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Angaben zur Veröffentlichung / Publication details:

Maußner, Alfred, and Julius Spatz. 2001. "Determinants of business cycles in small scale macroeconomic models: the German case." Augsburg: Volkswirtschaftliches Institut, Universität Augsburg.

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Determinants of Business Cycles in Small Scale Macroeconomic Models: The German Case

November 2001

Abstract

We identify measures of shocks to total factor productivity and preferences from two real business cycle models and subject them to Granger causality tests to see whether they can be considered exogenous to other plausible sources of the German business cycle in the mid nineteen seventies and nineteen eighties. We find no evidence to reject the exogeneity of our shock measures. This results contrasts with similar studies for other countries that question the exogeneity of either productivity or preference shocks.

Keywords Real Business Cycles, Solow Residual, Granger Causality

JEL Classification E32, O47

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I. Introduction

Small scale dynamic general equilibrium models have become the dominant tool of business cycle analysis. The driving force of these models are a few stochastic processes that mimic either demand or supply shocks that permanently hit the economy. These shocks trigger intra- and intertemporal substitution between leisure, consumption, and asset holdings that cause patterns similar to those found in macroeconomic time series. If this story is a credible explanation of the business cycle, the various shock measures used should be exogenous in the sense that they are not themselves caused by other variables that one might regard as alternative driving forces of the cycle.

Among the most prominent shock measures is the Solow residual, i.e., that part of output growth which is not due to increased use of labor and capital. The seminal papers of Kydland and Prescott (1982) and Long and Plosser (1983) aim to show that a great part of observed output fluctuations is explained by shocks to total factor productivity approximated by the Solow residual. Subsequent papers cast doubt on the validity of this approach since they show that the Solow residual is Granger caused by real and monetary variables (Evans (1992) for the US, Cozier and Gupta (1993) for Canada, and Holland and Scott (1998) for the UK). Among the explanations for this lack of exogeneity are variable utilization rates of capital and labor (Burnside, Eichbaum, and Rebelo (1993), Burnside and Eichenbaum (1996), Finn (1995), Paquet and Robidoux (2001)) and cyclical markups (Hornstein (1993), Rotemberg and Woodford (1992), and Hairault and Portier (1993)).

Holland and Scott (1998) introduce stochastic shifts of the marginal rate of substitution between leisure and consumption into an otherwise standard real business cycle model to capture demand shocks. Their measure of the preference shift parameter for the UK is Granger caused by the GDP deflator, the retail price index, the nominal and real price of oil.

In this paper we examine the issue of the exogeneity of technology and demand shocks for the West-German economy. We limit our investigation to quarterly data that cover the time period 76.i to 89.iv. We have chosen this period for the following reasons. Firstly, we want to exclude possible structural breaks associated with the German reunification in the fall of 1990. Secondly, considering the time between 1960 (from where onwards quarterly national accounts are available) and the mid nineteen seventies, there is evidence that the West-German economy was not on a long-run growth path. However, the calibration of the model's parameters that are necessary to identify the technology and the preference shock rely on the steady-state assumption. Using the Holland and Scott (1998) model as well as a more elaborate version allowing for oil price shocks, a variable utilization of capital, and a declining trend in working hours, we identify two different measures of the technology and preference shock and test within an error correction framework their exogeneity with respect to government consumption, taxes, M1 and M3, short and long-term interest rates, the trade balance, and the terms of trade. In the face of the existing evidence for other countries, our results are quite surprising: none of our shock measures can be predicted by past realizations of the above mentioned variables.

The remaining of the paper is structured as follows. The next section sets up the theoretical framework we use to identify our shock measures. In Section III, we derive the shocks from the data and test for Granger causality. Section IV concludes.

II. Theoretical Framework

1. A Basic Model

The basic real business cycle model with a technology and a preference shock consists of a representative household who solves at time t the following program:

$$\begin{aligned} &\max \qquad E_t \sum_{s=0}^{\infty} \beta^s u(C_{t+s}, 1 - N_{t+s}, \theta_{t+s}), \qquad \beta \in (0,1) \\ &\text{subject to } K_{t+s+1} \leq (1-\delta)K_{t+s} + F(N_{t+s}, K_{t+s}, A_{t+s}) - C_{t+s}. \end{aligned} \tag{II.1}$$

Utility u at period $t+s$ depends upon consumption C_{t+s}, leisure $1-N_{t+s}$, and the realization of the preference shock θ_{t+s}. Expected life-time utility at time t is the discounted flow of utilities u with discount factor β^s attached to utility obtained s periods hence. Output is a function F of working hours N_{t+s}, capital services K_{t+s}, and the stochastic level of technological progress A_{t+s}. Future capital K_{t+s+1} is equal to the stock of capital inherited from the previous period $(1-\delta)K_{t+s}$, where $\delta \in [0,1]$ is the rate of depreciation, plus investment $F(\cdot) - C_{t+s}$.

Given representations of both the current-period utility function u and the production function F, the usual procedure is to compute measures of θ_t and A_t from actual data using the first order conditions of (II.1).

As usual, we parameterize F as a constant-returns-to-scale Cobb-Douglas function

$$F(N_t, K_t, A_t) := B(A_t N_t)^{\alpha} K_t^{1-\alpha}, \quad \alpha \in (0,1), \quad B > 0, \tag{II.2}$$

where labor-augmenting technological progress A_t evolves according to

$$A_{t+1} = A_t e^{a+\epsilon_t^A}, \qquad a \geq 0, \quad \epsilon \sim N(0, \sigma^A). \tag{II.3}$$

To derive the preference shock, we specify u as

$$u(C_t, 1 - N_t, \theta_t) := \frac{C_t^{1-\eta}(1 - N_t)^{\theta_t(1-\eta)} - 1}{1 - \eta}, \qquad \eta > 0. \tag{II.4}$$

The first order conditions for (II.1) with respect to consumption and leisure at time t imply

$$\theta_t = \alpha \frac{1 - N_t}{N_t} \frac{Y_t}{C_t}. \tag{II.5}$$

Solving (II.2) for A_t provides

$$A_t = (Y_t/B)^{1/\alpha} K_t^{(\alpha-1)/\alpha} N_t^{-1}. \tag{II.6}$$

Equations (II.5) and (II.6) allow to derive the model's shocks from the national accounts. Before we proceed towards that goal, we develop a more elaborate version of this model that captures two distinctive features of the West-German economy: Firstly, working hours per member of the work force have steadily declined since the nineteen sixties. Secondly, West Germany depends on energy imports.

2. A More Elaborate Model

To account for the decline in working hours, we follow Lucke (1997) and assume that the disutility of labor increases with the level of technological knowledge. Therefore, we measure leisure as $1 - A_t^{\nu} N_t$, with $\nu > 0$, in the household's utility function u.

We use the device developed by Finn (1995) to model the dependence on energy imports and assume that output of period t, Y_t, is produced according to the following production function:

$$Y_t = B(A_t N_t)^{\alpha}(v_t K_t)^{1-\alpha}, \quad \alpha \in (0, 1), \quad B > 0, \tag{II.7}$$

where, as before, A_t is the level of labor-augmenting technical progress and N_t are working hours. Different from the basic model, we allow for less than full utilization of capital K_t and let v_t denote the respective utilization rate. The process that governs A_t is still given by (II.3).

Let W_t and R_t denote the real wage and the rental rate of capital services $v_t K_t$. Profit maximization on competitive markets implies:

$$W_t = \alpha \frac{Y_t}{N_t} = \alpha A_t B(A_t N_t)^{\alpha-1}(v_t K_t)^{1-\alpha}, \tag{II.8a}$$

$$R_t = (1 - \alpha) \frac{Y_t}{v_t K_t} = (1 - \alpha) B(A_t N_t)^{\alpha}(v_t K_t)^{-\alpha}. \tag{II.8b}$$

The household accumulates capital according to

$$K_{t+1} = (1 - \delta(v_t))K_t + I_t \tag{II.9a}$$

$$\delta(v_t) := v_t^{\omega}/\omega, \quad \omega \geq 1, \tag{II.9b}$$

where I_t denotes the household's investment expenditures. The dependence of the rate of depreciation δ on the utilization rate of capital v_t captures the idea that wear and tear increase with a more intense use of the capital equipment. This assumption dates back to papers by Taubman and Wilkinson (1970) and Greenwood, Hercowitz, and Huffman (1988), and was also employed by Finn (1995) and by Burnside and Eichenbaum (1996) to account for factor hoarding over the business cycle.

To account for the influence of energy prices, we follow Finn (1995) and assume that the higher capital utilization rate, the more energy per unit of capital is required. Specifically, we postulate:

$$\frac{Z_t}{K_t} = v_t^{\gamma}/\gamma, \quad \gamma \geq 1. \tag{II.10}$$

The household spends its net income, i.e., wages W_tN_t and capital rents $R_tv_tK_t$ less government taxes T_t, on energy imports p_tZ_t, consumption C_t, and investment I_t. Thus, its budget constraint reads:

$$I_t + C_t \leq w_tN_t + R_tv_tK_t - T_t - p_tZ_t. \tag{II.11}$$

The household seeks time profiles for consumption and leisure that maximize

$$E_t \sum_{s=0}^{\infty} \beta^s u(C_{t+s}, 1 - A_{t+s}^{\nu}N_{t+s}, \theta_{t+s})$$

subject to (II.9) and (II.11). The first order conditions for optimal time sequences read:

$$\Lambda_t = C_t^{-\eta}(1 - A_t^{\nu}N_t)^{\theta_t(1-\eta)}, \tag{II.12a}$$

$$A_t^{-\nu}W_t\Lambda_t = \theta_tC_t^{1-\eta}(1 - A_t^{\nu}N_t)^{\theta_t(1-\eta)-1}, \tag{II.12b}$$

$$R_t = v_t^{\omega-1} + p_tv_t^{\gamma-1}, \tag{II.12c}$$

$$\Lambda_t = \beta E_t\Lambda_{t+1}\Big(1 - (u_{t+1}^{\omega}/\omega) - p_{t+1}(u_{t+1}^{\gamma}/\gamma) + R_{t+1}u_{t+1}\Big), \tag{II.12d}$$

where Λ_t is the shadow price of capital. According to (II.12a), this shadow price equals the marginal utility of consumption. (II.12b) states that the marginal disutility of an additional hour of work has to be compensated by the increase of utility derived from spending the extra income generated on consumption. Equation (II.12c) balances the marginal costs and benefits of changing the utilization rate of capital. The rate of change of the price of new capital is determined in equation (II.12d). It balances the current utility loss of saving one extra euro, Λ_t, with its discounted expected future utility gain, the latter being equal to the discounted expected utility increase from spending the gross return $1 - \delta(u_{t+1}) - p_{t+1}(Z_{t+1}/K_{t+1}) + r_{t+1}u_{t+1}$ on consumption in the next period.

We model government expenditures as a pure transfer of resources from the private to the public sector without any feed-back effects that would arise if they were considered an argument of either the household's utility function or the economy's production function. This transfer grows deterministically at the same rate as output increases in the long run, so that the government does not contribute to economic fluctuations. These assumptions can be summarized in the following equations:

$$G_t = T_t,$$
$$G_t = e^{g_Y t} G_0,$$

where g_Y is the growth rate of output on a balanced growth path, which is derived in the following section.

3. Dynamics

We want to calibrate the model's key parameters from its implications for a deterministic balanced growth path. Our next task is, thus, to seek a transformation that yields new variables being constant on such a path. It is obvious from the utility function that $A_t^{\nu} N_t$ must be constant in the long run. Thus, we define $n_t := A_t^{\nu} N_t$. Furthermore, in the steady state both the capital-output ratio K_t/Y_t and the utilization rate of capital v_t should be constant. Thus, from equation (II.8b), the steady-state rental rate of capital R_t is constant. We can use this implication to look for an adequate transformation of the capital stock:

$$\begin{aligned} R_t &= (1-\alpha)B(A_t N_t)^{\alpha}(v_t K_t)^{-\alpha}, \\ &= (1-\alpha)B A_t^{\alpha(1-\nu)} n_t^{\alpha}(v_t K_t)^{-\alpha}. \end{aligned}$$

The last line tells us that ultimately the capital stock will grow at the rate of $g_K = e^{a(1-\nu)} - 1$. We use the transformation

$$k_t := \frac{K_t}{A_{t-1}^{1-\nu}},$$

which guarantees that the new variable is predetermined at the beginning of period t as a result of past realizations of the technology shock and past investment decisions. The equilibrium condition for the labor market (II.8a) may be written as

$$\begin{aligned} W_t &= \alpha B A_t^{\alpha+(1-\alpha)\nu} n_t^{\alpha-1}(v_t k_t)^{1-\alpha} A_{t-1}^{(1-\alpha)(1-\nu)}, \\ &= \alpha B A_t (A_t/A_{t-1})^{(\alpha-1)(1-\nu)} n_t^{\alpha-1}(v_t k_t)^{1-\alpha}, \end{aligned}$$

from which we see that $w_t := W_t/A_t$ is stationary. It is obvious from the household's budget constraint that in the long run consumption and government expenditures must grow at the same rate as the capital stock. Therefore, we define $c_t := C_t A_t^{\nu-1}$, $g_t := G_t A_t^{\nu-1}$ and derive the adequate transformation of the shadow price of new capital from equation (II.12a):

$$\lambda_t := \Lambda_t A_t^{\eta(1-\nu)} = \left(\frac{C_t}{A_t^{1-\nu}}\right)^{-\eta} (1-n_t)^{\theta_t(1-\eta)}.$$

Given these definitions, we combine equations (II.7), (II.8), (II.9), (II.11), and (II.12) and arrive at the following system of equations that governs the time paths of our transformed variables:

$$\lambda_t = c_t^{-\eta}(1-n_t)^{\theta_t(1-\eta)}, \tag{II.13a}$$

$$w_t = \theta_t \frac{c_t}{1-n_t}, \tag{II.13b}$$

$$w_t = \alpha B e^{(1-\alpha)(\nu-1)[a+\epsilon_{t-1}^A]} n_t^{\alpha-1}(v_t k_t)^{1-\alpha}, \tag{II.13c}$$

$$R_t = (1-\alpha) B e^{\alpha(1-\nu)[a+\epsilon_{t-1}^A]} n_t^{\alpha}(v_t k_t)^{-\alpha}, \tag{II.13d}$$

$$R_t = v_t^{\omega-1} + p_t v_t^{\gamma-1}, \tag{II.13e}$$

$$\begin{aligned} k_{t+1} &= B e^{(\alpha-1)(1-\nu)[a+\epsilon_{t-1}^A]} n_t^{\alpha}(v_t k_t)^{1-\alpha} \\ &\quad + (1-(v_t^{\omega}/\omega) - p_t(v_t^{\nu}/\nu))\, e^{(\nu-1)[a+\epsilon_{t-1}^A]} k_t - c_t - g_t, \end{aligned} \tag{II.13f}$$

$$\lambda_t = \beta e^{\eta(\nu-1)(a+\epsilon_t^A)} E_t \lambda_{t+1} \left(1-(u_{t+1}^{\omega}/\omega) - p_{t+1}(u_{t+1}^{\gamma}/\gamma) + u_{t+1}R_{t+1}\right). \tag{II.13g}$$

We get the deterministic counterpart of our model by replacing the technology shock, the preference shock, and the energy-price shock with their expected values of e^a, θ and p, respectively. This permits us to omit the expectation operator. If we further drop time indices, the system of equations (II.13) determines the model's long-run equilibrium. We use these relations to calibrate the model to West-German data.

If $n_t := A_t^{\nu} N_t$ is constant and $\nu > 0$, hours per capita decline at the rate $g_N = e^{-a\nu} - 1$ and output grows at the rate $g_Y = e^{a(1-\nu)} - 1$. Thus, we can use the long-run rate of output growth g_Y and the rate of change of hours per capita g_N to infer ν and a from

$$\begin{aligned} \ln(1+g_Y) &= a(1-\nu), \\ \ln(1+g_N) &= -a\nu. \end{aligned} \tag{II.14}$$

We set the long-run rate of capital depreciation δ equal to the average rate of capital depreciation and compute this rate from quarterly data of depreciation and the capital stock. We construct the latter from yearly data of the capital stock and quarterly data of net investment expenditures via the perpetual inventory method.

The Euler equation for the price of new capital

$$1 = \beta e^{-\eta(1-\nu)a} \underbrace{(1 - \delta - p(Z/K) + Ru)}_{q} \tag{II.15}$$

provides two options to infer the magnitude of the discount factor β. Given information on the long-run gross rate of return on equities q and the intertemporal elasticity of substitution $1/\eta$, we may compute β from

$$\beta = \frac{e^{\eta(1-\nu)a}}{q}. \tag{II.16}$$

Alternatively, using the stationary version of (II.8b), equation (II.15) may also be written as

$$1 = \beta e^{-\eta(1-\nu)a} \left(1 - \delta - \left[\frac{pZ}{Y} - (1-\alpha)\right] \frac{Y}{K}\right), \tag{II.17}$$

which allows us to derive β from the capital-output ratio K/Y, the fraction of output spent on energy imports (pZ/Y), and the elasticity of production with respect to labor α. As usual, the latter parameter is set equal to the long-run wage share.

We derive point estimates of γ, ω, and v from the fraction of output spent on energy imports $\zeta := (pZ/Y)$, the rate of capital depreciation δ, and the capital-output ratio K/Y. Notice that equations (II.13d) and (II.13e) imply

$$Rv = v^\omega + pv^\gamma = (1-\alpha)\frac{Y}{K},$$

which we arrange to read

$$\underbrace{\frac{pv^\gamma}{\gamma}\frac{K}{Y}}_{\zeta} = \frac{1-\alpha}{\gamma} - \frac{\omega}{\gamma}\frac{K}{Y}\underbrace{\frac{v^\omega}{\omega}}_{\delta}.$$

Thus, together with the definitions in (II.9b) and (II.10), the following system of equations jointly determines ω, γ, and v:

$$\begin{aligned} \zeta &= \frac{1-\alpha}{\gamma} - \frac{\omega}{\gamma}\frac{K}{Y}\delta, \\ \delta &= \frac{v^\omega}{\omega}, \\ \zeta\frac{Y}{K} &= p\frac{v^\gamma}{\gamma}, \end{aligned} \tag{II.18}$$

where p is the average relative price of imported energy.

III. Productivity and Preference Shocks

1. Identification of the Shocks

Given the model's deep parameters, we are able to construct the productivity and preference shocks from the model's equations and published data.

Equations (II.8b) and (II.12c) imply:

$$v_t^{\omega} + p_t v_t^{\gamma} = (1-\alpha)\frac{Y_t}{K_t}. \tag{III.1}$$

Together with the law for capital accumulation (II.9) and an initial value of the capital stock, this equation implies an empirical series for the utilization rate of capital v_t from published data on output Y_t, the relative price of imported energy p_t, and investment expenditures I_t. Given the series on v_t and K_t, we derive the level of technical progress from the production function using published data on working hours and output:

$$A_t = (Y_t/B)^{1/\alpha}(v_t K_t)^{(\alpha-1)/\alpha} N_t^{-1}. \tag{III.2}$$

We use the value of B to normalize $A_{t=1} \equiv 1$. Given this series we construct $n_t = A_t^{\nu} N_t$ and compute the preference shock from

$$\theta_t = \alpha \frac{1-n_t}{n_t}\frac{Y_t}{C_t} \tag{III.3}$$

using data on output and consumption. These equations are the counterparts to the simpler shock measures given in (II.6) and (II.5).

We use West-German quarterly data from 1976.i to 1989.iv for the following reasons. Between 1960 and the mid nineteen seventies, the consumption share in output has steadily increased, which is obviously at odds with the steady-state assumption. In addition, no spot-market data on oil prices and oil imports are available before the first quarter of 1976. We have not used post-1989 data since we want to exclude possible structural breaks due the German reunification in the fall of 1990. To take account of the representative agent character of our model, we use per-capita data on output, consumption, investment, capital, and working hours. If not otherwise mentioned, we use seasonally-adjusted time series from the database provided by the German Institute of Economic Research (DIW). Our measure of output is the gross domestic product per capita at factor prices, which grew at an average quarterly rate of $g_Y = 0.47\%$. Hours per capita declined at an average quarterly rate of $g_N = 0.08\%$. Using (II.14) we find $a = 0.0055$ and $\nu = 0.144$.

We combine the yearly data of the capital stock provided by the German Statistical Office (Statistisches Bundesamt) and quarterly data on depreciation and gross investment to compute a quarterly series of the capital stock. Let $\bar{K}_j$ and K_t denote the stock of capital at the beginning of year $j = 0, 1, ..., 13$ and at the beginning of quarter $t = 1, 2, ..., 56$, respectively. For $t = 4j+1$, we set $K_t = \bar{K}_j$, and for $t = 4j+1+s,\ s = 1, 2, 3$, we compute K_t from

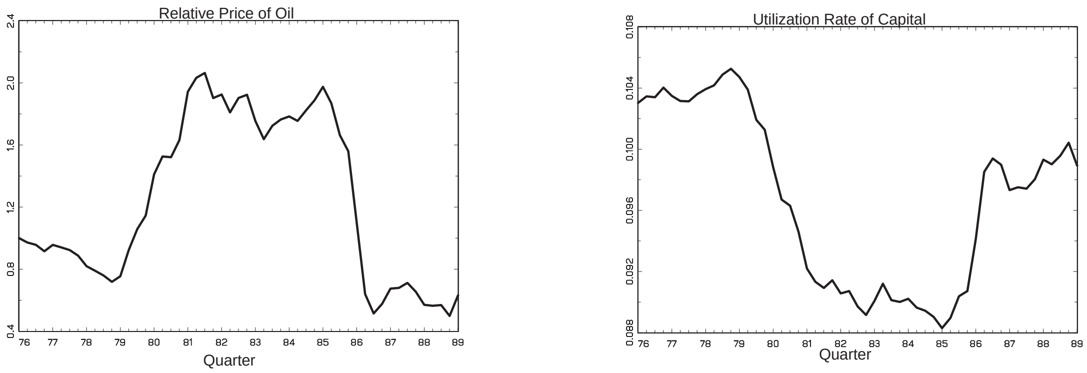
$$K_{t+1} = K_t + (I_t - D_t)\frac{\bar{K}_{j+1} - \bar{K}_j}{\sum_{i=1}^{4}(I_{4j+i} - D_{4j+i})},$$

where I_t and D_t are gross investment and depreciation of quarter t. Given this measure of capital we set δ equal to the average of D_t/K_t, which yields $\delta = 0.0108$.

The average expenditure on raw-oil imports as a fraction of the gross domestic product at factor prices is $\zeta = 0.0215$. The wages share in the gross domestic product at factor prices is $\alpha = 0.72$. We derived this figure assuming that the wage income of a self-employed person equals the average wage per employee.

Like many other authors, we assume that the coefficient of relative risk aversion equals $\eta = 2$, and use the average quarterly ex-post real interest rate on long-term bonds as measure of the rate of return on capital to compute β from (II.16). The value of β that is consistent with a yearly rate of return on capital of about 4.3 percent is $\beta = .9989$ and implies $Y/K = 0.084$ via (II.17).[1] The values assigned to α, δ, ζ, and Y/K imply via (II.18) the long-run utilization rate of capital $u = 0.10$, $\gamma = 2.45$, and $\omega = 1.73$.

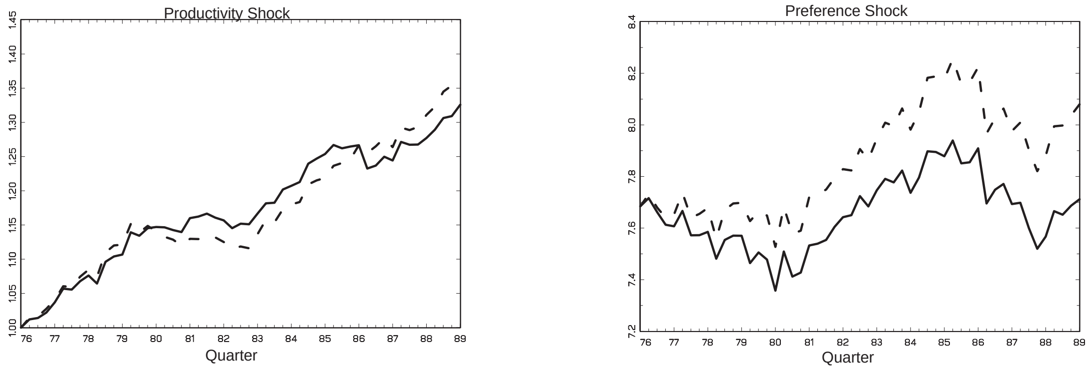
Figure III.1: Oil Price Shocks and Utilization Rate of Capital



[1]King, Plosser, and Rebelo (1982) use the average real return on equity to determine the value of β. In the period considered, the FAZ stock index implies a real return on West German equity of about 7% p.a.. Using the related value of β, we find no significant differences in our results. The shocks computed from the more elaborate model, which are the only ones affected by the choice of β, are almost perfectly correlated to those reported in the text.

We are now able to construct our measures of productivity and preference shocks. The left panel of Figure III.1 displays the relative price of oil, measured as the ratio of the price of imported oil to the GDP deflator. As can be seen from the right panel of Figure III.1, the price increase in the nineteen eighties let the utilization rate of capital drop sharply. As a consequence, the traditional method to compute the Solow Residual from

Figure III.2: Productivity and Preference Shocks



(II.6) systematically overestimates the productivity shocks in the first half of nineteen eighties and underestimates them in the second half. This can be seen from the left panel of Figure III.2, where the dashed line depicts the usual measure of the Solow Residual. The solid line shows the productivity shock computed from (III.2). The right panel of Figure III.2 displays the preference shock. If measured by (II.5), the decline in working hours shows up in an upward sloping trend of the dashed line. The solid line represents the preference shock measure from (III.3). We will refer to the shocks from the basic model as model-one shocks and to those from the more elaborate model as model-two shocks.

2. Granger Causality Tests

Exogenous Variables We investigate the exogeneity of our measures of the productivity and the preference shock in the framework of Granger causality tests. If these shocks are indeed the driving forces of the business cycle, it should be impossible to predict them from past realizations of other variables that are also exogenous to the model. Since we have assumed that government expenditures and, hence, tax revenues grow at a constant rate, we include measures of both variables in the set of plausible driving forces of the German business cycle. We capture monetary shocks with a narrow (M1) and a broad (M3) measure of money supply, as well as with a short-term and a long-term nominal interest rate. The trade balance and the terms of trade are used to indicate demand and supply side shocks that originate in the world market.

Figure III.3: Time Paths of Test Variables

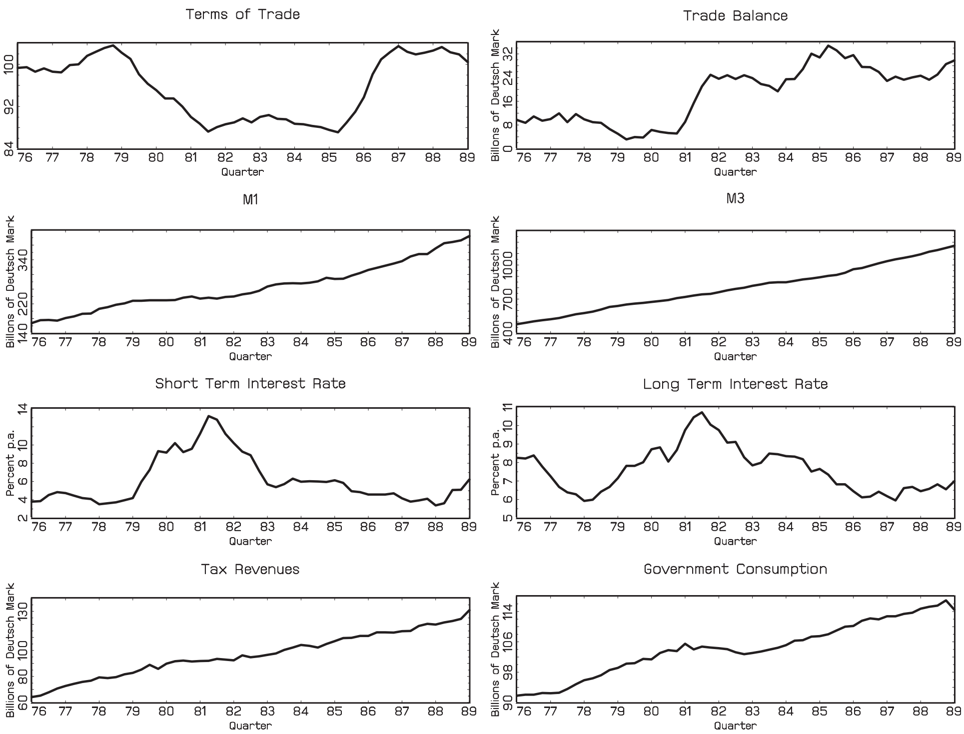


Figure III.3 depicts the time path of these variables. Like our shocks, they are either upward trending or display a highly persistent behavior. Before we can proceed with running regressions, we must determine the nature of this non-stationarity.

Unit Roots Tests Table III.1 displays the results from various unit-root tests. Column 2 refers to F-tests with the null of a random walk with drift versus the alternative of a trend-stationary process. We employ this test only for those variables that show a clear upward trend. At the five percent level, the test rejects the null only in the case of M3. However, neither the augmented Dickey-Fuller t-test nor the Phillips-Perron Z_t statistic reject the null of a unit root in any of the cases considered at the five percent level. Columns 5 and 6 show that none of our variables is integrated of order two. At the 5% level, the ADF t-statistic rejects the random walk model for the first differences for all but one variables considered, the Phillips-Perron Z_t for all of them. As a consequence, we accept that all of our variables are integrated of order one or are at least highly persistent. This latter conclusion is motivated by two observations. Firstly, per construction, our second measure

Table III.1:
Unit-Root Tests

Variable	Levels			First Differences	
	ADF-F	ADF-t	PP	ADF-t	PP
Productivity Shock (T)	1.351	−1.449	−1.590	−2.958**	−8.643***
Productivity Sho ck (OP)	4.261	−2.889	−2.729	−3.686***	−8.323***
Preference Shock (T)		−1.479	−2.362	−7.161***	−10.118***
Preference Shock (OP)		−1.984	−2.220	−6.952***	−9.943***
Government Expenditures	1.832	−1.055	−1.382	−7.264***	−7.396***
Taxes	6.675*	−3.453*	−3.462*	−7.921***	−7.923***
M1	1.635	−1.793	−1.976	−7.082***	−7.097***
M3	9.044**	−2.807	−2.692	−4.871***	−4.844***
Short Term Interest Rate		−1.934	−1.692	−4.423***	−4.393***
Long Term Interest Rate		−1.322	−1.698	−5.140***	−5.126***
Trade Balance	3.702	−2.673	−2.037	−4.499***	−6.167***
Terms of Trade		−1.787	−1.241	−1.947	−3.523**

Notes:
ADF-F: The augmented Dickey-Fuller F-statistic. The estimated model is

$$x_t = m + bt + rx_{t-1} + \sum_{i=1}^{q} a_i \Delta x_{t-i} + \epsilon_t \qquad \text{(i)}$$

and the true process is

$$x_t = \mu + x_{t-1} + \sum_{i=1}^{q} \alpha_i \Delta x_{t-i} + \epsilon_t, \qquad \text{(ii)}$$

where x_t refers to log of the variable in column 1, except in the case of the trade balance. The F-statistic refers to the null hypothesis $H_0 : b = 0, r = 1$. Lagged differences were included until the Box-Ljung statistic does not reject the null of no residual autocorrelation of up to fourth order at the 5% level. Critical values are from Dickey and Fuller (1981), p. 1063.

ADF-t: Augmented Dickey-Fuller t-statistic. The estimated and model and the true process are the same as in (i) and (ii). The test statistic is the t-statistic of the estimated r. Critical values are from MacKinnon (1991).

PP: Phillips-Peron Z_t statistic with lag truncation parameter equal to 3. The estimated model and the true process are as in (i) and (ii). The critical values are the same as those of the ADF t-statistic.

The ADF-t and PP tests for the first differences of the variables in column 1 estimate the model (i) without the time trend bt and assume (ii) without the drift term μ.

*, ** ,or *** denote rejection of the null at the 10%, 5%, or 1% level.

of the preference shock from (III.3) should be stationary. Secondly, it is well known that unit-root tests have small power against the alternative of a nearly integrated process. Nevertheless, it is save to proceed as if all series were integrated of order one.

Estimation Framework Under this proposition, we need to check whether variables that enter in a bivariate or multivariate vector autoregression are cointegrated. If so, the adequate framework to pursue Granger causality tests is the following autoregressive error correction model:

$$\begin{bmatrix}\Delta x_t\\ \Delta \mathbf{z}_t\end{bmatrix} = \begin{bmatrix}c_1\\ c_2\end{bmatrix} + \begin{bmatrix}A_{11}(L) & A_{12}(L)\\ A_{21}(L) & A_{22}(L)\end{bmatrix}\begin{bmatrix}\Delta x_{t-1}\\ \Delta \mathbf{z}_{t-1}\end{bmatrix} + \begin{bmatrix}\alpha_1(x_{t-1}-\beta_1\mathbf{z}_{t-1})\\ \alpha_2(x_{t-1}-\beta_2\mathbf{z}_{t-1})\end{bmatrix} \tag{III.4}$$

Here x_t is one of our shock measures and $\mathbf{z}_t$ is a subset of the variables that we consider to Granger cause the respective shock measure. The symbol Δ denotes first differences, A_{11} and A_{21} are polynomials in the lag operator L, whereas A_{12} and A_{22} are matrix polynomials in L that conform to the size of the vector $\mathbf{z}_t$. If the variables x_t and $\mathbf{z}_t$ are cointegrated, the expressions $(x_{t-1} - \beta_1\mathbf{z}_{t-1})$ and $(x_{t-1} - \beta_2\mathbf{z}_{t-1})$ capture deviations of the variables from their long-run equilibrium.

In this setting, the variables in $\mathbf{z}_t$ jointly Granger cause the shock measure x_t if the coefficients of A_{12} are significantly different from zero. We follow Holland and Scott (1998) and do not test whether or not the matrix α_1 is different from zero. The error correction term in the first equation captures the propagation of shocks but not their origin. Consider, e.g., a negative preference shock that temporarily lowers output growth. When tax revenues are tied to output, government expenditures will also fall below their trend path and help to predict future output growth.

Without cointegration, we have to drop the error correction term and estiamte the VAR in first differences.

Cointegration Tests We use the Johansen (1988, 1992) cointegration test. To select the appropriate VAR order, we use the Akaike (AIC) and Schwarz (SIC) information criteria together with the adjusted portmanteau statistic to check for the whiteness of the residual vector.[2] Since all three statistics are based on the estimated residuals which in turn depend upon the hypothesized number of cointegration relations, we based our decision on a two dimensional grid over the number of cointegration relations r and the number of lags q (see Patterson (2000), p.623f for this approach). We allowed for at most 9 lags in levels and selected the VAR order for each r either according to the AIC or according to the SIC under the restriction that the portmanteau statistic does not reject the null of whiteness of the residuals at the 5 percent level. There are a few cases where the portmanteau statistic rejects the null of whiteness for all lags considered. They comprise the two bivariate systems between the preference shocks and the terms of trade, and the system with our model-two measure of the preference shock and the trade balance. This may indicate that bivariate

[2]For a definition of these statistics in the framework of vector autoregressive models see, e.g., Lütkepohl (1991), equations (4.3.2), (4.3.9), and (4.4.23).

VARs are too small to capture the dynamics properly. In these cases we used the AIC and SIC alone to determine the lag length.

We find only two instances of cointegration. In the case of the VAR consisting of our model-two measure of the preference shock and the terms of trade the AIC chooses $q = 8$ and the trace statistic (but not the maximum eigenvalue statistic) rejects $r = 0$. Our model-one measure of the productivity shock is cointegrated with the short-term interest rate at lag length 1 (selected by both the AIC and SIC) according to both the trace and the maximum eigenvalue statistic.

In two other instances the cointegration tests indicate that the respective VAR is stationary in levels, which contradicts the results of our unit-root tests. For the systems involving our model-two preference shock and the short-term interest rate at $q = 1$, the trace statistic indicates $r = 2$, whereas the maximum eigenvalue statistic indicates $r = 0$. Both the trace and the maximum eigenvalue statistic imply stationarity in levels for the VAR(3) between our model-two preference shock and the long-term interest rate. Since both results confirm the intuition, we perform the Granger causality tests in levels.

For the remaining cases, we run causality tests using first differences alone. To check the sensitivity of the results with respect to VAR order selection, we also run test for lags from $q = 1$ to $q = 9$ and used the error correction representation whenever either the trace or the maximum eigenvalue statistic rejected $r = 0$ at that lag length.

Results of the Causality Tests Table III.2 presents the results of the bivariate Granger causality tests according to our criteria for lag-length selection. There is only one instance where we have to accept Granger causality: the model-one Solow residual is Granger caused by government expenditures, when the VAR order is equal to 7. Since the productivity shock is unable to predict government expenditures (the p–value for this null is 0.71), causality seems indeed to run from government expenditures to the productivity shock. Yet, in VARs with a smaller order we are unable to reject the null.

Table A.2 and A.3 in the Appendix show that our results are almost insensitive to VAR order selection. When we estimate VARs for orders 1 to 8 either in differences, levels, or error correction form, depending upon the respective results form the cointegration test, we find only 6 more instances in 272 tests that indicate Granger causality.

IV. Conclusion

The plausibility of small scale dynamic general equilibrium models of the business cycle driven by shocks to productivity and preferences depends upon whether or not these shocks

Table III.2:
Bivariate Granger Causality Tests

Model-One Shock Measures

Variable	Productivity						Preferences					
	AIC			SIC			AIC			SIC		
	q	r	p	q	r	p	q	r	p	q	r	p
Government Expenditures	7	0	0.00	2	0	0.86	3	0	0.50	1	0	0.89
Taxes	4	0	0.17	1	0	0.60	3	0	0.10	1	0	0.87
M1	1	0	0.47	1	0	0.47	1	0	0.15	1	0	0.15
M3	6	0	0.52	1	0	0.97	3	0	0.16	1	0	0.67
Short Term Interest Rate	1	1	0.70	1	1	0.70	1	0	0.33	1	0	0.33
Long Term Interest Rate	3	0	0.44	1	0	0.94	1	0	0.57	1	0	0.57
Trade Balance	8	0	0.81	1	0	0.21	6	0	0.14	6	0	0.14
Terms of Trade	5	0	0.76	4	0	0.66	8	0	0.37	1	0	0.39
Oil Price	4	0	0.83	1	0	0.48	3	0	0.76	2	0	0.55

Model-Two Shock Measures

Variable	Productivity						Preferences					
	AIC			SIC			AIC			SIC		
	q	r	p	q	r	p	q	r	p	q	r	p
Government Expenditures	4	0	0.74	1	0	0.79	4	0	0.43	2	0	0.33
Taxes	4	0	0.12	4	0	0.12	3	0	0.09	1	0	0.96
M1	1	0	0.99	1	0	0.99	1	0	0.11	1	0	0.11
M3	1	0	0.47	1	0	0.47	3	0	0.06	1	0	0.71
Short Term Interest Rate	4	0	0.90	1	0	0.59	1	2	0.33	1	2	0.33
Long Term Interest Rate	1	0	0.24	1	0	0.24	2	2	0.14	2	2	0.14
Trade Balance	7	0	0.71	2	0	0.40	8	0	0.19	1	0	0.20
Terms of Trade	5	0	0.58	5	0	0.58	8	1	0.31	1	0	0.54

Notes:

AIC: Lag length selected according to Akaike's information criterium

SIC: Lag length selected according to Schwarz's information criterium

q: number of lags considered, r: number of cointegration relations, where r=2 indicates that we run the regression in levels, p: marginal level of significance for the null that the shock measure is not Granger caused by the variable in column 1.

can be considered exogenous with respect to other possible shock measures such as government expenditures, tax rates, money supply, interest rates, foreign demand, or world market prices. We consider this question with respect to the West-German economy within the framework of two models. Model 1 is a standard real business cycle model whereas model 2 allows for variable capital utilization and the declining trend in West-German working hours per capita. We use these models to identify shocks to total factor productivity and the marginal rate of substitution between leisure and consumption. Our Granger causality tests do not reject the exogeneity of these shock measures for the period 76.i to 89.iv.

Appendix

Table A.1:
Cointegration Tests

Variable	Model-One Shock Measures															
	Productivity								Preferences							
	Var Order								Var Order							
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
Government Expenditures	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	4
Taxes	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0
M1	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0
M3	0	0	0	0	0	0	2	0	0	0	0	0	0	0	4	0
Short Term Interest Rate	2	0	0	0	0	0	0	0	0	0	0	0	2	2	1	0
Long Term Interest Rate	0	0	0	0	0	0	0	0	0	4	2	2	0	0	0	0
Trade Balance	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0
Terms of Trade	0	0	0	0	0	0	0	0	0	1	0	3	0	0	0	2
Oil Price	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Variable	Model-Two Shock Measures															
	Productivity								Preferences							
	Var Order								Var Order							
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
Government Expenditures	0	0	0	0	0	0	0	0	0	0	0	0	2	2	4	4
Taxes	0	0	2	0	0	0	2	2	0	0	0	0	0	0	0	0
M1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
M3	0	0	2	0	0	0	2	2	0	0	0	0	0	0	4	0
Short Term Interest Rate	0	0	0	0	0	0	0	0	4	2	4	1	4	4	4	0
Long Term Interest Rate	0	0	0	0	0	0	0	0	4	4	4	4	4	4	1	4
Trade Balance	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Terms of Trade	0	0	0	0	0	0	0	3	0	4	0	0	0	0	0	2

Notes:

Var Order: number of lagged differences included

0: r=0 according to both the trace and the maximum eigenvalue statistic at the 5 percent level

1: r=1 (r=0) according to the trace (maximum eigenvalue) statistic at the 5 percent level

2: r=1 according to both the trace and the maximum eigenvalue statistic at the 5 percent level

3: r=1 (r=0) according to the maximum eigenvalue (trace) statistic at the 5 percent level

4: r=2 either according to the maximum eigenvalue or the trace statistic

Table A.2:

Granger Causality Tests: Model-One Shock Measures

Variable	Productivity Var Order							
	1	2	3	4	5	6	7	8
Government Expenditures	0.73	0.86	0.35	0.57	0.58	0.25	0.00	0.05
Taxes	0.60	0.49	0.54	0.17	0.14	0.14	0.07	0.00
M1	0.47	0.10	0.11	0.34	0.23	0.35	0.34	0.02
M3	0.97	0.97	0.81	0.74	0.80	0.52	0.24	0.44
Short Term Interest Rate	0.70	0.48	0.14	0.20	0.20	0.09	0.03	0.16
Long Term Interest Rate	0.94	0.65	0.44	0.51	0.22	0.34	0.35	0.28
Trade Balance	0.21	0.25	0.46	0.32	0.44	0.43	0.63	0.81
Terms of Trade	0.63	0.40	0.66	0.66	0.76	0.81	0.91	0.92
Oil Price	0.48	0.81	0.78	0.83	0.90	0.95	0.91	0.94
Variable	Preferences Var Order							
	1	2	3	4	5	6	7	8
Government Expenditures	0.89	0.43	0.50	0.43	0.56	0.68	0.36	0.15
Taxes	0.87	0.18	0.10	0.15	0.27	0.35	0.58	0.76
M1	0.15	0.38	0.76	0.69	0.82	0.94	0.95	0.79
M3	0.67	0.09	0.16	0.22	0.34	0.55	0.06	0.78
Short Term Interest Rate	0.33	0.57	0.73	0.58	0.74	0.77	0.86	0.77
Long Term Interest Rate	0.57	0.14	0.96	0.83	0.87	0.92	0.92	0.88
Trade Balance	0.30	0.22	0.33	0.26	0.25	0.14	0.03	0.23
Terms of Trade	0.39	0.43	0.39	0.53	0.56	0.78	0.42	0.37
Oil Price	0.44	0.55	0.76	0.40	0.18	0.27	0.27	0.47

Notes:

Table entries are marginal levels of significance for the null of no Granger causality. The Var is estimated in differences, levels or error correction form depending upon r=0, r=4 or r=1,2,3, respectively, as indicated in Table A.1.

Table A.3:

Granger Causality Tests: Model-Two Shock Measures

Variable	Productivity Var Order							
	1	2	3	4	5	6	7	8
Government Expenditures	0.79	0.97	0.94	0.74	0.69	0.66	0.44	0.41
Taxes	0.58	0.62	0.03	0.12	0.07	0.11	0.01	0.00
M1	0.99	0.40	0.36	0.69	0.16	0.26	0.24	0.22
M3	0.47	0.79	0.87	0.60	0.75	0.74	0.39	0.29
Short Term Interest Rate	0.59	0.75	0.73	0.90	0.91	0.83	0.82	0.82
Long Term Interest Rate	0.24	0.38	0.50	0.79	0.65	0.71	0.80	0.71
Trade Balance	0.39	0.40	0.36	0.35	0.47	0.66	0.71	0.77
Terms of Trade	0.13	0.15	0.11	0.47	0.58	0.60	0.69	0.62

Variable	Preferences Var Order							
	1	2	3	4	5	6	7	8
Government Expenditures	0.84	0.33	0.46	0.43	0.61	0.71	0.06	0.15
Taxes	0.96	0.11	0.09	0.14	0.25	0.35	0.62	0.84
M1	0.11	0.32	0.59	0.49	0.66	0.86	0.91	0.72
M3	0.71	0.05	0.06	0.12	0.20	0.41	0.06	0.57
Short Term Interest Rate	0.33	0.41	0.67	0.45	0.74	0.74	0.92	0.74
Long Term Interest Rate	0.30	0.14	0.73	0.63	0.78	0.92	0.93	0.77
Trade Balance	0.20	0.12	0.14	0.16	0.13	0.08	0.02	0.19
Terms of Trade	0.54	0.33	0.19	0.20	0.30	0.53	0.21	0.31

Notes:

Table entries are marginal levels of significance for the null of no Granger causality. The Var is estimated in differences, levels or error correction form depending upon r=0, r=4 or r=1,2,3, respectively, as indicated in Table A.1.

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