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Abstract

This paper analyzes optimal incentive compatible debt contracts when lenders are risk averse. The decisive factor in this regard is that risk aversion requires to consider further sources of risk the lenders are exposed to. The main results derived in a setting of asymmetric information – the payment obligation of the optimal incentive compatible contract increases due to risk aversion of lenders which is reinforced by the introduction of a further source of risk – are shown to be in line with the results from the industrial organization approach of banking. Moreover, the result of the present paper are more general than the ones from the industrial organization approach.

Keywords: debt contracts, risk aversion, costly state verification, risk.

JEL classification: D4, D8, G14, G33

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This paper analyzes optimal incentive compatible debt contracts when lenders are risk averse. The decisive factor in this regard is that risk aversion requires to consider further sources of risk the lenders are exposed to. The main results derived in a setting of asymmetric information – the payment obligation of the optimal incentive compatible contract increases due to risk aversion of lenders which is reinforced by the introduction of a further source of risk – are shown to be in line with the results from the industrial organization approach of banking. Moreover, the result of the present paper are more general than the ones from the industrial organization approach.

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1 Introduction

The allocation of funds between economic agents in financial surplus and economic agents in financial deficit can be considered as the major function of the financial system of an economy (cf. Hellwig, 2000a, p. 3). In this regard the former – usually called lenders – wish to profitably invest their funds and the latter – usually called borrowers – require to obtain outside finance in order to operate profitable projects. However, there are several factors which complicate the efficiency of performing this function. Especially the level of available information regarding the contracting situation and the economic agents' attitudes towards risk are possibly the most important ones.

Traditionally, research focused on the interdependence between efficiency of the financial system and information availability. Especially the seminal work of Gale and Hellwig (1985) analyzed the influence of information asymmetries on optimal contracts between borrowers and lenders of funds. Gale and Hellwig (1985) use a costly state verification model similar to the one of Townsend (1979) to show that optimal incentive compatible contracts between risk neutral borrowers and lenders are standard debt contracts (SDC's). But, in fact, assuming risk neutrality of both contracting parties seems to be quite restrictive. Therefore, Gale and Hellwig (1985) consider also the case of risk aversion of the borrower. They point out that this is the more tractable case since with risk aversion of the lender one has to consider the other contractual arrangements of the lender when analyzing the optimal contract between borrower and lender (cf. Gale and Hellwig, 1985, p. 660f.). The meaning of this observation becomes clear when one considers the literature on choice under uncertainty with multiple sources of risk. In particular Kimball (1993) indicates that the behavior of risk averse decision makers changes when they are exposed to a further source of risk even when risks are statistically independent. Hence, since the several contractual arrangements of the lender can be expected to expose her to multiple sources of risk, the conjecture of Gale and Hellwig (1985) refers to changes in the lender's behavior in the manner described.

But why should there be risk aversion of lenders? Of course, when one considers lenders to be private households there are several reasons why they may behave risk averse. For example, private households usually do not have enough funds to achieve sufficient diversification of their risks arising from undertaking investment projects. Thus their personal wealth largely depends on just a few – or even a single – investment projects. As a result, one can expect private households to care about the investment project's level of risk. Therefore, Gale and Hellwig (1985) argue that lenders are meant to be banks or other financial institutions which behave as if they are risk neutral due to sufficiently diversified investment portfolios. In fact, this assumption seems obvious since there is empirical evidence that financial institutions are major actors in the savings-investment process (see Gorton and Winton, 2002, p. 5ff., for a survey of the empirical literature). However, recent theoretical work analyzed several reasons why the assumption of risk neutrality may not hold. Especially Froot et al. (1993) present a formal argument for a per se risk neutral firm to behave as if it were risk averse. The reason is that – starting out from the observation that externally obtained funds are more expensive than funds generated internally – undertaking investment projects requires to obtain external finance when internally generated funds do not suffice. Hence the firm has an incentive to reduce random variations of internally generated funds in order to minimize the need for expensive external finance. Froot and Stein (1998) show this reasoning to hold for the banking industry likewise. Furthermore they show that the endogenous risk aversion is decreasing in the amount of capital the bank holds. Another important reason for risk averse behavior of financial institutions is derived by Pausch and Welzel (2002) and Kürsten (2001). They show that capital adequacy regulation explains risk averse behavior of banks. The reason is as follows: capital adequacy regulation forces banks to hold the higher amounts of capital the higher the banks' exposure to risk. Since capital is costly, the regulation imposes costs on banks when they face higher levels of risk and, thus, makes banks care about the level of risk in this way. With these arguments risk aversion of lenders may be an obvious assumption.

Therefore, in this paper I analyze optimal incentive compatible debt contracts when lenders behave risk averse. The objective in this regard is twofold. First, it is necessary to determine the structure of the optimal contract with risk averse lenders when there is just a single source of risk. This enables a comparison of the optimal contract when lenders behave risk averse with the one in the standard model with risk neutral lenders of Gale and Hellwig (1985). I then introduce a further source of risk the lender is exposed to. The additional risk can be considered as the aggregate risk of all other contracts of the lender. In this way I provide a formal analysis of the conjecture of Gale and Hellwig (1985) that with risk averse lenders there is an interdependency between the optimal incentive compatible debt contract and all the other contracts of the lender. Second, I will point out that the results of the analysis presented in this paper, which are derived in a setting with asymmetric information, are closely related to the findings of the industrial organization approach of banking of e.g. Freixas and Rochet (1997), Wahl and Broll (2000) and Broll and Welzel (2002).

The remainder of the paper is organized as follows. In section 2 I present a simple base model and determine the optimal incentive compatible debt contract when lenders are risk averse and exposed to a single source of risk. However, the formal analysis differs from the one of Gale and Hellwig (1985) since their more intuitive derivation of optimal contracts is no longer applicable with risk averse lenders. As the main result I derive the optimal incentive compatible contract to be still a SDC. However, risk aversion of the lender increases the payment obligation of the optimal SDC compared to the situation with risk neutral lenders. In section 3 I introduce a further source of risk which can be thought of as the aggregate (random) repayment from the lender's other investment projects. The following analysis shows that, in fact, there exists an interdependency between the several contracts just like supposed by Gale and Hellwig (1985). I will argue that the further risk increases the lender's level of absolute risk aversion and, therefore, forces the borrower to further increase the payment obligation of the SDC. Section 4 concludes.

2 Optimal Incentive-Compatible Debt Contracts with Risk Averse Lenders

For the formal analysis I apply a model very similar to the ones of Gale and Hellwig (1985) and Townsend (1979). In particular, I consider an economy with two types of agents, entrepreneurs and investors. Entrepreneurs have the opportunity to undertake an investment project which generates a risky return of y per unit of funds invested but do not have funds at all. Thus, they turn to investors for external finance. In the following I will refer to entrepreneurs as borrowers as well. Borrowers are assumed to be risk neutral.

The assumption of risk neutral borrowers, one could complain, is not very obvious since it contradicts the arguments of Froot et al. (1993) which were presented in section 1. However, there are certain arguments why these arguments do not hold for the case of borrowers. Note, the setting stated above refers to the standard situation of limited liability of borrowers.[1] Limited liability, in any case, can be considered as a kind of insurance to the borrowers since in the state of bankruptcy the borrowers' loss is limited to the pre-specified amount. The reason is as follows: without limited liability the borrowers could be forced to use a part of their personal wealth for repayment obligation in case of bankruptcy. In this case there appears, thus, a kind of shock to the borrowers which makes the financing opportunities of the borrowers state-dependent since they need further external finance (out of personal wealth) in addition to the lenders' funds. With limited liability, however, their personal wealth is not affected (cf. Jensen and Meckling, 1976, p. 331). Hence limited liability can be considered as a put option on the borrowers' financing risk in the state of bankruptcy. Moreover it can be shown that put options reduce – or even eliminate – risk aversion of firms when there are state-dependent financing opportunities (cf. Froot et al., 1993, p. 1645 ff., for a formal proof).

In contrast to Gale and Hellwig (1985) investors may be considered as private households as well as financial institutions. In either case they wish to invest the funds they hold into one of the following investment opportunities. First, there is a security which generates a risk free repayment R per unit of funds at the end of the period. Second, investors can lend out funds to an entrepreneur. Therefore, in the following I will refer to investors as lenders as well. In this latter case lenders and borrowers have to negotiate a contract which specifies the amount to be repayed to lenders at the end of the period. Because of the reasons explained in the introduction, lenders are assumed to be risk averse with an increasing and strictly concave utility function U.

The difficulty with writing a contract arises because the lenders are not able to observe the return of the project to the borrowers without cost at the end of the period while the borrowers can do. Hence, the borrowers have an incentive to misreport the outcome of the investment project in order to reduce the repayment to the lenders. The lenders, in turn, have the opportunity to incur a fixed cost of c to buy a technology for verifying the project's outcome. When they buy verification they learn the outcome of the project without error. However, the information with respect to the outcome of the project remains private information of the lender who demands verification. As a result, the contract between a particular pairing of lender

[1] Instead of assuming that the borrower does not have any own funds, one could suppose the borrower to posses of any given level of equity.

and borrower must also specify when there is to be verification. This problem is well known in the literature as the costly state verification problem first analyzed by Townsend (1979). Furthermore, before writing the contract borrowers as well as lenders do not know the ex post realized outcome from the investment project. Ex ante both agents just know the range of possible outcomes $y \in [\underline{y}, \overline{y}]$ with $\underline{y} \geq 0$ and the corresponding probability distribution function $F(y)$ with $F(\underline{y}) = 0$ and $F(\overline{y}) = 1$. Further, let $f(y) = F'(y)$ the corresponding probability density function which is strictly positive as long as $y \in [\underline{y}, \overline{y}]$ and zero else.

For keeping things tractable I need some further assumptions. First, I assume that a lender's riskless return from securities is larger than the minimal project outcome ($R > \underline{y}$). Furthermore, I assume that the borrowers have all the bargaining power. To give a reason for this assumption one could argue as follows: When there is a large number of lenders in the market, the borrowers will be able to acquire all the gains form trade. This is true since in this case a single borrower can negotiate with several lenders and enter into a contract with the one offering best terms for the borrower. But this, in fact, is equivalent to a situation where a borrower offers a debt contract which maximizes his payoff from trade (cf. Gale and Hellwig, 1985, p. 650 f.).[2] Further, I normalize the funds the entrepreneur needs to one. As a consequence the borrower demands just one unit of funds from an investor. Thus, it is sufficient for the analysis to consider only a single representative pair of borrower and lender.

With the above assumptions, the game to be solved can be described as follows: in the first stage the borrower ex ante offers a contract which specifies the repayment (depending on the outcome of the project) to the lender $t(y)$ and a set $S \subset [\underline{y}, \overline{y}]$ which defines when verification occurs. I will refer to this contract as debt contract (DC). Let S' be the complement of S defining the states when there is no verification. Thereafter, the lender decides on accepting the contract or not. Afterwards, the borrower observes the outcome of the project and makes repayment to the lender, and the lender decides wether to carry out verification or not.

To find a solution of the game stated above I begin with the analysis of the last stage. This, in fact, is the most crucial one in the game since the problem of costly state verification affects the behavior of both agents in this stage. Remember that as long as the lender does not verify the outcome of the investment project the borrower has an incentive to report a bad outcome and make only a minimum repayment. But when the lender verifies he observes the realization of y and has to pay the fixed cost c. In the case of verification the investment project is liquidated and the lender gets a repayment which covers his opportunity costs. All other costs are borne by

[2] One could argue that in the case when lenders are financial institutions this is not a plausible assumption. However, the qualitative results do not change when lenders are assumed to have all the bargaining power since providing incentives to borrowers requires the same structure of the contract.

the borrower. Therefore, for a DC to work it has to be incentive compatible. That is, the contract has to meet the following conditions: (i) the ex post realized set when there is to be verification has to be the same as the ex ante specified one and (ii) the ex post realized repayment must be the same as the ex ante specified one (cf. Townsend, 1979, p. 270). In other words, neither contracting party may have an incentive to deviate from the terms of contract.

As is well known from the revelation principle, any allocation of wealth among borrowers and lenders which is contractually feasible is also feasible with direct and incentive compatible contracts (cf. Salaniè, 1997, p. 17). That means, one can confine attention to debt contracts in which the borrower truthfully reports outcome and which determine the lender's decision on verification and the repayment based on the outcome reported by the borrower. Thus, applying the revelation principle, one can state

Proposition 1 *With risk averse lenders, a debt contract is incentive compatible if and only if*

$$t(y) = \begin{cases} t_0 & ; y \geq t_0 \\ \overline{t(y)} + c < t_0 & ; y < t_0 \end{cases}$$

i.e. the risk neutral borrower has to pay a fixed repayment t_0 *as long as there is no verification* ($y \geq t_0$) *and an outcome dependent repayment* $\overline{t(y)}$ *which is smaller than* t_0 *else.*

Proof: For the proof of proposition 1 I adapt the one of Townsend (1979, p. 287). For any debt contract $(t(y), S)$, if the borrower reports $y \notin S$ the lender will not ask for verification. In this case the repayment of the borrower is $t(y) = \min_{y \notin S} \overline{t(y)}$ where $\overline{t(y)}$ is the ex ante specified repayment function for any outcome. Alternatively, if the borrower reports $y \in S$ the lender will ask for verification and the repayment will be $t(y) = \overline{t(y)}$. Furthermore, the borrower must bear the cost of verification c.

For proving that proposition 1 is necessary and sufficient for incentive compatibility of the DC define $t_0 = \min_{y \notin S} \overline{t(y)}$ which is constant for any $y \notin S$. On the one hand, suppose for some $y' \notin S$, $\overline{t(y')} > t_0$. If this y' were realized, the borrower had an incentive to report any $y \notin S$ other than y' since in this case there occurs no verification and the repayment is lower. Thus, for the DC to be incentive compatible $t(y) = t_0 \Leftrightarrow y \notin S$. On the other hand, when $y \in S$ the repayment of the borrower must be $\overline{t(y)} + c < t_0 \; \forall \; y \in S$ in order to be incentive compatible. Suppose there is some $y'' \in S$ which violates this condition. Then the borrower had an incentive to report any $y \notin S$ since there occurs no verification and the repayment is lower. Thus $t(y) = t_0 \;\Leftrightarrow\; y \notin S$ and $t(y) = \overline{t(y)} + c < t_0 \;\Leftrightarrow\; y \in S$ is necessary for incentive compatibility of the DC.

It is easy to see that both conditions are also sufficient since with $y \notin S$ there is no verification and t_0 is repayed and with $y \in S$ verification occurs and the repayment is $\overline{t(y)}$. Furthermore, since the borrower does not have any funds he can at most pay the realization of the outcome of the project, y, to the lender. As a result, for a DC to be incentive compatible the set when there is no verification is $S' = \{y : y \geq t_0\}$ and the set where there is verification is $S = \{y : y < t_0\}$.□

Note that Proposition 1 is exactly the same as Proposition 1 in Gale and Hellwig (1985, p. 653) which defines the structure of the incentive compatible debt contract for risk neutral borrowers and lenders. Thus, the risk aversion of the lenders does not influence the structure of the incentive compatible DC. In fact, this result is not surprising since incentive compatible DCs make borrowers residual claimants of the investment project and therefore create strong incentives for the borrowers to truthfully report the outcome of the investment project to lenders. In doing so, risk neutrality of borrowers ensures that it is not necessary to offer them a compensation for bearing all the risk of the investment project. As a result the incentive mechanism of DCs is not altered by risk aversion of lenders.

Now, with the result of stage 3 of the contracting game at hand the remaining stages can be analyzed: On stage two the lender decides whether to accept the contract offered from the borrower or not. The lender will accept the offer if and only if the levels of expected utilities from the DC and from buying securities are at least the same. Thus, with considering the structure of the incentive compatible DC, one can write the lender's participation constraint as follows:[3]

$$\int_{\underline{y}}^{t_0} U\left(\overline{t(y)}\right) dF(y) + \int_{t_0}^{\overline{y}} U\left(t_0\right) dF(y) \geq U\left(R\right). \tag{1}$$

With this information one can determine the optimal incentive compatible DC at the first stage of the game. Since the borrower ,by assumption, has all the bargaining power she can maximize her expected profit from the DC considering incentive compatibility, the lender's participation constraint, and the fact that her repayment when there is verification can be at most $y - c$. Thus, one can state the borrower's optimization problem as

$$\max_{\overline{t(y)}, t_0} \int_{\underline{y}}^{t_0} \left(y - \overline{t(y)} - c\right) dF(y) + \int_{t_0}^{\overline{y}} \left(y - t_0\right) dF(y)$$

[3]Note, by incentive compatibility (Proposition 1) the borrower has to pay $\overline{t(y)} + c$ to the lender in case of verification. Thus, since in this case the lender has to pay verification cost c his payoff is $\overline{t(y)} + c - c = \overline{t(y)}$. In other words, verification cost are shifted to the borrower.

$$\begin{aligned} \text{s.t.} \int_{\underline{y}}^{t_0} U\left(\overline{t(y)}\right) dF(y) + \int_{t_0}^{\overline{y}} U(t_0)\, dF(y) &\geq U(R) \\ \overline{t(y)} + c &\leq y \; \forall \; y < t_0 \\ \overline{t(y)}, t_0 &> 0. \end{aligned} \tag{2}$$

In the maximization problem (2) the incentive compatibility constraint (proposition 1) has already been incorporated. From the maximization problem (2) one can derive the following

Proposition 2 *The optimal incentive compatible debt contract with risk neutral borrowers and risk averse lenders is a standard debt contract.*

Proof: See the Appendix. □

Thus, the optimal repayment of the borrower when there occurs verification is $\overline{t(y)} = y - c$. As a result, the optimal incentive compatible DC is a contract with the following repayment function

$$t(y) = \begin{cases} t_0 & ; y \geq t_0 \\ y - c & ; y < t_0 \end{cases} \tag{3}$$

which is exactly a standard debt contract (SDC) as defined by Gale and Hellwig (1985, p. 654) where t_0 can be determined by equation (19). Since t_0 can be considered as the normal case of repayment I will refer to t_0 as the payment obligation of the SDC in the following.

So far, the results do not depart from the standard case of Gale and Hellwig (1985) with risk neutral borrowers and lenders. Therefore, one can go on comparing the SDC with risk neutral lenders with the one when lenders are risk averse.[4]

For this purpose consider the lender's participation constraint (1). The interpretation of this constraint was that the lender's expected utility from the repayment must cover at least the utility of the riskless investment opportunity. Furthermore, in the proof of Proposition 2 in the appendix it was shown that the participation constraint is binding in the optimum. Thus, one can rewrite the participation constraint (1) as follows:

$$\mathrm{E}\left(U\left(t(y)\right)\right) = U(R)\,. \tag{4}$$

Note that (4) is a very general representation of the participation constraint since depending on the specification of the utility function it holds for risk aversion as well as risk neutrality.

[4]In the following the indices "$_{RN}$" and "$_{RA}$" refer to the cases of risk neutral and risks averse lenders, respectively.

From the interpretation of (4) it is immediately clear that R is the so called certainty equivalent of the risky repayment $t(y)$.[5] Now, following e.g. Kimball (1990, p. 56) let π be the equivalent risk premium which is implicitly defined by

$$\mathrm{E}U\left(t(y)\right) = U\left(\mathrm{E}(t(y)) - \pi(t(y))\right), \tag{5}$$

where $\mathrm{E}(t(y))$ is the lender's expected repayment from the borrower. Thus, the equivalent risk premium is the certain reduction in from the expected repayment which constitutes the same level of utility as the risky DC. Combining (4) and (5) and using monotonicity of the utility function yields

$$R = \mathrm{E}(t(y)) - \pi(t(y)). \tag{6}$$

With (6) one can now compare the SDC with risk neutral (SDC_{RN}) and the SDC with risk averse lenders (SDC_{RA}). Since the return R from the riskless investment opportunity is the same for both situations the following condition must hold:

$$\mathrm{E}_{RN}(t(y)) - \pi_{RN}(t(y)) = \mathrm{E}_{RA}(t(y)) - \pi_{RA}(t(y)),$$

where risk neutrality implies a zero risk premium, i.e. $\pi_{RN}(t(y)) = 0$, and for the case of risk aversion there must be a strictly positive risk premium ($\pi_{RA}(t(y)) > 0$). Thus the expected repayment must be larger with risk averse lenders, i.e. $\mathrm{E}_{RA}(t(y)) > \mathrm{E}_{RN}(t(y))$. Thereby, the expected repayment of any standard debt contract can be computed as

$$\mathrm{E}(t(y)) = \int_{\underline{y}}^{t_0} (y - c)\, dF(y) + \int_{t_0}^{\overline{y}} t_0 dF(y). \tag{7}$$

From (7) it is immediately clear that differences in expected repayments of SDCs can only appear due to differences in the corresponding payment obligation of the SDCs t_0.

Now, since with risk averse lenders there must be a higher expected repayment compared to the case of risk neutral lenders I compute the change in expected repayment when the payment obligation is altered. In doing so I can state and prove

Proposition 3 *Risk aversion of lenders leads to a higher payment obligation of the optimal standard debt contract compared to a situation with risk neutral lenders. As a result the expected profit of the borrower decreases.*

Proof: See the appendix.□

The interpretation of Proposition 3 is straightforward. Since the borrower is not able to insure the risk averse lender against the risk of the investment project she

[5]For a detailed definition of the certainty equivalent see Kreps (1990), p. 84.

has to pay a risk premium to the lender. As a result, the expected profit of the borrower decreases. Furthermore, the rise of the payment obligation of the SDC causes that verification takes place more often compared to a situation with risk neutral lenders. This, in turn, increases the expected cost of verification borne by the borrower which further decreases her expected profit.

Moreover, proposition 3 states a result in a setting of asymmetric information which is analogous to the conclusion of the industrial organization approach of banking. E.g. Wong (1997) finds that the optimal loan rate of a risk averse bank increases compared to the risk-neutral case.[6] In fact, this result is confirmed by proposition 3 since a higher payment obligation of the optimal SDC due to risk aversion of lenders can be interpreted as a higher loan rate paid by the borrower. However, proposition 3 is more general than the results from the industrial organization approach since it also includes the case of direct lending, i.e. the case of borrowers issuing bonds in financial markets. Therefore proposition 3 can be considered as generalization of the seminal result of Jensen and Meckling (1976), who prove that for firms external finance is more expensive than internal finance due to agency costs. With proposition 3 it is easy to see that because of risk aversion of external financiers – i.e. lenders – agency costs increase and thus the reasoning of Jensen and Meckling (1976) is strengthened.

These results are very important for analyzing the interaction of SDC's to the other contractual arrangements of the lender in the next section.

3 Optimal Debt Contracts with Multiple Sources of Risk

The previous section analyzed optimal incentive compatible debt contracts with risk averse lenders. In this regard the debt contract has been the sole source of risk the lenders are exposed to. However, in the introduction I explained that risk aversion of the lenders can cause interdependencies between different contractual arrangements of a lender. That is, when there are multiple sources of risk the lenders are exposed to, one can expect interaction among them. Since the previous section ignored this problem, the present section will analyze the issue in more detail.

Therefore, the model presented in section 2 has to be modified. Again, consider one representative pairing of entrepreneur and investor. The borrower can still undertake the investment project known from the previous section. But in addition suppose there exists a further random variable $z \in [\underline{z}, \overline{z}]$ with $\underline{z} \geq 0$. Let $G(z)$ and

[6] Similar results are derived in Wahl and Broll (2000), Freixas and Rochet (1997), and Broll and Welzel (2002).

$g(z)$ be the corresponding cumulative distribution and probability density functions, respectively. Further, suppose the random variables y and z to be statistically independent, i.e. the joint probability density function $h(y,z)$ can be written as $h(y,z) = f(y) \cdot g(z)$. Without loss of generality, one can rewrite z as $z = \mathrm{E}(z) + \tilde{z}$ where $\mathrm{E}(z)$ is the mean of z and $\tilde{z} \in [\underline{\tilde{z}}, \overline{\tilde{z}}]$ is a zero-mean random variable (see Moschini and Lapan, 1995, p. 1029, for a similar argument). Let $\tilde{G}(\tilde{z})$ and $\tilde{g}(\tilde{z})$ be the cumulative distribution and the probability density function of $\tilde{z}$, respectively. With standard calculations one can easily see that given the assumptions above $G(z) = \tilde{G}(\tilde{z})$ and $g(z) = \tilde{g}(\tilde{z})$ must hold (see Larsen and Marx, 1986, p. 133ff.). Thus, the joint distribution of y and $\tilde{z}$ can be written as $\tilde{h}(y,\tilde{z}) = f(y) \cdot \tilde{g}(\tilde{z})$.

The additional assumptions stated above can be interpreted as follows: z can be considered as aggregate (random) repayment from all other contracts of the lender. Hence, $\mathrm{E}(z) > 0$ is the expected aggregate repayment from all other contracts. Furthermore, the zero-mean random variable $\tilde{z}$ adds noise to the expected repayment and, thus, exposes lenders to a further source of risk. Therefore, in the following I will analyze in which way the additional source of risk affects the optimal incentive compatible debt contract.

To answer the questions stated above, note first that the contracting game to be solved is basically the same as the one in section 2. That is, the contract to be negotiated between lender and borrower must maximize the borrower's expected profit considering the lender's participation constraint and incentive compatibility. However, due to the assumptions above, there appear changes with respect to the lender's participation constraint. Because of the introduction of an additional source of risk, it is now necessary to compute the lender's expected utility with respect to the joint distribution of both risky prospects. Thus the participation constraint can be rewritten as

$$\int_{\underline{y}}^{\overline{y}} \int_{\underline{\tilde{z}}}^{\overline{\tilde{z}}} U\left(t(y) + \mathrm{E}(z) + \tilde{z}\right) dG(\tilde{z}) dF(y) \geq \int_{\underline{\tilde{z}}}^{\overline{\tilde{z}}} U\left(R + \mathrm{E}(z) + \tilde{z}\right) dG(\tilde{z}). \tag{8}$$

Now, following Kihlstrom et al. (1981) and Eeckhoudt and Kimball (1991) one can transform the participation constraint. Therefore define a utility function $V\left(X + \mathrm{E}(z)\right)$ by integrating out the background risk (cf. Kihlstrom et al., 1981, p. 914, and Eeckhoudt and Kimball, 1991, p. 244), i.e.

$$V\left(X + \mathrm{E}(z)\right) = \int_{\underline{\tilde{z}}}^{\overline{\tilde{z}}} U\left(X + \mathrm{E}(z) + \tilde{z}\right) dG(\tilde{z}). \tag{9}$$

The intuition of this technique is to determine the lender's expected utility with respect to the risky prospect $\tilde{z}$. Thus, the effect of the additional source of risk is already contained in the utility function $V(\cdot)$.

Thus, with this definition at hand equation (8) can be written as

$$\int_{\underline{y}}^{\overline{y}} V\left(t(y)+\mathrm{E}(z)\right) dF(y) \geq V\left(R+\mathrm{E}(z)\right). \tag{10}$$

With this modifications, one can state the borrower's maximization problem as follows:

$$\begin{aligned}
&\max_{\overline{t(y)},t_0} \int_{\underline{y}}^{t_0} \left(y-\overline{t(y)}-c\right) dF(y) + \int_{t_0}^{\overline{y}} \left(y-t_0\right) dF(y) \\
&\text{s.t.} \int_{\underline{y}}^{t_0} V\left(\overline{t(y)}+\mathrm{E}(z)\right) dF(y) + \int_{t_0}^{\overline{y}} V\left(t_0+\mathrm{E}(z)\right) dF(y) \geq V\left(R+\mathrm{E}(z)\right) \\
&\qquad \overline{t(y)}+c \leq y \ \forall \ y<t_0 \\
&\qquad \overline{t(y)}, t_0 > 0.
\end{aligned} \tag{11}$$

A comparison of maximization problems (2) and (11) shows that they are formally equivalent. The only difference is in replacing $U(\cdot)$ in (2) by $V(\cdot)$ in (11). Further, the term $\mathrm{E}(z)$ is a constant not influencing the qualitative results. In particular, the arguments regarding incentive compatibility of the contract are not affected. As a result, one can apply the approach of section 2 to derive the structure of the optimal incentive compatible contract which is, again, a SDC. Therefore, even when the lender bears further background risk, the optimal incentive compatible debt contract can be characterized by the repayment function displayed in equation (3).

As before, to compare two SDCs it is sufficient to compare the corresponding payment obligations of both contracts. For this, one can, in principle, proceed as in section 2: applying the definition of Kimball (1990) of an equivalent risk premium with utility function $V(\cdot)$ yields

$$\mathrm{E}V\left(t(y)+\mathrm{E}(z)\right) = V\left(\mathrm{E}_V\left(t(y)\right)+\mathrm{E}(z)-\pi_V\left(t(y),\mathrm{E}(z)\right)\right) \tag{12}$$

where $\pi_V\left(t(y),\mathrm{E}(z)\right)$ is the equivalent risk premium with respect to utility function $V(\cdot)$ for any given $\mathrm{E}(z)$ and $\mathrm{E}_V\left(t(y)\right)$ denotes the expected repayment when the lender's utility function is $V(\cdot)$.

Combining equations (10) and (12) one derives

$$\mathrm{E}_V\left(t(y)\right)-\pi_V\left(t(y),\mathrm{E}(z)\right)=R. \tag{13}$$

A similar relation can be deduced for a given level of $\mathrm{E}(z)$ when there is no background risk $\tilde{z}$:

$$\mathrm{E}_U\left(t(y)\right)-\pi_U\left(t(y),\mathrm{E}(z)\right)=R \tag{14}$$

where $\mathrm{E}_U\left(t(y)\right)$ and $\pi_U\left(t(y),\mathrm{E}(z)\right)$ are the expected repayment and the equivalent risk premium given $\mathrm{E}(z)$ with respect to the utility function $U(\cdot)$, respectively.[7]

[7]Note, when there is no background risk it is not necessary to calculate a derived utility function by integrating out the influence of the additional risk. Therefore, in this situation $U(\cdot)$ is the adequate utility function.

Since I aim at a comparison of $\mathrm{E}_V(t(y))$ and $\mathrm{E}_U(t(y))$ and since the right hand side of (13) equals the right hand side of (14) it is necessary to compare the risk premia in both cases. As shown in the appendix, under certain conditions the following relation holds:

$$\pi_V(t(y), \mathrm{E}(z)) \geq \pi_U(t(y), \mathrm{E}(z)). \tag{15}$$

The arguments for (15) to hold can be summarized as follows: As Kimball (1993) points out "[...] bearing one risk should make an agent less willing to bear another risk, even when the two risks are independent" (p. 598). That is, adding a further risk should increase an individual's risk aversion, even when the risks the individual is exposed to are independent. Kimball (1993) introduces the term *standard risk aversion* to name this fact. It is easy to see that standard risk aversion represents a quite natural assumption. Kimball (1993) further shows that two conditions are necessary and sufficient for standard risk aversion. These are decreasing absolute risk aversion in the sense of Arrow and Pratt and decreasing absolute prudence where prudence "[...] is meant to suggest the propensity to prepare and forearm oneself in the face of uncertainty [...]"(Kimball, 1990, p. 54). Now, ensuring that the utility function $U(\cdot)$ meets these requirements, one can show $V(\cdot)$ to exhibit a higher degree of absolute risk aversion in the Arrow – Pratt sense than $U(\cdot)$ for any given $\mathrm{E}(z)$. As a result the risk premium with respect to $V(\cdot)$ must be larger than the one with respect to $U(\cdot)$ for any $\mathrm{E}(z)$ (cf. Kihlstrom et al., 1981, p. 911f).

Therefore, combining (13) and (14) and applying (15) yields

$$\mathrm{E}_V(t(y)) \geq \mathrm{E}_U(t(y)) \tag{16}$$

for any given value of $\mathrm{E}(z)$. Now, applying the arguments of section 2, for comparing the SDCs with and without background risk it is sufficient to compare the corresponding payment obligations. Therefore, the reasoning presented in the proof of proposition 3 still holds in an analogue way. Thus, I can state

Proposition 4 *If the risk averse lender faces a further independent risk in addition to the one of the standard debt contract with the borrower, the payment obligation of the SDC has to increase for any given level of* $\mathrm{E}(z)$. *The expected profit of the borrower decreases in this situation.*

The interpretation of proposition 4 is simple. When there are further contractual arrangements of the lender except the SDC with the borrower which expose the lender to risk, then the lender is less willing to bear an additional risk. Since – by assumption – the borrower does not have any own funds he is not able to fully insure the lender against the risk of the investment project. Thus, the SDC further exposes the lender to risk causing her risk aversion to increase. As a result the borrower has to pay a higher risk premium which requires the payment obligation of the SDC to increase. However, for these arguments to hold one must assume that $\mathrm{E}(z)$ is given and constant in both cases. Hence $\mathrm{E}(z)$ represents the lender's initial wealth generated from the other contractual arrangements. Comparing situations where there are different levels of $\mathrm{E}(z)$ is difficult. When this is the case a tradeoff appears: On the one hand, increasing the initial wealth decreases the lender's risk aversion due to the assumption of decreasing absolute risk aversion. On the other hand, exposing the lender to an additional independent risk makes her risk aversion to rise. Thus, the net effect is ambiguous.

Again, one can find analogous results from the industrial organization approach of banking. E.g. Wong (1997, p. 257f.) shows that the optimal loan rate of a bank increases as the bank becomes more risk averse. The result of increasing payment obligation of the SDC of the present analysis can be interpreted in this way. However, Wong (1997) applies the notion of strongly more risk aversion in the sense of Ross (1981) to derive this result which is a stronger concept than the one of a higher degree risk aversion in the Arrow–Pratt sense. Note, for proposition 4 to hold the stronger concept of higher risk aversion of Ross (1981) is not necessary. Therefore, the result of the present paper holds for a wider range of utility functions. Furthermore, the above analysis shows that the higher level of absolute risk aversion is generated endogenously by the lender's additional source of risk. Thus, the results presented above are more general the the ones of the industrial organization approach. Moreover, since the results of the present section also hold for the case of direct lending, it further generalizes the findings of Jensen and Meckling (1976). It is easy to see that when outside financiers are exposed to further risks in addition to those of the debt contract their level of absolute risk aversion increases and, thus, make external finance even more expensive for borrowers. Therefore, agency costs can be expected to rise due to risk averse lenders with multiple sources of risk.

4 Conclusion

In their seminal work Gale and Hellwig (1985) pointed out that in a situation of costly state verification for risk neutral borrowers and lenders the optimal incentive compatible debt contract is a standard debt contract. I extended the analysis of Gale and Hellwig (1985) by assuming risk aversion with lenders. As a result, the structure of the optimal incentive compatible debt contract does not change. However, lender's risk aversion causes the payment obligation of the optimal incentive compatible SDC to rise. Hence the expected repayment of the borrower increases since with risk aversion of the lender the borrower has to pay a risk premium which, in turn, lowers the expected profit.

Furthermore, with the assumption of risk averse lenders the contracting situation gets more complicated. Gale and Hellwig (1985) argue that with the assumption of risk aversion one has to consider all the other contractual arrangements of the lenders. Therefore, this paper provides a formal analysis of this subject. The result can be summarized as follows: as long as there exists any further contract except the SDC which exposes the lender to risk there appears an interaction with both contracts even when their risks are independent. As a result, risk aversion of lenders increases the payment obligation of the optimal incentive compatible debt contract when the lenders are exposed to further sources of risk. That is, a further risk increases the lender's degree of absolute risk aversion in the Arrow–Pratt sense and therefore forces the borrower to pay an even higher risk premium when offering the SDC.

These results are in line with those of the industrial organization approach of banking. Moreover, the present paper derives results in a setting of asymmetric information which are analogous to those of the industrial organization approach. In particular, the increase in payment obligation of the optimal SDC due to risk aversion of lenders and adding a further source of risk can be interpreted as raising the loan rate in these situations. But this, in fact, is the result of the industrial organization approach of banking. However, the analysis of the present paper was shown to be more general. That is, on the one hand the above results hold for a wider range of utility functions. On the other hand since the results hold for financial institutions as well as for direct external finance one gains further insights with respect to the formation of agency costs in the sense of Jensen and Meckling (1976).

However, there is a number of remaining questions. In particular, the assumption of risk neutrality of borrowers appears questionable in some cases. It would, therefore, be important to know whether the results of the present paper are still valid with risk averse borrowers. The answer is by no means obvious since on the one hand the literature on costly state verification suggests that this is the case (cf. Townsend, 1979, p. 268ff). On the other hand the work of Hellwig (2000b) and Hellwig (2001) as well as the literature on incentive contracts (cf. Salaniè, 1997, ch. 5) points out that with risk aversion of borrowers and lenders there appears a trade-off between finance and insurance leading to contracts other than SDCs. Another question regards situations when risks are not statistically independent. Note, from the literature on choice under uncertainty it is known that, e.g., there exist circumstances when adding a further source of risk could be desirable from the lenders' point of view (cf. Kihlstrom et al., 1981, p. 913). Thus, it is not immediately clear whether the results of the analysis of this paper still hold in these cases. Therefore, more research is needed in these fields.

Appendix

Proof of Proposition 2: Consider the Lagrangean of maximization problem (2)

$$\begin{aligned}\mathcal{L} &= \int_{\underline{y}}^{t_0} \left(y - c - \overline{t(y)}\right) dF(y) + \int_{t_0}^{\overline{y}} (y - t_0)\, dF(y) + \\ &+\lambda \left(\int_{\underline{y}}^{t_0} U\left(\overline{t(y)}\right) dF(y) + \int_{t_0}^{\overline{y}} U(t_0)\, dF(y) - U(R) \right) + \\ &+\mu \left(y - c - \overline{t(y)}\right). \qquad (17)\end{aligned}$$

The corresponding first order necessary conditions with respect to $\overline{t(y)}$, t_0, λ, and μ applying the Kuhn-Tucker-Theorem are:

$$\frac{\partial \mathcal{L}}{\partial \overline{t(y)}} = -f(y) + \lambda U'\left(\overline{t(y)}\right) f(y) - \mu = 0 \quad \forall\, y < t_0 \qquad (18)$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial t_0} &= \left(t_0 - c - \overline{t(t_0)}\right) f(t_0) - (1 - F(t_0)) + \\ &+\lambda \left(U'(t_0)(1 - F(t_0)) - f(t_0)\left(U(t_0) - U\left(\overline{t(t_0)}\right)\right)\right) = 0 \qquad (19)\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \lambda} &= \int_{\underline{y}}^{t_0} U\left(\overline{t(y)}\right) dF(y) + \int_{t_0}^{\overline{y}} U(t_0)\, dF(y) - U(R) \geq 0\ ;\ \lambda \geq 0 \\ &\frac{\partial \mathcal{L}}{\partial \lambda}\lambda = 0 \qquad (20)\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = y - c - \overline{t(y)} \geq 0\ ;\ \mu \geq 0\ ;\ \frac{\partial \mathcal{L}}{\partial \mu}\mu = 0 \quad \forall\, y < t_0 \qquad (21)$$

Now, equations (18) and (19) can be transformed into

$$\lambda = \frac{f(y) + \mu}{U'\left(\overline{t(y)}\right) f(y)} \quad \forall\, y < t_0 \qquad (22)$$

$$\lambda = \frac{(1 - F(t_0)) - f(t_0)\left(t_0 - c - \overline{t(t_0)}\right)}{U'(t_0)(1 - F(t_0)) - f(t_0)\left(U(t_0) - U\left(\overline{t(t_0)}\right)\right)}. \qquad (23)$$

From (22) one can observe $\lambda > 0$ due to $f(y) > 0$, $\mu \geq 0$, and $U'(\overline{t(y)}) > 0\ \forall\, \overline{t(y)}$. Thus, from (20) the lender's participation constraint must be binding in the optimum. Now, let t_0^* be the optimal value of t_0 which satisfies (19) for the optimal repayment function $\overline{t(y)}$. Therefore, one can use (23) to calculate the corresponding value of λ which has to be constant. Given this information, one can ask what the optimal repayment function $\overline{t(y)}\ \forall\ y < t_0$ looks like. To answer this question I look at equation (22). There are two possible cases: In the first case, where $\mu = 0\ \forall\ y < t_0$, the Kuhn-Tucker-Theorem implies that $y - c > \overline{t(y)}\ \forall\ y < t_0$ must hold. In the second case, where $\mu > 0\ \forall\ y < t_0$, it must be true that $y - c = \overline{t(y)}\ \forall\ y < t_0$ by the same reasoning.

Consider the first case. With $\mu = 0\ \forall\ y < t_0$ equation (22) can be simplified to

$$\lambda = \frac{1}{U'\left(\overline{t(y)}\right)} = \text{constant} \quad \forall\ y < t_0.$$

For this equation to hold it must be true that $U'(\overline{t(y)})$ is also constant for all $y < t_0$ which in turn holds if and only if $\overline{t(y)} = \text{constant}\ \forall\ y < t_0$ due to $U''(\overline{t(y)}) < 0\ \forall\ \overline{t(y)}$. But, in fact, since the borrower does not have any own funds he can pay at most $y - c\ \forall\ y < t_0$. Therefore, the repayment in the case of verification can not be constant for all realizations of $y < t_0$ – a contradiction.

Now, consider the second case. First, note that $\mu > 0\ \forall\ y < t_0$ means that for every realization of $y < t_0$ there has to be a constant $\mu > 0$. But for two different realizations $y_1 < t_0$ and $y_2 < t_0$ the parameter μ need not be constant. Thus, since the borrower can pay at most $y - c\ \forall\ y < t_0$ the repayment $\overline{t(y)}$ need not be constant if and only if $\mu > 0$ and (22) still holds. But with $\mu > 0$ it follows from (21) that in optimum $y - c = \overline{t(y)}\ \forall\ y < t_0$.

Thus the optimal repayment function when there is verification is $\overline{t(y)} = y - c$.
□

Proof of Proposition 3: Differentiating (7) with respect to t_0 yields

$$\frac{d\mathrm{E}(t(y))}{dt_0} = (1 - F(t_0)) - cf(t_0). \tag{24}$$

Due to the properties of the cumulative distribution function $F(y)$ and the probability density function $f(y)$ and because of $c > 0$ the sign of (24) is ambiguous.

Now, consider the first order necessary condition (23) above. With the optimal repayment function when there is verification, (23) can be rewritten as

$$\lambda = \frac{1 - F(t_0)}{U'(t_0)(1 - F(t_0)) - f(t_0)(U(t_0) - U(t_0 - c))}. \tag{25}$$

Note, with the optimal repayment function $\overline{t(y)} = y - c\ \forall\ y < t_0$ the repayment converges to $t_0 - c$ as $y \to t_0\ \forall\ y < t_0$. Further, as pointed out in the proof of proposition 2 in optimum $\lambda > 0$. Thus, because of $1 - F(t_0) > 0$ due to the properties of cumulative distribution functions the following condition must hold:

$$U'(t_0)(1 - F(t_0)) - f(t_0)(U(t_0) - U(t_0 - c)) > 0 \tag{26}$$

Furthermore, I apply a Taylor series expansion of the utility function $U(t)$ around t_0 up to order one to get

$$U(t) = U(t_0) + U'(t_0)\,(t - t_0) + \tfrac{1}{2}U''(t^*)\,(t - t_0)^2\,, \tag{27}$$

where t^* is some value between t_0 and t.[8] With (27) one derives for $t = t_0 - c$:

$$U(t_0 - c) = U(t_0) - U'(t_0)c + \tfrac{1}{2}U''(t^*)\,(-c)^2\,. \tag{28}$$

Replacing $U(t_0 - c)$ in (26) using (28) and rearranging terms yields

$$U'(t_0)\,((1 - F(t_0)) - cf(t_0)) + \tfrac{1}{2}U''(t^*)\,(-c)^2 > 0. \tag{29}$$

Due to $U' > 0$ and $U'' < 0$ equation (29) is satisfied if and only if $(1 - F(t_0)) - cf(t_0) > 0$ in optimum. But this is exactly the expression derived for $d\mathrm{E}(t(y))/dt_0$ above. Thus, in optimum it must be true that

$$\frac{\mathrm{E}(t(y))}{dt_0} > 0$$

and therefore an increase of the expected repayment requires the payment obligation t_0 of the SDC to rise.

Furthermore, the expected profit of the borrower with SDC

$$\mathrm{E}(P) = \int_{t_0}^{\overline{y}} (y - t_0)\,dF(y)$$

decreases when t_0 rises since

$$\frac{d\mathrm{E}(P)}{dt_0} = -\,(1 - F(t_0)) < 0.$$

□

Proof of equation (15): For proving equation (15) I adopt the technique of Eeckhoudt and Kimball (1991, p. 244 f.). First, consider the definition of the derived utility function (9) which can be rewritten as

$$\begin{aligned} V\,(t(y) + \mathrm{E}(z)) &= \int_{\underline{\tilde{z}}}^{\overline{\tilde{z}}} U\,(t(y) + \mathrm{E}(z) + \tilde{z})\,dG(\tilde{z}) \\ &= U\,(t(y) + \mathrm{E}(z) - \pi_U\,(\tilde{z}, t(y), \mathrm{E}(z))) \end{aligned} \tag{30}$$

[8]Note, the Taylor series expansion is not an approximation of the utility function. This is true since the term $\frac{1}{2}U''(t^*)(t-t_0)^2$ represents the error of the linear approximation of $U(t)$. Therefore, t^* is not an arbitrary value, rather it is the point where the utility function must be evaluated to correctly calculate the error of approximation (see Chiang, 1984, p. 256ff., for more details).

due to the definition of the equivalent risk premium of Kimball (1990, p. 56). From (30) one derives

$$\frac{\partial V(\cdot)}{\partial t(y)} = V'\left(t(y)+\mathrm{E}(z)\right) = U'\left(t(y)+\mathrm{E}(z)-\pi_U\left(\tilde{z},t(y),\mathrm{E}(z)\right)\right)\left(1-\frac{\partial \pi_U(\cdot)}{\partial t(y)}\right)$$
$$= U'\left(t(y)+\mathrm{E}(z)-\psi_U\left(\tilde{z},t(y),\mathrm{E}(z)\right)\right) \tag{31}$$

where the second line of (31) follows from Kimball (1990, p. 55). That is, $\psi(\cdot)$ is the equivalent precautionary premium which is defined as the certain reduction from $t(y)+\mathrm{E}(z)$ that has the same effect on the optimal value of $t(y)$ as the addition of the random variable $\tilde{z}$ (cf. Kimball, 1990, p. 55). Furthermore, from differentiating (31) with respect to $t(y)$ one yields

$$\frac{\partial^2 V(\cdot)}{(\partial t(y))^2} = V''\left(t(y)+\mathrm{E}(z)\right) = U''\left(t(y)+\mathrm{E}(z)-\psi_U\left(\tilde{z},t(y),\mathrm{E}(z)\right)\right)\left(1-\frac{\partial \psi_U(\cdot)}{\partial t(y)}\right). \tag{32}$$

With (31) and (32) the Arrow – Pratt measure of absolute risk aversion of the utility function $V(\cdot)$ can be calculated as

$$-\frac{V''(\cdot)}{V'(\cdot)} = -\frac{U''\left(t(y)+\mathrm{E}(z)-\psi_U\left(\tilde{z},t(y),\mathrm{E}(z)\right)\right)}{U'\left(t(y)+\mathrm{E}(z)-\psi_U\left(\tilde{z},t(y),\mathrm{E}(z)\right)\right)}\left(1-\frac{\partial \psi_U(\cdot)}{\partial t(y)}\right). \tag{33}$$

To compare the levels of absolute risk aversion for the two utility functions $V(\cdot)$ and $U(\cdot)$ some further assumptions are needed. These assumptions regard to the decision maker's behavior when there appears an additional risk. Kimball (1993, p. 589) states that an individual who is already exposed to risk should be less willing to bear an additional one, even when both risks are independent. Kimball (1993) refers to this behavioral rule as *standard risk aversion* an shows that decreasing absolute risk aversion in the Arrow – Pratt sense and decreasing absolute prudence are necessary and sufficient therefore. The notion of prudence means "[...] the propensity to prepare and forearm oneself in the face of uncertainty [...]" (Kimball, 1990, p. 54) for which the measure of *absolute prudence* is an adequate one. Kimball (1990, p. 54ff.) defines the measure of absolute prudence as $-\frac{U'''(\cdot)}{U''(\cdot)}$ and shows the equivalent precautionary premium $\psi_U(\cdot)$ to be proportional to this measure. Thus, standard risk aversion requires $U'''(\cdot) > 0$ and therefore $\psi_U(\cdot) > 0$ and $\frac{\partial \psi_U(\cdot)}{\partial t(y)} \leq 0$ (cf. Eeckhoudt and Kimball, 1991, p. 240f.).

For the following arguments I suppose the utility function $U(\cdot)$ to exhibit standard risk aversion. Thus, from (33) one can derive the following relations to hold:

$$-\frac{V''\left(t(y)+\mathrm{E}(z)\right)}{V'\left(t(y)+\mathrm{E}(z)\right)} \geq -\frac{U''\left(t(y)+\mathrm{E}(z)-\psi_U\left(\tilde{z},t(y),\mathrm{E}(z)\right)\right)}{U'\left(t(y)+\mathrm{E}(z)-\psi_U\left(\tilde{z},t(y),\mathrm{E}(z)\right)\right)}$$
$$\geq -\frac{U''\left(t(y)+\mathrm{E}(z)\right)}{U'\left(t(y)+\mathrm{E}(z)\right)}. \tag{34}$$

In (34) the first inequality follows from $\frac{\partial \psi_U(\cdot)}{\partial t(y)} \leq 0$ and the second one appears since $U'''(\cdot) > 0$ (cf. Eeckhoudt and Kimball, 1991, p. 245). Therefore, the utility function $V(\cdot)$ exhibits a larger degree of absolute risk aversion in the Arrow – Pratt sense than $U(\cdot)$ for any $\mathrm{E}(z)$. As a result it must be true that

$$\pi_V\left(t(y), \mathrm{E}(z)\right) \geq \pi_U\left(t(y), \mathrm{E}(z)\right)$$

holds for any given value of $\mathrm{E}(z)$. □

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