The Limits of Abstraction
By KIT FINE
Clarendon Press, 2002. x + 204 pp. £18.99

Kit Fine's recent book is dedicated to developing a theory of abstraction along broadly Fregean lines. Its aim lies strictly within the province of the philosophy of mathematics: it is to provide a neo-Fregean foundation of arithmetic and analysis. In keeping with the Fregean orientation, set theory is not assumed by Fine as the general theoretical framework. Fine, in one place (p. 35), points out that it has not been his aim "to be faithful to Frege's thought", that his interest has rather been "to see whether the ideas themselves can be sustained". It seems to me, however, that Fine's connection to Frege is rather tenuous (not to say nonexistent) even if we stick to the 'ideas themselves'. I will try to give some evidence for this below.

The book has four parts, three of which are more or less technical. The first part is (in the author's own words (p. ix)) "devoted to philosophical matters and serves to explain the motivation for the technical work and its significance". In that part, the author deals with three main questions: "What are the correct principles of abstraction? In what sense do they serve to define the abstracts with which they deal? To what extent can they provide a foundation for mathematics?" (p. ix)

It is impossible to give in the little space here allowed an adequate impression of the richness of Fine's ideas. I will stick to making a few remarks, which seem to me not entirely marginal.

According to the author (p. 3), "[a]ny abstraction principle can be stated in the [following] form:

the abstract of α = the abstract of β iff . . . α . . . β . . . , where the variables ‘α’ and ‘β’ range over the items (be they objects or concepts) on which the abstraction is to be performed, the phrase ‘the abstract of’ stands in for an abstraction operator, such as ‘the number of’ or the ‘direction of’, and the clause ‘. . . α . . . β . . .’ represents a criterion of identity, such as equinumerosity or coextensiveness”.

If this is understood as a specification of what it is to be *formally* an abstraction principle, then Frege’s famous Law V is *formally* an abstraction principle:

> the extension of F = the extension of G iff $\forall$x(Fx $\equiv$ Gx) (or in other words: . . . iff F and G are *coextensive*, ‘F’ and ‘G’ being variables for concepts).

And the following, equally famous Fregean principle—called ‘Hume’s Law’ by Fine—is also *formally* an abstraction principle:

> the number of F = the number of G iff F and G are *equinumerous* (‘F’ and ‘G’ being variables for concepts).

Both these *formal* abstraction principles (which aslo *seem* to be *genuine* ones) play an important role throughout Fine’s book. In contrast, it is understandable that he accords no attention to the following principle, which is also a *formal* abstraction principle according to Fine’s specification of what it is to be *formally* an abstraction principle:

> the mother of a = the mother of b iff $\forall$x(x is a mother of a $\equiv$ x is a mother of b) (or in other words: . . . iff a and b are *equimothered*, ‘a’ and ‘b’ being variables for human beings).

This principle is *certainly* true. But, just as certainly, it is *not* a *genuine* principle of abstraction.

It is, in fact, also rather doubtful whether Law V is a genuine (and not merely formal) principle of abstraction, leaving quite aside the question of how it may be restricted so as to make it true (it was *not* an abstraction principle for Frege; see below). In general, given an equivalence relation R (for example, coextensionality, equinumerosity, equimotheredness) the equivalence classes with respect to R can *adequately* serve as the R-abstracts and be rightfully called ‘abstracts’ (though an intuitive identification for them may be lacking). But *other* objects (*mothers*, for example, and apparently also *extensions*, which certainly seem to be, usually, other objects than classes of coextensional concepts) can *formally* serve as the R-abstracts (the equimotheredness-abstracts, the coextensionality-abstracts) if only they are (in whatever manner) one-to-one correlated with the equivalence classes with respect to R. Those other objects, however, are not rightfully called ‘abstracts’, and a principle that displays *them* as the R-abstracts is not a *genuine* principle of abstraction.

Fine makes no effort to distinguish genuine from non-genuine (merely formal) abstraction principles; according to him (p. 5), “it would be very difficult to say what makes a principle a genuine principle of abstraction”. Frege, it seems to me, had something to offer with regard to this question. Fine, however, dismisses the classical theory of abstraction—according to which (genuine) abstracts are equivalence classes with respect to an equivalence relation—as non-Fregean, because it gives no adequate account of conceptual abstraction (see pp. 2 and 3). The reasons for this alleged failure are, according to Fine, that "the members of the [equivalence] classes are objects, not concepts" and that "the classes in question will not arise from a principle of abstraction of the sort envisaged by Frege" (p. 3). As if there were no equivalence classes of concepts! Frege certainly recognised such classes (howsoever they arose for him) and would not have found it particularly objectionable if his position had been described to him as the view that natural numbers are certain equivalence classes (i. e., equinumerosity classes) of concepts—although of course he ultimately decided not to speak of classes but of *value-courses* ("Wertverläufe"). But, as is well known, the ontological difference between classes and the value-courses of monadic (Fregean) functions whose values are truth-values *is negligible*. Except for an inessential difference (the difference between an equivalence class and the corresponding value-course), the classical theory of abstraction is just the Fregean one. In accordance with this assertion, the value-courses of monadic concepts—isomorphically: the classes—are, for Frege, *not* abstractions from concepts, but the ontological prerequisite of all abstractions from concepts or objects. Hence Law V—contrary to Fine (p. 2)—cannot be regarded as a (genuine, not merely formal) principle of conceptual abstraction, at least not from a Fregean point of view.

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