Mini-workshop: entropy, information and control: introduction by the organisers

Fritz Colonius, Tomasz Downarowicz, Christoph Kawan, Girish Nair

Angaben zur Veröffentlichung / Publication details:

Colonius, Fritz, Tomasz Downarowicz, Christoph Kawan, and Girish Nair. 2018. "Mini-workshop: entropy, information and control: introduction by the organisers." *Oberwolfach Reports* 15 (1): 531–33. https://doi.org/10.4171/owr/2018/10.

Mathematisches Forschungsinstitut Oberwolfach

Report No. 10/2018

DOI: 10.4171/OWR/2018/10

Mini-Workshop: Entropy, Information and Control

Organised by
Fritz Colonius, Augsburg
Tomasz Downarowicz, Wroclaw
Christoph Kawan, Passau
Girish Nair, Melbourne

4 March – 10 March 2018

Abstract. This mini-workshop was motivated by the emerging field of networked control, which combines concepts from the disciplines of control theory, information theory and dynamical systems. Many current approaches to networked control simplify one or more of these three aspects, for instance by assuming no dynamical disturbances, or noiseless communication channels, or linear dynamics. The aim of this meeting was to approach a common understanding of the relevant results and techniques from each discipline in order to study the major, multi-disciplinary problems in networked control.

Mathematics Subject Classification (2010): 93B52, 93C41, 37A35, 94A17.

Introduction by the Organisers

Sixteen researchers from Europe, North and South America and the Asia-Pacific area participated in this mini-workshop, including the organisers Fritz Colonius, Tomasz Downarowicz, Christoph Kawan, and Girish Nair. In total, twenty-two talks were featured over 5 days. Due to the varying backgrounds of the participants, these included several tutorial-style and overview talks on relevant material from control theory (Colonius), dynamical systems entropy (Downarowicz), information theory (Sahai), dimension and entropy (Gelfert), data rate theorems (Franceschetti), invariance entropy (Kawan) and nonstochastic information (Nair). These talks provided important background to the presentations of new and recent research results, and exposed participants from each discipline to the main techniques and terminology of the others. In addition, there was an evening "Open

Problems" session, a closing discussion, and an impromptu joint session with the mini-workshop on "Deep Learning and Inverse Problems".

In broad terms, the research presentations and Open Problems followed two themes. The first theme focused on entropy-related concepts for controlled or uncontrolled dynamical systems without channel or system noise. Liberzon discussed the notion of *estimation entropy* for such uncontrolled systems, and showed that it characterises the minimum bit rate needed to be able to estimate the system states with a specified exponential speed. In a similar setting, Pogromsky proposed the concept of *restoration entropy*, and proved that this characterised the minimum errorless bit rates needed to estimate system states in *regular* and *fine* senses. Gelfert discussed the box dimension of skew-product systems with partially hyperbolic and hyperbolic structure, and showed that it can be determined in terms of *pressure*, a notion closely related to entropy and inspired by thermodynamics. Santana proposed extensions of the pressure concept to control systems, via the notions of *invariance pressure* and *outer invariance pressure*. Serafin discussed a notion of *universal* topological systems, and showed that the class of dynamical systems with measure-theoretic entropies lying in any given nonnegative interval always admits a universal topological system. Downarowicz described the notion of *symbolic extension entropy*, which characterises the minimum bit rate needed to estimate the initial state of a topological dynamical system with error approaching zero with time, and showed how it is related to the *entropy structure* of the topological system. Colonius proposed an extension of invariance entropy to measure-theoretic systems governed by quasi-stationary measures, and showed that this measure-theoretic invariance entropy is invariant under measurable transformations and determined by the control sets of the system. The question of whether there is a variational principle linking this new concept to invariance entropy remains open.

The second theme focused on the information-theoretic aspects of systems with noise or uncertainty. Yüksel spoke about the control of nonlinear stochastic systems via noisy channels, under stability notions such as asymptotic mean stationarity, ergodicity and Harris recurrence, and gave characterizations of the largest class of channels for which there exist coding and control policies so that the closed-loop system can be made stochastically stable. In a similar setting, Kawan discussed the relationship between the channel capacity needed to achieve a given objective, and a measure-theoretic entropy-like quantity defined by representing the control loop as a random dynamical system. Franceschetti considered the problem of asymptotically stabilizing a linear system via an errorless channel with independent random delays. He showed that it is necessary for the entropy of the linear system to be no greater than the ordinary Shannon capacity of the delay viewed as a timing channel, a condition which is also sufficient when the delays are exponentially distributed. A related problem was discussed by Linsenmayer, who considered the containability of linear systems controlled via a digital channel with uncertain delay, and derived sufficient conditions in terms of the delay bound, the number of bits per packet, and the system dynamical parameters. Ranade considered the stabilization in probability of linear systems with multiplicative random noise, and showed that the stabilizability condition could be viewed in terms of a new concept called *control capacity*. She also discussed the open problem of determining tight conditions for the stabilizability and optimal nonlinear control of scalar linear plants with multiplicative measurement noise. Kostina spoke about the control of noisy linear systems via noisy channels and presented fundamental trade-offs between the Marko-Massey's directed information, equivalent to the expected bit-rate, sent across a noisy channel, and control performance as measured by the mean square state. Ishii considered a similar class of systems and showed that if the additional requirement of asymptotic second-order (i.e. wide-sense) stationarity was also imposed, then fundamental trade-offs between directed information and the disturbance rejection performance could be derived in frequency domain in terms of generalised Bode integrals. Sahai described the classical problem of the optimal decentralized control of Gaussian linear systems under a quadratic performance cost, and discussed connections to rate-distortion theory, dirty paper coding and sphere-packing bounds in information theory.

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of a junior researcher in the workshop by the grant DMS-1641185, "US Junior Oberwolfach Fellows".