case. As a result, it must be assumed that the validity of hypothesis 8 should be discarded.

Simultaneous integration of supply-side drivers

In the previous paragraphs, Hotelling's basic model was expanded by a cet. par. consideration of each single determinant. Subsequently theory-based hypothesis were derived and considered in relation to existing empirical literature. Both the model of Hotelling and many of the empirical studies focus primarily on the question of how prices of finite resources develop over time. This perspective will shift in the following section to the question of what drives resources prices. In order to analyze the impact of each single driver on the price of a finite resource, it will be shown how different drivers can be integrated into an overall model. Only on this basis does the investigation of driver-specific effects appear reasonable. This approach thus provides a starting point for a multivariate linear analysis and a directly testable equation.

After relating the impact of the previously-analyzed supply-side drivers of extraction costs (i.e.,: technical progress, degradation, market power, risk, backstop technology, as well as recycling by means of cet. par. observations in relation to the basic Hotelling model), existing theoretical model approaches (Krautkraemer 1998; Fisher 1981) will now be integrated. We will formally show how the *simultaneous* appearance of several drivers that are observable in practice can be described. If extraction costs are assumed to exist in a monopolistic market structure which also includes the risk determinant, backstop technology and recycling, a Lagrangian function can generated. By solving the resulting system of equations analogous to the basic Hotelling model, the optimal price path can be derived (see Appendix 2 for the detailed derivation of the equation)

$$p(y,t) = \left(\left(\frac{\partial p_0(y_0)}{\partial (y_0)} y_0 (1 + \gamma_{ra})^t - \frac{\partial p(y,t)}{\partial y(t)} y(t) + p_0 (1 + \gamma_{ra})^t \right) - c_0 (1 + \gamma_{ra})^t + c(t) + \Delta c_{tp,t} - \Delta c_{dg,0} (1 + \gamma_{ra})^t + \Delta c_{dg,t} + \Delta c_{re,0} (1 + \gamma_{ra})^t - \Delta c_{re,t} \right) \cdot p_{bs,t}$$
(X)

If we compare these implications relating to the addition of a single driver with the combined calculations, it becomes clear that the drivers in each case are additively linked with each other. The converse argument is that this allows – at least theoretically – the possibility of separating the various drivers having an effect on the price path of a finite resource. In turn, this represents a necessary condition for compliance with the call made by Slade und Thille (2009, p. 33): "We must be able to separate the signal from the noise."

Change in Demand

However, changes in *demand* for finite resources are not acknowledged in the Hotelling approach, even though the recent huge increase in worldwide demand for finite resources, and in particular for new resources, has become ever more significant (Ahumada and Cornejo (2014). One reason for the supply side being largely ignored may be due to the fact that the raw materials inventory \overline{y} – and thereby the *quantities* of raw materials available on the market and demanded – are not afforded any significance in the Hotelling model when calculating the optimal price path (see above). As a result, the effects of demand are currently only rarely or not at all integrated into a closed model environment.

In order to have an integrated approach, it is therefore necessary to also consider these drivers more closely. Demand for raw materials is now driven by many new applications and will continue to grow strongly in the future. This is due to global population growth, increasing global prosperity, and to the correspondingly higher need for raw materials (Skinner 1989, p. 270; Pollard et al. 2010, p. 9). Technical progress has brought a range of new products to market which need not

only large amounts of additional raw materials for their manufacture, but also use many additional elements and compounds, due to their specific material properties (Krohns et al. 2011). This trend can be clearly seen in the semi-conductor industry, which in the 1980s used only 12 elements, increased to 16 in the 1990s, and has rocketed to over 60 elements just a decade later (Shadman 2006, p. 9).

In the following section we will briefly discuss some papers that emphasize the importance of the demand. In 1981, Berck presented an equilibrium model, based on seven assumptions and explicitly includes an increase in demand (assumptions 1 and 6). His work, however, refers only to renewable resources and also does not refer the Hotelling model. With the goal of predicting future price development of finite resources, Radetzki et al. (2008) develop a simple model, which includes the increase in demand. On this basis, they come to the realization that demand-driven price increases may turn into the opposite within a few years.

In an empirically-based study Eggert (1991) examines the reasons for price fluctuations of finite resources, and firstly identifies business cycles as a driver affecting demand for commodities. Secondly, he emphasizes consumers' inflexibility to vary their demand depending on the commodity price. In contrast, he identifies that the mining capacity of raw material suppliers is fixed in short and medium-term perspectives and thus affects prices. Also, Rosenau-Tornow et al. (2009) conduct an empirical study on commodity prices and include both the supply and the demand side. With respect to the demand side, they stress the importance of the following drivers: new technologies influencing growth in demand, substitution, GDP, industrial production, population or migration into cities, regulatory or other public policy changes (p. 136).

These approaches collectively underline the need for the study of supply and demand sides. However, they lack connection to a self-contained overall model including a formal relation between the supply side and the demand side. In contrast, Faucheux and Noël (1995) make a proposal on how the missing relation between development of the demand and associated change in the price path can be established. To this end, the authors use a four-quadrant model (cp. Fig. 7). Analogous to Hotelling's basic model, it is assumed that the demand for raw materials – as well as the remaining amount – is zero at the end point T. Again, \bar{y} stands for the available amount of raw material, p for price, and t for time.

First, the optimal price path of the Hotelling basic model is shown by the solid line in quadrant (I). In the second quadrant (II), the corresponding demand function is depicted. This is a normal demand function, as quantity demanded decreases with increasing prices of finite resources. Only a bisector is plotted in quadrant (III), which helps to establish a connection to quadrant (IV). The function in this quadrant illustrates the delivered quantity depending on *t*. If there is a cet. par. increase in demand, the demand curve in quadrant (II) shifts in

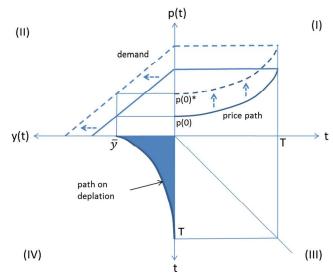


Fig. 7 Variation of the optimal price path of Hotelling's basic model due to an increase in demand (own figure based on Faucheux and Noël 1995, p. 155)

the direction of arrow to the left (dashed line). This in turn leads to a shift of the price path price in arrow direction upwards, which is shown with a dashed line in quadrant (I).

Based on these considerations, the following hypothesis can be formulated:

Hypothesis 9 The greater the demand, the higher the price path climbs.

On the basis of empirical literature examining the impact of changes in demand on the price of finite resources, conclusions about the validity of this hypothesis will now be derived. As part of his research on commodity prices since the Second World War, Radetzki (2006) analyses three phases (1950–51, 1973-74 and from 2004) in which prices of different resources (metals as well as energy, food and agricultural raw materials) have dramatically increased. He states that "demand shocks predominated as triggers to the commodity price rises" (p. 56). These demand shocks were each preceded by a strong growth in global GDP and a sharp rise in industrial production (p. 62). Similar results can be derived from the examination of copper, as there is a clear correlation between the increase in demand and its price increase: While global copper mining amounted to only 495,000 tons in 1900, the amount has been rising steadily to over 16 million tons in 2010 (i.e., an increase by a factor of 33). During the same period, the price rose - except for small, short-term fluctuations - steadily from \$ 357 / t to \$ 7,680 / t (i.e., an increase by a factor of 22) (USGS 2011). Studying copper prices in the years 1994–2003,

Cerda (2007) derives similar results and proves that increases in demand, particularly from Asia, lead to increases in raw material prices. In their empirical study, focusing on the years 1960-2010, Ahumada and Cornejo (2014) investigated the evolution of the price of eight commodities. They come to the conclusion that the demand-pull of China's economy has a positive impact on the development of commodities. Also Klotz et al. (2014), whose study refers to the period of 1998– 2012, conclude that the economic boom in China and the associated increased demand is a reason for the energy and industrial metal price increase. In contrast, Papież et al. (2014) examined the time period from 1997 to 2013. Referring only to the Euro area, they cannot find a relationship between the demand for raw materials and their prices. Given the fact that demand fluctuations within the Euro zone, especially compared to demand fluctuations of emerging countries, are relatively small, this study cannot be regarded as a clear indication that demand changes have no impact on prices of finite resources in general. Although the validity of hypothesis 9 thus cannot be confirmed for studies with only a regional focus, it may nevertheless be ascertained- on the basis of the above-discussed studies - that an increase in demand leads to price increases. As the demand side plays an increasingly important role for a comprehensive understanding of price changes of finite resources, it is important to model this driver into an expansion of the Hotelling model.

Summary of results

In paragraph 2 and 3, the basic Hotelling model and its expansions were discussed. Ergo, the following determinants were integrated into Hotelling's model: Extraction costs, technical progress, degradation, effect of risk, backstop technology, recycling, and changes in demand. Their impact on the price path of finite resources was additionally examined. Furthermore, theory-based hypothesis were derived and related to the results of empirical literature. The derived hypotheses and results of a qualitative review are summarized in Table 1.

Summarizing the results, four of our eight hypotheses can basically be considered proven. After testing the validity of the basic Hotelling model (with hypothesis 1), contradictory results were obtained. Our empirical findings lead to the result that hypothesis 2, i.e., the influence of technical progress, can be confirmed. Due to a lack of empirical studies, no indication of hypothesis 3 could be made. In contrast, the influence of both extraction cost and supplier-side market power can be taken as substantiated. No verifiable statement can made about the influence of raw materials risk, backstop

Table 1 Results of general theoretical hypotheses

No.	Hypothesis	Proven	Inconsistent	Not proven
Hypothesis 1	The later a raw material is extracted, the higher its price	1	2	3
Hypothesis 2	Technical progress influences the price development of a finite resource.	3	0	0
Hypothesis 3	Degradation negatively influences the price development of a finite resource.	-	-	-
Hypothesis 4	Extraction costs influence the price development of a finite resource.	7	0	0
Hypothesis 5	Supplier-side market power influences the price development of a finite resource.	3	0	0
Hypothesis 6	The higher the raw materials risk, the faster the resources are extracted.	0	3	0
Hypothesis 7	If a backstop technology exists, the price of an affected finite resource does not exceed the price of the backstop technology.	2	0	1
Hypothesis 8	The higher the recycling rate of a finite resource, the flatter is its price path.	1	0	2
Hypothesis 9	The greater the demand, the higher the price path climbs.	5	1	0

technologies or the influence of recycling on the price development of a finite resource. Hypothesis 9, which is the only one referring to the extension of Hotelling's model to the demand side, can basically be considered as confirmed.

As part of our qualitative review, we also established a relationship between existing empirical studies and an extended version of Hotelling's model. Doing so, we aimed to integrate both perspectives into a "big picture". By means of the proposed simultaneous approach, which implies the additivity of each driver, it is shown how the results of single empirical studies can be related to theoretical model considerations, when referring to the prices of finite resources.

Discussion

With the objective of allowing a comprehensive analysis of the drivers of raw materials prices, this paper first of all introduces the most common drivers having an effect on the price path, and suggests how to achieve simultaneous integration of these drivers in the basic Hotelling model.

The publication of comprehensive quality reviews related to Hotelling's model and its empirical tests date back already several years (Krautkraemer 1998; Tilton 2002). In the meantime new (especially demand-driven) trends affecting the price of raw materials can be observed and, in addition, new empirical studies are available. Against this background, it is appropriate that this qualitative review takes account of these developments.

Analyzing empirical studies, which form the basis of our qualitative review, the following points partially reduce the significance of single studies and limit their generality: Empirical analyses often concentrate only on a single finite resource or a specific commodity group. In this case, it remains unclear to what extent the derived results can also be applied to other finite resources. Sometimes only short periods of time are analyzed, raising the concern that the validity of these results is limited particularly to long-term considerations. Other studies only refer to single countries or regions. In such cases it cannot be ensured that these findings are valid worldwide. In addition, several empirical studies are limited due to univariate consideration of singular drivers. Dependencies on other variables are neglected in this case, resulting in a limited general validity. In view of the above, the assessment of these single empirical analyses has proven to be of little use (apart from studies examining the demand side). Until now, wideranging and integrated empirical studies not affected by these restrictions have been missing.

In this paper, different variables are examined. In addition, their impact on the price of finite resources is discussed. Here it is implicitly assumed that these variables act as drivers of the prices of finite resources. In principle, however, it would also be possible that a reverse interrelationship exists; namely, that at first a change in the price of a finite resource occurs, which then leads to an adjustment of the variables studied. Against this background, the problem of endogeneity is briefly addressed here. Due to the use of appropriate methods, such as vector autoregression (VAR) and the Granger causality test recent studies (e.g., Ahumada and Cornejo 2014; Chai et al. 2011; Klotz et al. 2014; Papież et al. 2014), one can distinguish between causes and effects. This is not the case in older studies, where it remains unclear whether a changing driver leads to the adaption of the price of a finite resource, or rather whether a changing price entails the putative driver's adaption. At present, the consideration of endogeneity is already widespread for monetary policy variables (in particular for interest rates) as well as in the context of global economic demand. In contrast, endogeneity problems only gradually find their way into empirical studies related to the finite resource drivers.

Conclusion and outlook

This paper initially addressed the question "What drives prices of finite resources?" from the point of view of a theoretical model. Then individual supply-side drivers like extraction costs, technical progress, degradation, market structure, risk, backstop technology and recycling were examined for their impact on the price of raw materials. In addition, we identified the need to distinguish between short- and long-term effects, and showed their different results depending on each specific determinant. Corresponding hypotheses were derived and tested by a

qualitative review based on empirical findings. Consequently, four of our eight hypotheses can basically be considered as proven: Our empirical findings lead to the result that the influence of technical progress on price development of a finite resource can be affirmed. In addition, the influence of both extraction cost and supplier-side market power can be taken as confirmed. By further expanding of the basic Hotelling model to the demand-side, we also showed that the hypothesis "price path is positively dependent on demand" to generally be supported.

Further research should focus on developing an integrated, theory-based approach that answers the question of what factors drive the price of finite resources, and brings together supply and demand drivers into one closed, economic model. Such a theoretically sound model should take a holistic approach, especially integrating the current fundamental changes in demand, which until now have been mainly ignored by studies related to Hotelling's theoretical model. Based on the examined literature it can be assumed that each driver has a different impact on the price development of each specific finite resource. Against this background, the model to be developed should be calibrated and tested separately for each finite resource.

To examine these effects in detail, we presented first an approach on how different drivers can be simultaneously integrated into an overall model. Based on this approach, a multivariate linear analysis seems to be a useful next step. In this context, additional empirical studies are required – particularly concerning the impact of the determinants backstop technology and recycling.

To extend the present qualitative review, further research needs can also be identified. Additional drivers like exchange rates, government interventions (taxes, subsidies, mining regulations and trade restrictions) and their impact on the price of finite resources could be integrated in further review articles. The effects of substitution possibilities between different raw materials and the role of income-sensitive demand on finite raw materials are additional aspects which could be integrated both in further empirical as well as qualitative review articles.

A better understanding of these drivers and the way they work together can help the quantification of raw materials price determinants and availability risks. Results of research in this area could also contribute to the implementation of appropriate governance strategies, especially those aimed at a fairer international and inter-generational distribution of finite resources.

Appendix 1

Derivation of the price path of a finite resource: Basic Hotelling model

$$G(y_t) = \sum_{t=0}^{T} \frac{p_t y_t}{(1+\gamma)^t}$$

In order to solve the equation – with the constraint of the finiteness of the resource – Lagrange's approach can be applied. Therefore, the Lagrange function

$$L(y,t) = \sum_{t=0}^{T} \frac{p_t y_t}{\left(1+\gamma\right)^t} + \lambda \quad \overline{y} - \sum_{t=0}^{T} y_t$$

and the corresponding system of equations

$$\begin{split} \frac{\partial L}{\partial y_0} &= p_0 - \lambda = 0 \\ \frac{\partial L}{\partial y_1} &= \frac{p_1}{1 + \gamma} - \lambda = 0 \\ \frac{\partial L}{\partial y_T} &= \frac{p_T}{\left(1 + \gamma\right)^T} - \lambda = 0 \end{split}$$

including the constraint

$$\overline{y} = \sum_{t=0}^{T} y_t$$

can be constructed. Solving this set of equations, it can be easily derived that maximum profit is reached if discounted prices of each period of time are equal:

$$p_0 = \frac{p_1}{1+\gamma} = \frac{p_2}{(1+\gamma)^2} = \dots = \frac{p_T}{(1+\gamma)^T} = \lambda$$

Transforming this equation, the price path of a finite resource can be derived as follows:

$$p_t = p_0 (1 + \gamma)^t$$

Appendix 2

Derivation of the price path of a finite resource: Simultaneous integration of supply-side drivers

If extraction costs are assumed to exist in a monopolistic market structure which also includes the risk determinant, backstop technology and recycling, the following Lagrangian function can be derived:

$$L(y,t) = \sum_{t=0}^{T} \frac{p(y,t)y(t)}{(1+\gamma_{ra})^{t}} - \sum_{t=0}^{T} \frac{c(t)y(t)}{(1+\gamma_{ra})^{t}} + \sum_{t=0}^{T} \frac{\Delta c_{tp}(t)y(t)}{(1+\gamma_{ra})^{t}} - \sum_{t=0}^{T} \frac{\Delta c_{dg}(t)y(t)}{(1+\gamma_{ra})^{t}} + \sum_{t=0}^{T} \frac{\Delta c_{re}(t)y(t)}{(1+\gamma_{ra})^{t}} - \lambda \ \overline{y} - \sum_{t=0}^{T} y(t)$$

The corresponding system of equations can be set as

$$\begin{split} \frac{\partial L}{\partial y_0} &= \frac{\partial p_0(y_0)}{\partial y_0} y_0 + p_0(y_0) - c_0 + \Delta c_{tp,0} - \Delta c_{dg,0} + \Delta c_{re,0} - \lambda = 0 \\ \frac{\partial L}{\partial y_1} &= \frac{\frac{\partial p_1(y_1)}{\partial y_1} y_1}{1 + r_{ra}} + \frac{p_1(y_1)}{1 + r_{ra}} \frac{c_1}{1 + r_{ra}} + \frac{\Delta c_{tp,1}}{1 + r_{ra}} \frac{\Delta c_{dg,1}}{1 + r_{ra}} + \frac{\Delta c_{re,1}}{1 + r_{ra}} - \lambda = 0 \\ &\vdots \\ \frac{\partial L}{\partial y_T} &= \frac{\frac{\partial p_T(y_T)}{\partial y_T} y_T}{(1 + r_{ra})^T} + \frac{p_T(y_T)}{(1 + r_{ra})^T} - \frac{c_T}{(1 + r_{ra})^T} + \frac{\Delta c_{tp,T}}{(1 + r_{ra})^T} - \frac{\Delta c_{dg,T}}{(1 + r_{ra})^T} + \frac{\Delta c_{re,T}}{(1 + r_{ra})^T} - \lambda = 0 \end{split}$$

including the constraint

$$\overline{y} = \sum_{t=0}^{T} y_t.$$

Solving this system of equations analogously to the abovementioned basic Hotelling model, the following optimal price path can be derived, representing the simultaneous appearance of extraction costs — including adjustments caused both by technical progress and degradation — as well as monopolistic market power, risk, backstop technology and recycling:

$$\begin{split} p(y,t) &= \min \Big(\Big(\frac{\partial p_0(y_0)}{\partial (y_0)} y_0 (1 + \gamma_{ra})^t - \frac{\partial p(y,t)}{\partial y(t)} y(t) \\ &+ p_0(y_0) (1 + \gamma_{ra})^t - c_0 (1 + \gamma_{ra})^t + c(t) \\ &+ \Delta c_{tp,0} (1 + \gamma_{ra})^t - \Delta c_{tp,t} - \Delta c_{dg,0} (1 + \gamma_{ra})^t \\ &+ \Delta c_{dg,t} + \Delta c_{re,0} (1 + \gamma_{ra})^t - \Delta c_{re,t} \Big) p_{bs,t} \Big) \end{split}$$

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