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ALGORITHMS FOR THE DETERMINATION OF ORIENTATION-TENSORS FROM THREE DIMENSIONAL µ-CT IMAGES WITH VARIOUS MICROSTRUCTURES

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Abstract

Modelling of composite materials is essential for establishing mass production of automotive or aerospace components. Fiber-orientation statistics therefore deliver important data for the quantitative description of a composite and its microstructure. Numerous methods exist for describing the orientation of reinforcing structures in composite materials. The rapid development of computer tomography concerning resolution and computing capacity allows for the extraction of orientation data directly from three dimensional images. This offers an improvement in detailed 3D orientation analysis compared to methods such as stereology with less consumption of computing resources. Within this work, three algorithms, based on the concepts of anisotropic Gaussian filtering, Hessian matrix and structure tensor were implemented and evaluated. Finally the introduction of an error criterion allows for the optimization and comparison of the three algorithms for which first results are presented. Furthermore, the methods were adapted to compute the orientation of either fibrous composites or planar reinforcements, e.g. from freeze-cast materials.

The results of the computations were used to determine the elastic properties of the material using a homogenization method by Mori-Tanaka and validated with experimental data. The results are in good agreement with the experimental data. The introduced error criterion shows that the calculation via structure tensor offers the most reliable data, especially for fibrous composites in regions where fibers cross or end.

1 INTRODUCTION

Composite materials like fiber reinforced polymers offer a high potential for lightweight design. The characterization of the orientation of the structures is essential to understand their mechanical properties. There are many methods to determine the orientation of structural materials. A relative old technique is the stereological measurement in micro-sections where orientations can be calculated from the elliptical cross section of the fibers [1]. A disadvantage of this method is the ambiguity of these ellipses. It is not possible to differ between orientations that are rotated 180° around the orthogonal axis to the investigated plane. Furthermore, the fiber orientation of thin plates from 1 mm to 4 mm can be measured by polarized microwaves [2], but this method offers merely two-dimensional data. To extract orientation data from µ-CT images, there are many ways to analyse discrete three-dimensional images. A very simple one is the skeletonization [3], where fibers are thinned to their medial axis. The image is filtered by an anisotropic diffusion filter to smooth it along the local fiber orientation in a first step. Subsequently, the image is binarized by a threshold operator and skeletonized in respect to the method of Lee et al. [4]. With this data, it is possible to follow every single thinned fiber and calculate its orientation. It is also possible to determine the orientation without having a look on every single fiber. The method implemented by Illerhaus [5] calculates a corresponding value in respect to a ray tracing algorithm, which inspects a limited number of orientations by summing the voxels along a ray. The first method, which is investigated in this paper, is based on a anisotropic Gaussian filter that is rotated in various directions similar to the method mentioned before [6]. The second tested method was implemented by Daniels et al. [7] for the orientation analysis of collagen. It is based on the Hessian matrix, which represents the curvature of the grey values of the image in a region. Krause et al. [8] determinate fiber orientations of reinforced concrete with a technique that uses the structure tensor as an image descriptor to estimate the fiber orientation. This method has already found its way into commercial CT analysis software and is therefore the third algorithm that is optimized and compared in the present paper.

2 MATERIAL AND METHODS

2.1 Artificial Test Image Generation

The comparison and evaluation of the different algorithm was carried out on artificial fiber architectures to allow the separation of several overlapping problems or errors indicated by each algorithm. Synthetic test images with one or two fibers were produced using a self-programmed tool. Most important input arguments are the number of fibers, diameter, angle Θ, distance d and image size. For every tested configuration, two images were generated: A binary image to test the orientation

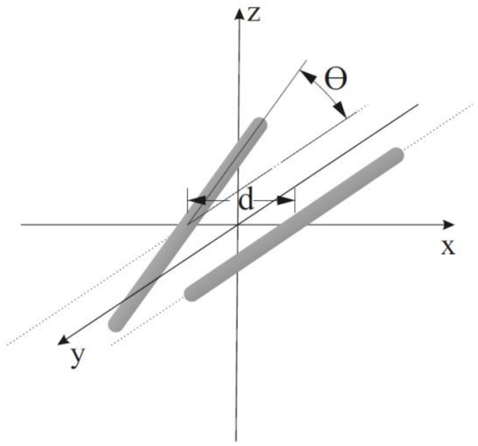

Figure 1 – Synthetic fiber generation

algorithms and another one of vectorial data with the correct orientation for each voxel to validate the resulting images in a later step.

2.2 Image Acquisition

The algorithms have also been tested on real µ-CT images. Therefore, specimen were scanned in an Yxlon-CT precision computed tomography system containing an open micro-focus X-ray transmission tube with tungsten target and a 2048 x 2048 pixel flat panel detector from Perkin Elmer.

2.3 Algorithms for orientation analysis

In the present work, the following methods were tested on binary images to compare the detected orientations directly with the real orientation in the artificial images. Binary images have been depicted to exclude gray value variation effects from the analysis although it is possible to use all algorithms as well on gray value data.

Anisotropic Gaussian filtering is a simple method to calculate orientations within an image. The filter mask or the image is rotated in many steps to specific orientations and filtered by an anisotropic Gaussian Filter [9]. At these pixels, where the filter is aligned to the direction e.g. of a fiber, the response will be a high value. The grey ellipsoids in Figure 2 illustrate two filter masks. Fiber A is badly aligned and the filter returns a low value while fiber B has the same orientation as the filter mask and the filter will result in a high response value. During the calculation, only the orientation and the value of the highest filter response of all observed directions are saved to separate images. In the present investigation, 200 discrete orientations selected by the method of Leopardi [10] were observed over half the unit sphere.

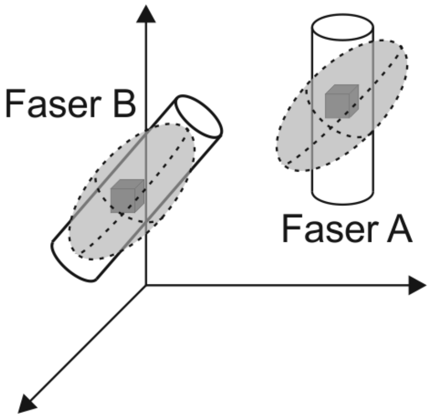

Figure 2 – Example of a anisotropic Gaussian filter

Orientation analysis via the **Hessian Matrix** is based on the curvature of the gray values in an image. Gray values do not change along fibers, but orthogonal to them so that the curvature parallel to the fiber is significant smaller. The algorithm smoothes the image with a Gaussian filter to create a gradient in the inner of the structures (Figure 3 a to b). After this, the eigenvalues of the Hessian matrix (Equation 2) is calculated in each voxel. The eigenvector belonging to the smallest eigenvalue describes the orientation in the current point. Figure 3 (c) illustrates this connection in a 3D-diagram of a 2D-image with the grey value on the z-axis. The eigenvector to the smallest eigenvalue at the red marked point in (a) and (b) is depicted by the red arrow in (c).

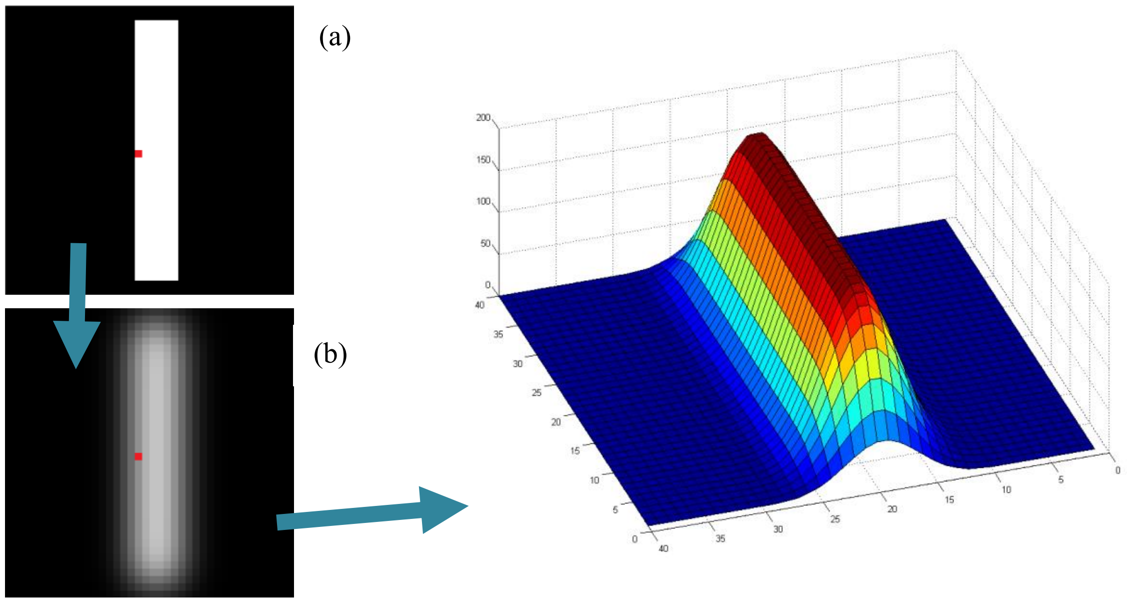

Figure 3 – Orientation analysis via the Hessian matrix

$$\nabla^2 I = \begin{bmatrix} \frac{\partial^2 I}{\partial x^2} & \frac{\partial^2 I}{\partial x \partial y} & \frac{\partial^2 I}{\partial x \partial z} \\ \frac{\partial^2 I}{\partial y \partial x} & \frac{\partial^2 I}{\partial y^2} & \frac{\partial^2 I}{\partial y \partial z} \\ \frac{\partial^2 I}{\partial z \partial x} & \frac{\partial^2 I}{\partial z \partial y} & \frac{\partial^2 I}{\partial z^2} \end{bmatrix} \tag{1}$$

Structure Tensor analysis is a more complicated method than these mentioned before. In a first step, the binary image is smoothed by a Gaussian filter with the input parameter σ similar to the method based on the Hessian matrix to produce gradients in the structures. Subsequently, the structure tensor, shown in Equation 2, is calculated for each voxel in the image. This tensor uses the squared gradients to avoid the cancellation of gradients which are aligned in opposite directions in the later smoothing process [11]. After computing every voxel in the image, it is necessary to average the tensors to determine the orientation in a variable region around every point. This is realized by a final Gaussian smoothing of the tensors with a width of ρ.

$$J = \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x} \cdot \frac{\partial I}{\partial y} & \frac{\partial I}{\partial x} \cdot \frac{\partial I}{\partial z} \\ \frac{\partial I}{\partial y} \cdot \frac{\partial I}{\partial x} & \left(\frac{\partial I}{\partial y}\right)^2 & \frac{\partial I}{\partial y} \cdot \frac{\partial I}{\partial z} \\ \frac{\partial I}{\partial z} \cdot \frac{\partial I}{\partial x} & \frac{\partial I}{\partial z} \cdot \frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial z}\right)^2 \end{bmatrix} \tag{2}$$

2.4 Rating and optimization of orientation analysis algorithms

The quality of each algorithm was rated by evaluating the absolute misorientation angle θ_f between the real and the calculated orientation at each of n voxels in the image and summing them up to the global error θ_g. It should be noted, that the orientation is not influenced by the direction of a vector and hence the global error cannot be larger than 90° at a fully wrong determined orientation analysis. Consequently, vectors with $\theta_f > 90°$ were negated and calculated again.

$$\theta_g = \frac{\sum_0^n \theta_f}{n} \tag{3}$$

This error estimate can be evaluated for every artificial generated image with given parameter set of angle, diameter and distance. To transfer the global error to a more application-oriented case, several configurations of fiber distance and angle were investigated and averaged to another error θ_{FA} that is based on the fiber architecture. In this work, a random fiber orientation was assumed. Hence, angles from 0 to 90 degrees in ten steps and fiber distances from zero to one fiber diameter in six steps were observed. The global orientation errors θ_g of each configuration were summed up and divided by the number of observed configurations to receive the fiber architecture dependent error criterion θ_{FA}. This criterion is used to find the best working parameters for each method in dependence of the fiber diameter and to compare the orientation analysis algorithms against each other.

2.5 Elasticity Modeling

Using the orientation data gathered with the aforementioned algorithms it is possible to model the elastic behavior of composite materials. Therefore the orientation data has to be approximated by orientation distribution functions (ODF), which represent the fiber orientations within the region of interest (ROI). One example of such an ODF is the rotationally symmetric ODF (Equation 4) as used by Schjødt-Thomsen et al. [12,13] to model the elastic behavior of composites by a Mori-Tanaka approach for non-dilute fiber volume contents. In the application investigated herein the model has been applied to the rotational symmetric fiber distribution of a glass fiber reinforced sandwich face sheet. The fiber distribution function was extracted from CT-image data by the anisotropic Gaussian filtering method.

$$g(\theta) = \frac{\sin(\theta)^{2P-1}\cos(\theta)^{2Q-1}}{\int_{\theta_a}^{\theta_b}\sin(\theta)^{2P-1}\cos(\theta)^{2Q-1}\,d\theta} \tag{4}$$

3 RESULTS

3.1 Orientation analysis

The result of an orientation analysis can be illustrated in various ways to get a first impression of its accuracy. Figure 5 shows a color-coded orientation analysis of carbon fiber in air and a freeze cast material while Figure 4 depicts the relation between color and direction as a colormap on a half sphere. The fibrous material is calculated by the anisotropic Gaussian filter just as described in Figure 2, the freeze cast material with planar structures was filtered with a Gaussian lenticular filter mask to determinate the orthogonal vectors to the reinforcing alumina platelets.

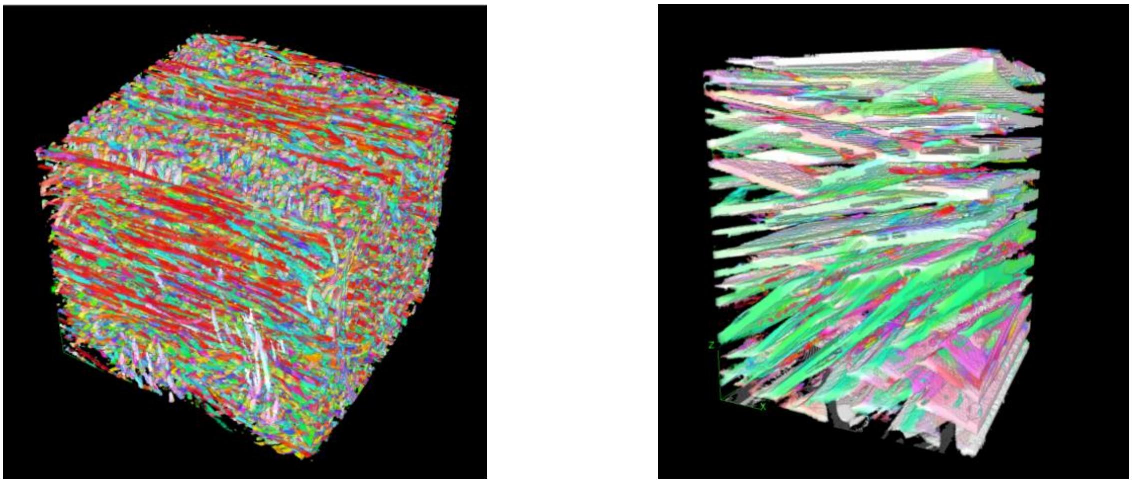

Figure 5 - Orientation of carbon-fibres (left) and of a freeze cast alumina preform (right) coded with colour

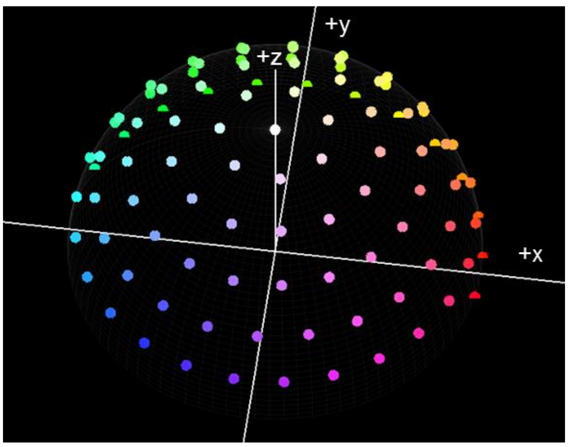

Figure 4 – Colormap of orientations

The optimal settings for each method were found by comparing all combinations of parameters in the range of interest and determine the one with smallest θ_{FA}. The analysis by anisotropic Gaussian filtering offers the best results at $\sigma = 0.6 \cdot d_f$. Note that the aspect ratio of the filter mask was chosen to two [6] although with straight fibers it is possible to get even better results at higher values but is not applicable in realistic fiber architectures. Hessian matrix analysis delivers the smallest fiber architecture dependent error at circa $\sigma = 0.6 \cdot d_f$. Optimal parameters for the method based on the structure tensor were observed by varying both σ and ρ. The first smoothing worked best at $\sigma =$ 0.8 px for all cases while ρ should be to approximately $0.6 \cdot d_f$.
As mentioned before, the fiber architecture dependent error θ_{FA} was used to rate the adequacy of the tested methods for different fiber diameters. Tests were performed with fiber diameters of 10 and 5 pixels, a fiber length of 80 pixel and an image size of 100^3 voxels. Table 1 shows the resulting error criterion for all tested methods with optimum parameter sets.

Method	$\boldsymbol{d_f}$ =10 px	$\boldsymbol{d_f}$ =5 px
Anisotropic Gaussian Filter	8.19°	8.48°
Hessian Matrix	10.31°	10.31°
Structure Tensor	1.68°	2.18°

Table 1 - θ_{FA} for different fiber diameters and methods

3.2 Elastic Modeling

To show the applicability of the aforementioned optimal parameter setting for the image analysis the anisotropic Gaussian diffusion was used to determine fiber orientation in a sandwich face sheet component as mentioned before. The ODF was fit to the relative directional fiber volume fraction (Figure 6). Therefore, the spherical orientation distribution was assumed to be rotational symmetric around the thickness direction of the face sheet.

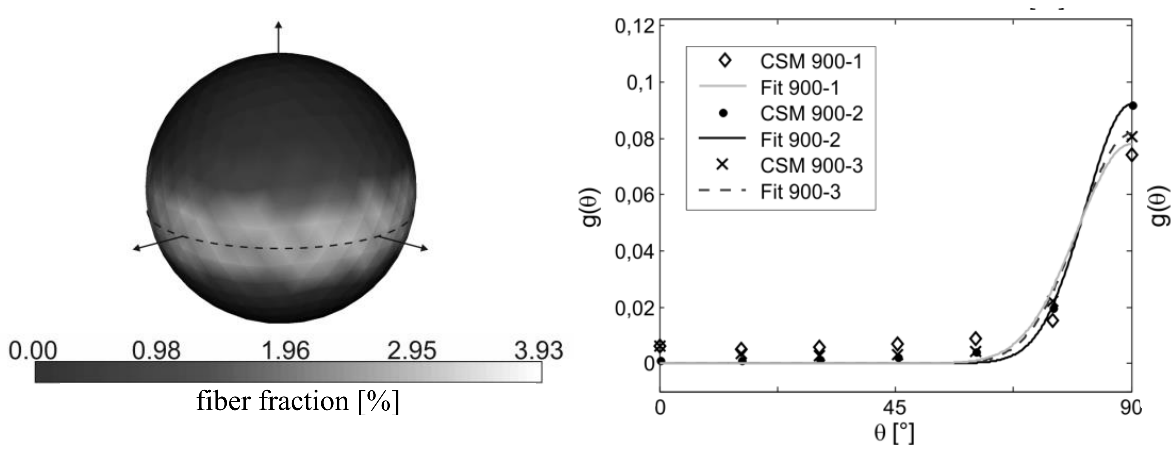

Figure 6 – 3D orientation histogram (left) with according reduced ODF (right)

The fitted ODF was then used in the Mori-Tanaka approach to represent fiber directions and allowed for a precise estimation of the elastic properties. The respective Youngs modulus for several ODFs representing different face sheet structures is designated in Figure 7. It can be seen clearly that using the fiber orientations from CT data allows for a precise modelling of elastic properties. Especially the transition from the low fiber content ODF (dashed green line) to the higher fiber content ODF (solid dark black line) in the Youngs modulus prediction corresponds well to the experimental data from bending tests.

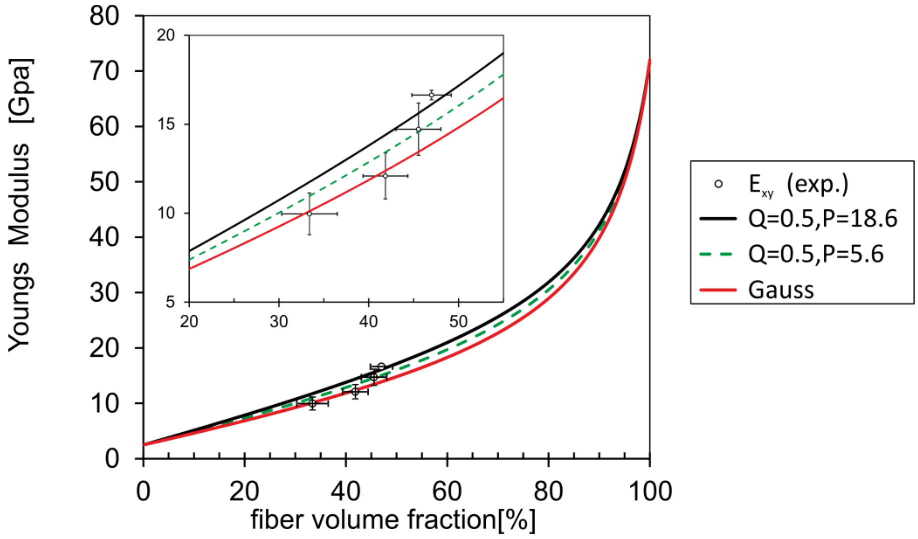

Figure 7 – Youngs modulus for different fiber contents in the sandwich face sheet approximated with according ODFs in the Mori-Tanaka model

4 DISCUSSION

The method using the structure tensor offers considerably the best results of the investigated approaches. Indeed, this method requires a higher computing effort than the technique based on the hessian matrix, but offers a misorientation that is nearly five times lower on the artificial images. Although the anisotropic Gaussian filter has a relative high fiber architecture dependent error, it has some unused potential as well. The error could be minimized by a higher number of investigated orientations but results also in a linear increase of runtime. On the other hand, the algorithm is economical in memory consumption for increasing quantification steps which is a great advantage especially for handling of large volumes of data. Furthermore, it is theoretically possible to detect multiple orientation e.g. in crossing points of open foams. The orientation analysis via the Hessian matrix is the fastest algorithm but offers the worst results regarding the errors investigated herein.
The ODF based elastic Mori-Tanaka model is in good agreement with experimental data of sandwich face sheets being able to validate orientation dependent processing and property interactions. Therefore the application of optimized fiber orientation analysis algorithms proofs to be an essential tool for a realistic modeling of composite microstructures.

5 CONCLUSION

In the present work, three algorithms for orientation analysis were tested, optimized and rated due to their accuracy. The structure tensor based method offered by far the best results. The technique using the anisotropic Gaussian filtering has its right to exist owing to its low requirement of memory consumption.
In future applications it should be interesting to investigate other artificial generated fiber architectures like unidirectional rovings, weavings or curved fibers or other microstructures to check the superiority of the structure tensor based method in general and to fully exploit the structural information contained in the orientation descriptors of CT images.

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