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Abstract

The industrial organization approach to banking is applied to analyze the effects of the introduction of joint credit and interbank rate risk on the optimal decisions on deposits and loans of a competitive bank. It is found that due to the introduction of both sources of risk there appear direct effects as well as portfolio effects which jointly determine changes in the bank's behavior. Moreover, it is shown that there is an interaction between the effects of the introduction of risk and economies or diseconomies of scope in the bank's business which determines the extend of behavioral changes.

Keywords: bank, risk, risk aversion, decisions with multiple sources of risk.

JEL classification: D21, D81, G21

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The industrial organization approach to banking is applied to analyze the effects of the introduction of joint credit and interbank rate risk on the optimal decisions on deposits and loans of a competitive bank. It is found that due to the introduction of both sources of risk there appear direct effects as well as portfolio effects which jointly determine changes in the bank's behavior. Moreover, it is shown that there is an interaction between the effects of the introduction of risk and economies or diseconomies of scope in the bank's business which determines the extend of behavioral changes.

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1 Introduction

It is generally accepted that risk is a fact of life and hence firms have to account for risk when making decisions on output levels or prices. This is particulary true for the banking industry. Since banks are traditionally in the business of issuing loans and taking deposits they are – in contrast to "normal" firms – exposed to a number of very special risks. In this regard credit risk – i.e. the risk of borrower default – and the risk of changes of market prices for certain financial products and services maybe considered as the most important ones.[1] Moreover, it is acknowledged that by issuing loans and taking deposits banks perform a number of very important functions in the economy – e.g. allocation of funds and provision of liquidity to name just a few.[2] However, if the existence of risks induces banks to change decisions on

[1]See e.g. Jorion (1997) and Nelson (1989) for a more comprehensive discussion of risk in the banking industry.

[2]See Hellwig (2000) for a detailed discussion of several functions.

deposits and loans compared to the riskless case one could expect risks to influence the performance of the functions stated above as well. It is thus very important to know in which way risks affect banks' optimal decisions on deposits and loans and hence banks' assets and liabilities management. And this is the subject to be analyzed in the present paper.

Of course, there have been a number of papers which are also concerned with this matter. For example Wahl and Broll (2000) and Wong (1997) show that the introduction of risk changes bank behavior fundamentally. Both papers apply the so-called industrial organization approach to banking for their analysis.[3] In the model of Wahl and Broll (2000) the banking firm participates in an interbank market in which any amount of funds can be lent or borrowed in addition to standard lending and deposit taking. Risk in this setting appears in the way that the interbank market interest rate is uncertain when the bank makes decisions. The main result of Wahl and Broll is that due to the risky interbank rate the optimal level of loans increases and the optimal level of deposits decreases as long as the bank is a net lender in the interbank market. However, as stated above, in reality banks are exposed to multiple sources of risk which may occur at the same time and influence each other. Therefore, the model of Wahl and Broll (2000) needs to be generalized in order to draw inferences in the more realistic situations.

The work of Wong (1997) extends the setting of Wahl and Broll (2000) in considering two sources of risk which need not be statistically independent. In particular Wong assumes that there exists credit risk as well as uncertainty regarding the deposit rate. However, in Wong's model the bank does not choose deposits and loans simultaneously. The level of deposits is given exogenous and thus the bank only determines the profit maximizing amount of loans which is shown to decrease due to the introduction of risk. Thus, although there are multiple sources of risk in Wong's model, the assumption of an exogenous amount of deposits seems not very realistic. Moreover, the effects of statistical dependence of risks are not explicitly figured out.

With the animadversion on previous papers at hand, the objective of this paper is to analyze the impact of multiple sources of risk on banks' behavior regarding optimal levels of deposits and loans. For this purpose the industrial organization approach to banking is applied. The model to be presented considers a risk averse competitive bank which is exposed to two sources of risk – namely credit and interbank rate risk. The bank simultaneously chooses the levels of deposits and loans so as to maximize profit. The main results, which are derived in this setting, can be summarized as follows. With multiple sources of risk there appear direct as well as portfolio effects in the bank's lending and deposit taking activities. While direct effects represent the impact of the introduction of risks on optimal decisions on deposits and loans which arise due to risk aversion, portfolio effects account for

[3] See (Freixas and Rochet, 1997, ch. 3) for a presentation of this approach.

the kind of statistical dependence of risks. In both cases the bank's net position in the interbank market is crucial for the direction of action. In addition portfolio effects depend on the kind of statistical dependence of both sources of risk. As a result, the interaction of direct and portfolio effects determines the changes in the bank's assets and liabilities management due to the introduction of joint credit and interbank rate risk as follows: On the one hand, when the bank is a net borrower in the interbank market and risks are positively correlated all effects will be negative. That is, in this constellation risks reenforce each other. When this is the case the optimal level of deposits increases and the profit maximizing level of loans decreases compared to the riskless case. On the other hand, I derive certain conditions when there appear positive portfolio effects in the sense that risks ameliorate each other. Unfortunately, in this situation it is not possible to make unambiguous predictions on changes in bank behavior compared to the riskless case.

Moreover, the analysis of the present paper does not assume that there are no economies or diseconomies of scope in simultaneously performing lending and deposit taking activities as is usual in the literature.[4] Therefore, the model of the present paper allows to determine the role of these forces in the context of the impact of risk on the bank's decision making process. It will be shown that economies and diseconomies of scope interact with both direct and portfolio effects. The general result in this regard is that economies of scope weaken the predicted adjustments in deposit and loan level whereas diseconomies of scope strengthen adjustments when they are strong enough.

The paper is related to the work of Kihlstrom et al. (1981) and Kimball (1993) since it applies and generalizes some of their concepts for decision making with multiple sources of risk. There is also a close relationship to the papers of Wong (1996) and Eeckhoudt and Kimball (1991) who analyze production decisions of a competitive firm and demand for insurance when there are multiple sources of risk, respectively. However, in the papers of Wong (1996) and Eeckhoudt and Kimball (1991) there are two sources of risk where one of them is an additive background risk. In both cases the actors are not able to influence the level of the background risk. In the present paper the bank is able to influence the exposure to both sources of risk. Therefore, the analysis in the present paper generalizes the ones of Wong (1996) and Eeckhoudt and Kimball (1991)

The remainder of the paper is organized as follows. Section 2 presents the base model without risk and thus serves as point of reference. In section 3 two sources of risk – credit risk and interbank rate risk – are introduced. Further assumptions which are necessary in order to perform the analysis are explained. Section 4 compares the bank's behavior in the situations of sections 2 and 3. For this purpose, in a first step the first order necessary conditions from sections 2 and 3 are compared (section

[4] Usually authors who apply the industrial organization approach to banking – including Wahl and Broll (2000) and Wong (1997) – assume that there are no economies or diseconomies of scope.

4.1). Thereafter, from the second order sufficient conditions the role of economies and diseconomies of scope is derived (section 4.2). At last the insights from 4.1 and 4.2 are used to compare optimal decisions with and without risk and results are interpreted (section 4.3). Section 5 concludes.

2 The Base Model without Risk

As explained in the introduction, I apply the industrial organization approach to banking (cf. Freixas and Rochet, 1997, ch. 3) in order to perform the analysis. In particular, consider a one period setting with a small banking firm which is active in the business of issuing loans ($L > 0$) and taking deposits ($D > 0$). Loans as well as deposits are assumed to be homogenous at a time. That is, I assume perfect competition in both, the market for deposits and the market for loans. Thus loan and deposit rates – r_L and r_D, respectively – can be considered as exogenously given.

Furthermore, there are costs associated with the bank's business which depend on the amount of deposits taken as well as the amount of loans issued – i.e. $C = C(D, L)$. The costs function is supposed to be twice continuously differentiable and increasing and convex in both, deposits and loans, i.e. $\partial C/\partial D = C_D(D, L) > 0$, $\partial C/\partial L = C_L(D, L) > 0$, $\partial^2 C/\partial D^2 = C_{DD}(D, L) > 0$, and $\partial^2 C/\partial L^2 = C_{LL}(D, L) > 0$. Note that there are no explicit assumptions with respect to $\partial^2 C/\partial D \partial L = C_{DL}(D, L)$. Thus, $C_{DL}(D, L)$ may be positive, negative, or zero which refers to the cases of diseconomies of scope, economies of scope, and no economies of scope, respectively.[5]

Moreover, the bank participates in an interbank market where it can lend or borrow any amount of funds at a certain interest rate r. That is, the interbank rate is given exogenously, too. To give a reason for this assumption, one could imagine the interest rate to be set by the central bank or to be the result of perfect competition among participants in the interbank market. As a result, the bank's balance sheet constraint can be written as

$$L + M = (1 - \alpha)D + K$$

where M represents the bank's net position in the interbank market, K denotes a certain level of equity, and α is the share of deposits which have to be held at no interest with the central bank, i.e. α is a reserve requirement. Note that M is not constrained and represents the excess ($M > 0$) or shortage ($M < 0$) in the bank's

[5] From the theoretical literature on the existence of banks there is strong evidence that economies of scope is a very important reason for the formation of banks. Therefore, $C_{DL}(D, L) < 0$ may be considered as the "natural" case. See for example Diamond (1984) and Krasa and Villamil (1992) for details.

liabilities over bank's assets. As a result, the bank is a net lender in the interbank market when $M > 0$ or a net borrower in the interbank market in case of $M < 0$. However, the most interesting point regarding M – and thus of the availability of an interbank market – is that it separates both sides of the bank's balance sheet. Hence the bank can realize any desired combination of amounts of deposits and loans without violating the balance sheet constraint.

With these assumptions the bank's profit can be written as

$$\Pi = (1-\theta)r_L L + rM - r_D D - C(D, L) \tag{1}$$

where θ is a certain share of non-performing loans at the end of the period. Thus, at the beginning of the period the bank solves the following maximization problem:

$$\max_{D,L} \Pi = ((1-\theta)r_L - r)\, L + ((1-\alpha)r - r_D) + rK - C(D, L). \tag{2}$$

In (2) the definition of the bank's profit (1) was rewritten using the balance sheet constraint. The corresponding first order necessary conditions (FONCs) are

$$(1-\alpha)r - r_D - C_D(D^*, L^*) \;=\; 0 \tag{3}$$
$$(1-\theta)r_L - r - C_L(D^*, L^*) \;=\; 0. \tag{4}$$

where D^* denotes the optimal level of deposits and L^* denotes the optimal amount of loans in the riskless case. Further, note that for D^* and L^* the FONCs (3) and (4) must hold simultaneously. This is true since both FONCs may be interdependent via the marginal costs – remember that $C_{DL}(D, L)$ need not be zero.

3 The Model with Multiple Sources of Risk

In the introduction it was pointed out that in reality banks are heavily exposed to several sources of risk. Therefore, in the present section the base model from section 2 is extended by introducing risk into the model. In particular, I consider two sources of risk which are credit risk, on the one hand, and interbank rate risk, on the other hand.

For modelling credit risk I follow Wong (1997): Let $\tilde{\theta} \in [0, 1]$ denote the share of non-performing loans at the end of the period. Since this share is not known to the bank at the beginning of the period – i.e. at the time when decisions on D and L are made – it must be treated as random variable.[6] Without loss of generality the share of non-performing loans can be represented as

$$\tilde{\theta} = \theta + \tilde{\epsilon}$$

[6]Throughout the paper all random variables will be marked by a " ˜ ".

where $\theta = \mathrm{E}(\tilde{\theta})$ and $\tilde{\epsilon}$ is a zero-mean noise term. (cf. Moschini and Lapan, 1995, p. 1029)

Similarly, the interbank rate risk may be considered as an ex ante uncertain interbank market interest rate – i.e. $\tilde{r}$. (cf. Wahl and Broll, 2000, p. 217). From the same arguments stated with credit risk $\tilde{r}$ can be rewritten as

$$\tilde{r} = r + \tilde{\mu}$$

where, again, $r = \mathrm{E}(\tilde{r})$ and $\tilde{\mu}$ is a zero-mean noise term. Hence, the bank's profit can be rewritten as

$$\Pi(\tilde{\epsilon}, \tilde{\mu}) = \Pi + \tilde{\mu}M - \tilde{\epsilon}r_L L \tag{5}$$

where Π is given by (1) in section 2.

While the bank does not know ex ante the particular realizations of $\tilde{\theta}$ and $\tilde{r}$ at the time when decisions on D and L are made, the probability distribution of both risks is known. Let $f(\tilde{\theta}, \tilde{r}) = f(\tilde{\epsilon}, \tilde{\mu})$ denote the joint probability density function of both sources of risk with $F(\tilde{\theta}, \tilde{r}) = F(\tilde{\epsilon}, \tilde{\mu})$ as the joint probability distribution function. Since, as explained in the introduction, $\tilde{\theta}$ and $\tilde{r}$ and hence $\tilde{\epsilon}$ and $\tilde{\mu}$ need not be statistically independent let $f(\tilde{\epsilon} \mid \tilde{\mu})$ $[f(\tilde{\mu} \mid \tilde{\theta})]$ and $F(\tilde{\epsilon} \mid \tilde{\mu})$ $[F(\tilde{\mu} \mid \tilde{\epsilon})]$ denote the conditional probability density functions and conditional probability distribution functions of $\tilde{\epsilon}$ $[\tilde{\mu}]$ given $\tilde{\mu}$ $[\tilde{\epsilon}]$, respectively. Furthermore, let $g(\tilde{\epsilon})$ $[h(\tilde{\mu})]$ and $G(\tilde{\epsilon})$ $[H(\tilde{\mu})]$ denote the marginal probability density and probability distribution functions of $\tilde{\epsilon}$ $[\tilde{\mu}]$, respectively.

For the purpose of explicitly considering the structure of the statistical dependence of $\tilde{\epsilon}$ and $\tilde{\mu}$ I apply the concept of regression dependence of Lehmann (1966) which he shows to be even more general than one would expect at first glance (p. 1144). Lehmann (1966, p. 1143f) states that two random variables X and Y are positively (negatively) regression dependent when $P(Y \leq y \mid X = x)$ is non-increasing (non-decreasing) in x. Furthermore he shows that positive (negative) regression dependence implies a positive (negative) covariance of both random variables. This concept of dependence is adopted by Wong (1996) to the case of continuous random variables. He states that X is positively (negatively) regression dependent on Y if, and only if, the conditional probability distribution function of X improves (deteriorates) in the sense of first-degree stochastic dominance as Y increases, i.e. $dF(X \mid Y)/dY = F_Y(X \mid Y) \leq (\geq) 0 \ \forall \ x$. If the former relation holds with equality, both random variables are independent. Adopting the arguments at hand, I can state

Definition 1 *$\tilde{\epsilon}$ is positively (negatively) regression dependent on $\tilde{\mu}$ if, and only if, the conditional probability distribution function of $\tilde{\epsilon}$ improves (deteriorates) in the sense of first-order stochastic dominance, i.e. $dF(\tilde{\epsilon} \mid \tilde{\mu})/d\tilde{\mu} = F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq (\geq) 0 \ \forall \ \tilde{\epsilon}$. $\tilde{\epsilon}$ and $\tilde{\mu}$ are said to be independent if the former relationship holds with equality.*

Of course, due to risk the bank does no longer maximize its profit. Rather it will consider an expected variable for optimization. However, in this regard the bank is assumed to use a von Neumann-Morgenstern utility function U which exhibits risk aversion – i.e. $U' > 0$ and $U'' < 0$ – to evaluate profit before taking expectations.[7] Moreover, I assume $U''' > 0$ which implies "prudence" in the behavior of the bank. The notion of prudence was introduced by Kimball (1990) who defines that "[...] 'prudence' is meant to suggest the propensity to prepare and forearm oneself in the face of uncertainty [...]"(p. 54). Furthermore, let $U'''' < 0$ which indicates decreasing absolute prudence in the sense of Kimball (1990). That is, the incentive for taking precautions in the face of risk decreases in the bank's wealth.

Therefore, the optimization problem of the bank in the case of multiple sources of risk can be written as

$$\max_{D,L} \mathrm{E}\left(U(\Pi(\tilde{\epsilon},\tilde{\mu}))\right) = \int_{\tilde{\mu}}\int_{\tilde{\epsilon}} U\left(\Pi(\tilde{\epsilon},\tilde{\mu})\right) f(\tilde{\epsilon},\tilde{\mu}) d\tilde{\epsilon} d\tilde{\mu} \tag{6}$$

where $\Pi(\tilde{\epsilon},\tilde{\mu})$ is given by (5).

From (6) one can derive the corresponding first order necessary conditions:

$$\int_{\tilde{\mu}}\int_{\tilde{\epsilon}} U'\left(\Pi^{**}(\tilde{\epsilon},\tilde{\mu})\right)\left((1-\alpha)r - r_D - C_D(D^{**},L^{**}) + (1-\alpha)\tilde{\mu}\right) f(\tilde{\epsilon},\tilde{\mu}) d\tilde{\epsilon} d\tilde{\mu} = 0 \tag{7}$$

$$\int_{\tilde{\mu}}\int_{\tilde{\epsilon}} U'\left(\Pi^{**}(\tilde{\epsilon},\tilde{\mu})\right)\left((1-\theta)r_L - r - C_L(D^{**},L^{**}) - \tilde{\mu} - \tilde{\epsilon} r_L\right) f(\tilde{\epsilon},\tilde{\mu}) d\tilde{\epsilon} d\tilde{\mu} = 0. \tag{8}$$

Note, in (7) and (8) D^{**}, L^{**}, and $\Pi^{**}(\cdot)$ denote the optimal levels of deposits, loans, and profit. Furthermore, just like in section 2 (7) and (8) must simultaneously hold for D^{**} and L^{**}. However, the interpretation of the FONCs in the risky case differs from the one of the situation without risk. While in the latter case the FONCs indicate that the optimal levels of deposits and loans ensure that marginal revenues equal marginal costs, in the former case the optimal values require the expected marginal utilities to be zero.

4 Comparison of Optimal Decisions

In order to compare optimal decisions on deposits and loans with and without risk – i.e. to compare D^* with D^{**} and L^* with L^{**} – the comparison of the FONCs in

[7] See for example Myers and Majluf (1984) and Froot et al. (1993) for rationals for risk aversion of firms in general and Froot and Stein (1998) and Pausch and Welzel (2002) for risk aversion of banks in particular.

the respective cases is necessary. For this purpose the FONCs of the risky situation (7) and (8) will be evaluated at D^* and L^*, i.e. at the place of the optimal amounts of deposits and loans of the riskless case in section 2. Thus, the FONCs of section 3 can be rewritten as

$$(1-\alpha)\int_{\tilde{\mu}}\int_{\tilde{\epsilon}} U'\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right)\tilde{\mu}dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu}) \tag{9}$$

$$-\int_{\tilde{\mu}}\int_{\tilde{\epsilon}} U'\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right)(\tilde{\mu}+\tilde{\epsilon}r_L)dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu}) \tag{10}$$

where I have used the fact that $f(\tilde{\epsilon},\tilde{\mu}) = f(\tilde{\epsilon}\mid\tilde{\mu})h(\tilde{\mu})$ and the first order necessary conditions from section 2.

To perform the comparison I proceed as follows: In a first step I derive relationships between the modified first order necessary conditions (9) and (10) and the corresponding ones from section 2 – i.e (3) and (4), respectively. Thereafter it will be shown that the second order sufficient conditions are essential for the desired comparison in addition to the relationships determined in step number one. At the end, both aspects will be taken together and results will be derived.

4.1 Comparing First Order Necessary Conditions

The idea of how to perform the comparison of the first order necessary conditions in the risky case of section 3 to the ones of the riskless case of section 2 was at first presented in Kraus (1979) and can be summarized as follows: In section 2 it was derived that D^* and L^* satisfy the first order conditions in this case (3) and (4) with equality. Therefore, if, on the one hand, D^* and L^* could be shown to satisfy the first order conditions (7) and (8) of section 3 as well, D^* and L^* would also be the optimal levels of deposits and loans in the risky case and hence no changes in the bank's behavior appear due to risk. If, on the other hand, D^* and L^* do not satisfy (7) and (8) one could draw conclusions on the changes in the optimal decisions of the bank from the knowledge whether (7) and (8) are larger or less than zero. Or, stated in other words, if (9) and (10) could be shown to be equal to, larger than, or less than zero, one could determine changes in the optimal decisions of the bank due to the introduction of risk.

Bearing this in mind and considering the deposit side of the bank's optimization problem at first, I state and prove the following

Lemma 1 *For equation (9) the following relation holds:*

$$(1-\alpha)\int_{\tilde{\mu}}\int_{\tilde{\epsilon}} U'\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right)\tilde{\mu}dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu}) \begin{array}{c} > \\ = \\ < \end{array} 0 \ \textit{iff} \begin{array}{l} M<0 \ \textit{and}\ F_{\tilde{\mu}}(\tilde{\epsilon}\mid\tilde{\mu})\leq 0 \\ M=0 \ \textit{and}\ F_{\tilde{\mu}}(\tilde{\epsilon}\mid\tilde{\mu})=0 \\ M>0 \ \textit{and}\ F_{\tilde{\mu}}(\tilde{\epsilon}\mid\tilde{\mu})\geq 0 \end{array} \cdot$$

In any other case the sign of (9) is ambiguous.

Proof: To prove lemma 1 I first rewrite (9) in the following way:

$$(1-\alpha)\int_{\tilde{\mu}} V'\left(\Pi^*(\tilde{\mu})\mid\tilde{\mu}\right)\tilde{\mu}dH(\tilde{\mu}). \tag{11}$$

For the derivation of (11) I have used a technique which was initially introduced in Kihlstrom et al. (1981) and successfully applied in Eeckhoudt and Kimball (1991) and Wong (1996). The idea is to determine a so-called derived utility function by "integrating out" (Eeckhoudt and Kimball, 1991, p. 244.) the second source of risk. That is, define a function $V(\Pi+\tilde{\mu}M\mid\tilde{\mu})$ such that

$$V\left(\Pi+\tilde{\mu}M\mid\tilde{\mu}\right)=\int_{\tilde{\epsilon}} U\left(\Pi+\tilde{\mu}M-\tilde{\epsilon}r_L L\right)dF(\tilde{\epsilon}\mid\tilde{\mu}). \tag{12}$$

The intuition of this procedure is straightforward: by integrating out the second source of risk – $\tilde{\epsilon}$ in the example at hand – one derives a utility function which already considers both, the effect of the second risk alone and the effect of the statistical interdependence of both sources of risk. If one further defines $\Pi+\tilde{\mu}M=\Pi(\tilde{\mu})$, one observes immediately:

$$\frac{dV(\Pi(\tilde{\mu})\mid\tilde{\mu})}{d\Pi(\tilde{\mu})}=V'(\Pi(\tilde{\mu})\mid\tilde{\mu})=\int_{\tilde{\epsilon}} U'\left(\Pi(\tilde{\mu})-\tilde{\epsilon}r_L L\right)dF(\tilde{\epsilon}\mid\tilde{\mu}) \tag{13}$$

which clarifies the manipulation necessary to transform (9) into (11).

Furthermore, I adopt techniques from Kraus (1979) and Wong (1996) to go on in the proof of lemma 1: From (13) one derives by integration by parts:

$$V'(\Pi^*(\tilde{\mu})\mid\tilde{\mu})=U'(\Pi^*(\tilde{\mu})-\bar{\epsilon}r_L L)+r_L L\int_{\tilde{\epsilon}} U''\left(\Pi^*(\tilde{\mu})-\tilde{\epsilon}r_L L\right)F(\tilde{\epsilon}\mid\tilde{\mu})d\tilde{\epsilon} \tag{14}$$

where $\bar{\epsilon}$ is the highest possible realization of $\tilde{\epsilon}$. Now, differentiation of this latter equation with respect to $\tilde{\mu}$ yields:

$$\frac{d}{d\tilde{\mu}}V'(\Pi^*(\tilde{\mu})\mid\tilde{\mu})=MV''\left(\Pi^*(\tilde{\mu})\mid\tilde{\mu}\right)+r_L L\int_{\tilde{\epsilon}} U''\left(\Pi^*(\tilde{\mu})-\tilde{\epsilon}r_L L\right)F_{\tilde{\mu}}(\tilde{\epsilon}\mid\tilde{\mu})d\tilde{\epsilon}.$$

From this equation one can observe that the following relations hold:

$$\frac{d}{d\tilde{\mu}}V'(\Pi^*(\tilde{\mu}) \mid \tilde{\mu}) \begin{array}{c} > \\ = \\ < \end{array} 0 \text{ iff } \begin{array}{l} M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0 \\ M = 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) = 0 \\ M > 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \geq 0 \end{array} \tag{15}$$

since $V'' < 0$ due to risk aversion and $\int_{\tilde{\epsilon}} U''\left(\Pi^*(\tilde{\mu}) - \tilde{\epsilon} r_L L\right) F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) d\tilde{\epsilon}$ has the opposite sign of $F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu})$ due to $U''(\cdot) < 0 \; \forall \; \tilde{\epsilon}$. Note, in any other case the sign of $\frac{d}{d\tilde{\mu}}V'(\Pi^*(\tilde{\mu}) \mid \tilde{\mu})$ is ambiguous. Therefore, one should bear in mind that one is not able to derive unambiguous results except for the stated situations.

For determining the sign of (11) consider $\mu = 0$ as a certain realization of $\tilde{\mu}$. That is, in the riskless case of section 2 the noise term, which causes the interbank rate to be risky, is equal to zero. Suppose now, on the one hand, μ to increase – i.e. $\mu > 0$. Then from (15) one can observe

$$V'(\Pi^*(\mu) \mid \mu) \begin{array}{c} > \\ = \\ < \end{array} V'(\Pi^*(0) \mid 0) \text{ iff } \begin{array}{l} M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0 \\ M = 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) = 0 \\ M > 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \geq 0 \end{array} .$$

Thus, multiplying $\mu > 0$ to both sides of the equation at hand yields

$$V'(\Pi^*(\mu) \mid \mu)\mu \begin{array}{c} > \\ = \\ < \end{array} V'(\Pi^*(0) \mid 0)\mu \; iff \; \begin{array}{l} M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0 \\ M = 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) = 0 \\ M > 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \geq 0 \end{array} .$$

If, on the other hand, μ decreases – i.e. $\mu < 0$ – from (15) it follows immediately that

$$V'(\Pi^*(\mu) \mid \mu) \begin{array}{c} < \\ = \\ > \end{array} V'(\Pi^*(0) \mid 0) \text{ iff } \begin{array}{l} M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0 \\ M = 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) = 0 \\ M > 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \geq 0 \end{array} .$$

Thus, multiplying $\mu < 0$ to both sides of the equation at hand yields

$$V'(\Pi^*(\mu) \mid \mu)\mu \begin{array}{c} > \\ = \\ < \end{array} V'(\Pi^*(0) \mid 0)\mu \text{ iff } \begin{array}{l} M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0 \\ M = 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) = 0 \\ M > 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \geq 0 \end{array} .$$

As a result

$$V'(\Pi^*(\mu) \mid \mu)\mu \begin{array}{c} > \\ = \\ < \end{array} V'(\Pi^*(0) \mid 0)\mu \text{ iff } \begin{array}{l} M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0 \\ M = 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) = 0 \\ M > 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \geq 0 \end{array} .$$

holds for any realization of $\tilde{\mu}$. Hence with taking expectations with respect to $\tilde{\mu}$ on both sides of the above relation one yields

$$\int_{\tilde{\mu}} V'\left(\Pi^*(\tilde{\mu}) \mid \tilde{\mu}\right) \tilde{\mu} dH(\tilde{\mu}) \begin{array}{c} > \\ = \\ < \end{array} 0 \text{ iff } \begin{array}{l} M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0 \\ M = 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) = 0 \\ M > 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \geq 0 \end{array} .$$

where the right hand side follows from the fact that $V'(\Pi^*(0) \mid 0)$ is constant for any realization of $\tilde{\mu}$ and from the assumption that $\tilde{\mu}$ has a zero mean. At the end, multiplying $(1-\alpha) > 0$ to both sides of the above relation completes the proof of lemma 1. □

From the proof of lemma 1 one can observe some interesting insights regarding the impact of risk on the bank's decisions: on the right hand side of (14) there appear two terms which jointly specify the consequences of the introduction of interbank rate risk in the presence of credit risk on the bank's optimal decision on deposits. The first term, $U'(\Pi^*(\tilde{\mu}) - \bar{\epsilon} r_L L)$, represents the direct effect on marginal utility in this regard. However, as explained in the previous section the potential statistical dependence between both sources of risk causes a portfolio effect to appear. This last effect is represented by the second term on the right hand side of (14). Therefore, in order to compare optimal decisions on deposits one has to take into account both effects.

Further calculations in the proof of lemma 1 showed that the sign of the direct effect depends on the bank's net position in the interbank market while the sign of the portfolio effect is determined by the statistical dependence of both sources of risk. Hence, unambiguous results about the change in marginal utility due to the introduction of risk are available if and only if both, direct and portfolio effects, reenforce each other, i.e., if and only if they have the same sign. Thus, (15) captures all the cases when unambiguous results regarding the change in marginal utility due to the introduction of risk can be derived.

After having analyzed the deposits side of the bank's optimization problem, the loans side needs to be considered. And indeed, just like in the case of deposits it is of special interest whether L^* satisfies the corresponding first order necessary condition (8) in section 3 or not. Or, stated in other words, what has to be known is the sign of (10). Thus, I can state and prove

Lemma 2 *For equation (10) the following relation holds:*

$$-\int_{\tilde{\mu}}\int_{\tilde{\epsilon}} U'\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right)(\tilde{\mu}+\tilde{\epsilon}r_L)dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu})<0 \text{ iff } M<0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon}\mid\tilde{\mu})\leq 0.$$

In any other case the sign of (10) is ambiguous.

Proof: For the proof of lemma 2 it seems advantageous to transform (10) at first. Standard calculations and application of the derived utility function from the proof of lemma 1 yields:

$$-\int_{\tilde{\mu}} V'\left(\Pi^*(\tilde{\mu})\mid\tilde{\mu}\right)\tilde{\mu}dH(\tilde{\mu})-r_L\int_{\tilde{\mu}}\int_{\tilde{\epsilon}} U'\left(\Pi^*+\tilde{\mu}M-\tilde{\epsilon}r_L L\right)\tilde{\epsilon}dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu}). \quad (16)$$

Note that the first term in (16) is exactly minus (11). Therefore, the proof of lemma 1 can be directly applied to arrive at

$$-\int_{\tilde{\mu}} V'\left(\Pi^*(\tilde{\mu})\mid\tilde{\mu}\right)\tilde{\mu}dH(\tilde{\mu}) \begin{array}{c} < \\ = \\ > \end{array} 0 \text{ iff } \begin{array}{l} M<0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon}\mid\tilde{\mu})\leq 0 \\ M=0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon}\mid\tilde{\mu})=0 \\ M>0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon}\mid\tilde{\mu})\geq 0 \end{array}. \quad (17)$$

For the second term of (16) the definition of the derived utility function and thus of the proof of lemma 1 is not adequate. However, interchanging random variables yiels

$$-\int_{\tilde{\epsilon}}\tilde{\epsilon}\int_{\tilde{\mu}} U'\left(\Pi^*+\tilde{\mu}M-\tilde{\epsilon}r_L L\right)dF(\tilde{\mu}\mid\tilde{\epsilon})dG(\tilde{\epsilon})$$

which allows the definition of a further derived utility function which is similar to the one of the proof of lemma 1: Let

$$\mathcal{V}(\Pi(\tilde{\epsilon})\mid\tilde{\epsilon})=\int_{\tilde{\mu}} U(\Pi(\tilde{\epsilon})+\tilde{\mu}M)dF(\tilde{\mu}\mid\tilde{\epsilon})$$

where $\Pi(\tilde{\epsilon})=\Pi-\tilde{\epsilon}r_L L$. Therefore, with calculations analogous to those presented in the proof of lemma 1 – i.e. integrating by parts the new derived utility function, differentiation with respect to $\Pi(\tilde{\epsilon})$, and differentiation of the result with respect to $\tilde{\epsilon}$ – yields

$$\frac{d}{d\tilde{\epsilon}}\mathcal{V}'(\Pi^*(\tilde{\epsilon})\mid\tilde{\epsilon})=-r_L L\mathcal{V}''(\Pi^*(\tilde{\epsilon})\mid\tilde{\epsilon})-M\int_{\tilde{\mu}} U''(\Pi^*(\tilde{\epsilon})+\tilde{\mu}M)F_{\tilde{\epsilon}}(\tilde{\mu}\mid\tilde{\epsilon})d\tilde{\mu}.$$

From similar arguments as in the proof of lemma 1 – i.e. $-r_L L<0$, $\mathcal{V}''(\Pi^*(\tilde{\epsilon})\mid\tilde{\epsilon})<0$ due to risk aversion, and $\int_{\tilde{\mu}} U''(\Pi^*(\tilde{\epsilon})+\tilde{\mu}M)F_{\tilde{\epsilon}}(\tilde{\mu}\mid\tilde{\epsilon})d\tilde{\mu}$ having the opposite sign of $F_{\tilde{\epsilon}}(\tilde{\mu}\mid\tilde{\epsilon})$ – one easily observes from the above equation:

$$\frac{d}{d\tilde{\epsilon}}\mathcal{V}'(\Pi^*(\tilde{\epsilon})\mid\tilde{\epsilon})>0 \quad \text{iff} \quad M<0 \text{ and } F_{\tilde{\epsilon}}(\tilde{\mu}\mid\tilde{\epsilon})\leq 0 \text{ or } M>0 \text{ and } F_{\tilde{\epsilon}}(\tilde{\mu}\mid\tilde{\epsilon})$$
$$\text{and ambiguous else.} \quad (18)$$

With this result at hand, it is immediately verified that

$$\mathcal{V}'(\Pi^*(\epsilon) \mid \epsilon)\epsilon > \mathcal{V}'(\Pi^*(0) \mid 0)\epsilon \;\forall\; \epsilon = \tilde{\epsilon} \quad \text{iff} \quad \begin{array}{l} M < 0 \text{ and } F_{\tilde{\epsilon}}(\tilde{\mu} \mid \tilde{\epsilon}) \leq 0 \\ \text{or } M > 0 \text{ and } F_{\tilde{\epsilon}}(\tilde{\mu} \mid \tilde{\epsilon}). \end{array}$$

Note, the argument applies $\epsilon = 0$ as a certain realization of $\tilde{\epsilon}$ as point of reference since, just like in the case of $\tilde{\mu}$, in the riskless situation of section 2 $\epsilon = 0$. Thus, taking expectations with respect to $\tilde{\epsilon}$ and multiplying $-r_L$ to both sides of the above relation yields

$$-r_L \int_{\tilde{\epsilon}} \mathcal{V}'(\Pi^*(\tilde{\epsilon}) \mid \tilde{\epsilon})\tilde{\epsilon} dG(\tilde{\epsilon}) < 0 \text{ iff } M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0 \text{ or } M > 0 \text{ and } F_{\tilde{\epsilon}}(\tilde{\mu} \mid \tilde{\epsilon}) \tag{19}$$

where the fact has been used that $\tilde{\epsilon}$ has a zero mean.

At the end, from (17) and (19) it can be observed that the sign of (16) is unambiguously negative if, and only if, $M < 0$ and $F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0$ and ambiguous else. □

In fact, the proof of lemma 2 shows that there appear direct as well as portfolio effect which jointly determine the impact of risk on the bank's optimal choice of the level of loans. However, from (16) one can observe that the situation is more complicated now. That is, the direct effect of the introduction of credit risk in the presence of interbank risk can be decomposed into two parts. This is true since the level of loans not only determines the bank's exposure to credit risk. Rather, via M the volume of loans also affects the bank's exposure to interbank rate risk. Therefore, the direct effect of the introduction of risk on the optimal level of loans has to consider both aspects.

The portfolio effect, however, has the same interpretation as in the case of deposits. And, again, M is a crucial determinant of the sign of both effects as was shown in the calculations of the proof of lemma 2. Thus, just like in the analysis of the deposit side of the bank's decision, unambiguous results can be derived if and only if all associated effects reenforce each other, i.e., if and only if all associated effects have the same sign.

4.2 Second Order Effects: The Role of Economies of Scope

To perform the comparison of the optimal levels of deposits and loans with and without risk it is necessary to have a closer look at the second order sufficient conditions. This is true because of two reasons: At first, only from the additional consideration of the second order sufficient conditions one can be sure to have found a maximum of the bank's optimization problem. Secondly, while the comparison of the first order necessary conditions of the previous subsection indicates whether the optimal amounts of deposits and loans from the setting without risk remain optimal in the presence of joint credit and interbank rate risk, the second order sufficient conditions are necessary to make predictions about the direction of (eventual) changes. Thus, in the present subsection I will consider the first reason only in short and concentrate on the second one.

In order to ensure that one has derived a maximum for the considered optimization problem, the following conditions have to hold simultaneously in the optimum (cf. Chiang, 1984, p. 317)

$$\frac{\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)}{\partial {D^*}^2} < 0 \tag{20}$$

$$\frac{\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)}{\partial {L^*}^2} < 0 \tag{21}$$

and

$$\frac{\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)}{\partial {D^*}^2}\frac{\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)}{\partial {L^*}^2} > \left(\frac{\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)}{\partial D^* \partial L^*}\right)^2. \tag{22}$$

For the riskless situation of section 2 the validity of (20) and (21) is easily verified since

$$\frac{\partial^2 \Pi}{\partial {D^*}^2} = -C_{DD}(D^*, L^*) < 0 \text{ and } \frac{\partial^2 \Pi}{\partial {L^*}^2} = -C_{LL}(D^*, L^*) < 0$$

due to $C_{DD}(D, L), C_{LL}(D, L) > 0$ by assumption. The third sufficiency condition (22) is, however, not verifiable. Rather, from (22) one can derive further restrictions to be imposed on the optimization problem in order to ensure a unique maximum. For the problem of section 2 it can be easily shown that this restriction has to be

$$C_{DD}(D^*, L^*)C_{LL}(D^*, L^*) > C_{DL}(D^*, L^*)^2.$$

This latter equation has a natural interpretation: In order to ensure a unique maximum the direct effects of a change in optimal values on first order necessary conditions – i.e. any economies or diseconomies of scale – have to be larger than the indirect effects – i.e. economies or diseconomies of scope – in absolute terms.

For the setting of section 3, where the bank is exposed to joint credit and interbank rate risk, similar conditions can be derived. The only difference to the ones of section 2 above is that they now have to hold in expectations. However, I do not explicitly give second order sufficient conditions for the optimization problem of section 3 for the optimal levels D^{**} and L^{**}.[8] Rather, I derive second order conditions evaluated at D^* and L^* since they have been the point of reference of the earlier considerations.

[8] In doing so, one can easily show that (20) and (21) hold because of the assumptions of section 3. As in the case of section 2, (22) is not verifiable explicitly. Rather, one can derive further restrictions for the uniqueness of the maximum. The interpretation of this restrictions is analogous to the one in the riskless case.

Therefore, differentiating (9) and (10) with respect to D^* and L^*, respectively, yields

$$\begin{aligned}\frac{\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)}{\partial {D^*}^2} &= (1-\alpha)^2 \int_{\tilde{\mu}}\int_{\tilde{\epsilon}} U''\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right)\tilde{\mu}^2 dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu}) - \\ &\quad - \int_{\tilde{\mu}}\int_{\tilde{\epsilon}} C_{DD}(D^*,L^*)U'\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right)dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu}) < 0 \qquad (23)\\ \frac{\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)}{\partial {L^*}^2} &= \int_{\tilde{\mu}}\int_{\tilde{\epsilon}} U''\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right)(\tilde{\mu}+\tilde{\epsilon}r_L)^2 dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu}) - \\ &\quad - \int_{\tilde{\mu}}\int_{\tilde{\epsilon}} C_{LL}(D^*,L^*)U'\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right)dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu}) < 0 \qquad (24)\end{aligned}$$

due to $(1-\alpha) > 0$, $U'(\cdot) > 0$, $U''(\cdot) < 0$, $C_{DD}(\cdot) > 0$, $C_{LL}(\cdot) > 0$, $\tilde{\mu}^2 > 0 \;\forall\; \tilde{\mu}$, and $(\tilde{\mu}+\tilde{\epsilon}r_L)^2 > 0 \;\forall\; \tilde{\mu},\tilde{\epsilon}$.

Furthermore, differentiating (9) or (10) with respect to L^* or D^*, respectively, yields

$$\begin{aligned}\frac{\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)}{\partial D^*\partial L^*} &= -(1-\alpha)\int_{\tilde{\mu}}\int_{\tilde{\epsilon}} U''\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right)\tilde{\mu}^2 dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu}) - \\ &\quad -(1-\alpha)\int_{\tilde{\mu}}\tilde{\mu}r_L\int_{\tilde{\epsilon}} U''\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right)\tilde{\epsilon}dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu}) - \\ &\quad - \int_{\tilde{\mu}}\int_{\tilde{\epsilon}} C_{DL}(D^*,L^*)U'\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right)dF(\tilde{\epsilon}\mid\tilde{\mu})dH(\tilde{\mu}). \qquad (25)\end{aligned}$$

From (25) I can now state and prove

Lemma 3 *For equation (25) the following relation holds:*

$$\frac{\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)}{\partial D^*\partial L^*} \geq 0 \textit{ iff } M < 0 \textit{ and } F_{\tilde{\mu}}(\tilde{\epsilon}\mid\tilde{\mu}) \leq 0 \textit{ and } C_{DL}(D^*,L^*) \leq 0.$$

In any other case the sign of (25) is ambiguous.

Proof: The first line of the right hand side of (25) is unambiguously non-negative due to $(1-\alpha) > 0$, $U''(\cdot) < 0$, and $\tilde{\mu}^2 > 0 \;\forall\; \tilde{\mu}$.

The sign of the third line of (25) depends on the sing of $C_{DL}(\cdot)$ and is positive (negative) iff $C_{DL}(\cdot) < (>) 0$ due to $U'(\cdot) > 0$. In case of $C_{DL}(\cdot) = 0$ the third line of (25) diminishes.

To complete the proof of lemma 3 the sign of the second line of (25) has to be determined. But although the integral term looks quite familiar, the derived utility approach cannot be applied. Note, there is no transformation which separates the utility function from the rest of the random variables.

Therefore, I generalize the derived utility approach as follows: Define a function $B(\Pi(\tilde{\mu}) \mid \tilde{\mu})$ such that

$$B(\Pi^*(\tilde{\mu}) \mid \tilde{\mu}) = r_L \int_{\tilde{\epsilon}} U'(\Pi^*(\tilde{\mu}) - \tilde{\epsilon} r_L L)\tilde{\epsilon} dF(\tilde{\epsilon} \mid \tilde{\mu}) \tag{26}$$

holds. I will refer to $B(\cdot)$ as derived expected marginal utility since $B(\cdot)$ can be obtained by differentiating the bank's expected utility function with respect to L^* taking $\tilde{\mu}$ as given. The interpretation of this function is similar to the one of the derived utility: The derived expected marginal utility function already considers the effect of the second risk – $\tilde{\epsilon}$ in this example – as well as the effect of the statistical interdependence of both sources of risk on marginal utility.

From the definition of $B(\cdot)$ it is easy to see that the interior integral of the second line of (25) can be derived from differentiating $B(\cdot)$ with respect to $\Pi^*(\tilde{\mu})$:

$$\begin{aligned} \frac{dB(\Pi^*(\tilde{\mu}) \mid \tilde{\mu})}{d\Pi^*(\tilde{\mu})} &= B'(\Pi^*(\tilde{\mu}) \mid \tilde{\mu}) = r_L \int_{\tilde{\epsilon}} U''(\Pi^*(\tilde{\mu}) - \tilde{\epsilon} r_L L)\tilde{\epsilon} dF(\tilde{\epsilon} \mid \tilde{\mu}) \\ &= r_L U'' \left(\Pi^*(\tilde{\mu}) - \bar{\epsilon} r_L L\right) \bar{\epsilon} + \\ &\quad + r_L \int_{\tilde{\epsilon}} \left(U'''(\Pi^*(\tilde{\mu}) - \tilde{\epsilon} r_L L)\tilde{\epsilon} r_L L - U''(\Pi^*(\tilde{\mu}) - \tilde{\epsilon} r_L L)\right) F(\tilde{\epsilon} \mid \tilde{\mu}) d\tilde{\epsilon} \end{aligned} \tag{27}$$

where the equality in the second line of (27) follows from integration by parts and $\bar{\epsilon}$ is the highest possible realization of $\tilde{\epsilon}$.

Basically, one can now apply the technique used in the proofs of lemma 1 and lemma 2 in order to determine the sign of the second line of (25). However, for this purpose the sign of $\frac{dB'(\Pi^*(\tilde{\mu})|\tilde{\mu})}{d\tilde{\mu}}$ has to be known. Therefore, I differentiate (27) with respect to $\tilde{\mu}$ to yield

$$\begin{aligned} \frac{dB'(\Pi^*(\tilde{\mu}) \mid \tilde{\mu})}{d\tilde{\mu}} &= MB''(\Pi(\tilde{\mu}) \mid \tilde{\mu}) + \\ &+ \; r_L \int_{\tilde{\epsilon}} \left(U'''(\Pi^*(\tilde{\mu}) - \tilde{\epsilon} r_L L)\tilde{\epsilon} r_L L - U''(\Pi^*(\tilde{\mu}) - \tilde{\epsilon} r_L L)\right) F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) d\tilde{\epsilon}. \end{aligned} \tag{28}$$

In (28) $B''(\Pi(\tilde{\mu}) \mid \tilde{\mu})$ is non-negative: By definition

$$B''(\Pi(\tilde{\mu}) \mid \tilde{\mu}) = r_L \int_{\tilde{\epsilon}} U'''(\Pi(\tilde{\mu}) - \tilde{\epsilon} r_L L)\tilde{\epsilon} dF(\tilde{\epsilon} \mid \tilde{\mu}).$$

Since by assumption $U''''(\cdot) < 0$ it must be true that

$$\frac{d}{d\tilde{\epsilon}} U'''(\Pi(\tilde{\mu}) - \tilde{\epsilon} r_L L) = -U''''(\Pi(\tilde{\mu}) - \tilde{\epsilon} r_L L) r_L L > 0.$$

Therefore, the following relation holds for any realization ϵ of $\tilde{\epsilon}$:

$$r_L U'''(\Pi(\tilde{\mu}) - \epsilon r_L L)\epsilon \geq r_L U'''(\Pi(\tilde{\mu}))\epsilon$$

where by the well known reasoning $\epsilon = 0$ serves as the point of reference. Thus, taking expectations with respect to $\tilde{\epsilon}$ given $\tilde{\mu}$ on this last equation, one derives

$$B''(\Pi(\tilde{\mu}) \mid \tilde{\mu}) = r_L \int_{\tilde{\epsilon}} U'''(\Pi(\tilde{\mu}) - \tilde{\epsilon} r_L L)\tilde{\epsilon} dF(\tilde{\epsilon} \mid \tilde{\mu}) \geq 0$$

due to $\tilde{\epsilon}$ to exhibit a zero mean by assumption.

For determining the sign of the second term on the right hand side of (28) consider the following feature of the first-order stochastic dominance: If a probability distribution function F statistically dominates a probability distribution function G to the first order, then for any non-decreasing function A

$$\int AdF \geq \int AdG$$

holds. (cf. Wolfstetter, 1999, p. 137f)

Now, define $A(\tilde{\epsilon}) = -U''(\Pi^*(\tilde{\mu}) - \tilde{\epsilon} r_L L)\tilde{\epsilon}$ [9] and consider without loss of generality two realizations μ_1 and μ_2 of $\tilde{\mu}$. Then, by the feature of the first-order stochastic dominance stated above,

$$\int_{\tilde{\epsilon}} A(\tilde{\epsilon})dF(\tilde{\epsilon} \mid \mu_1) - \int_{\tilde{\epsilon}} A(\tilde{\epsilon})dF(\tilde{\epsilon} \mid \mu_2) \geq (\leq) 0$$

depending on $F(\tilde{\epsilon} \mid \mu_1)$ stochastically dominates (is stochastically dominated from) $F(\tilde{\epsilon} \mid \mu_2)$ to order one.

Using integration by parts, one derives:

$$\begin{aligned}
&\int_{\tilde{\epsilon}} A(\tilde{\epsilon})dF(\tilde{\epsilon} \mid \mu_1) - \int_{\tilde{\epsilon}} A(\tilde{\epsilon})dF(\tilde{\epsilon} \mid \mu_2) \\
&= A(\bar{\epsilon}) - \int_{\tilde{\epsilon}} F(\tilde{\epsilon} \mid \mu_1)dA(\tilde{\epsilon}) - A(\bar{\epsilon}) + \int_{\tilde{\epsilon}} F(\tilde{\epsilon} \mid \mu_2)dA(\tilde{\epsilon}).
\end{aligned}$$

From the latter equation it follows that if $F(\tilde{\epsilon} \mid \mu_1)$ stochastically dominates (is stochastically dominated from) $F(\tilde{\epsilon} \mid \mu_2)$ to order one

$$\int_{\tilde{\epsilon}} \left(F(\tilde{\epsilon} \mid \mu_1) - F(\tilde{\epsilon} \mid \mu_2)\right) dA(\tilde{\epsilon}) \leq (\geq) 0.$$

Dividing both sides of the relation at hand by $\mu_1 - \mu_2 \neq 0$ and taking limits $\mu_2 \longrightarrow \mu_1$, one yields

$$\begin{aligned}
\lim_{\mu_2 \to \mu_1} \int_{\tilde{\epsilon}} \frac{F(\tilde{\epsilon} \mid \mu_1) - F(\tilde{\epsilon} \mid \mu_2)}{\mu_1 - \mu_2} dA(\tilde{\epsilon}) \quad &\leq (\geq) \quad \lim_{\mu_2 \to \mu_1} 0 \\
\Leftrightarrow \int_{\tilde{\epsilon}} \frac{dF(\tilde{\epsilon} \mid \mu)}{d\mu} dA(\tilde{\epsilon}) \quad &\leq (\geq) \quad 0
\end{aligned}$$

[9] Using the well known approach form the proof of lemma 1 it is easily verified that $A(\tilde{\epsilon})$ is non-decreasing in $\tilde{\epsilon}$.

if $F(\tilde{\epsilon} \mid \mu_1)$ stochastically dominates (is stochastically dominated from) $F(\tilde{\epsilon} \mid \mu_2)$ to order one.

Note, since the concept of regression dependence introduced in section 3 is based on first-order stochastic dominance and since

$$dA(\tilde{\epsilon}) = \frac{dA(\tilde{\epsilon})}{d\tilde{\epsilon}} d\tilde{\epsilon} = \left(U'''(\Pi^*(\tilde{\mu}) - \tilde{\epsilon} r_L L)\tilde{\epsilon} r_L L - U''(\Pi^*(\tilde{\mu}) - \tilde{\epsilon} r_L L)\right) d\tilde{\epsilon}$$

it follows from the above relation that the second term on the right side of (28) has the same sign as $F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu})$. Therefore,

$$\frac{dB'(\Pi^*(\tilde{\mu}) \mid \tilde{\mu})}{d\tilde{\mu}} \begin{array}{c} > \\ = 0 \text{ iff} \\ < \end{array} \begin{array}{l} M > 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \geq 0 \\ M = 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) = 0 \\ M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0 \end{array} .$$

.

With the results at hand, one can now apply the standard approach to derive for the second line of (27):

$$-(1-\alpha)\int_{\tilde{\mu}} \tilde{\mu} r_L \int_{\tilde{\epsilon}} U''\left(\Pi^*(\tilde{\epsilon},\tilde{\mu})\right) \tilde{\epsilon} dF(\tilde{\epsilon} \mid \tilde{\mu}) dH(\tilde{\mu}) \begin{array}{c} < \\ = 0 \text{ iff} \\ > \end{array} \begin{array}{l} M > 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \geq 0 \\ M = 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) = 0 \\ M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0 \end{array} .$$

This completes the proof of lemma 3. □

Inspection of (25) shows that economies or diseconomies of scope are not the only factor which determines the influence of one decision variable on the FONC regarding the other one. In particular, the first and the second line of (25) represent the "cross effects" arising from the introduction of risk and risk aversion of the bank. Moreover, the calculations of the proof of lemma 3 show that the first and the second line of (25) reveal the impact of a change in L or D on the direct and portfolio effects on expected marginal utilities in the respective cases. It is, therefore, not surprising that unambiguous results with respect to the first two lines of (25) can be derived if and only if direct as well as portfolio effects reenforce each other. This becomes immediately clear from the interpretations of the results of section 4.1.

Furthermore, from (25) one can observe that the effects arising from the introduction of risk and risk aversion interact with economies or diseconomies of scope. That is, the sign of (25) is unambiguous if and only if all effects – i.e. risk effects as well as effects due to economies or diseconomies of scope – reenforce each other. Otherwise effects of risk may outweigh effects arising from economies or diseconomies of scope and vice versa.

4.3 Comparing Optimal Decisions

With the results of sections 4.1 and 4.2 a direct comparison of the optimal levels of deposits and loans of the risky and the riskless case is available. Thus, I can state and prove

Proposition 1 *The optimal level of deposits increases due to the introduction of joint credit and interbank rate risk, iff the bank is a net borrower in the interbank market and risks are positively regression dependent. The optimal amount of deposits decreases, iff the bank is a net lender in the interbank market and risks are negatively regression dependent. If the bank does not participate in the interbank market and risks are statistically independent, the optimal level of deposits will remain constant. In any other case the change in the optimal level of deposits is ambiguous.*

Proof: It follows immediately from lemma 1 and $\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)/\partial D^{*2} < 0$ that

$$D^* \begin{array}{c} < \\ = \\ > \end{array} D^{**} \text{ iff } \begin{array}{l} M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0 \\ M = 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) = 0 \\ M > 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \geq 0 \end{array} .$$

In any other case the impact of the introduction of risk on the optimal level of deposits is ambiguous. □

The intuition for proposition 1 is straightforward. On the one hand, the size of the net position in the interbank market determines the bank's exposure to interbank rate risk. Inspection of the profit function (5) in section 3, therefore, shows that an increase of the interbank rate decreases, does not change, or increases the bank's profit if, and only if, the net position in the interbank market is negative, zero, or positive, respectively. Thus, M may be regarded as the determinant for the direct effect of interbank rate risk on the bank's profit and hence on the decision on optimal deposit levels. Note, in section 4.1 the role of M in this regard has already been emphasized. On the other hand, in the proof of lemma 1 there also appeared an indirect effect of the introduction risk which has been labeled portfolio effect. In section 4.1. it has been explained that both effects jointly determine the impact of the introduction of risk on the optimal decision on deposits of the bank. However, in contrast to the standard models of portfolio choice, the above arguments showed that the overall effect of the introduction of risk, which is the total of direct and portfolio effect, in the present model is determined endogenous. That is, the overall effect depends on the statistical dependence of random variables as well as on the net position in the interbank market. As a result, by taking together both, the direct and the indirect effects of risks on decision variables, unambiguous results can be derived only when both effects aggravate each other. It is, thus, not surprising that results in any other cases are ambiguous. That is, when direct and indirect effects ameliorate each other, the overall effect cannot be uniquely determined and thus no unambiguous results are available.

Similarly, for the loan business I can state and prove

Proposition 2 *The optimal level of loans decreases due to the introduction of joint credit and interbank rate risk, iff the bank is a net borrower in the interbank market and risks are positive regression dependent. In any other case the change in the optimal loan rate is ambiguous.*

Proof: It follows immediately from lemma 2 and $\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)/\partial L^{*2} < 0$ that

$$L^* > L^{**} \text{ iff } M < 0 \text{ and } F_{\tilde{\mu}}(\tilde{\epsilon} \mid \tilde{\mu}) \leq 0.$$

In any other case the impact of the introduction of risk on the optimal decision on loans is ambiguous. □

The interpretation of proposition 2 is analogous to the one of proposition 1. Again, there appear direct and indirect effects of risk where the latter ones represent portfolio effects. However, as can be observed from the transformed version (16) of the first order necessary conditions (10) in the loan business both effects are more complicated. Note, changing L not only changes the bank's exposure to credit risk. Rather, by changing L also M is changed and hence the bank's exposure to interbank rate risk. Therefore, the portfolio effect with respect to L consists of two parts. The first part refers to the statistical dependence between both sources of risk. The second part, moreover, considers the impact of L on the statistical dependence via M. This is the interpretation of (16). And just like in the previous case, unambiguous results can be derived only when all effects aggravate each other.

But as explained in section 4.2, in order to compare the bank's optimal decisions on deposits and loans one needs to consider the second order sufficient conditions, too. In parts this has already been done in the proofs of propositions 1 and 2 by applying $\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)/\partial D^{*2} < 0$ and $\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)/\partial L^{*2} < 0$. However, it should be recognized that from inspection of $\partial^2 \mathrm{E}\left(U(\Pi^*(\tilde{\epsilon},\tilde{\mu}))\right)/\partial D^*\partial L^*$ – i.e. equation (25) – further insights can be derived. I can thus state and proove

Proposition 3 *Iff there are economies of scope in the bank's business and the bank is a net borrower in the interbank market and risks are positively regression dependent, the bank's incentive to change optimal levels of deposits and loans is reduced.*

Proof: The proof of proposition 3 follows immediately from lemma 3 and the fact that due to second order sufficient conditions for a unique maximum the direct effects of decision variables on first order necessary conditions need to be larger than indirect effects in absolute terms (see section 4.2 for details). □

Proposition 3 indicates that there may be forces which reduce the bank's propensity to change the optimal level of deposits and loans after the introduction on joint credit and interbank rate risk. These forces become apparent if one has a closer look at (25) in section 4.2: Note first that the third line of (25) accounts for economies or diseconomies of scope. From this term one can observe that economies of scope may be one of the forces stated above. The reason for this is clear: with economies of scope increasing the levels of deposits – which happens under the conditions of proposition 3 – decreases marginal costs in the loan business and thus creates an incentive to raise loans. However, note further that in contrast to the corresponding relation in the riskless case (25) contains terms in addition to the one which refers to the costs function. They appear due to risk aversion and represent the effect of the impact of D, say, on the expected marginal utility with respect to L, say, and vice versa. And, of course, this latter effect depends on the net position in the interbank market and the statistical depencence of the risks as has been shown in the proof of lemma 3. Therefore, from (25) one can observe an interaction between risk and cost effects. And in order to derive unambiguous results these effects need to aggravate each other which only appears under the conditions of proposition 3. In any other case no unambiguous conclusions can be drawn since then the net effect depends on the relative strength of the several components. Therefore, even with diseconomies of scope the incentive to change optimal decisions on deposits and loans due to the introduction of risk may be weakened. This is true since with the bank being a net borrower in the interbank market and the risks being positively regression dependent the effects arising from risk may outweigh the effects of diseconomies of scope. This, in fact, is a direct result of the arguments presented in section 4.2.

5 Conclusion

This paper analyzes the assets and liabilities management of a competitive bank which is exposed to joint credit and interbank rate risk. It is derived that under certain conditions the optimal levels of deposits and loans change compared to the riskless case due to the introduction of risks. In particular, a bank which is a net borrower in the interbank market increases the optimal level of deposits and decreases the optimal level of loans in case of risks being positively regression dependent. Driving forces of these changes are direct effects of the introduction of risk on expected utility caused by a certain decision variable on the one hand, and indirect effects in the form of portfolio effects due to statistical dependence of risks on the other hand. It is further shown that unambiguous conclusions can only be drawn when all associated effects aggravate each other.

Furthermore, it is shown that there are interdependencies between decisions on deposits and loans. That is, the bank is not able to change the volume of loans without regarding the level of deposits and vice versa. It is further shown that this kind of interdependency comes not simply from economies or diseconomies of scope in the business of deposit taking and lending as one might expect. Rather, there are additional effects which appear from risk aversion an depend on the interdependence of both sources of risk which, in turn, can be influenced by the decisions of the bank. Moreover, both – i.e. the effect arising from economies of scope and the one arising from risk aversion – are found to be interrelated. Thus, for the case of a bank which is a net borrower in the interbank market these interdependencies can be shown to be such that the incentives to alter optimal levels of deposits and loans are weakened as long as diseconomies of scope are not too strong provided both risks are positively regression dependent.

However, one might argue that considering a perfectly competitive banking sector may not be very realistic. In fact, real banks can be considered to exhibit some degree of market power. Thus an oligopolistic setting may be more adequate. While this argument is, indeed, correct it needs to be mentioned that the results found in the competitive context also hold with a monopolistic bank. The only difference is that calculations are more complex and formal expressions are more extensive. There are no additional insights in doing so. But although a setting with a monopolistic bank can be considered as a pre-stage of the analysis of an oligopolistic banking sector, it does not account for strategic effects which usually arise in an oligopoly. Therefore more research is needed in this field.

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